Lattice Evidence for Bound Heavy Tetraquarks

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Abstract. We investigate the possibility of $qq'\bar{Q}\bar{Q}'$ tetraquark bound states using $n_f = 2 + 1$ lattice QCD with pion masses $\simeq$ 164, 299 and 415 MeV. Two types of lattice interpolating operator are chosen, reflecting first diquark-antidiquark and second meson-meson structure. Performing variational analysis using these operators and their mixings, we determine the ground and first excited states from the lattice correlators. Using non-relativistic QCD to simulate the bottom quarks and the Tsukuba formulation of relativistic heavy quarks for charm quarks, we study the $ud\bar{b}\bar{b}$, $\ell s\bar{b}\bar{b}$ as well as $ud\bar{c}\bar{b}$, channels with $\ell = u, d$. In the case of the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ channels unambiguous signals for $J^P = 1^+$ tetraquarks are found with binding energies 189(10) and 98(7) MeV below the corresponding free two-meson thresholds at the physical point. These tetraquarks are therefore strong-interaction stable, implying they are stable under strong as well as electromagnetic interactions while they can decay weakly. So far these are the first exotic hadrons predicted to have this feature. Further evidence for binding is found in the $ud\bar{c}\bar{b}$ channel, whereby the binding energy broadly straddles the electromagnetic stability threshold. Studying further the quark mass dependence we vary the heavy quark mass in $ud\bar{Q}\bar{Q}$, $\ell s\bar{Q}\bar{Q}$ as well as $ud\bar{Q}\bar{b}$, $\ell s\bar{Q}\bar{b}$ between roughly 0.7 and 6.3 times the bottom quark mass. The observed mass dependence of these four flavor channels closely follows a behaviour argued from phenomenological considerations of the heavy baryon spectrum.

1 Phenomenological introduction

The observed heavy hadron spectrum suggests a phenomenological binding mechanism from a combination of "good" diquark configurations and heavy quark spin symmetry considerations that should lead to stable $J^P = 1^+$ tetraquarks. These exotic hadrons would be made up of two light quarks and two heavy anti-quarks, see [1–4] and [10] for a review. Considering $ud\bar{b}'\bar{b}'$, $ud\bar{b}'\bar{b}$, $\ell s\bar{b}'\bar{b}'$ and $\ell s\bar{b}'\bar{b}$ flavor combinations the underlying physical picture leads to the following model ansaetze for the heavy quark mass dependence of the binding energies

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$\Delta E = E_{\text{tetra}} - E_{\text{thresh.}}$ of these states [4], where $r = m_b/m_{b'}$:

$$\Delta E_{ud\bar{b}'\bar{b}',\ell s\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{ud,\ell s} + C_2^{ud,\ell s}(2r) + (C_{\bar{b}'\bar{b}'}^{ud,\ell s})\, r, \tag{1}$$

$$\Delta E_{ud\bar{b}'\bar{b},\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud,\ell s} + C_2^{ud,\ell s}(1+r) + (C_{\bar{b}'\bar{b}}^{ud,\ell s}(r)) \text{ for } r < 1, \tag{2}$$

$$\Delta E_{ud\bar{b}'\bar{b},\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud,\ell s} + C_2^{ud,\ell s}(1+r) + (C_{\bar{b}'\bar{b}}^{ud,\ell s}(r)) \text{ for } r > 1. \tag{3}$$

The first terms here parametrize the color Coulomb attraction between the two heavy anti-quarks and the attraction associated with the flavor $\bar{3}$, colour $\bar{3}$ good light diquark configuration, while $C_{\bar{b}'\bar{b},\bar{b}'\bar{b}'}^{ud,\ell s}(r)$ encapsulates threshold contributions and is fixed by the observed meson spectrum. The form of the first term, which reflects the expectation that the heavy anti-diquark colour Coulomb interaction will be proportional to the reduced mass of the heavy anti-diquark system, is such that the binding will increase with increasing $\bar{b}$ or $\bar{b}'$ quark mass. Based on heavy baryon splittings, the contribution to the binding in the good light-diquark configuration should also increase with decreasing light quark mass.

In the following we test the effectiveness and the understanding of the binding mechanism underlying these ansaetze using lattice QCD calculations. We find the predictions are confirmed and the motivated forms above describe the heavy quark mass dependence of both $\{bb\}$ and $\{b'b\}$ channels well. In addition, direct calculations provide clear evidence of strong-interaction stable, bound $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks, as well as first evidence for binding also in the $ud\bar{c}\bar{b}$ channel.

2 First-principles energies from lattice QCD

To obtain robust, first-principles results for the tetraquark binding energies the central paradigm we follow is to compute and study the finite volume energy spectrum of the $qq'\bar{Q}'\bar{Q}$ tetraquarks. The tetraquark and corresponding lowest two-meson threshold energies can be determined from current-current correlation functions in Euclidean time (t) calculated using lattice QCD.

With the phenomenological picture in mind we choose two local operators, one with flavour-color-spin structure of a diquark-antidiquark,

$$D = \left(q_a^T C\gamma_5 q_b' \right) \times \left[\bar{Q}_a C\gamma_i \bar{Q}_b'^T - \bar{Q}_b C\gamma_i \bar{Q}_a'^T \right] , \tag{4}$$

and one with flavor-color-spin structure of a meson-meson:

$$\begin{aligned} Q = Q' : \quad & M = (\bar{Q}_a\gamma_5 q_a')(\bar{Q}_b\gamma_i q_b) - (\bar{Q}_a\gamma_5 q_a)(\bar{Q}_b\gamma_i q_b') , \\ Q \neq Q' : \quad & M_1 = (\bar{Q}_a\gamma_5 q_a)(\bar{Q}'_b\gamma_i q_b') - (\bar{Q}_a\gamma_5 q_a')(\bar{Q}'_b\gamma_i q_b) , \\ & M_2 = (\bar{Q}'_a\gamma_5 q_a)(\bar{Q}_b\gamma_i q_b') - (\bar{Q}'_a\gamma_5 q_a')(\bar{Q}_b\gamma_i q_b) . \end{aligned} \tag{5}$$

Taking all possible combinations of source and sink operator correlation functions we arrive at the correlation matrices:

$$F_2(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix} \quad \text{and} \quad F_3(t) = \begin{pmatrix} G_{DD}(t) & G_{DM_1}(t) & G_{DM_2}(t) \\ G_{M_1D}(t) & G_{M_1M_1}(t) & G_{M_1M_2}(t) \\ G_{M_2D}(t) & G_{M_2M_1}(t) & G_{M_2M_2}(t) \end{pmatrix}. \tag{6}$$

From these matrices we form a generalized eigenvalue problem, GEVP. The ground state energy (ΔE_0) is then obtained from the exponent of the lowest-lying eigenvalue λ_0,

$$F_j(t)\nu = \lambda(t)F_j(t_0)\nu \Rightarrow \lambda_i(t) = Ae^{-\Delta E_i(t-t_0)} . \tag{7}$$

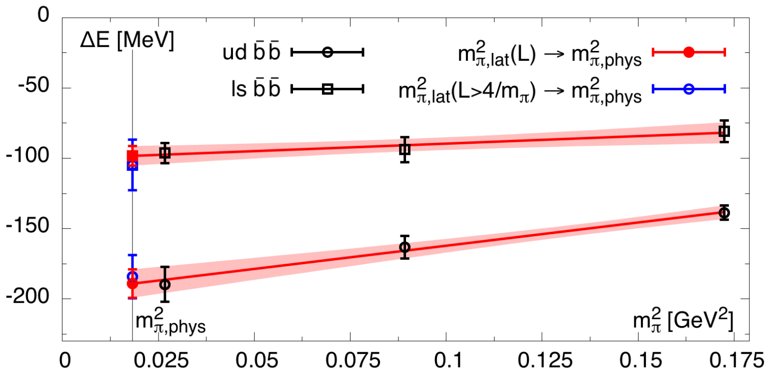

Figure 1. Chiral extrapolations of the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ binding energies. Red lines and points show the extrapolations using all three ensembles, the blue points those using $m_\pi = 415$ MeV and $m_\pi = 299$ MeV.

In the following, we will focus on presenting results from a combination of correlators giving access to the binding energy directly, $G_{O_1O_2}(t) = C_{O_1O_2}(t)/C_{PP}(t)C_{VV}(t)$. However, there is a possible underestimated systematic here due to the different excited state characteristics in the threshold and tetraquark components of this ratio. A careful analysis of this systematic is performed in our full paper [4] and given the length restrictions of this proceedings contribution we refer the reader there for further details. Choosing to show the results of the binding correlator here is a consequence of presentational considerations as the binding energy can be immediately read off from a single figure.

We note, that by determining the finite volume energies we gain access to the ground state and also its scattering properties in a systematically improvable way. The approach does not rely on the interpretation of our data in terms of phenomenological input or a potential. Similar calculations following this strategy have recently been reported in [5, 6], generally in good agreement with the work shown here. Further calculations in the static approximation, where a potential ansatz is required, performed in [7–9] also found signals of bound tetraquark states of the type studied here.

Our calculations are performed on three gauge ensembles with $n_f = 2 + 1$ dynamical quark flavors at fixed lattice spacing, $a^{-1} = 2.194(10)$ GeV, with pion masses $m_\pi = 415$, 299 and 164 MeV. The configurations were generated by the PACS-CS collaboration [11]. To calculate the hadron correlations functions we use gauge-fixed wall sources for all valence quark flavors. A relativistic heavy quark effective action with non-perturbatively tuned parameters is used for the charm and lattice NRQCD for the bottom quark [3, 12].

2.1 Bound $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks

In [3] we performed the first direct lattice calculation of the binding energies ΔE for the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquark candidates, as they are the most promising channels for binding given the phenomenological intuition outlined above. The finite volume binding energies are determined by fitting the exponential behavior of the ground state solutions to the above GEVPs, see [3] for more details. They are shown for all three pion masses and both channels in Fig. 1.

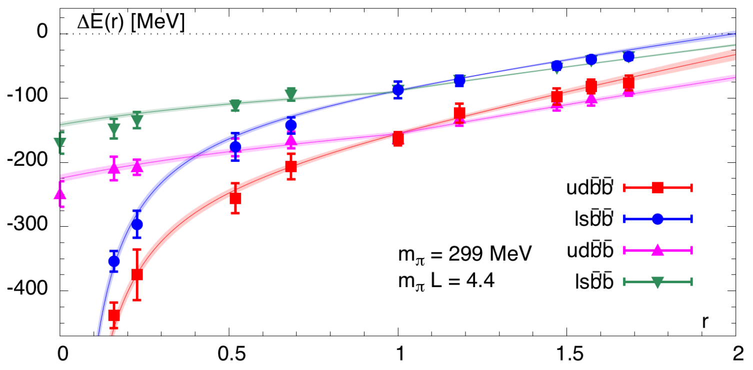

Figure 2. The dependence on the heavy-quark mass ratio, $r = m^{b}_{\text{bare}}/m^{b'}_{\text{bare}}$, of the binding energies for the $ud\bar{b}'\bar{b}$, $ud\bar{b}'\bar{b}'$, $\ell s b' b$ and $\ell s b' b'$ channels. The solid lines show the fits obtained using Eqs. (1)-(3).

Comparing the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ binding energies, as well as their dependence in the light quark mass implicitly given by m_π, we find confirmation of the expectation that the binding energies should increase with decreasing light quark mass.

Individually, the results are extrapolated to physical pion mass, once using all ensembles (red) and once cutting the result with $m_\pi L < 4$ (blue). We find clear evidence for strong-interaction stable tetraquarks with binding energies $\Delta E_{ud\bar{b}\bar{b}} = -189(10)(3)$ MeV and $\Delta E_{\ell s\bar{b}\bar{b}} = -98(7)(3)$ MeV. We note, the good overlap of the blue and red extrapolations to the physical point, is a signal that the finite volume systematic is under good control in this calculation and the error estimates reflect this uncertainty. Such deep binding energies imply both states can decay only through weak interactions and we conclude they are strong interaction stable.

2.2 Heavy quark mass dependence of heavy-light tetraquarks

To further increase our understanding of the underlying binding mechanism we study the heavy quark mass dependence of the four $J^P = 1^+$ tetraquark channels $ud\bar{b}'\bar{b}'$ $ud\bar{b}'\bar{b}$, $\ell s\bar{b}'\bar{b}'$ and $\ell s\bar{b}'\bar{b}$ using a variable unphysical heavy quark b' with $m_{b'}/m_b \sim 6.29, 4.40, 1.93, 1.46, 0.85, 0.68, 0.64$ and 0.60. This enables us to examine the reliabilty of the expectations for the heavy- and light-quark-mass dependence of the binding of these states reflected in the ansaetze above, and, finding them reliable, infer from the behavior as $m_{b'}$ is decreased below m_b which channels (if any) are likely to support bound states when one or both of the b quarks is replaced by a c quark. This allows us to focus direct simulation efforts on such favorable channels, which represents a practical advantage since simulations with our charm prescription are more computationally expensive than NRQCD simulations of bottom.

The binding energies are determined similarly to above (see [4, 12]) and we show the results in Fig. 2. As observed before the ℓs diquark tetraquark candidates exhibit smaller binding energies compared to their ud counterparts. At the same time the binding energies increase with the reduced mass of the heavy diquark component. This confirms all of the expectations of the proposed phenomenological binding mechanism. Furthermore, fitting the phenomenological model ansaetze, Eqs. (1)-(3), we achieve a good description of the data. This is an encouraging validation of phenomenological intuition. The results also clearly suggest that the channel most likely to support a bound tetraquark state with one or both of the heavy quarks being charm is the $ud\bar{c}\bar{b}$.

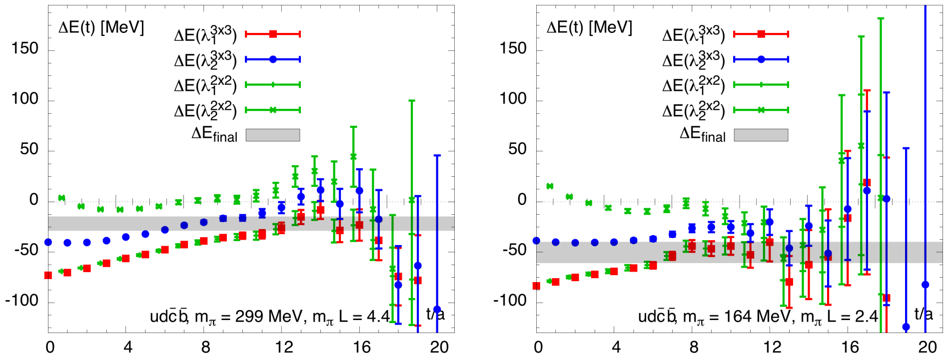

Figure 3. $ud\bar{c}\bar{b}$ tetraquark results for binding energies at $m_\pi = 299$ MeV (left) and $m_\pi = 164$ MeV (right). Red squares and blue circles denote 3×3 GEVP ground and first excited state results, respectively, green vertical dashes and green diagonal crosses ground and first excited state 2×2 GEVP results. The grey bands depict the final binding energies, derived from single-exponential fits to the first eigenvalues. Further details may be found in [4], 2×2 GEVP results are offset slightly in t.

2.3 Indications of a bound $ud\bar{c}\bar{b}$ tetraquark

With the phenomenological study of the previous section identifying the $ud\bar{c}\bar{b}$ channel as the most likely to support a strong interaction stable $J^P = 1^+$ tetraquark with non-zero charm, we focus our resources on a direct calculation in this channel. In Fig. 3 we show our results for the effective binding energies on the ensembles with pion masses $m_\pi = 299$ MeV (left) and 164 MeV (right). The first two finite volume state binding energies for both the 2×2 (green, offset in t/a) and 3×3 (colors) GEVPs are given. We fit the correlators obtained from the GEVPs (not shown here) to single exponentials. An in-depth discussion is given in [4]. For both lattice setups we observe ground state energies below the two-meson threshold, thereby providing evidence of the existence of a $ud\bar{c}\bar{b}$ tetraquark. We note in passing, that in our full paper we also analyse the $m_\pi = 415$ MeV ensemble, where we do not find signs of a bound tetraquark candidate. Recall, we chose to focus on binding correlators in this proceedings contribution, while in [4], whose final results are quoted, a complete analysis based also on the individual energy levels is performed.

Taking the upper bound of the $m_\pi = 299$ MeV and lower bound of the $m_\pi = 164$ MeV results to provide an estimate for the $ud\bar{c}\bar{b}$ binding energy we find -61 MeV $< \Delta E_{ud\bar{c}\bar{b}} <$ -15 MeV. Such a charmed-bottom tetraquark should be easier to detect experimentally than the more deeply bound doubly bottom tetraquarks. The dominant systematic uncertainties in this calculation are finite volume effects, making a future detailed finite volume study desirable.

3 Summary

We performed a first-principles study of $J^P = 1^+$ tetraquarks with flavor content $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$. In these channels we find strong interaction stable states below the corresponding thresholds with binding energies $\Delta E_{ud\bar{b}\bar{b}} = -189(10)(3)$ MeV and $\Delta E_{\ell s\bar{b}\bar{b}} = -98(7)(3)$ MeV at physical quark mass.

Going further to study the flavor channels $ud\bar{b}'\bar{b}'$, $ls\bar{b}'\bar{b}'$, $ud\bar{b}'\bar{b}$ and $ls\bar{b}'\bar{b}$, whereby we varied the b' quark mass in the range $m_{b'}/m_b = 6.29 \to 0.60$. Our lattice QCD results are seen to conform well to a phenomenological model of the underlying binding mechanism based on good diquark configurations and heavy quark spin symmetry considerations. The calculation results confirm the expectations that tetraquarks with lighter qq' diquark components have deeper binding energies and also that the binding energies increase with heavier $\bar{Q}'\bar{Q}$ masses.

Based on the knowledge gained from varying the b' quark mass we find the most likely channel to support a bound tetraquark in the region where at least one of the heavy quarks is a charm quark is $ud\bar{c}\bar{b}$. Performing direct calculations for this channel we observe evidence for tetraquark states below the corresponding free two-meson thresholds for $m_\pi = 164$ MeV and $m_\pi = 299$ MeV. Taking the upper bound of the latter and lower bound of the former result as providing the best assessment of the probable range of binding, we find a likely binding energy of -61 MeV $< \Delta E_{ud\bar{c}\bar{b}} < -15$ MeV for the $ud\bar{c}\bar{b}$ tetraquark ground state relative to the $\bar{D}B^*$ threshold.

In the future we hope to address the finite volume systematic, which represents our largest uncertainty at this time, by calculating the volume dependence of the binding energies, thereby pinning down their infinite volume values as well as giving further hints to their stability properties.

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References

[1] M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, no. 20, 202001 (2017) doi:10.1103/PhysRevLett.119.202001 [arXiv:1707.07666 [hep-ph]].

[2] E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, no. 20, 202002 (2017) doi:10.1103/PhysRevLett.119.202002 [arXiv:1707.09575 [hep-ph]].

[3] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017) doi:10.1103/PhysRevLett.118.142001 [arXiv:1607.05214 [hep-lat]].

[4] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, arXiv:1810.10550 [hep-lat].

[5] P. Junnarkar, N. Mathur and M. Padmanath, arXiv:1810.12285 [hep-lat].

[6] P. Junnarkar, M. Padmanath and N. Mathur, EPJ Web Conf. **175**, 05014 (2018) doi:10.1051/epjconf/201817505014 [arXiv:1712.08400 [hep-lat]].

[7] P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015) doi:10.1103/PhysRevD.92.014507 [arXiv:1505.00613 [hep-lat]].

[8] P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017) doi:10.1103/PhysRevD.95.034502 [arXiv:1612.02758 [hep-lat]].

[9] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017) doi:10.1103/PhysRevD.96.054510 [arXiv:1704.02383 [hep-lat]].

[10] J. M. Richard, Few Body Syst. **57**, no. 12, 1185 (2016) doi:10.1007/s00601-016-1159-0 [arXiv:1606.08593 [hep-ph]].

[11] S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. D **79**, 034503 (2009) doi:10.1103/PhysRevD.79.034503 [arXiv:0807.1661 [hep-lat]].

[12] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, EPJ Web Conf. **175**, 05023 (2018) doi:10.1051/epjconf/201817505023 [arXiv:1711.03380 [hep-lat]].