MPS/Int.LIN 66-2
14 March, 1966

TRANSVERSE PROPERTIES OF THE PS-LINAC BEAM

C.S. Taylor and P. Têtu

Abstract

The report contains an experimental description of the transverse behaviour of the PS-Linac beam. The behaviour is described in terms of the hypervolume projection on to a two-dimensional transverse phase plane. The distribution of particles in this projection is seen to be determined at energies less than 10 MeV. The rôles of phase coupling and quadrupole imperfections are discussed in relation to beam blow-up.

* * *

Contents

Introduction
1. Approach
2. Experimental Background
3. Effect of R.F. and Energy
4. Comparison of 500 KeV and 50 MeV Beams
5. Blow-up
6. Other Topics
7. Summary and Conclusions

TRANSVERSE PROPERTIES OF THE PS-LINAC BEAM

Introduction

The purpose of this report is to describe the transverse behaviour of the proton beam in the 50 MeV Linac up to currents of 60 mA. It is essentially an interim report on work in progress, discussing the situation as it appears in March, 1966, and many questions therefore remain unanswered.

The experimental work was carried out entirely with the R.F. ion source and conventional accelerating column.

1. Approach

At the present moment, one can only study the 6-dimensional hypervolume experimentally by studying its projection on the (p_x, x), (p_y, y) and (p_z, z) planes.1),2) In an uncoupled linear system this would not be a limitation, but in an actual accelerator which is both coupled and non-linear, our measurement yields a phase space area which has been integrated over the range of the coupled variables, and integrated also in the sense that discontinuities due to filamentation in the longtitudinal plane are smoothed out by present measuring techniques.

One can, however, take the view that departures from invariance in measured phase space in one plane are useful indications of the presence of couplings or non-linearities, and this viewpoint is in fact basic to the present study.

---

1) Rather like trying to describe a statue by its shadows

2) x, y, and z are longtitudinal, horizontal and vertical directions respectively

Experimental results are therefore discussed here in terms of one transverse plane only, but in many cases the measurements were performed in both transverse planes in order to check that no useful information was being lost.

2. Experimental Background

Since 1959, when the 50 MeV Linear Accelerator first went into operation as an injector for the P.S., there has been an accumulation of data concerning the transverse properties of the beam, derived both from routine measurements and observations, and from development work. Some of this information has not fitted easily into a consistent picture of the machine's behaviour, and we describe here four of these observations as an introduction to the problem.

1. Early in the operation of the Linac, it was noticed that the 500 KeV steering coils produced a steering effect on the 50 MeV beam. Fig. 1 shows the measured position dependence of the beam centres[1]. This behaviour led to the supposition that between the measuring points there was something approaching a 2 x 2 matrix[2].

2. Total emittance measurements appeared to be sensitive to R.F. accelerating conditions[3]. Further evidence for the effect of R.F. conditions appeared at higher beam currents, when the emittance envelope was seen to rotate during the passage of the beam, due to beam loading[4].

3. A small element of 500 KeV phase space was found to occupy 50 times its normalised 500 KeV area when measured at 50 MeV.[3]

4. The P.S. intensity was sensitive to the ion source quality.

It appeared from these observations that the output beam was closely related to the input beam, that there was blow-up, and that the R.F. played an important rôle. The connection between these facts was not obvious, however.

---

1) This subject has been explored in more detail recently on the 15 MeV Nimrod injector, and is reported by N.D. West: Machine Physics Experiments on the Nimrod Injector NIRL/R/84 1965

2) The A-48 accelerator at Livermore was said to produce an image of the source exit holes on the target

3) C.S. Taylor: International Conference on High Energy Accelerators Dubna 1963 pp. 482, 484
The paper by P. Têtu, "Emittance du faisceau Linac", MPS/Int.LIN 63-5, gives more details of the measurement technique

4) C.S. Taylor and Y. Dupuis: 1964 Linear Accelerator Conference, Mura, pp.242, 245

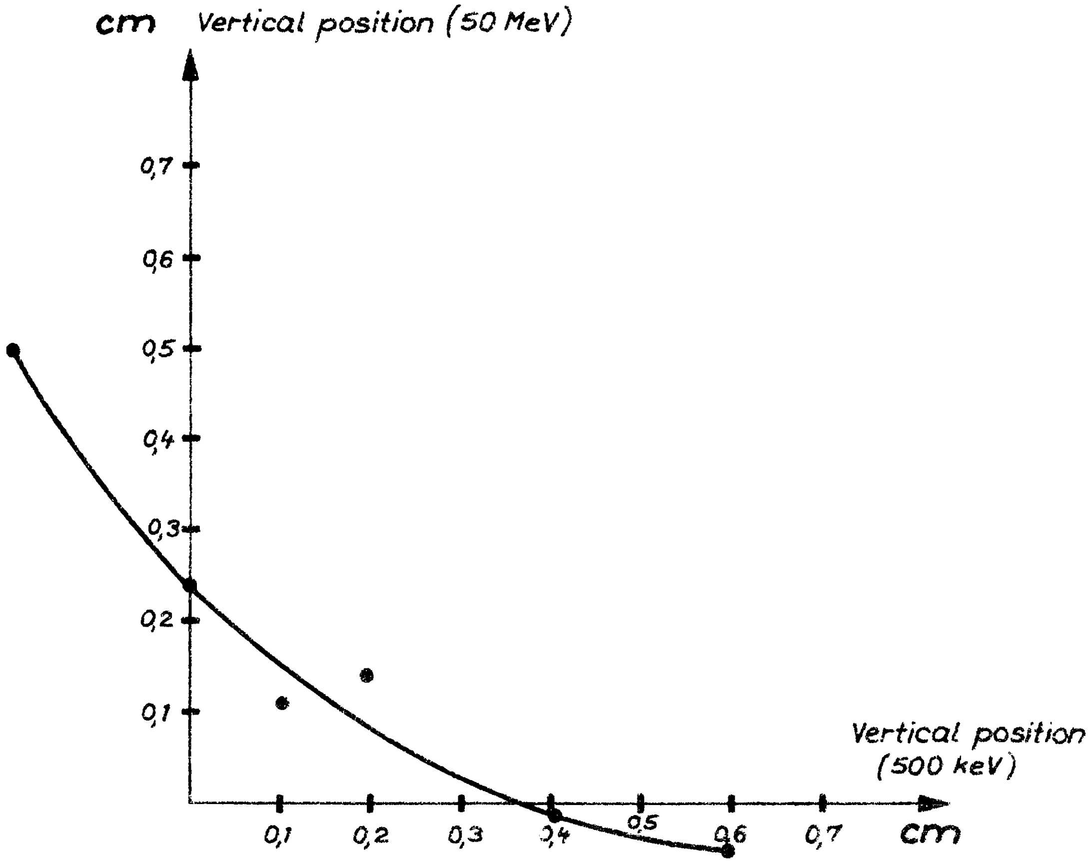

Fig. 1

Dependence of 50 MeV vertical position
on 500 keV vertical position (25-2-60)

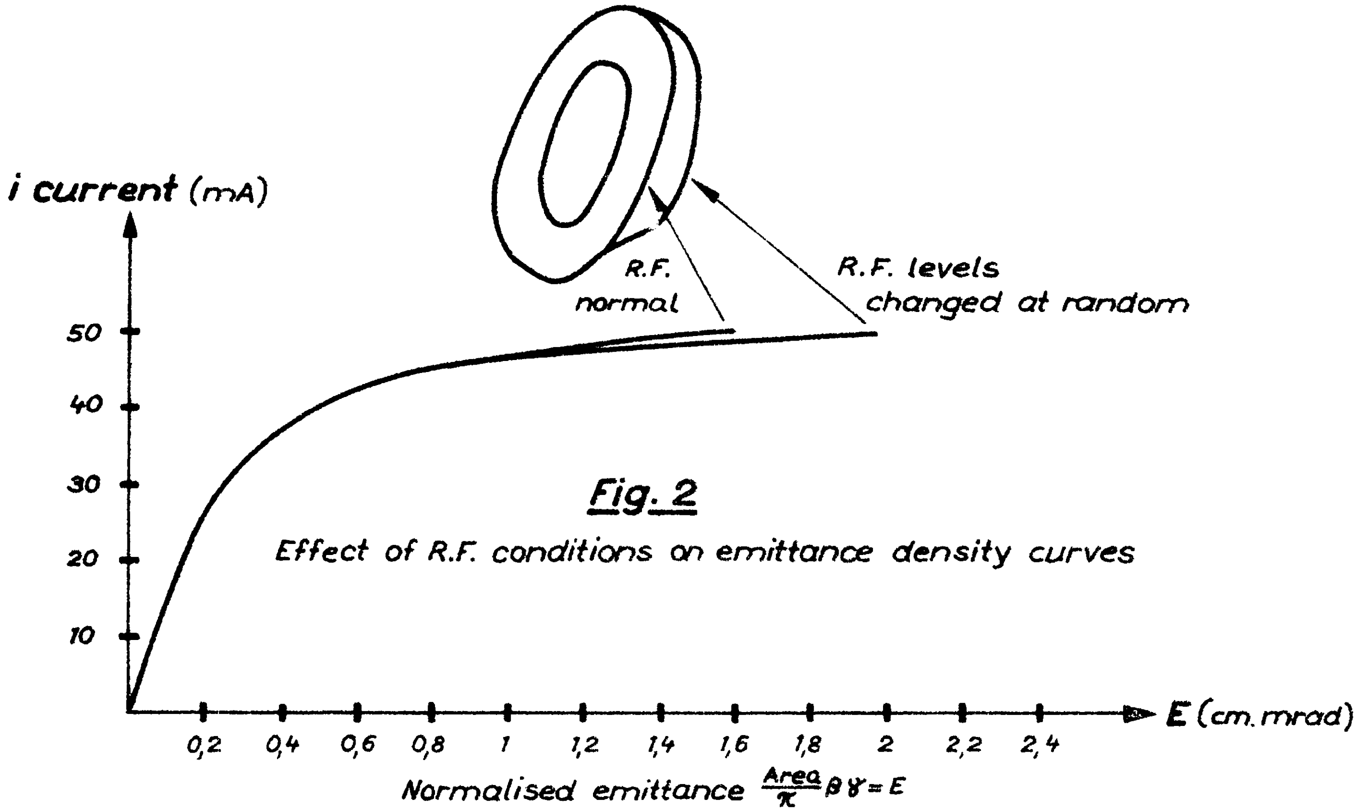

Fig. 2

Effect of R.F. conditions on emittance density curves

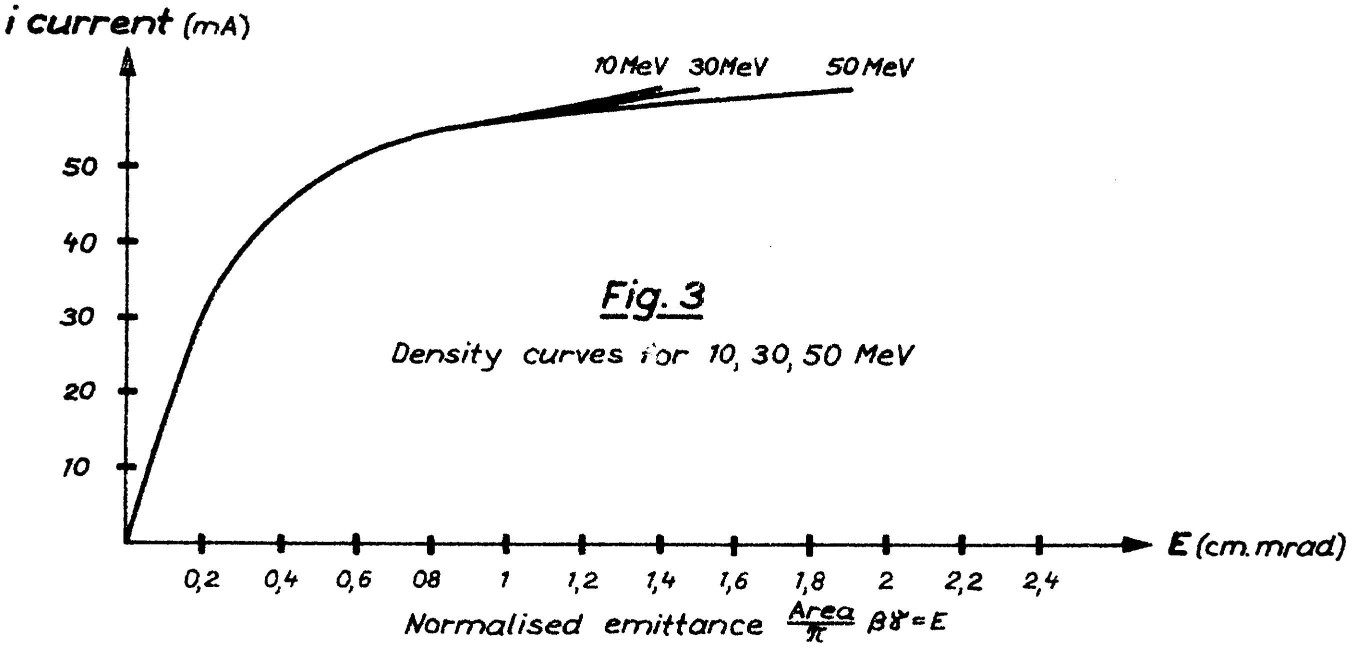

Fig. 3

Density curves for 10, 30, 50 MeV

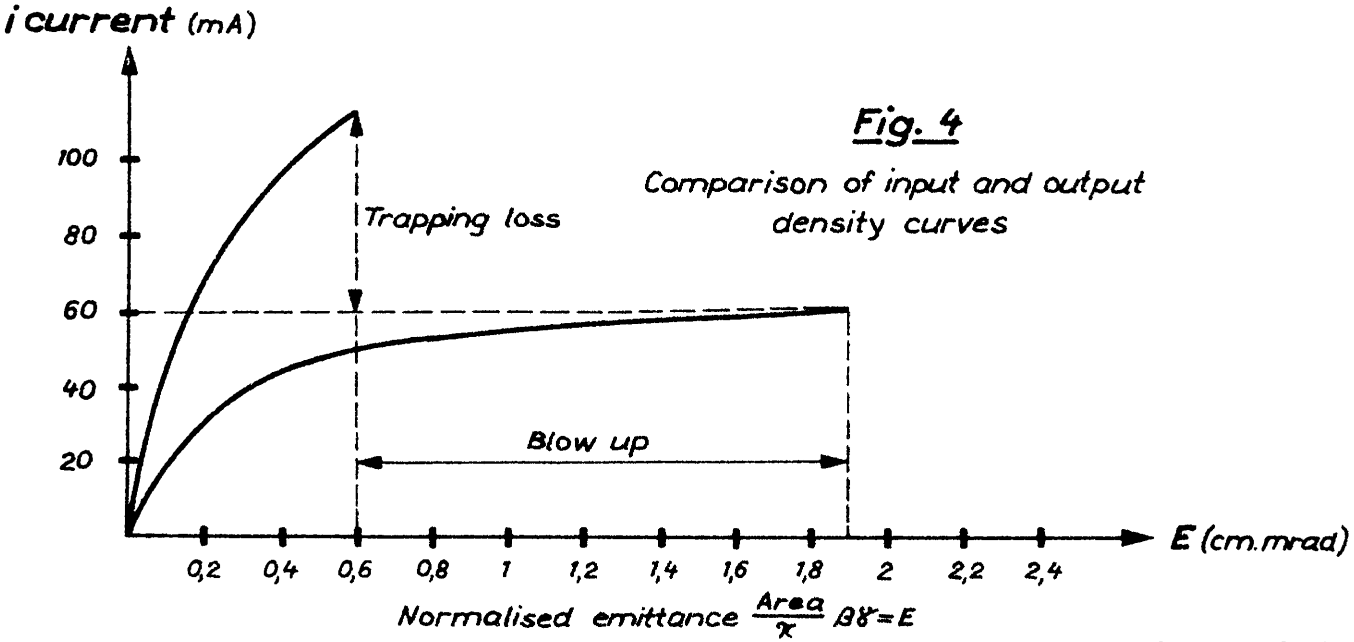

Fig. 4

Comparison of input and output density curves

3. Effects of R.F. and Energy

Some clarification of the problem followed when current density distributions across the phase plane were measured for different machine conditions. This information was plotted as a current density curve, or 'density curve', that is a curve of the current included within a given emittance[1] $\left(\frac{\text{Area}}{\pi}\beta\gamma\right)$, and these density curves revealed that changes in the tank R.F. levels, while having a marked effect on the outer regions of phase space, had a negligible effect in the regions where the charge was concentrated (Fig. 2). This meant that the total emittance observations (Section 2.2) were illusory as far as the density of the beam core was concerned.

The next step was to drift the 10 MeV and 30 MeV beams through the succeeding tanks and to measure their density curves at the 50 MeV measuring point. Comparison of the 10, 30 and 50 MeV curves then showed that the differences were insignificant except, again, towards the outer regions of phase space, within the present limits of experimental precision (Fig. 3). We assume, until further measurements can be made, that quadrupole imperfections in the second and third tanks have a negligible effect.

4. Comparison of 500 KeV and 50 MeV Beams

With the development of suitable instrumentation in the summer of 1965, it became possible to scan the 500 KeV phase plane with a slit - lens - slit - transformer combination[2] and to derive from these measurements the 500 KeV density curve. Comparing the 500 KeV and 50 MeV curves (plotted from data collected in the same experimental session), the most striking feature is the extent of the blow-up (Fig. 4). The total emittance is seen to increase by a factor of 3. From Section 3, we recall that this blow-up occurs below 10 MeV.

---

1) Area within a given equi-current density contour

2) Dubna 1963 Op. cit. p. 482

In order to form a clearer idea of the behaviour, we then reduced the 500 KeV beam intensity and emittance in successive steps$^{1)}$ and plotted the corresponding 50 MeV beam characteristic (Fig. 5). These curves proved to be illuminating. They show that:

1) adding phase space and therefore current to the outer portions of the 500 KeV phase plane increases the current density over the whole of the 50 MeV phase space, including the centre;

2) the total emittance blow-up is more pronounced for small beams than for large beams (Section 2.3);

3) the limit of acceptance for the first tank, defined experimentally as the emittance beyond which there is no increase in 50 MeV current, corresponds closely to the calculated figure of 180 mm mrad, or 0.61 cm mrad normalised. It should be mentioned that there was no limitation of the orthogonal radial motion in these experiments;

4) if we plot the locus of the total emittance for increasing input current, and the loci of the emittances corresponding to 95% of the total current, 80% and 50%, we see (Fig. 6) that current increases linearly with emittance near the centre, but in a non-linear fashion towards the outside, sweeping up to suggest a limiting value of emittance around 60 mm mrad, or 2 cm mrad normalised;

5) without the blow-up we would find the 50 mA at 50 MeV within the 500 KeV acceptance value. We can express the effect of blow-up by stating how much accelerated current lies outside of the input acceptance, in this case 20%;

6) simple scaling enables us to predict curve C' reasonably well from curve D', over the central region. This suggests that the blow-up is not a function of accelerated current from 30 to 50 mA. Confirmation of this comes from a separate measurement made with the aid of a sieve, which when placed across the 500 KeV beam, reduces the 500 KeV current while not appreciably changing the 500 KeV emittance. One sees from the curves (Fig. 7) that the total emittance blow-up is independent of current. It is interesting to note in the same figure, the 50 MeV density curves resulting from limitation of the 500 KeV beam by apertures, which confirm that a higher density 500 KeV beam produces a higher density 50 MeV beam (Section 2.4).

---

1) Effected by rectangular limitation in the phase plane, scaled to the original axis ratios

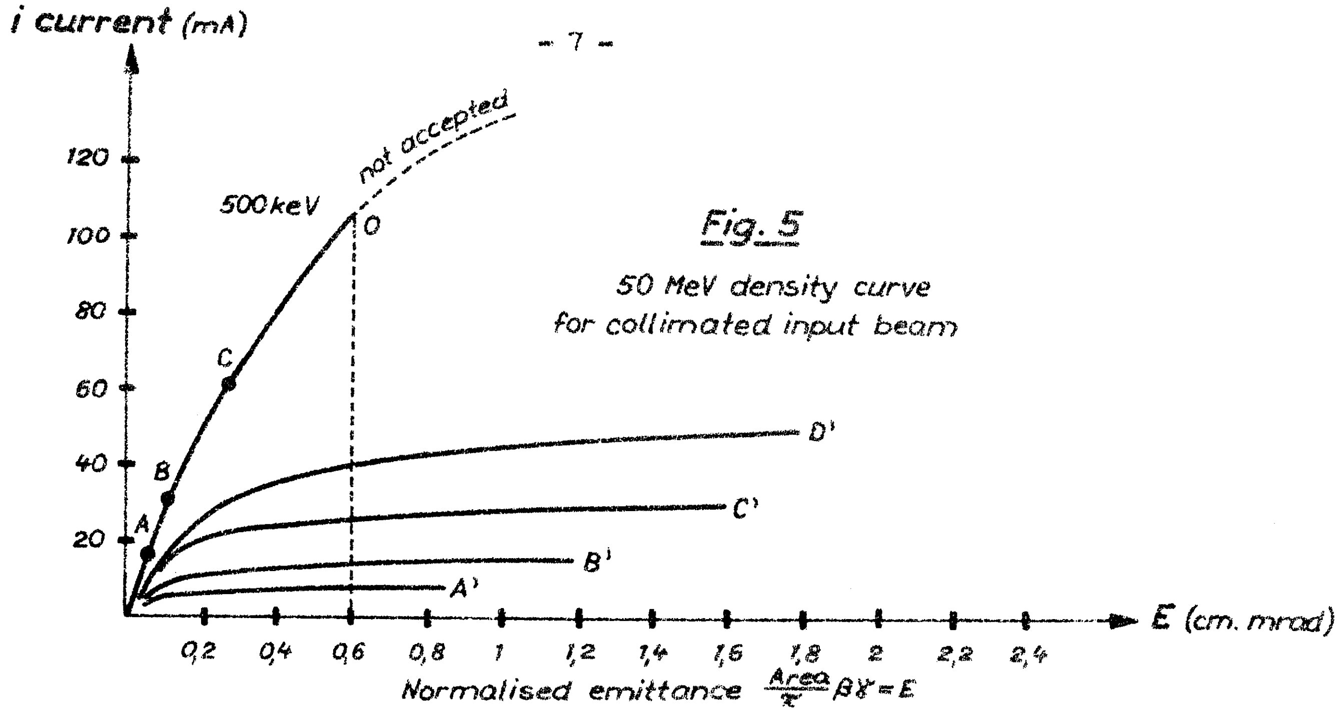

Fig. 5

50 MeV density curve
for collimated input beam

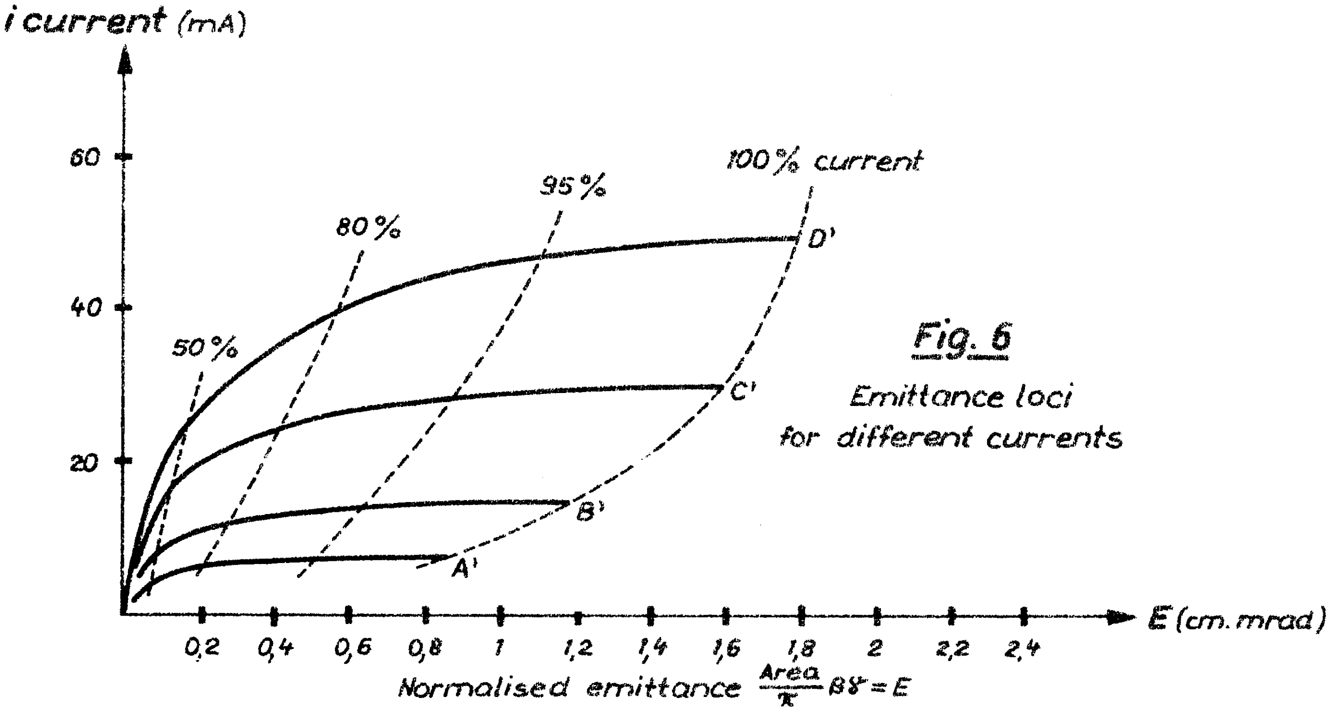

Fig. 6

Emittance loci
for different currents

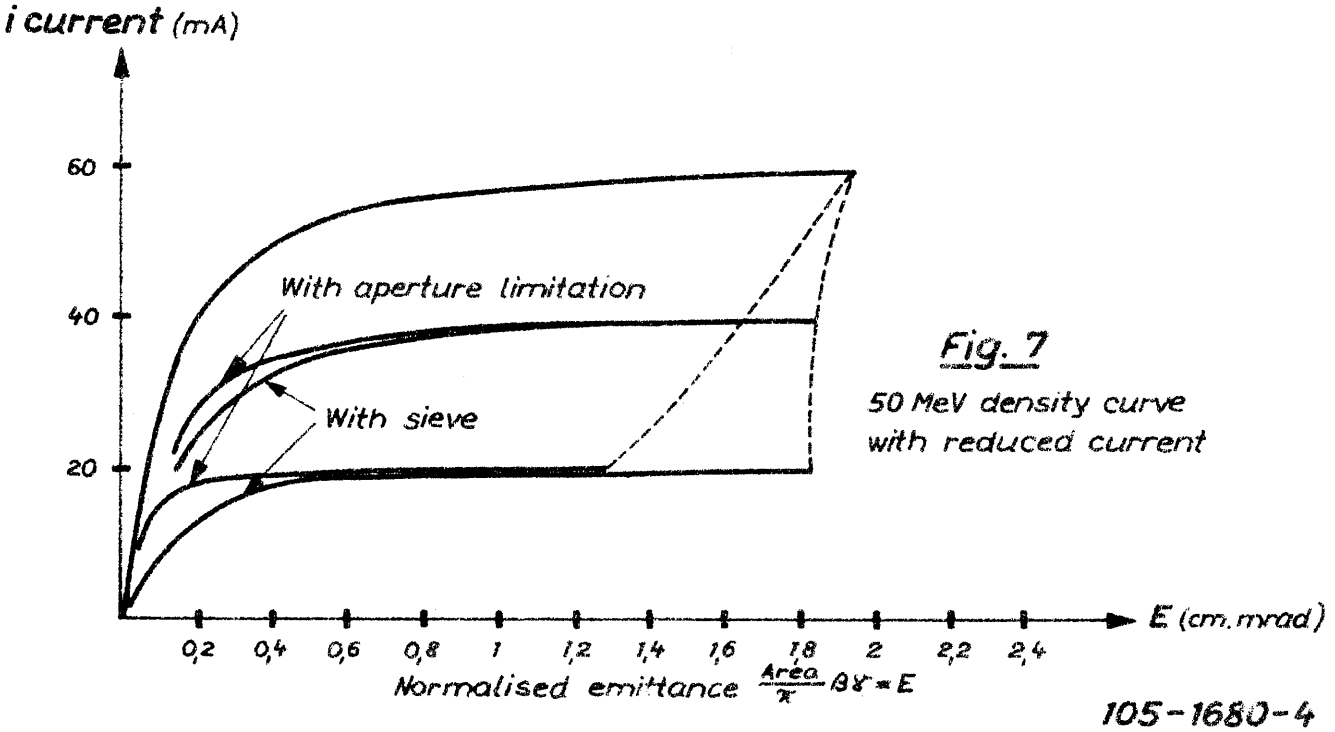

Fig. 7

50 MeV density curve
with reduced current

5. Blow-up

In this section we shall discuss some of the possible causes of the blow-up described. The obvious a priori possibilities are filamentation, misalignment, phase coupling and quadrupole coupling. We shall only mention filamentation and misalignment in passing, dismissing filamentation as unlikely in the transverse direction[1], and assuming that misalignments will intensify the problems of phase coupling and quadrupole coupling discussed below.

5.1 Phase coupling

The dependence of the transverse motion of a particle upon the R.F. field in the gap is seen from the equation of motion,[2] where we note the presence of a phase term and a velocity or energy term. Imagining the radial motion along the accelerator to be determined by ideal quadrupole focussing and perturbed by the phase coupling, we expect this perturbation to be most serious in the regions of maximum phase and energy oscillation amplitudes at the beginning of the accelerator. The effect of this perturbation then on the particle distribution across the transverse phase plane should depend on the distribution across the energy phase plane at the input.

While we have no means of measuring the distribution as yet, nor even of deducing it from control settings and indications,[3] we have been able to demonstrate the importance of this coupling experimentally by adjusting the buncher voltage and phase away from the nominal settings while maintaining the original value of 50 MeV current[4]. It was somewhat surprising to find that one setting of phase and voltage reduced the output emittance by 10% and increased the central phase space density at 50 MeV by 30% (Fig. 8)

---

[1] Unlikely that lens non-linearities could develop filamentation of structure fine enough to be missed by the scanning element whose area is about 1/100 of the space scanned

[2] See for example Lloyd Smith: Linear Accelerators Encyclopedia of Physics 1959

[3] Instrumentation limitation

[4] It is known that this is possible over fairly wide limits. See West Op.cit. p.8

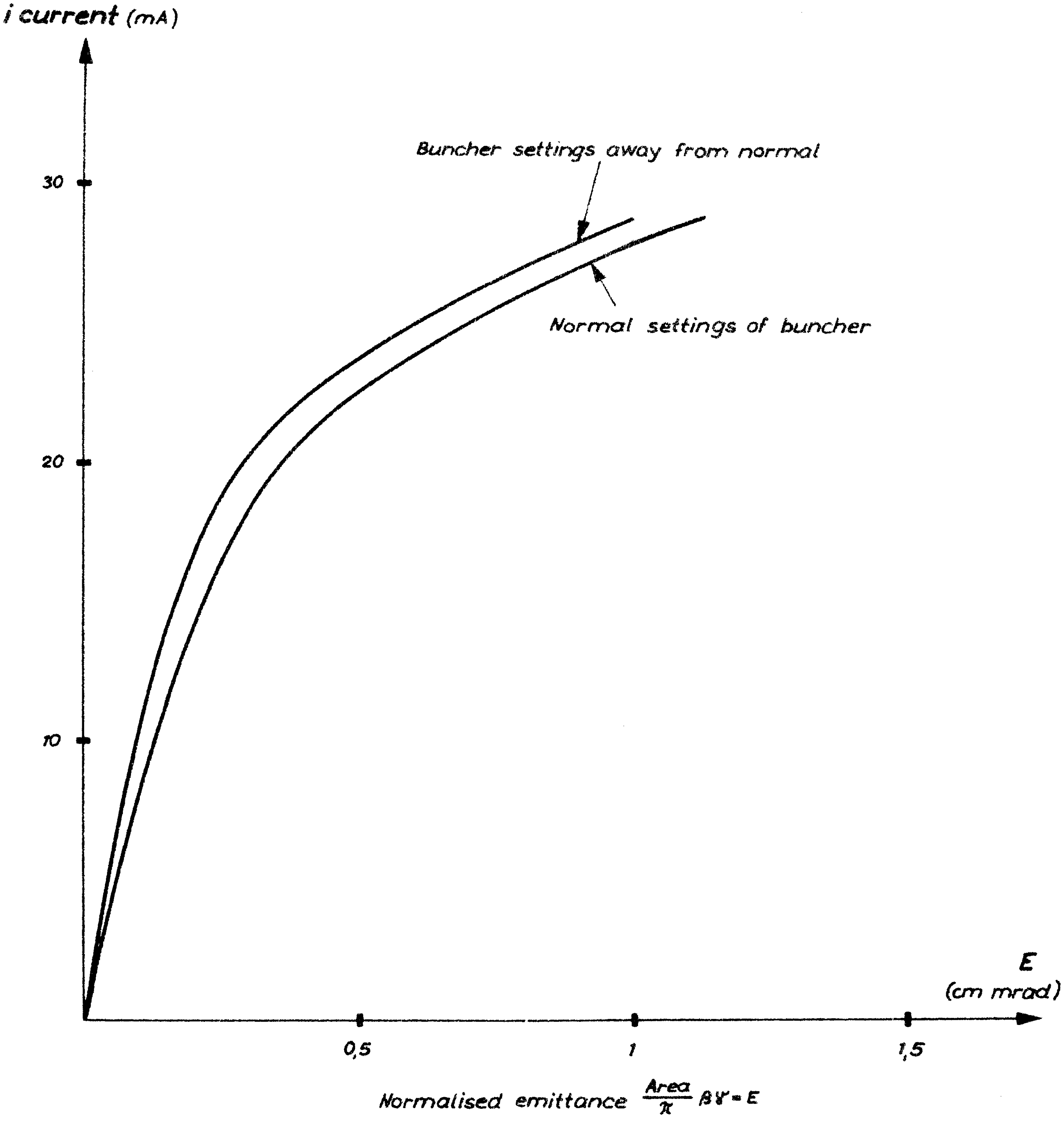

Fig. 8

Effect of input energy-phase distribution on output density

105-1681-4
18.3.66 RA

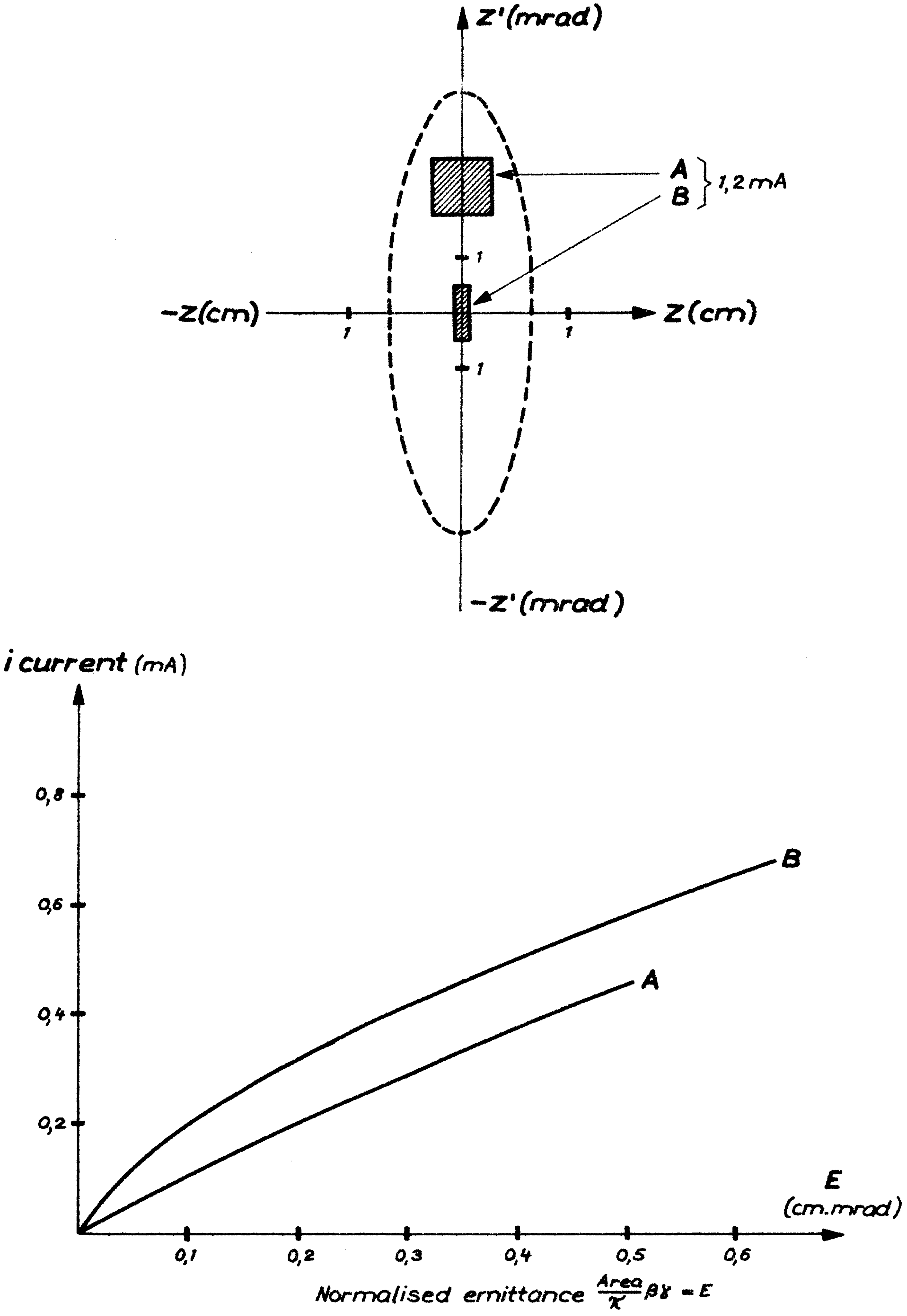

Fig. 9

Density yy' for two different regions zz'

The immediate conclusion is that the phase coupling has an important effect on output density, and that buncher conditions should therefore be optimised on output density and not just on total accelerated current. This result also suggests that tighter control of phase bunching would be profitable.

5.2 Quadrupole coupling

Numerical programmes for transverse motion through a linear accelerator normally treat the drift tube quadrupole as a region in which the radial force on a particle is proportional to its radial displacement, although it is well known that the quadrupole gradient is not usually constant over the aperture, nor does the field start and stop abruptly at the quadrupole extremities. It is worthwhile in fact reminding ourselves that in place of $F_y = ky$, we should write $F_y = f(x, y, z)$.

At the University of Rennes, P. Tanguy and E. Regenstreif are engaged in an analytical and numerical study of quadrupole inperfections in collaboration with this laboratory, and it seems clear already that for short lenses such as one is obliged to use in proton linacs at the lower energies, the end field non-linearities have more serious effects on the beam than do the departures from linearity within the lenses themselves. Coupling between the radial motions has been demonstrated by Tanguy and Regenstreif as a dispersion of points in one radial phase plane as a function of position in the other radial phase plane at the input.

In the case of measured distribution in a linac beam, we would expect coupling to manifest itself as a dependence of the density curve in, say, the yy' plane upon the position in the zz' plane from which the particles are selected. Fig. 9 shows this effect measured at 50 MeV, and we see that a given current selected from the centre of the zz' plane has a higher yy' density than the same number of milliampères selected from the outside of the zz' plane. Similar indications came from a measurement made of the 4-dimensional density curve at 50 MeV, where the measured curve showed higher central densities than did a 4-dimensional curve derived from the separate 2-dimensional measurements.

In order to demonstrate that this coupling arises in the linear accelerator however, we need information from the 500 KeV beam which is unfortunately not available.

It is worth adding that while some further attempts were being made to demonstrate coupling in the machine experimentally, it was found that the tank focussing conditions which yielded maximum output current also yielded the maximum 50 MeV density.

6. Other topics

6.1 Steering and Matching

It should be noted that the density curve is a generalisation, and says nothing about steering or matching effects represented by displacements, rotations or elongations of the phase diagram. In discussing current densities in phase space, however, it is convenient to treat these separately as problems of relative movement between beam emittance and acceptance.

To illustrate this point we show in Fig. 10 some vertical emittance contours, measured early and late in the beam pulse for conditions of inadequate beam-loading compensation. It is obvious that the density curves derived will depend upon whether one computes the current within a fixed acceptance or permits it to follow the beam centre. The first method would be more representative of the operational situation, but less informative for the present study.

6.2 Optimisation

In all of the experiments mentioned in this report, initial conditions were set up by the habitual process of optimisation of ion source conditions, pre-injector energy, matching and steering, buncher level and phase, and tank levels and focussing, using the 50 MeV energy spread and the total 50 MeV current and pulse shape as criteria.

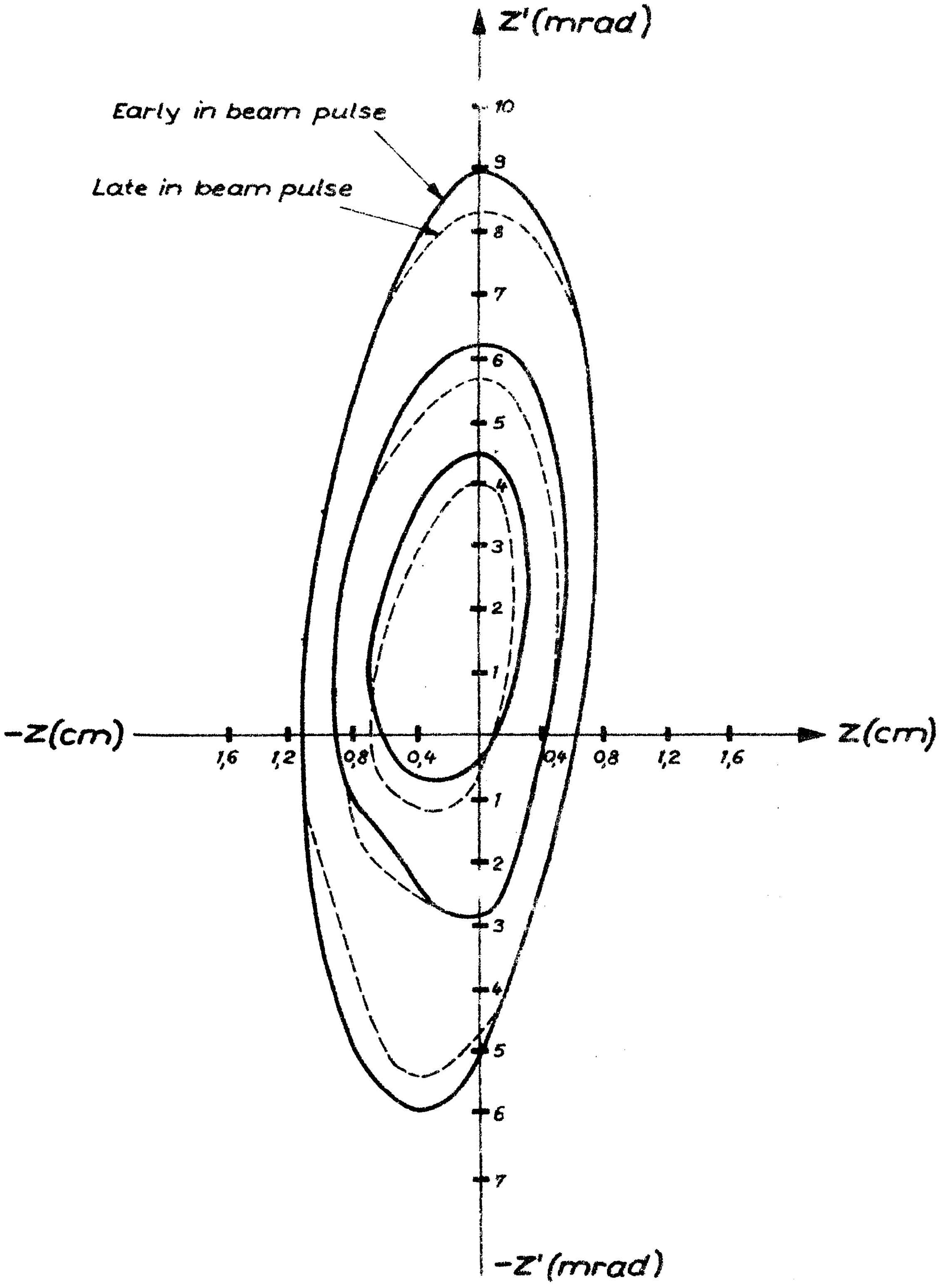

Fig. 10

Time variation of emittance contours

6.3 Space charge

Evidence was presented in Section 4 for the current independence of the blow-up process. We now discuss briefly another possible aspect of space charge.

It has been observed that the Tank I quadrupole currents arrived at by the process of optimisation have increased as the beam currents have increased, notably at the low energy end. Inspection of the log records since 1960 show the following trends:

Beam current mA	Tank quad. currents at input. **Arbitrary unit:**
10	0.75
21	0.9
50 - 65	0.95 - 1.05

This increase could mean that higher quadrupole strength is required in order to overcome space charge defocusing, or that there is an effect related to the larger diameters of the more intense beams. There is some further evidence which supports the space-charge explanation, in that the reductions by sieve of the 500 KeV beam (4.6) also required reduced quadrupole excitation in passing from 50 mA to 30 mA accelerated, although the beam size remained approximately constant.

Concerning possible future increases in beam current, we note that it is possible to reach quadrupole currents of 1.20 - 1.25 (arbitrary units) before yoke saturation effects become evident in the excitation current wave form.

6.4 Statistical Model

Consideration of the results shown in Fig. 5 led to a fairly simple statistical model, in which the 50 MeV curve was derived from the 500 KeV curve by a process of constant trapping combined with random scattering in the phase plane, the variance also remaining constant over the phase plane considered. This model expresses the physical idea that particles are in effect subjected to statistical perturbations in the phase plane due to couplings, etc.

One advantage of the model is that it gives us a simple function to describe a complicated process.

This work will be described in a subsequent paper.

Summary and Conclusions

We can summarise the results presented in this report as follows:

1. The current density in phase space in the useful central region of the 50 MeV beam can be affected by
    - i) 500 KeV transverse density
    - ii) 500 KeV longtitudinal density.
2. This 50 MeV density is determined below 10 MeV and is thereafter unaffected by acceleration.
3. There are indications that quadrupole couplings at the low energies contribute to the 50 MeV beam density.
4. There are indications of significant space-charge defocusing in Tank I.
5. Blow-up accounts for a beam loss of 20%.

Bearing in mind that a 20% loss into a 2-dimensional acceptance will mean nearer 40% into a 4-dimensional synchrotron acceptance, we conclude that it would be well worthwhile to look more closely at bunching, and that probably there is something to be gained in quadrupole design. Point 2) suggests that designers of long linacs have nothing to fear; "wild" particles can probably be scraped away before doing any damage. We note also that phase coupling gives us some control of output density and that this may be of use both experimentally and in operation.

Finally, the most serious limitation at the moment appears to be the Tank I quadrupoles, but this should not prevent us from obtaining higher densities and intensities in excess of 100 mA at 50 MeV.

Acknowledgements

We gratefully acknowledge the co-operation of our Linac Group colleagues in this work.

C.S. Taylor P. Têtu

Distribution:

MPS Scientific Staff
Linac Group
MCR Operators

W. Hardt
K. Johnson
P. Lapostolle
B. Montague

PS/5290/je