b and c spectroscopy at LHCb

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2022/10/06

On behalf of the LHCb collaboration

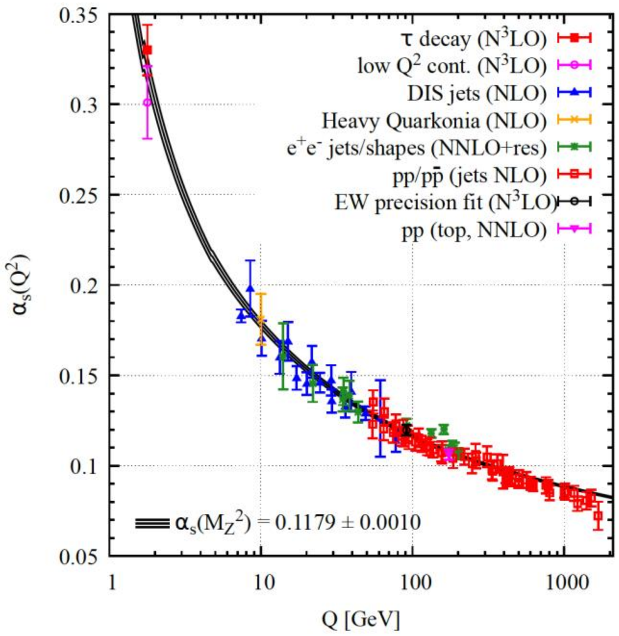
LHC Days in Split

3-8 Oct 2022, Split, Croatia

Introduction

PDG2021

- Two regimes of QCD
  - Asymptotic free
  - Confinement
- Experimental probes @ LHCb
  - Hard scatterings
  - Hadron spectroscopy

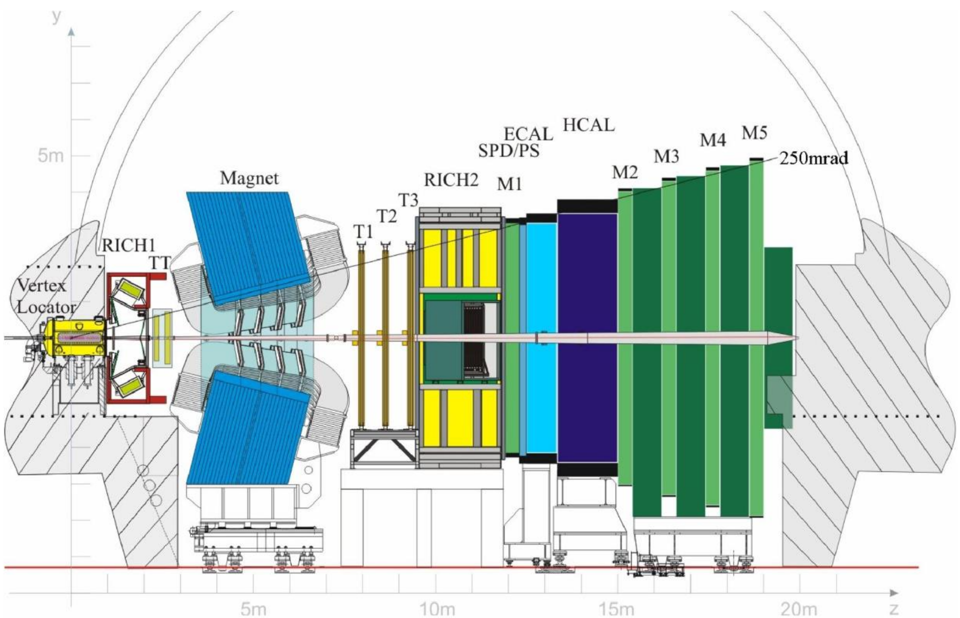


Confinement ⟷ Asymptotic free

- Spectroscopy
  - Test QCD-motivated effective theories / lattice QCD results at low energies (QCD non-perturbative)
  - **Conventional hadrons**:
    - Conventional mesons $q\bar{q}$, baryons qqq
  - **Exotic hadrons:**
    - Tetraquarks $q\bar{q}q\bar{q}$, pentaquarks $qqqq\bar{q}$ ……

The LHCb detector

- Optimized for heavy-flavor studies @ LHC
  - Single-arm and forward spectrometer, $\eta \in (2, 5)$



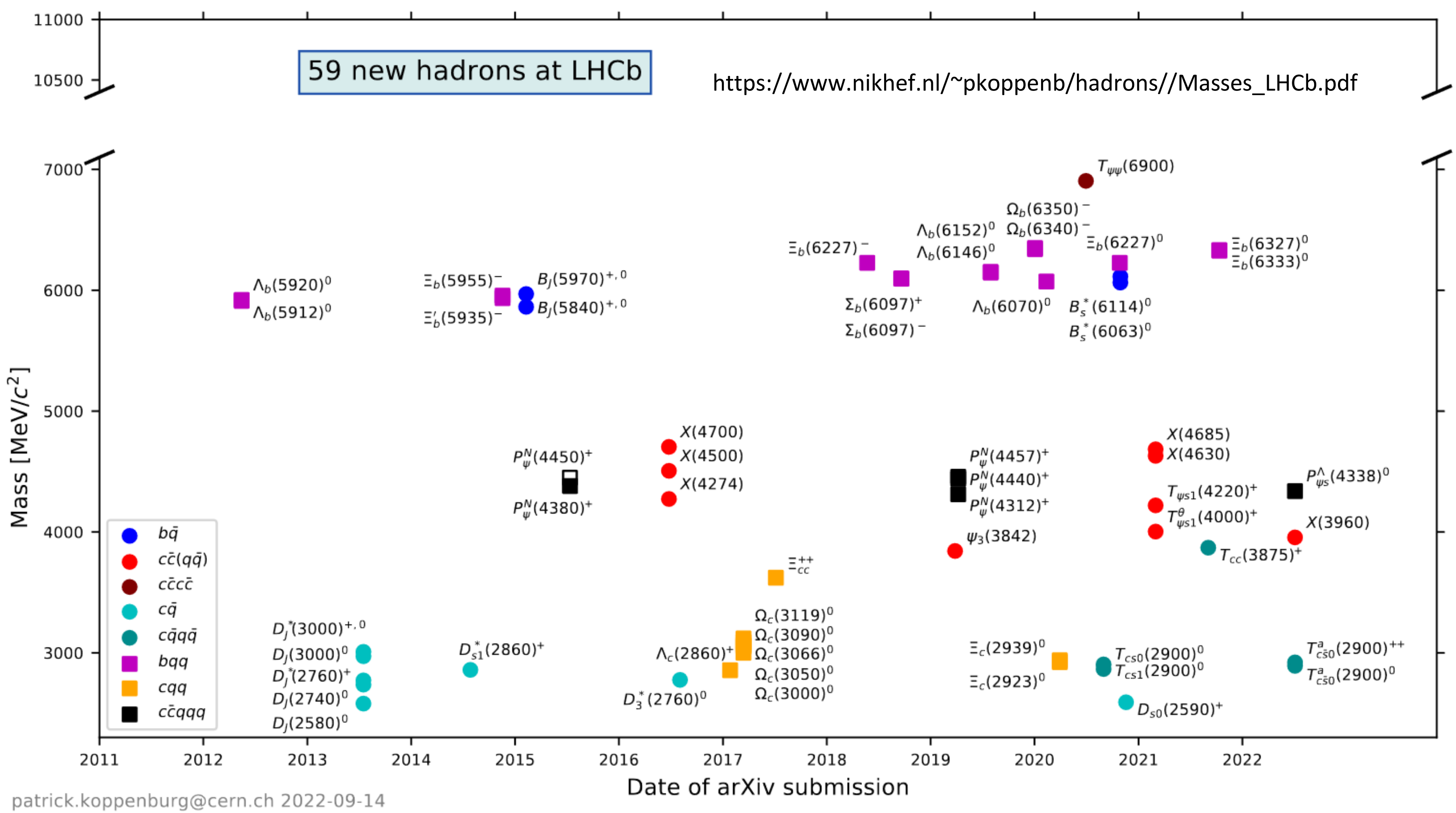
JINST 3 (2008) S08005, IJMPA 30 (2015) 1530022

- Powerful particle identification

  $\boldsymbol{\epsilon(K \to K) \sim 95\%}$ **mis-ID** $\boldsymbol{\epsilon(\pi \to K) \sim 5\%}$
  $\boldsymbol{\epsilon(\mu \to \mu) \sim 97\%}$ **mis-ID** $\boldsymbol{\epsilon(\pi \to \mu) \sim 1 - 3\%}$

- Good momentum resolution
  - $\frac{\delta p}{p} \sim (0.5 - 1.2)\%$
- Precise vertex resolution
  - $\sigma_{\mathrm{IP}_x} < 100\mu m$
  - $\sigma_\tau \sim 50\mathrm{fs}$

Hadrons observed at LHCb



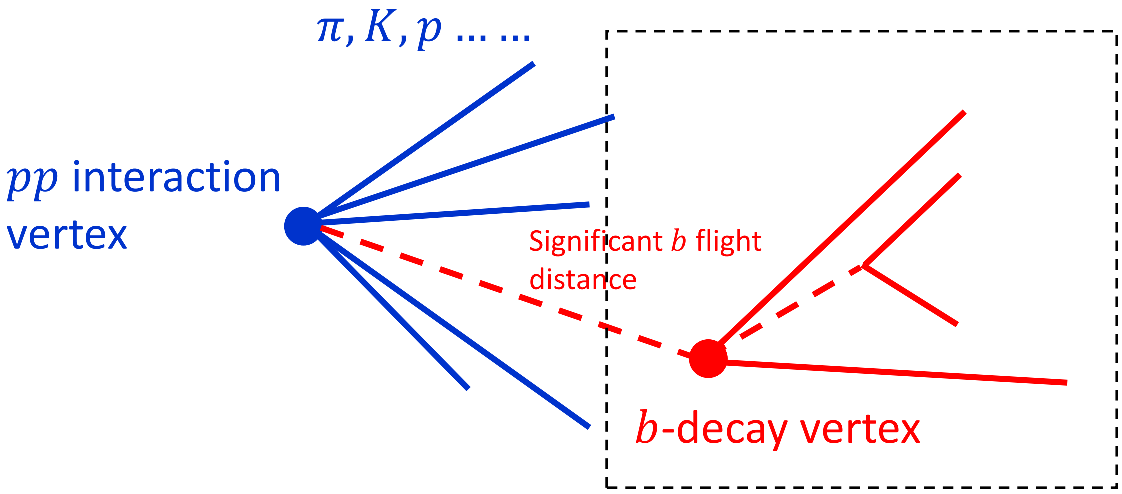
- Following "Exotic hadron naming convention" proposed by LHCb recently arXiv: 2206.15233

Selected results from LHCb

- Study of intermediate states in b decays
  - Excited Ξ_c^0 in $B^- \to \bar{\Lambda}_c^- \boldsymbol{\Lambda_c^+ K^-}$ LHCb-PAPER-2022-028, in preparation
  - Tetraquark candidates in $B^0 \to \bar{D}^0 \boldsymbol{D_s^+ \pi^-}$ and $B^+ \to D^- \boldsymbol{D_s^+ \pi^+}$ decays LHCb-PAPER-2022-026, LHCb-PAPER-2022-027, in preparation
  - Pentaquark candidates in $B^- \to \boldsymbol{J/\psi \Lambda} p$ LHCb-PAPER-2022-031, in preparation

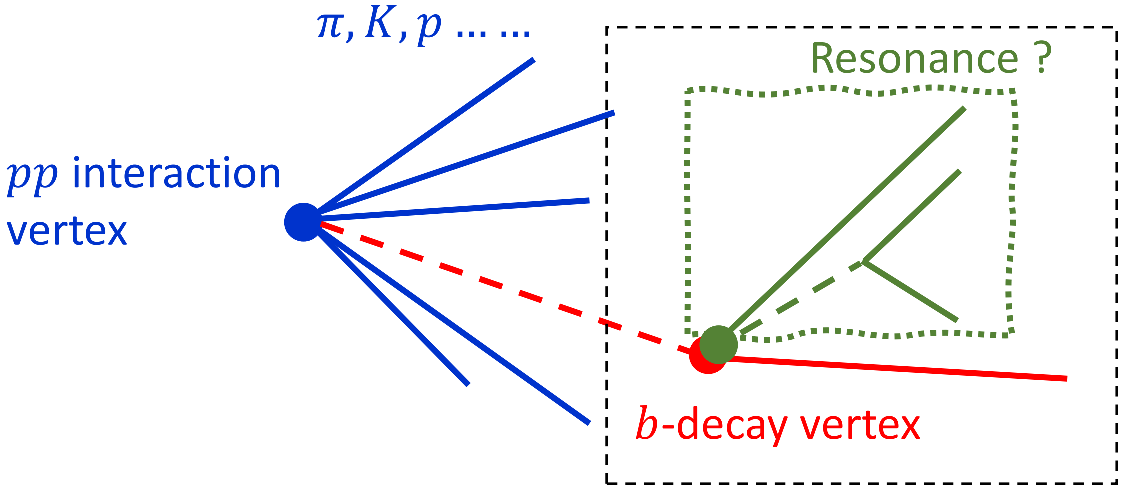
- Study the amplitude of three-body c decays：
  - $D^+ \to \pi^- \pi^+ \pi^+$ arXiv:2208.03300
  - $D_s^+ \to \pi^- \pi^+ \pi^+$ arXiv:2209.09840
  - $\Lambda_c^+ \to p K^- \pi^+$ arXiv:2208.03262

Spectroscopy study in b decays



Well-identified exclusive b-decays

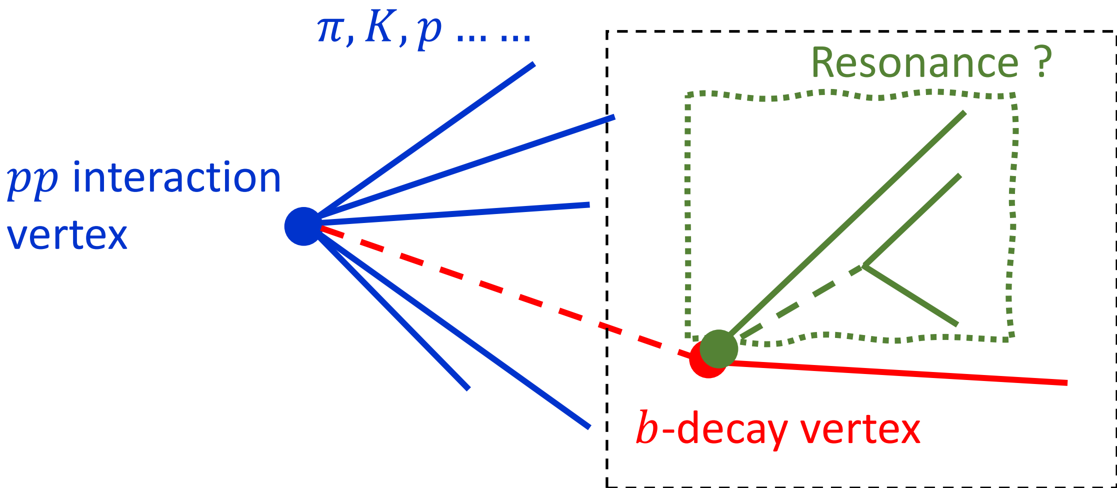
Spectroscopy study in b decays



Well-identified exclusive b-decays:

Low-background environment for spectroscopy study in final-state combinations

Spectroscopy study in b decays

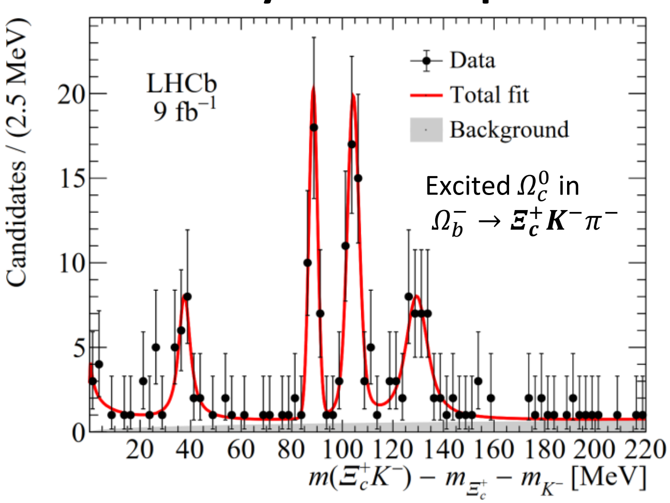


Well-identified exclusive b-decays:

Low-background environment for spectroscopy study in final-state combinations

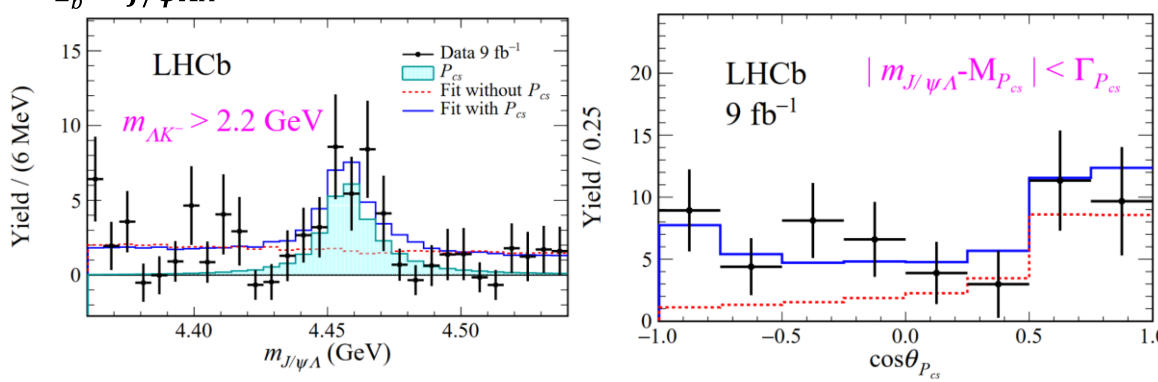
- Techniques:

Study the mass peaks



PRD 104(2021)L091102

Pentaquark evidence in $\Xi_b^- \to J/\psi \Lambda K^-$ **Amplitude analysis** (mass+angles)



Science Bulletin 66(2021)1278

Enables: J^P measurement; well describe **reflections**

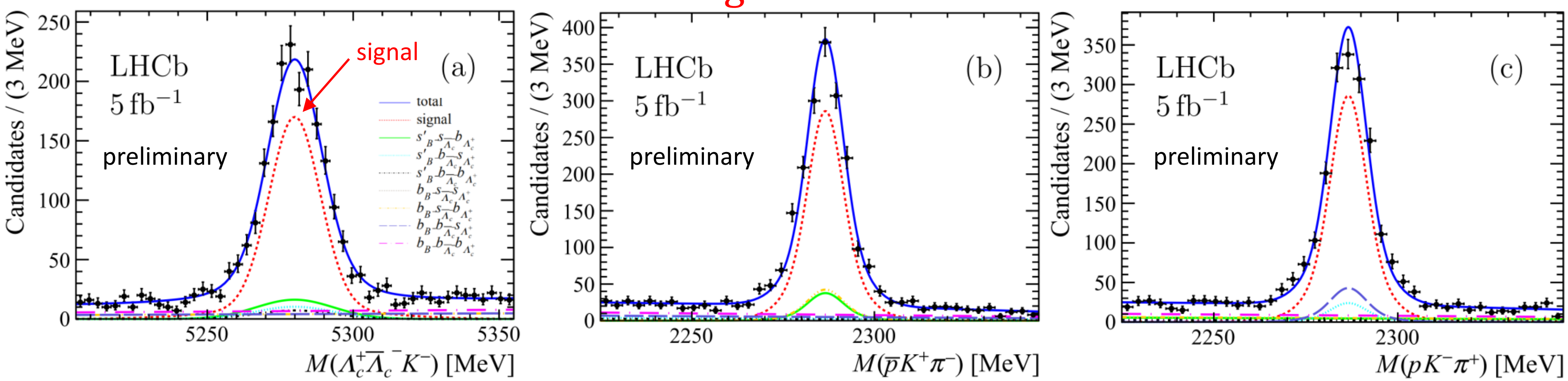
More & more important for **exotic hadron studies**

LHCb-PAPER-2022-028, in preparation

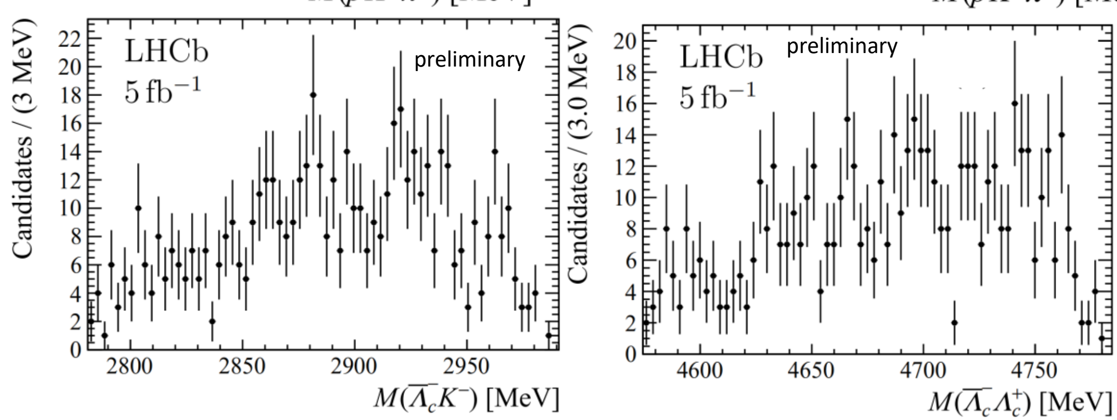
Study of $B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-$

The $B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-$ decay

- Interesting for conventional & exotic studies
    - $\boldsymbol{\Xi_c^{0**} \to \Lambda_c^+ K^-}$; exotic hadrons in $\boldsymbol{\Lambda_c^+ \bar{\Lambda}_c^-}$ and $\boldsymbol{\bar{\Lambda}_c^- K^-}$?
- High-purity sample, with $N_{\mathrm{sig}} = 1365 \pm 42$



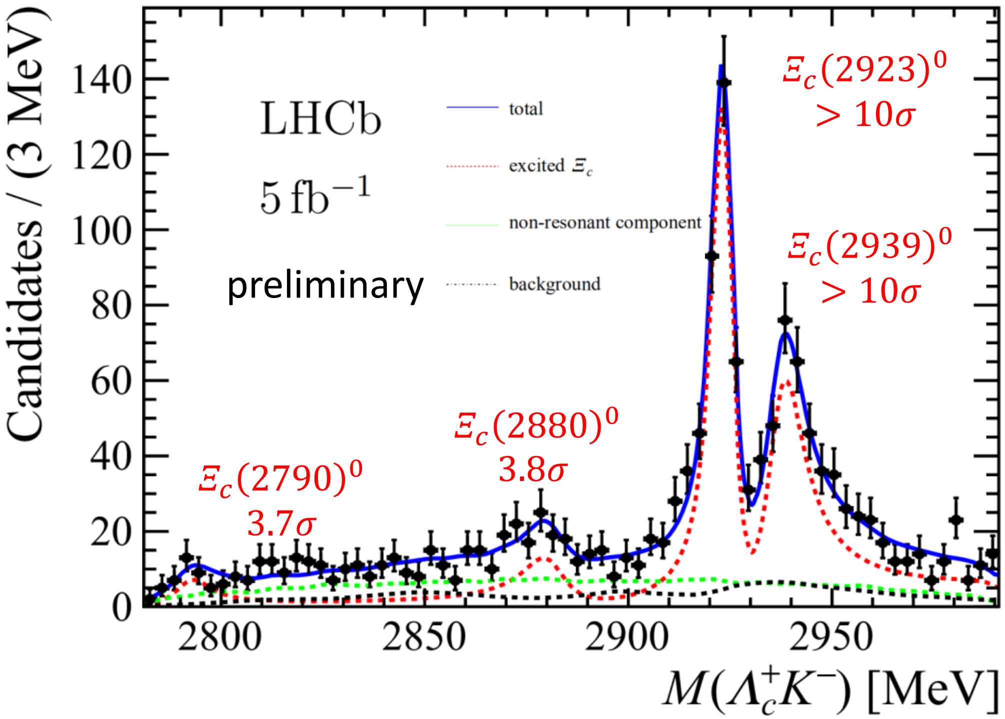
- No significant hint for exotic hadrons in $\boldsymbol{\Lambda_c^+ \bar{\Lambda}_c^-}$ or $\boldsymbol{\bar{\Lambda}_c^- K^-}$ mass spectra



LHCb-PAPER-2022-028, in preparation

$\Lambda_c^+ K^-$ mass spectrum study

- Fit $m(\Lambda_c^+ K^-)$ spectrum
  - 4 excited Ξ_c^0 states
  - Relativistic Breit-Wigner amplitude; interference considered



PRL 124(2020)222001

$\Xi_c(2923)^0$ & $\Xi_c(2939)^0$ were observed in prompt production, **confirm existence** in different production mechanism

Evidence of **new decay mode** for $\Xi_c(2790)^0$
Evidence of **new state** $\Xi_c(2880)^0$

State	Mass (MeV)	Width (MeV)
$\Xi_c(2880)^0$	$2881.8 \pm 3.1 \pm 8.5$	$12.4 \pm 5.2 \pm 5.8$
$\Xi_c(2923)^0$	$2924.5 \pm 0.4 \pm .1.1$	$4.8 \pm 0.9 \pm 1.5$
$\Xi_c(2939)^0$	$2938.5 \pm 0.9 \pm 2.3$	$11.0 \pm 1.9 \pm 7.5$

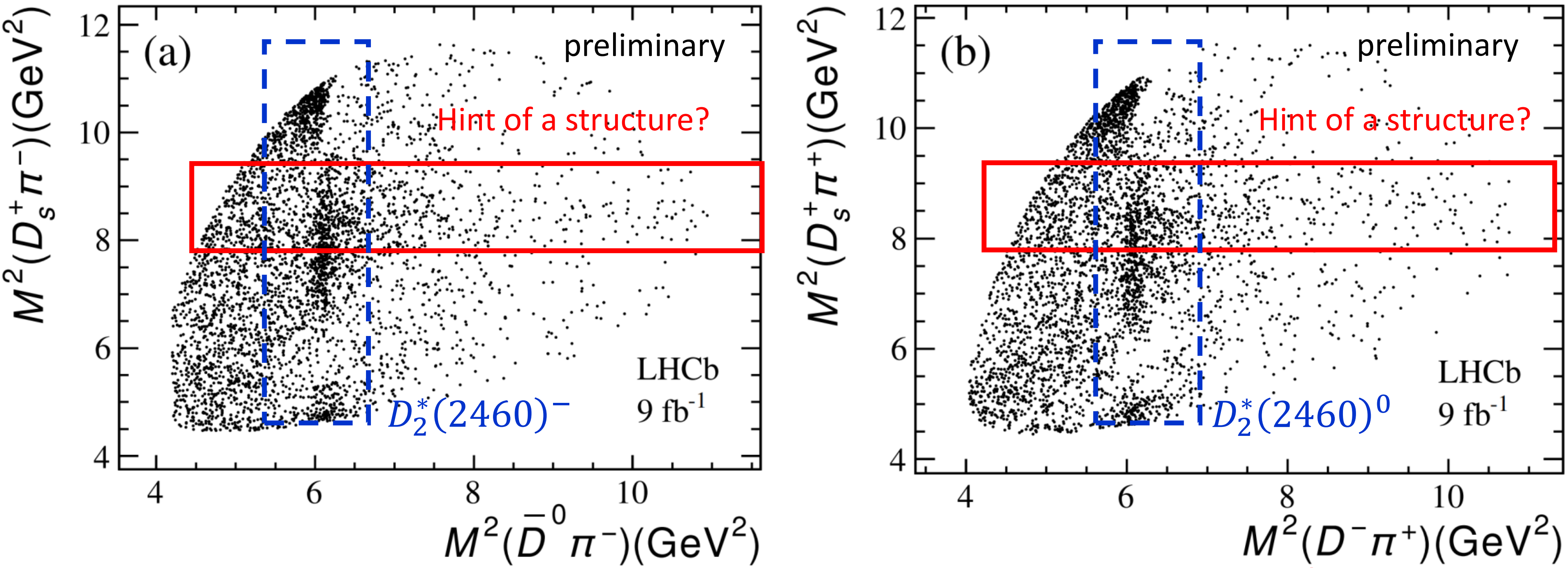
LHCb-PAPER-2022-026, LHCb-PAPER-2022-027, in preparation

Amplitude analysis of $B^0 \to \bar{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$ decays

LHCb-PAPER-2022-026, LHCb-PAPER-2022-027, in preparation

Study of $B^0 \to \overline{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$

- Full Run1+2 LHCb data, $\mathcal{L} = 9\ \mathrm{fb}^{-1}$
  - $B^0 \to \overline{D}^0 D_s^+ \pi^-$: **4420** candidates, with purity of $\mathbf{90.7\%}$
  - $B^+ \to D^- D_s^+ \pi^+$: **3940** candidates, with purity of $\mathbf{95.2\%}$

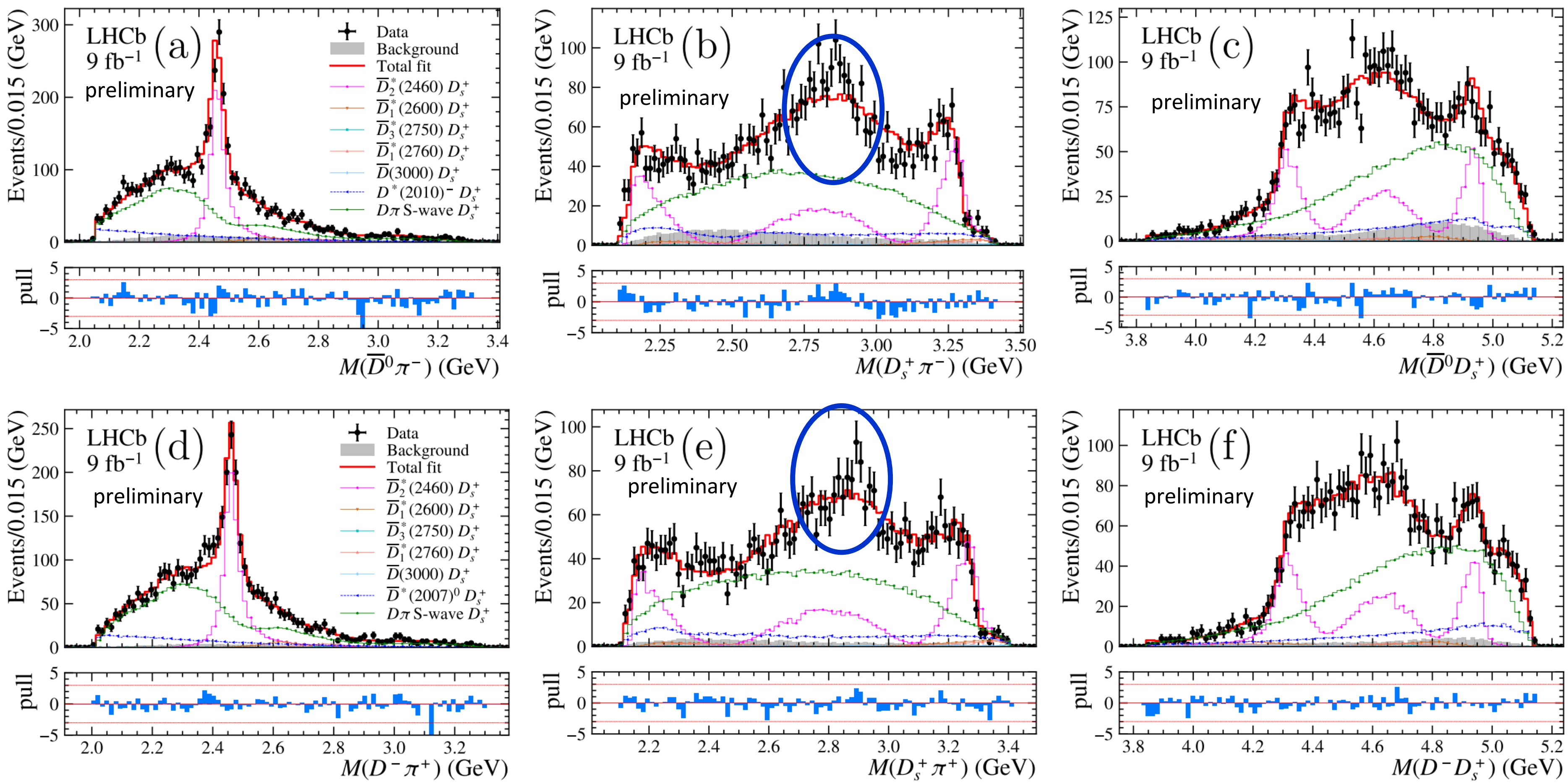


Amplitude analysis to investigate potential exotic $D_s^+\pi^\pm$ states

(joint study of two decays considering isospin symmetry)

LHCb-PAPER-2022-026, LHCb-PAPER-2022-027, in preparation

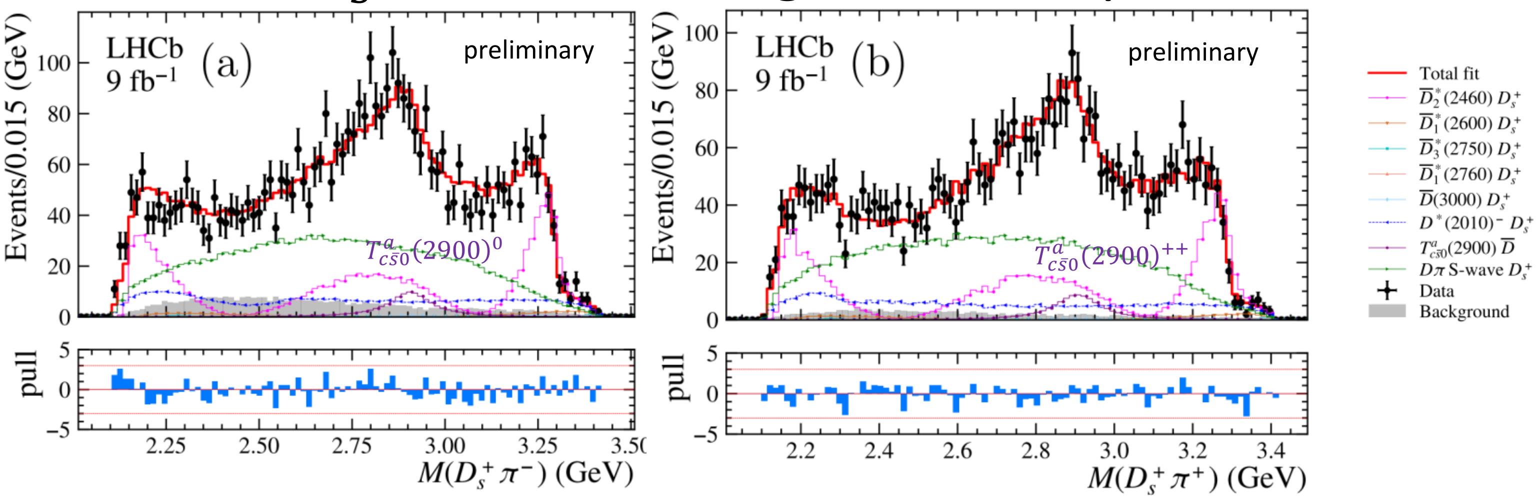
D^{**} −only fit



Cannot well describe $M(D_s^+\pi^\pm)$ spectrum at 2.85GeV when only considering conventional $D^-\pi^+$ and $\overline{D}^0\pi^-$ structures

Observation of $T^a_{c\bar{s}0}(2900)^{0/++}$

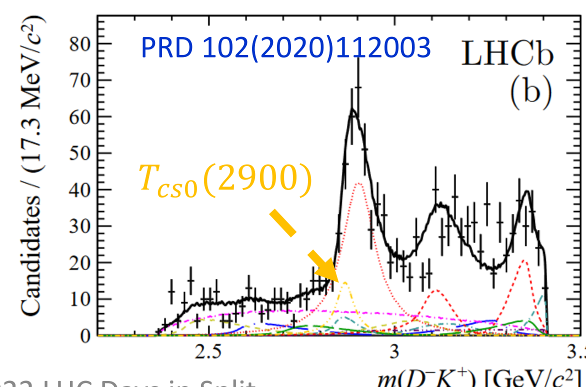
- Fit with two $D_s^+\pi$ states sharing resonance parameters



- $T^a_{c\bar{s}0}(2900)^0 \to D_s^+\pi^-$ & $T^a_{c\bar{s}0}(2900)^{++} \to D_s^+\pi^+$ **significance $> 9\sigma$**
- $\boldsymbol{J^P = 0^+}$ is preferred: $\boldsymbol{7.5\sigma}$ over other hypotheses

$$M = 2.908 \pm 0.011 \pm 0.020\,\text{GeV}$$
$$\Gamma = 0.136 \pm 0.023 \pm 0.011\,\text{GeV}$$
(RBW)

Flavor partner of $\boldsymbol{T_{cs0}(2900)}$ observed in $B^+ \to D^+\boldsymbol{D^-K^+}$?



$\boldsymbol{T_{cs0}(2900)}$:
$\bar{c}\bar{s}ud$
$\boldsymbol{T^a_{c\bar{s}0}(2900)^0}$:
$c\bar{s}\bar{u}d$
$\boldsymbol{T^a_{c\bar{s}0}(2900)^{++}}$:
$c\bar{s}u\bar{d}$

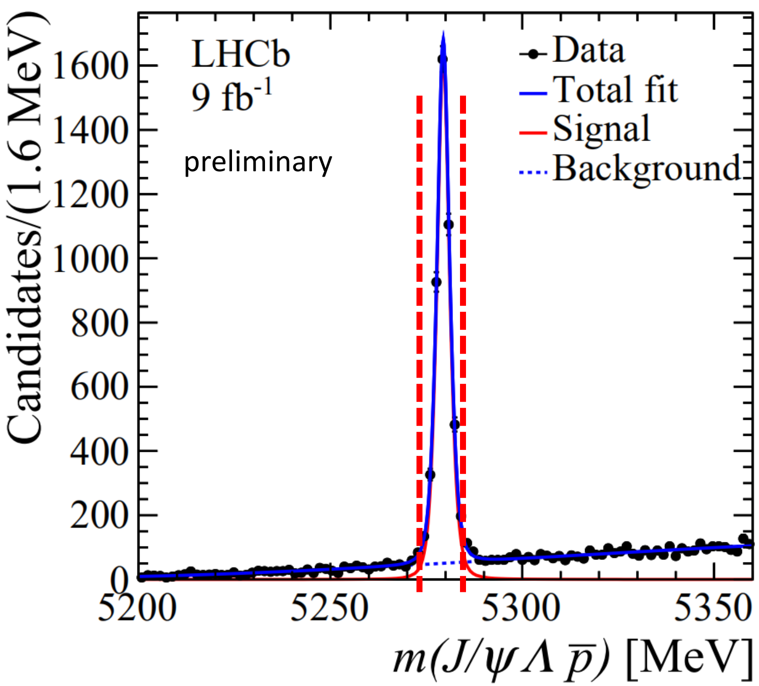
LHCb-PAPER-2022-031, in preparation

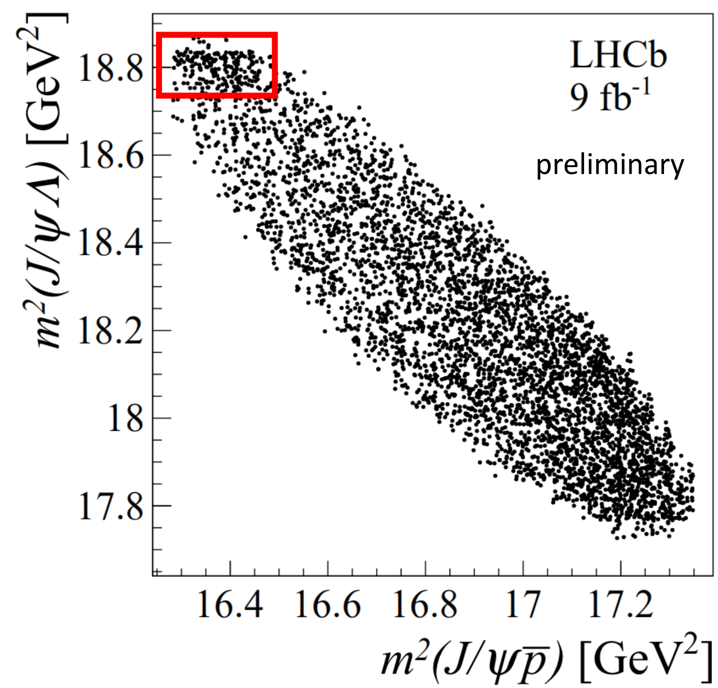
Amplitude analysis of $B^- \to J/\psi \Lambda \bar{p}$ decays

LHCb-PAPER-2022-031, in preparation

Study of $B^- \to J/\psi\Lambda\bar{p}$

- Search for pentaquark in $J/\psi p$ & $J/\psi\Lambda$
- Run1+Run2 LHCb data, $\mathcal{L} = 9\ \mathrm{fb}^{-1}$





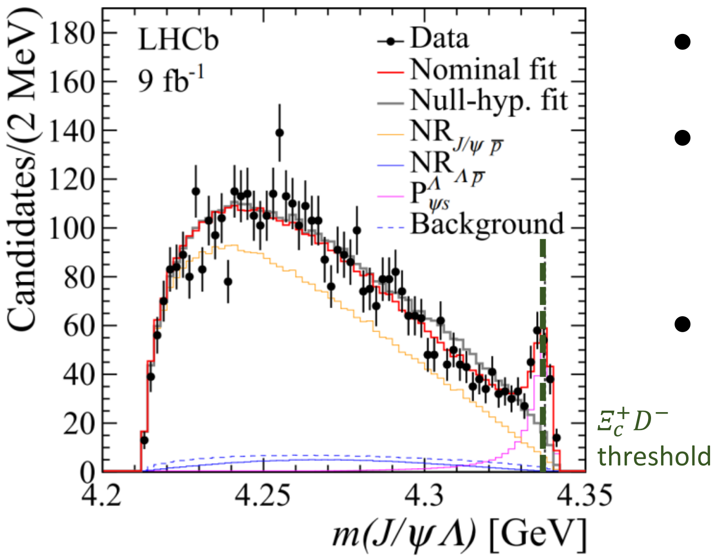
$N_{\mathrm{sig}} = \mathbf{4617 \pm 73}$

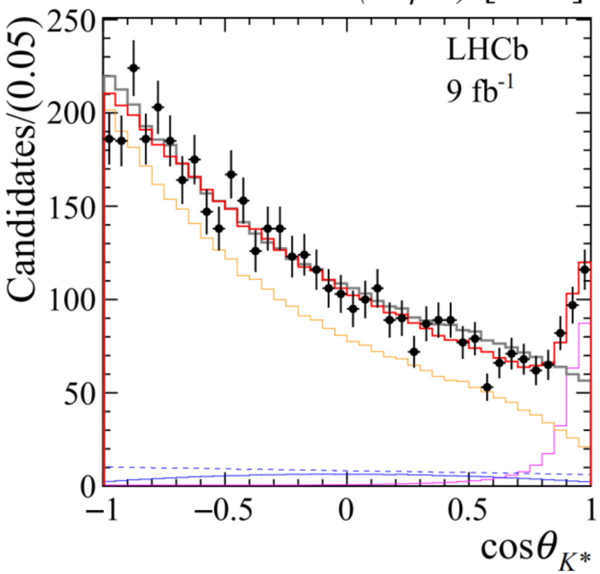
Purity in signal region : 93%

Horizontal band at $m^2(J/\psi\Lambda) \sim 18.8\mathrm{GeV}^2$

Further confirmed by amplitude analysis

$B^- \to J/\psi\Lambda\bar{p}$ amplitude analysis





- Helicity-based decay amplitude
- Components in nominal model:
  - Non-resonant $J/\psi p, p\Lambda$ + $P_{\psi s}^{\Lambda}$ in $J/\psi\Lambda$
- $P_{\psi s}^{\Lambda}$ significance $> 10\sigma$
  - 1st observation of pentaquark candidate with strangeness, $c\bar{c}uds$
- $J^P = \frac{1}{2}^-$ preferred, $J^P = \frac{1}{2}^+$ rejected under 90% $\mathrm{CL_s}$
- Mass, width (RBW) measured :

$$M\left(P_{\psi s}^{\Lambda}\right) = 4338.2 \pm 0.7 \pm 0.4\ \mathrm{MeV}$$
$$\Gamma\left(P_{\psi s}^{\Lambda}\right) = 7.0 \pm 1.2 \pm 1.3\ \mathrm{MeV}$$

- Near $\Xi_c^+ D^-$ threshold (4.337 GeV), in S-wave

Amplitude analysis of three-body c decays

LHC: good charm factory
LHCb: good at reconstruction of charm decays

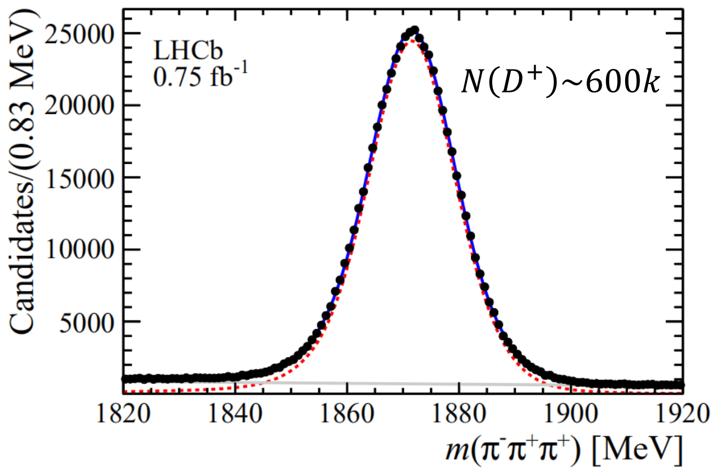
→ Investigate charm-decay amplitudes with competitive precisions

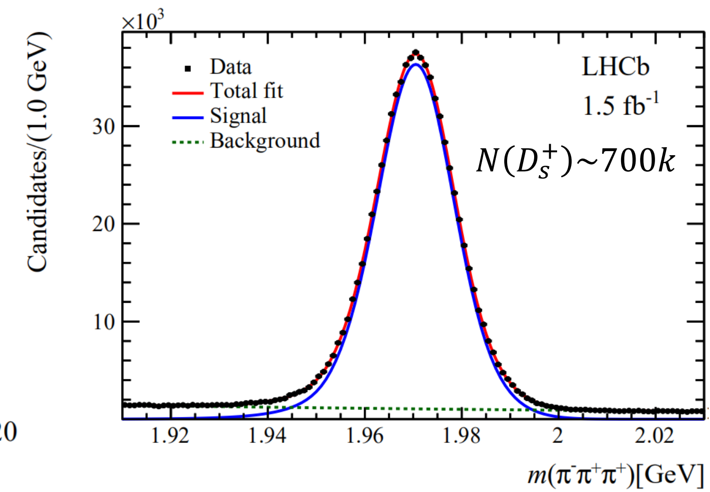
arXiv:2208.03300, arXiv:2209.09840

$$D^+(D_s^+) \to \pi^-\pi^+\pi^+$$

Amplitude analysis of $D^+(D_s^+) \to \pi^-\pi^+\pi^+$

- 2012 data, $\mathcal{L} = 1.5\ \mathrm{fb}^{-1}$. Promptly produced D mesons



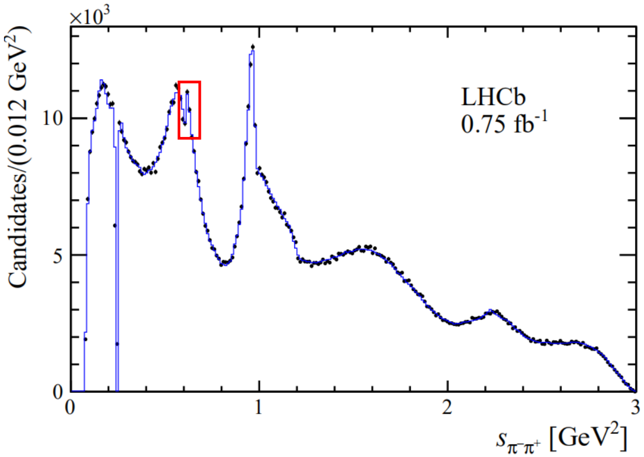


  - **Large statistic** & **High purity**
  - **Amplitude analysis** to enlighten knowledge of their dynamics
- Similar methodology for amplitude construction
  - **S-wave**: Quasi-Model Independent approach (**QMIPWA**)
    - Generic complex function determined by fit to data
  - **Isobar model for spin-1, spin-2 components**

arXiv:2208.03300, arXiv:2209.09840

Amplitude analysis of $D^+(D_s^+) \to \pi^-\pi^+\pi^+$

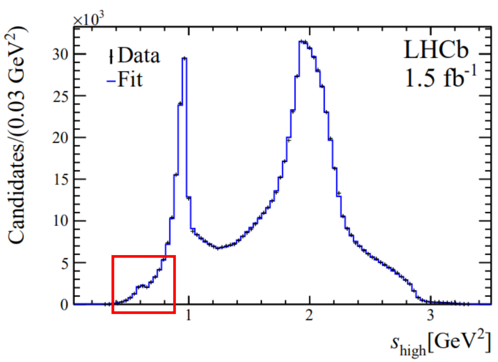
- $D^+ \to \pi^-\pi^+\pi^+$

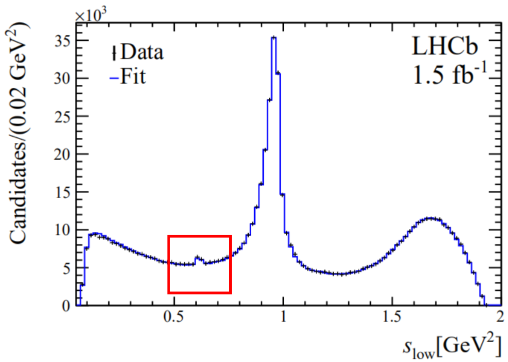


Component	Magnitude	Phase [°]	Fit fraction [%]
$\rho(770)^0\pi^+$	1 [fixed]	0 [fixed]	$26.0 \pm 0.3 \pm 1.6 \pm 0.3$
$\omega(782)\pi^+$	$(1.68 \pm 0.06 \pm 0.15 \pm 0.02) \times 10^{-2}$	$-103.3 \pm 2.1 \pm 2.6 \pm 0.4$	$0.103 \pm 0.008 \pm 0.014 \pm 0.002$
$\rho(1450)^0\pi^+$	$2.66 \pm 0.07 \pm 0.24 \pm 0.22$	$47.0 \pm 1.5 \pm 5.5 \pm 4.1$	$5.4 \pm 0.4 \pm 1.3 \pm 0.8$
$\rho(1700)^0\pi^+$	$7.41 \pm 0.18 \pm 0.47 \pm 0.71$	$-65.7 \pm 1.5 \pm 3.8 \pm 4.6$	$5.7 \pm 0.5 \pm 1.0 \pm 1.0$
$f_2(1270)\pi^+$	$2.16 \pm 0.02 \pm 0.10 \pm 0.02$	$-100.9 \pm 0.7 \pm 2.0 \pm 0.4$	$13.8 \pm 0.2 \pm 0.4 \pm 0.2$
S-wave			$61.8 \pm 0.5 \pm 0.6 \pm 0.5$
$\sum_i \mathrm{FF}_i$			112.8
χ^2/ndof (range)	[1.47 - 1.78]		$-2\log\mathcal{L} = 805622$

Dominated by S-wave, followed by $\rho(770)^0\pi^+$ and $f_2(1270)^0\pi^+$
Contribution from $(\omega(782) \to \pi^+\pi^-)\pi^+$ observed for the first time

- $D_s^+ \to \pi^-\pi^+\pi^+$



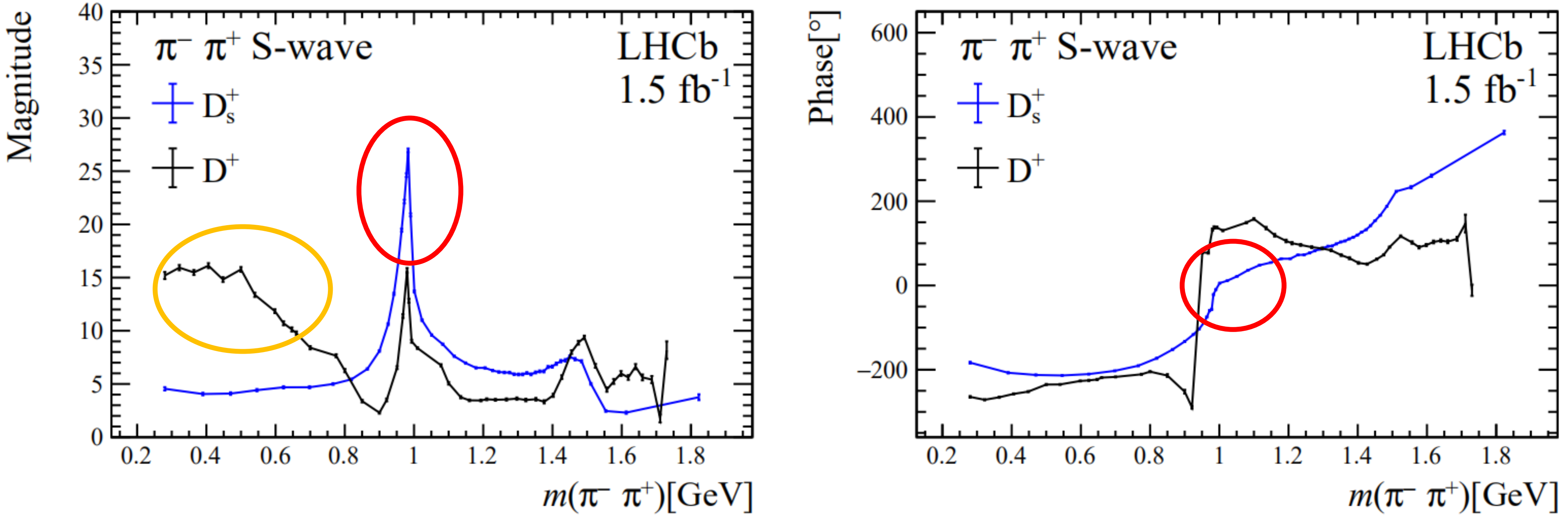


Resonance	Magnitude	Phase [°]	Fit fraction (FF) [%]
S-wave			84.97 ± 0.14
$\rho(770)^0$	0.1201 ± 0.0030	79.4 ± 1.8	1.038 ± 0.054
$\omega(782)$	0.04001 ± 0.00090	-109.9 ± 1.7	0.360 ± 0.016
$\rho(1450)^0$	1.277 ± 0.026	-115.2 ± 2.6	3.86 ± 0.15
$\rho(1700)^0$	0.873 ± 0.061	-60.9 ± 6.1	0.365 ± 0.050
combined	–	–	6.14 ± 0.27
$f_2(1270)$	1 (fixed)	0 (fixed)	13.69 ± 0.14
$f_2'(1525)$	0.1098 ± 0.0069	178.1 ± 4.2	0.0455 ± 0.0070
sum of fit fractions			104.3
χ^2/ndof (range)	$[1.45 - 1.57]$		

Dominated by S-wave, followed by spin-2 resonances
Contribution from $(\omega(782) \to \pi^+\pi^-)\pi^+$observed for the first time

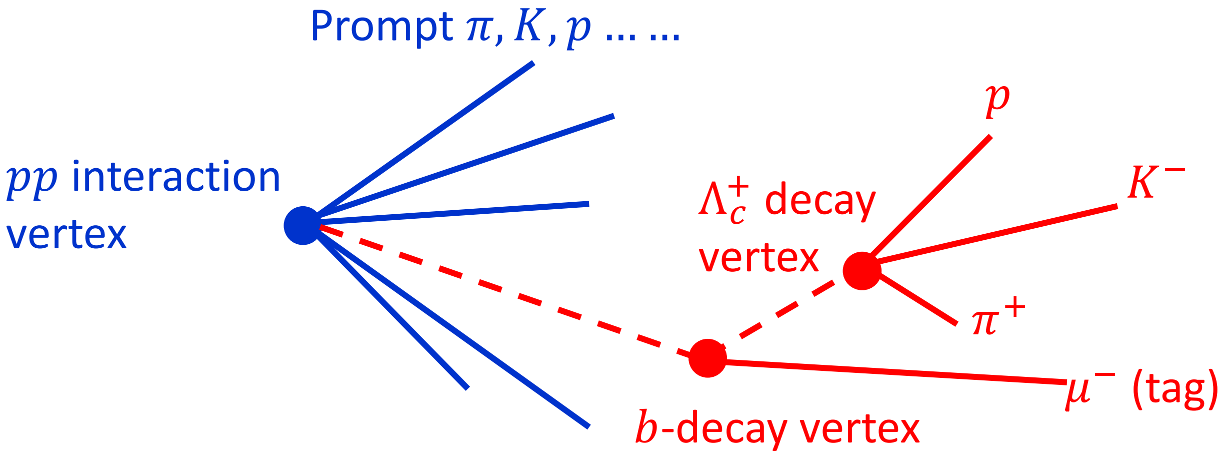
arXiv:2208.03300, arXiv:2209.09840

Comparison of S-wave amplitudes



- $f_0(980)$ is most prominent contribution in D_s^+, while $f_0(500)$ in D^+
  - D_s^+ @ 1GeV: peak in magnitude; sharp variation in phase
  - D^+ near threshold enhancement
- Indicate different $\pi^+\pi^-$ production process
  - Example: unitary chiral model, different intermediate rescattering channels
    Int. J. Mod. Phys. **25**(2016)1630001

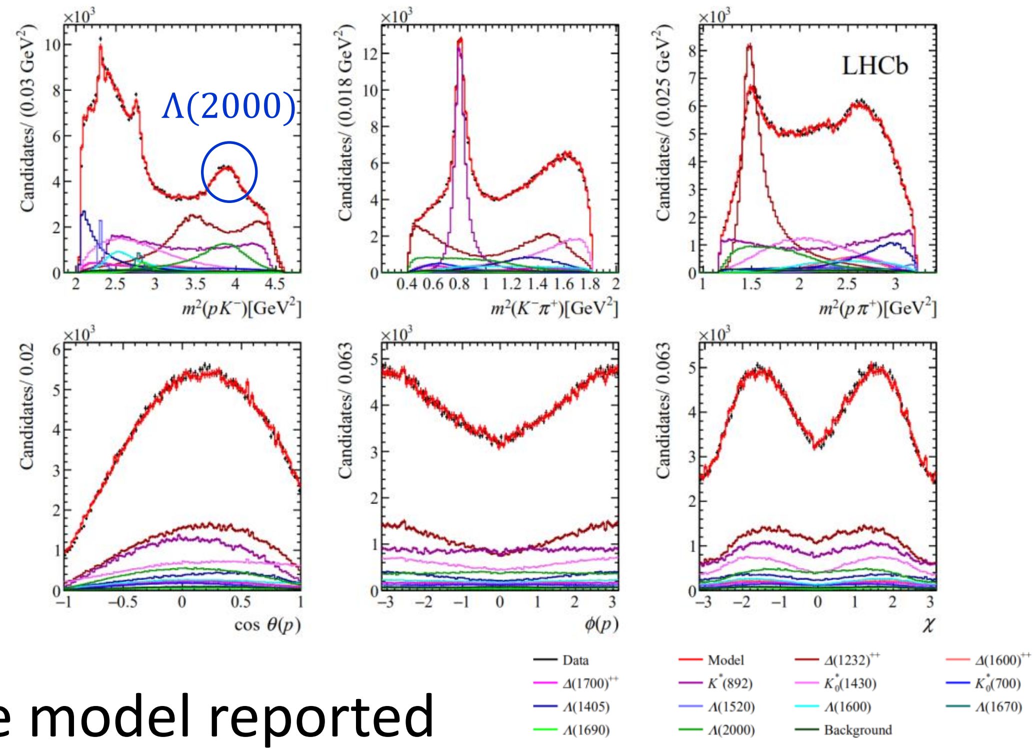
$$\Lambda_b^0 \to (\Lambda_c^+ \to pK^-\pi^+)\mu^-\bar{\nu}_\nu X$$



$\Lambda_c^+ \to pK^-\pi^+$ amplitude analysis

- 2016 data, $\mathcal{L} = 1.7\ \mathrm{fb}^{-1}$; $\sim$400k signals;

Resonance	J^P	Mass (MeV)	Width (MeV)
$\Lambda(1405)$	$1/2^-$	1405.1	50.5
$\Lambda(1520)$	$3/2^-$	$1515-1523$	$10-20$
$\Lambda(1600)$	$1/2^+$	1630	250
$\Lambda(1670)$	$1/2^-$	1670	30
$\Lambda(1690)$	$3/2^-$	1690	70
$\Lambda(2000)$	$1/2^-$	$1900-2100$	$20-400$
$\Delta(1232)^{++}$	$3/2^+$	1232	117
$\Delta(1600)^{++}$	$3/2^+$	1640	300
$\Delta(1700)^{++}$	$3/2^-$	1690	380
$K_0^*(700)$	0^+	824	478
$K^*(892)$	1^-	895.5	47.3
$K_0^*(1430)$	0^+	1375	190



- All parameters of amplitude model reported
- Mass and width of $\Lambda(2000)$ determined

$$m = 1988 \pm 2 \pm 21\,\mathrm{MeV} \qquad \Gamma = 179 \pm 4 \pm 16\,\mathrm{MeV}$$

Summary

- LHCb keeps making important contributions to spectroscopy study
- Achievement about physics:
  - Excited $\boldsymbol{\Xi_c^0}$ **states** in $B^- \to \boldsymbol{\Lambda_c^+} \bar{\Lambda}_c^- \boldsymbol{K^-}$
  - New exotic states: $\boldsymbol{T_{c\bar{s}0}^{a}(2900)^{0/++}}$, $\boldsymbol{P_{\psi s}^{\Lambda}}$
  - Improved knowledge to: $\boldsymbol{D^+(D_s^+) \to \pi^-\pi^+\pi^+, \Lambda_c^+ \to pK^-\pi^+}$

Summary

- LHCb keeps making important contributions to spectroscopy study
- Achievement about physics:
  - Excited $\boldsymbol{\Xi_c^0}$ **states** in $B^- \to \boldsymbol{\Lambda_c^+ \bar{\Lambda}_c^- K^-}$
  - New exotic states: $\boldsymbol{T_{c\bar{s}0}^{a}(2900)^{0/++}}$, $\boldsymbol{P_{\psi s}^{\Lambda}}$
  - Improved knowledge to: $\boldsymbol{D^+(D_s^+) \to \pi^-\pi^+\pi^+, \Lambda_c^+ \to pK^-\pi^+}$

A lot of new results from **amplitude analysis**

Making use of generalized & advanced analysis tools

https://tf-pwa.readthedocs.io/en/latest/ by Yi Jiang et. al.

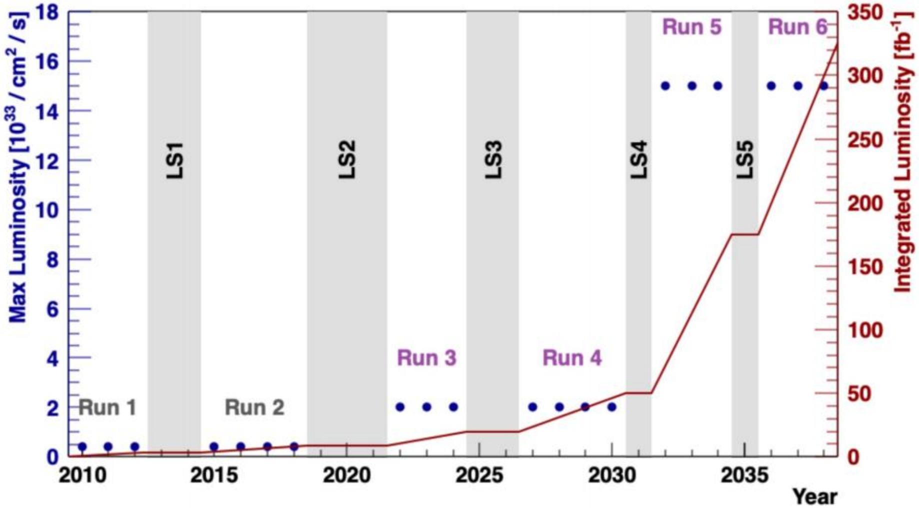
& with improved knowledge about general formalism of decay amplitudes

PRD 101(2020)034033, Adv.High Energy Phys. 2020(2020)6674595, Chinese Phys. C 45(2021)063103

Prospects

- Upcoming Run3 data
  - Improved integrated Luminosity
  - New detector, fully software-based trigger

A huge wave of high-quality data is coming !



More exciting results are to come!

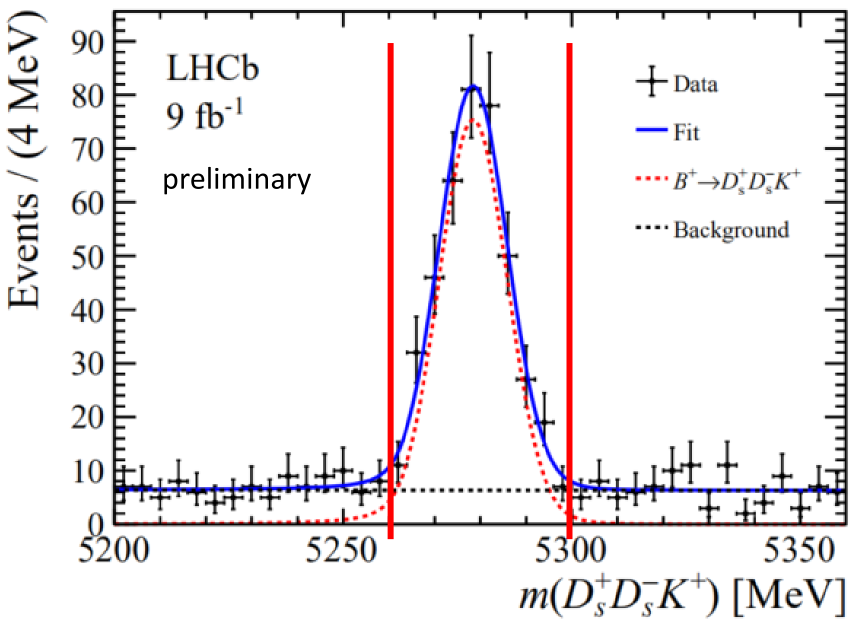
- Run3 data + improvement of analysis skills, boosting LHCb spectroscopy studies to a new level

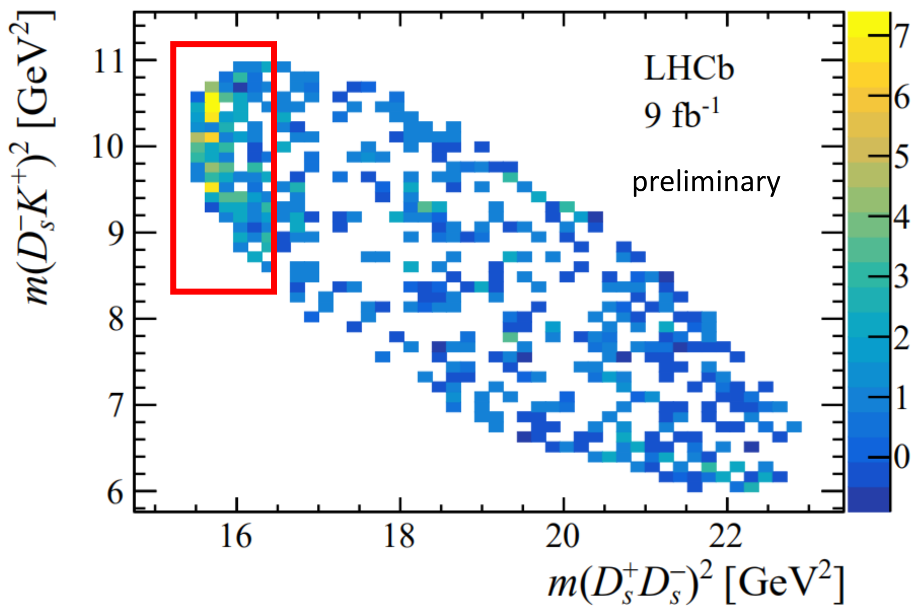
Thank you for your attention !
Any questions or comment ?

Back up

Study of $B^+ \to D_s^+ D_s^- K^+$

- Run1+Run2 LHCb data, $\mathcal{L} = 9\ \text{fb}^{-1}$





$N_{\text{sig}} = \mathbf{360 \pm 22}$

(1st observation of $B^+ \to D_s^+ D_s^- K^+$)

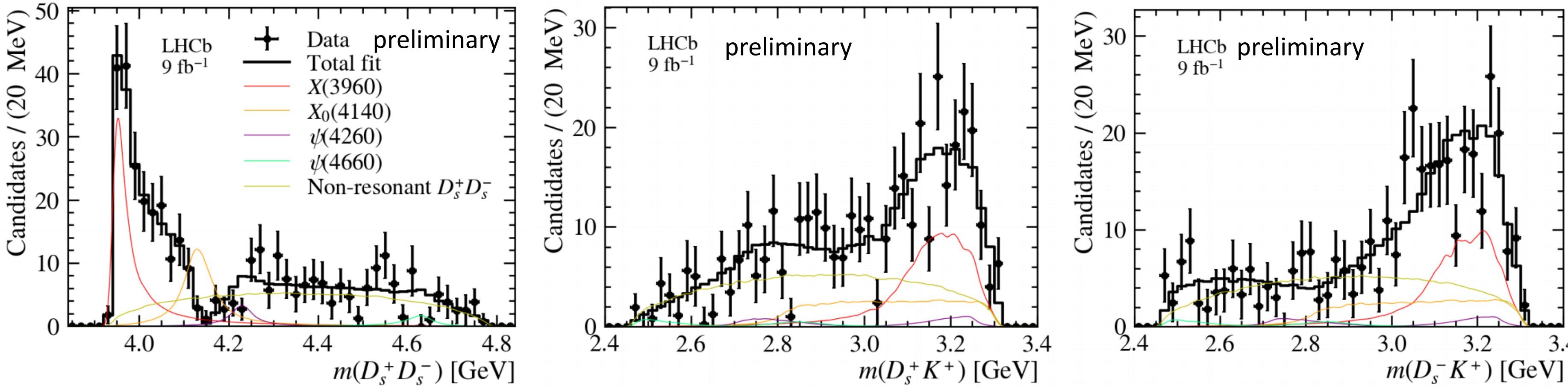
Purity in signal region : 84%

Hint of near-threshold structure in $D_s^+ D_s^-$ mass spectrum

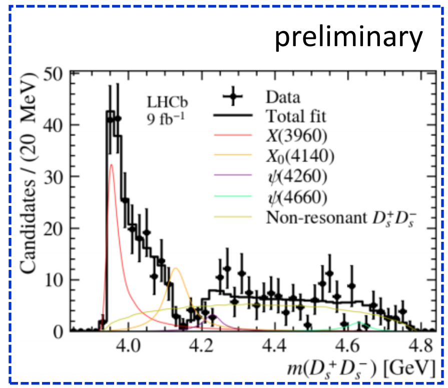
Amplitude analysis to investigate its properties

Observation of $X(3960) \to D_s^+ D_s^-$

- Base-line model with several $D_s^+ D_s^-$ structures well describes data



  - 0^{++}: $X(3960)$ $(> 10\sigma)$, $X_0(4140)$ (3.9σ), Non-resonant
  - 1^{--}: $\psi(4260), \psi(4660)$
- $X(3960)$: threshold enhancement
  - $J^{PC} = 0^{++}$ preferred by $> 9\sigma$
  - **1st observation, with J^P determined**
- $X_0(4140)$: for dip at 4.14GeV (interference)
  - $J^{PC} = 0^{++}$ preferred by 3.5σ



Alternative way to model the dip:
$J/\psi\phi \to D_s^+ D_s^-$ rescattering

$X(3960)$ and $\chi_{c0}(3930)$

	M [MeV]	Γ [MeV]	J^{PC}
$X(3960)$ preliminary	$3955 \pm 6 \pm 12$	$48 \pm 17 \pm 10$	0^{++}
$\chi_{c0}(3930)$	3924 ± 2	17 ± 5	

PRD **102**(2020)112003

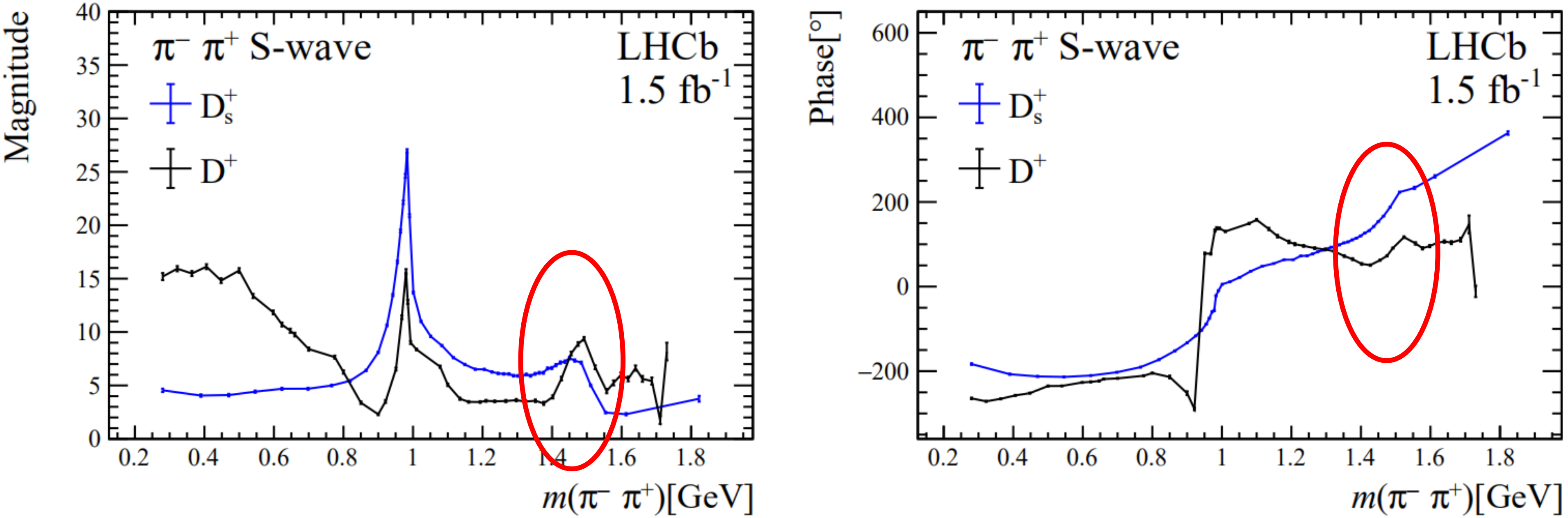
- No obvious candidate within conventional charmonium multiple for $X(3960)$ and $\chi_{c0}(3930)$: **exotic nature; same particle ?**
- If they are the same particle

$$\frac{\Gamma(X \to D^+ D^-)}{\Gamma(X \to D_s^+ D_s^-)} = 0.29 \pm 0.09 \pm 0.10 \pm 0.08$$

  - Enhancement of $D_s^+ D_s^-$ partial width indicate a intrinsic $s\bar{s}$ content
    - Generation of $s\bar{s}$ from vacuum more difficult than $u\bar{u}, d\bar{d}$
    - Smaller phase-space in $D_s^+ D_s^-$ mode
  - An exotic nature of the X state: **Candidate for $c\bar{c}s\bar{s}$**

arXiv:2208.03300, arXiv:2209.09840

S-wave amplitudes: Similarity



- Hint of scalar resonance at 1.5GeV for both modes
  - Sharp variation in both magnitude and phase

$D_s^+ \to \pi^+\pi^-\pi^+$ from BESIII

- BESIII

arXiv: 2108.10050, submitted in 2021

TABLE II. Fit fractions, magnitudes and phases from our baseline fit. The uncertainties are statistical and systematic, respectively.

Decay mode	Fit fraction (%)	Magnitude	Phase (radians)
$f_2(1270)\pi^+$	$10.5 \pm 0.8 \pm 1.2$	1. (Fixed)	0. (Fixed)
$\rho(770)\pi^+$	$0.9 \pm 0.4 \pm 0.5$	$0.13 \pm 0.03 \pm 0.04$	$5.44 \pm 0.25 \pm 0.62$
$\rho(1450)\pi^+$	$1.3 \pm 0.4 \pm 0.5$	$0.91 \pm 0.16 \pm 0.22$	$1.03 \pm 0.32 \pm 0.51$
$\mathcal{S}$ wave	$84.2 \pm 0.8 \pm 1.3$	Table III	Table III
Total	$96.8 \pm 2.4 \pm 3.5$		

- LHCb

arXiv:2209.09840, submitted in 2022

Resonance	Magnitude	Phase [°]	Fit fraction (FF) [%]
S-wave			84.97 ± 0.14
$\rho(770)^0$	0.1201 ± 0.0030	79.4 ± 1.8	1.038 ± 0.054
$\omega(782)$	0.04001 ± 0.00090	-109.9 ± 1.7	0.360 ± 0.016
$\rho(1450)^0$	1.277 ± 0.026	-115.2 ± 2.6	3.86 ± 0.15
$\rho(1700)^0$	0.873 ± 0.061	-60.9 ± 6.1	0.365 ± 0.050
combined	–	–	6.14 ± 0.27
$f_2(1270)$	1 (fixed)	0 (fixed)	13.69 ± 0.14
$f_2'(1525)$	0.1098 ± 0.0069	178.1 ± 4.2	0.0455 ± 0.0070
sum of fit fractions			104.3
χ^2/ndof (range)	$[1.45 - 1.57]$		

$D^+ \to \pi^+\pi^-\pi^+$ previous results

- Other experiments: (tables taken from Phys.Rev.D 76 (2007) 012001)

PRL. 86(2001)770 PLB 585(2004)200

Mode	E791 [3]	FOCUS [4]
$\sigma\pi^+$	46.3 ± 9.2	
$f_0(980)\pi^+$	6.2 ± 1.4	
$f_0(1370)\pi^+$	2.3 ± 1.7	
S wave π^+	54.8 ± 9.5	56.0 ± 3.9
$\rho^0(770)\pi^+$	33.6 ± 3.9	30.8 ± 3.9
$f_2(1270)\pi^+$	19.4 ± 2.5	11.7 ± 1.9

TABLE II: Results of the isobar model analysis of the $D^+ \to \pi^-\pi^+\pi^+$ Dalitz plot. For each contribution the relative amplitude, phase, and fit fraction is given. The errors are statistical and systematic, respectively.

Mode	Amplitude (a.u.)	Phase (°)	Fit fraction (%)
$\rho(770)\pi^+$	1(fixed)	0(fixed)	20.0±2.3±0.9
$f_0(980)\pi^+$	1.4±0.2±0.2	12±10±5	4.1±0.9±0.3
$f_2(1270)\pi^+$	2.1±0.2±0.1	−123±6±3	18.2±2.6±0.7
$f_0(1370)\pi^+$	1.3±0.4±0.2	−21±15±14	2.6±1.8±0.6
$f_0(1500)\pi^+$	1.1±0.3±0.2	−44±13±16	3.4±1.0±0.8
σ pole	3.7±0.3±0.2	−3±4±2	41.8±1.4±2.5

Phys.Rev.D 76 (2007) 012001

- LHCb

arXiv:2208.03300

Component	Magnitude	Phase [°]	Fit fraction [%]
$\rho(770)^0\pi^+$	1 [fixed]	0 [fixed]	$26.0 \pm 0.3 \pm 1.6 \pm 0.3$
$\omega(782)\pi^+$	$(1.68 \pm 0.06 \pm 0.15 \pm 0.02) \times 10^{-2}$	$-103.3 \pm 2.1 \pm 2.6 \pm 0.4$	$0.103 \pm 0.008 \pm 0.014 \pm 0.002$
$\rho(1450)^0\pi^+$	$2.66 \pm 0.07 \pm 0.24 \pm 0.22$	$47.0 \pm 1.5 \pm 5.5 \pm 4.1$	$5.4 \pm 0.4 \pm 1.3 \pm 0.8$
$\rho(1700)^0\pi^+$	$7.41 \pm 0.18 \pm 0.47 \pm 0.71$	$-65.7 \pm 1.5 \pm 3.8 \pm 4.6$	$5.7 \pm 0.5 \pm 1.0 \pm 1.0$
$f_2(1270)\pi^+$	$2.16 \pm 0.02 \pm 0.10 \pm 0.02$	$-100.9 \pm 0.7 \pm 2.0 \pm 0.4$	$13.8 \pm 0.2 \pm 0.4 \pm 0.2$
S-wave			$61.8 \pm 0.5 \pm 0.6 \pm 0.5$
$\sum_i \mathrm{FF}_i$			112.8
χ^2/ndof (range)	[1.47 - 1.78]		$-2\log\mathcal{L} = 805622$