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BEAM DYNAMICS CONSTRAINTS ON BOOSTER CYCLE TIME

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INTRODUCTION

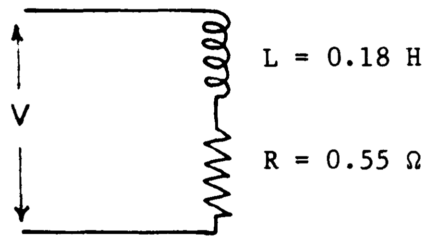
There is a strong interest to reduce the present 1.2 sec cycle time of the complex Linac-Booster-PS to around 0.6 sec to provide more protons for the SPS and 25 GeV physics[1]. For the Booster, this means a higher B, which raises the questions: can the present 12 kV RF system maintain a sufficient bucket area ? Is the transverse space-charge limit lowered ? Do longitudinal space-charge effects become more serious ? These questions are studied in this report. The conclusions are:

1. For intensities up to 5×10^{12} protons per ring, the present RF system is adequate provided the cycle is properly shaped*,

2. Transverse space-charge effects will be the same as present.

CYCLES

The simplified magnet circuit



$B = 2.15 \times 10^{-4}$ Tesla/A

FIG. 1

is used in the following. The present Booster cycle[2] is shown in Fig. 2.

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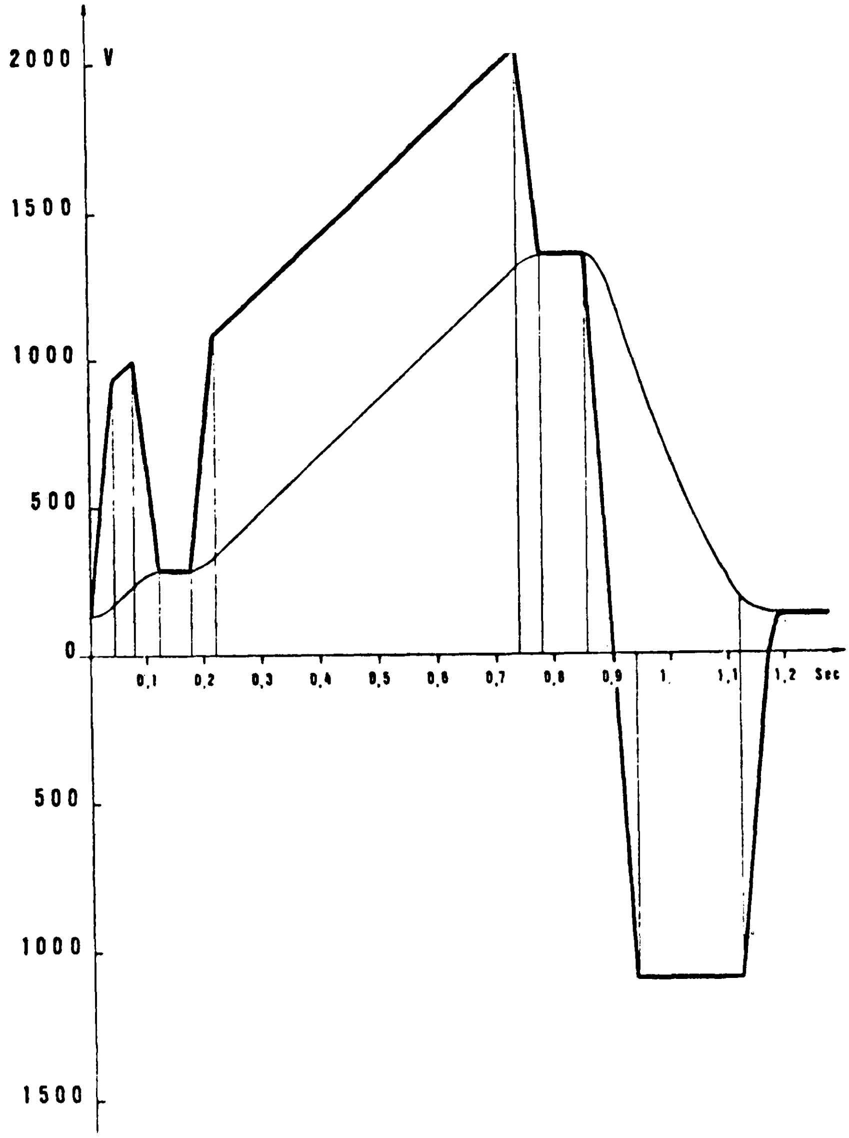
*) F. Pedersen suggested this, and made the first calculations.



<u>Fig. 2</u> : voltage and current for present 1.2 sec Booster cycle.

The same cycle (without the low-current waiting period) is shown in the I, $\dot{I}$ phase plane in Fig. 3.

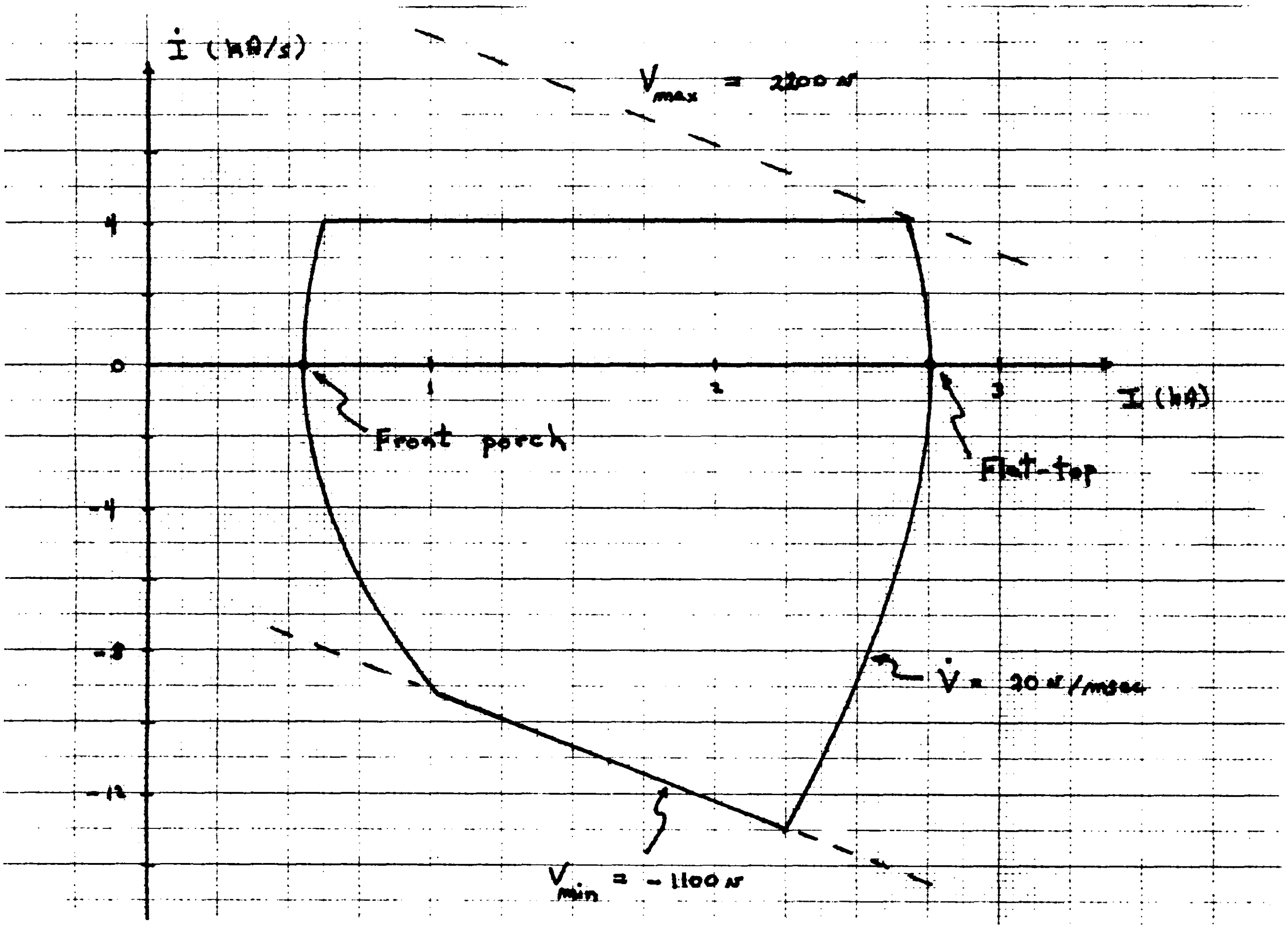


FIG. 3 : Normal cycle

It consists of an initial 'acceleration' ($\ddot{I}$ = constant) with $\dot{V}$ = 20 v/msec, a period of constant $\dot{I}$, then deceleration ($\dot{V}$ = - 20 v/msec) to the ejection flat-top at I = 2760 A. A further deceleration to the minimum voltage limit, then reacceleration brings the current back to the front porch.

The curved acceleration segments are approximately parabolas,

$$\dot{I} = \sqrt{\frac{2\dot{V}}{L}\,\Delta I} \qquad (1)$$

provided their duration is short compared with L/R = 327 msec so that dissipation can be neglected (Fig. 1). The time to go along a parabola from zero to a given $\dot{I}$ is

$$t = \frac{L\dot{I}}{\dot{V}} \ . \qquad (2)$$

To speed up the cycle, one must accelerate and decelerate faster, and maintain a higher $\dot{I}$, subject to the power supply limits. An additional constraint is the minimum allowable bucket area

$$A = \sqrt{\frac{\gamma}{|\eta|}} \ \alpha(\Gamma) A_o \qquad (3)$$

which can also be drawn on the $I,\dot{I}$ diagram since γ and η are functions of energy or I, while $\alpha(\Gamma)$ is a function of $\dot{I}$, and A_o = 10,2 mrad for V_{RF} = 12 kV (Appendix). These constraints are sketched in Fig. 4. The fastest cycle is evidentally the boundary curve.

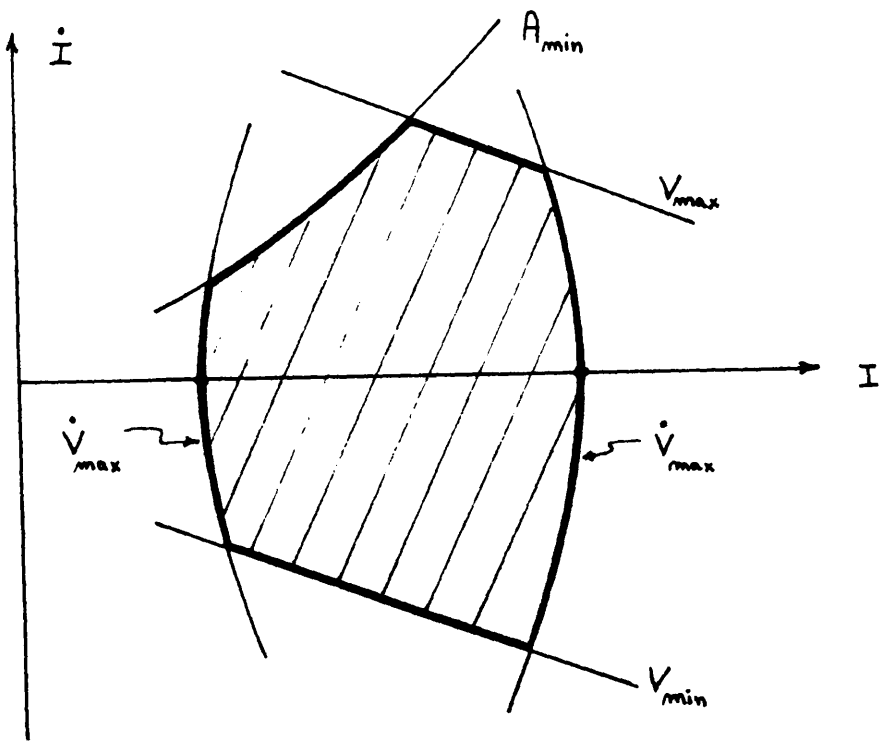


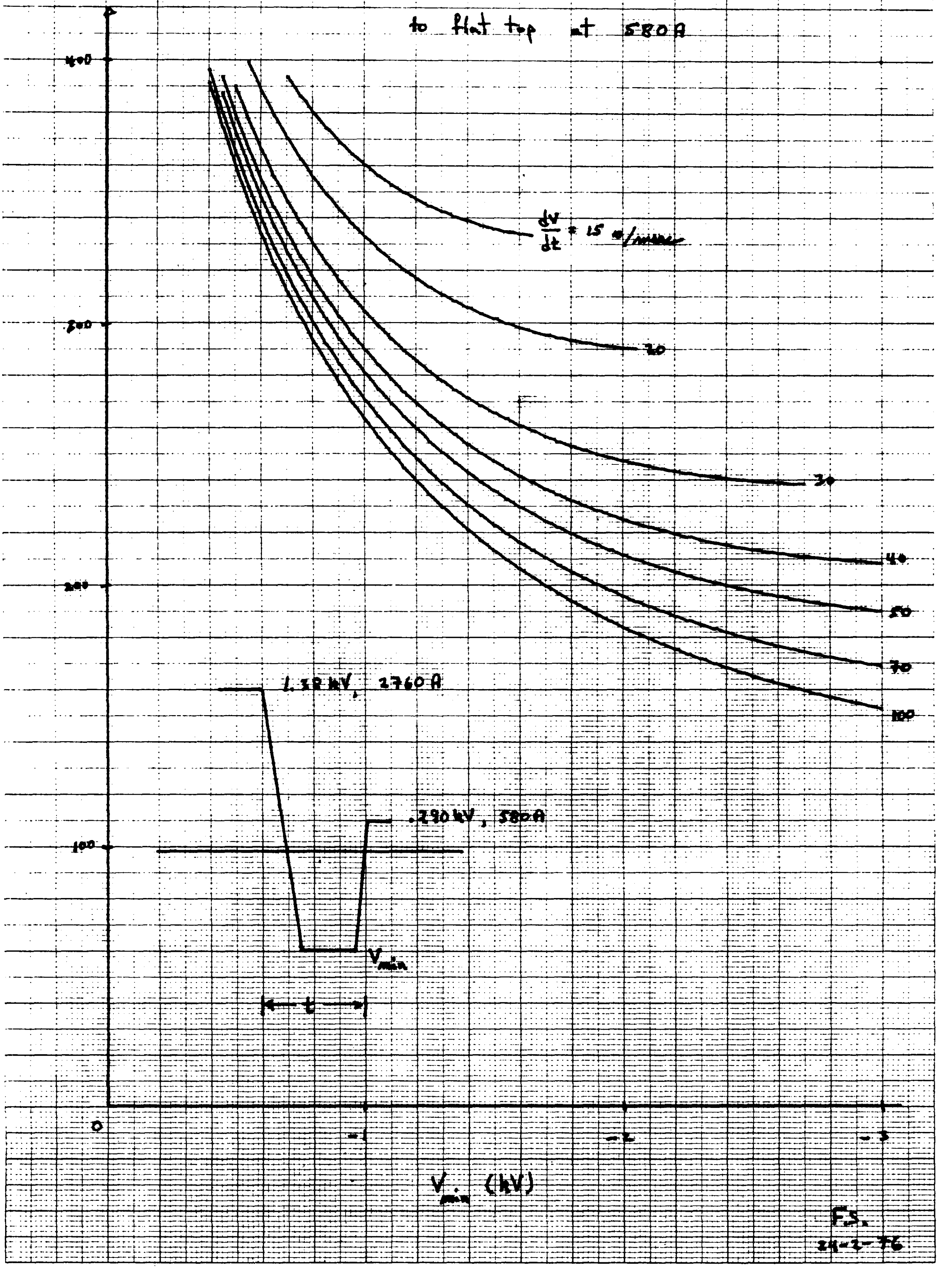
FIG. 4 : Constraints on cycle.

For the present cycle, the front porch occurs about 18 A below the injection current of 583 A, corresponding to 50 MeV. For simplicity in the following, we assume a higher front porch at 580 A with injection nearer to $\dot{B} = 0$. This saves 5 to 10 msec on cycle time and avoids the few millimeters of inward spiraling of the beam before it is captured by the RF.

Time for descent. The time to return from the ejection flat-top to the front porch is shown in Fig. 5 as a function of V_{min} for various deceleration rates $\dot{V}$. Völker[3] estimates that $V_{min} = -2200$ v and $\dot{V} = 60$ v/msec is possible with an additional rectifier group, and one finds

$$\text{Descent time} \simeq 200 \text{ msec.}$$

FIG. 5: Time from flat top at 2760A to flat top at 580A



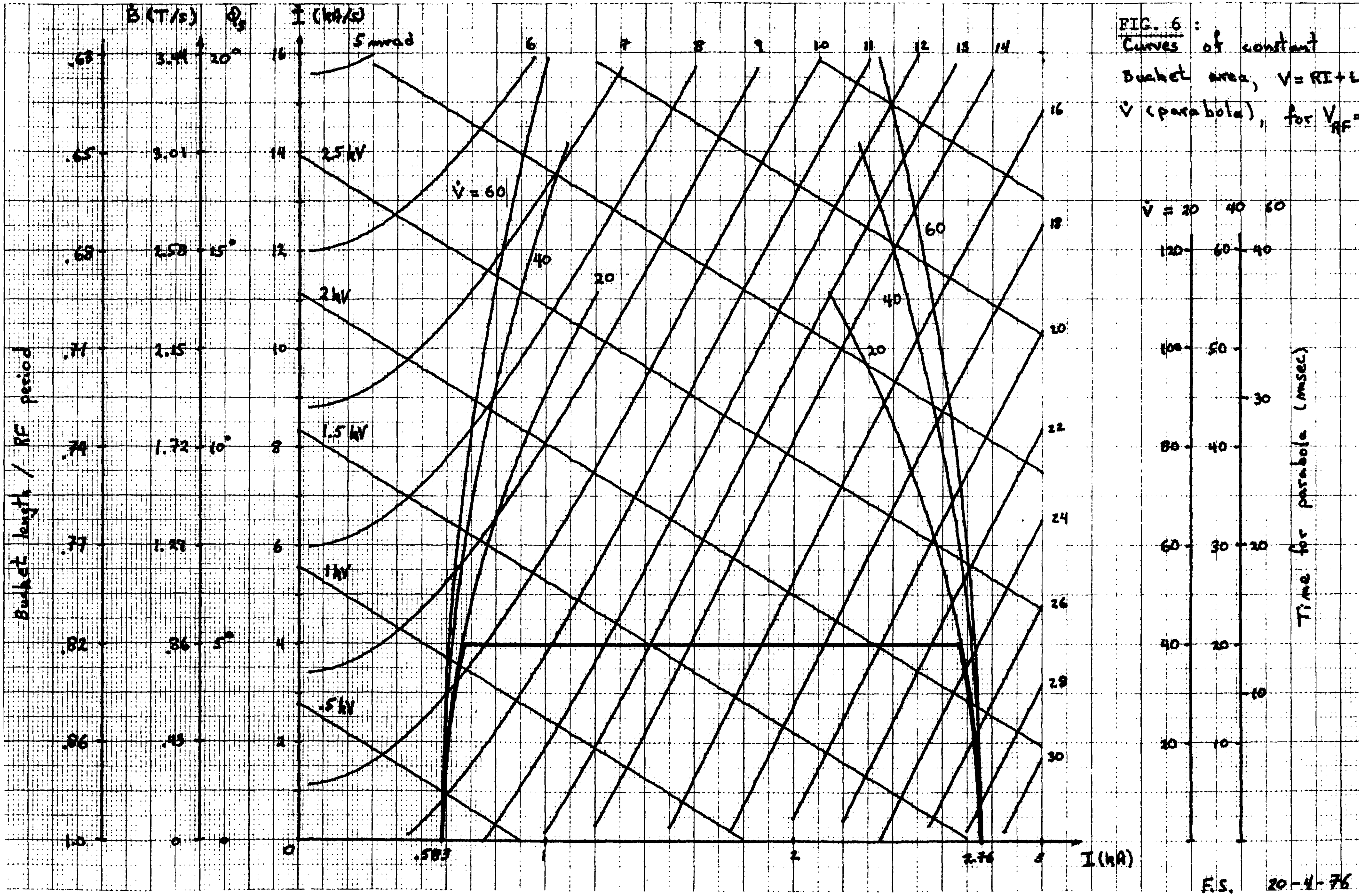


FIG. 6: Curves of constant bucket area, $V = RI + L\dot{I}$, $\dot{V}$ (parabola), for $V_{RF} = 12$ kV

Time for acceleration. The various constraints during the rising part of the cycle are shown in Fig. 6. Three acceleration rates, $\dot{V}$ = 20, 40, 60 v/msec are shown together with the time for each on the right-hand scales. The left-hand scales give the synchronous phase ϕ_s, $\dot{B}$, and bunching factor for a full bucket. Bucket areas and power supply voltages can also be read from the figure. For the present cycle, with injection half-way up the parabola, 20 msec after the start (STBI), the bucket area is 10.5 mrad. This shrinks to 9.7 mrad at the maximum $\dot{B}$,20 msec later, and thereafter grows smoothly to about 30 mrad before ejection. The maximum power supply voltage of 2200 v is reached just before the descent (WFT) to the ejection flat top.

For a faster cycle, one would follow the parabola $\dot{V}$ = 40 v/msec, for example, up to the minimum bucket area allowed, say 9 mrad, then along this curve to the maximum power supply voltage, say 3 kV, which is followed to the descending parabola, and down to the flat-top. The time required for such a cycle is shown in Fig. 7 for different bucket areas from 7 to 10 mrad, for $\dot{V}$ = 30 and 40 v/msec, and for different V_{max}. As mentioned before, it makes very little difference whether one injects on the front porch or at $\dot{B}$ = 0.4 T/s.

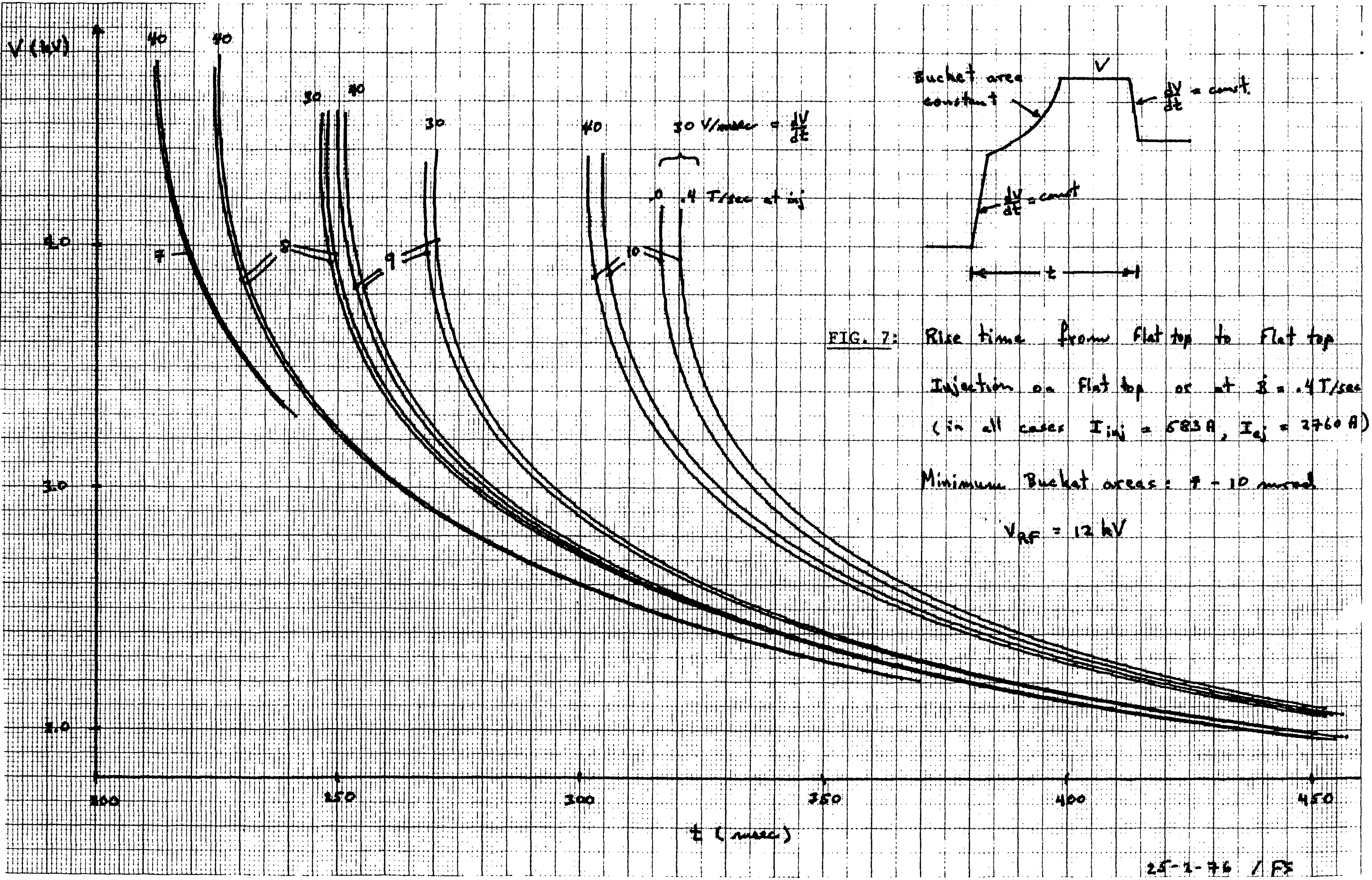


FIG. 7: Rise time from flat top to flat top

Injection on flat top or at $\dot{B}$ = .4 T/sec

(in all cases I_{inj} = 583 A, I_{ej} = 2760 A)

Minimum Bucket areas: 7 - 10 mrad

V_{RF} = 12 kV

Power supply voltages above about 3.6 kV give little or no gain in time since they are reached just before one starts on the descending parabola, or are not reached at all.

As an example, with an additional power supply group, V_{max} = 3600 v, and taking $\dot{V}$ = 40 v/msec (Völker[3]), gives a rise time of 306 msec for a 10 mrad bucket. Since a $\dot{V}$ close to 60 v/msec seems possible, and a somewhat smaller bucket area is acceptable, rise times of 300 msec are possible, with a total cycle time of

<u>500 msec + flat-tops</u>.

If one doubles the RF voltage from 12 to 24 kV, the bucket areas are increased by $\sqrt{2}$. The curve labeled 7 mrad becomes 10 mrad and one gains about 75 msec, or a larger bucket.

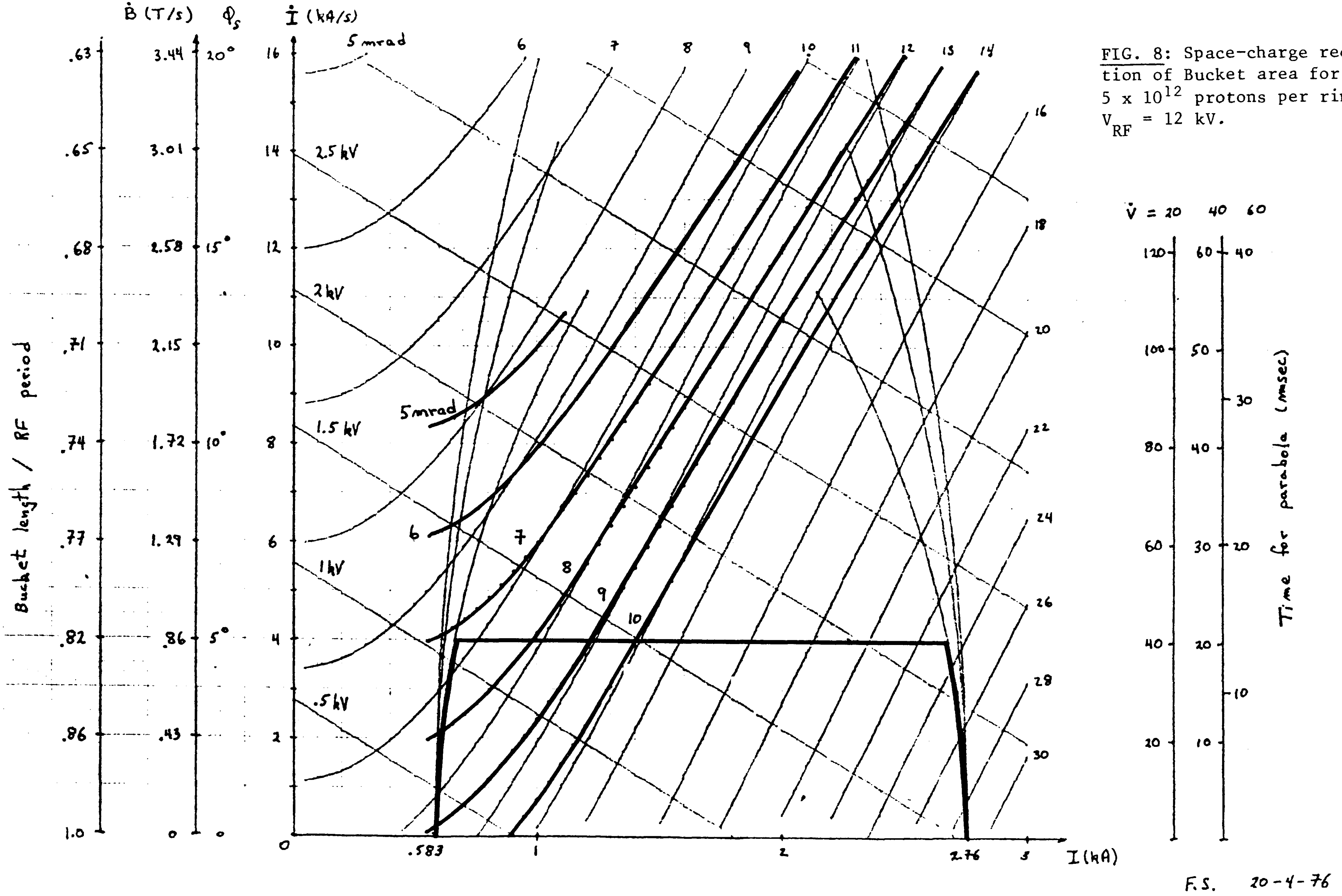


FIG. 8: Space-charge reduction of Bucket area for 5×10^{12} protons per ring, V_{RF} = 12 kV.

SPACE CHARGE EFFECTS

Longitudinal. For normal operating intensities of 3 to 5 x 10^{12} protons trapped per ring, space-charge forces substantially reduce the bucket area. Fig. 8 shows the reduction for 5 x 10^{12} protons/ring, which is a reasonable design aim (Appendix). For the normal cycle, the area is reduced from 9.7 to 7.1 mrad, which corresponds to an energy spread from the Linac of ± 170 keV.

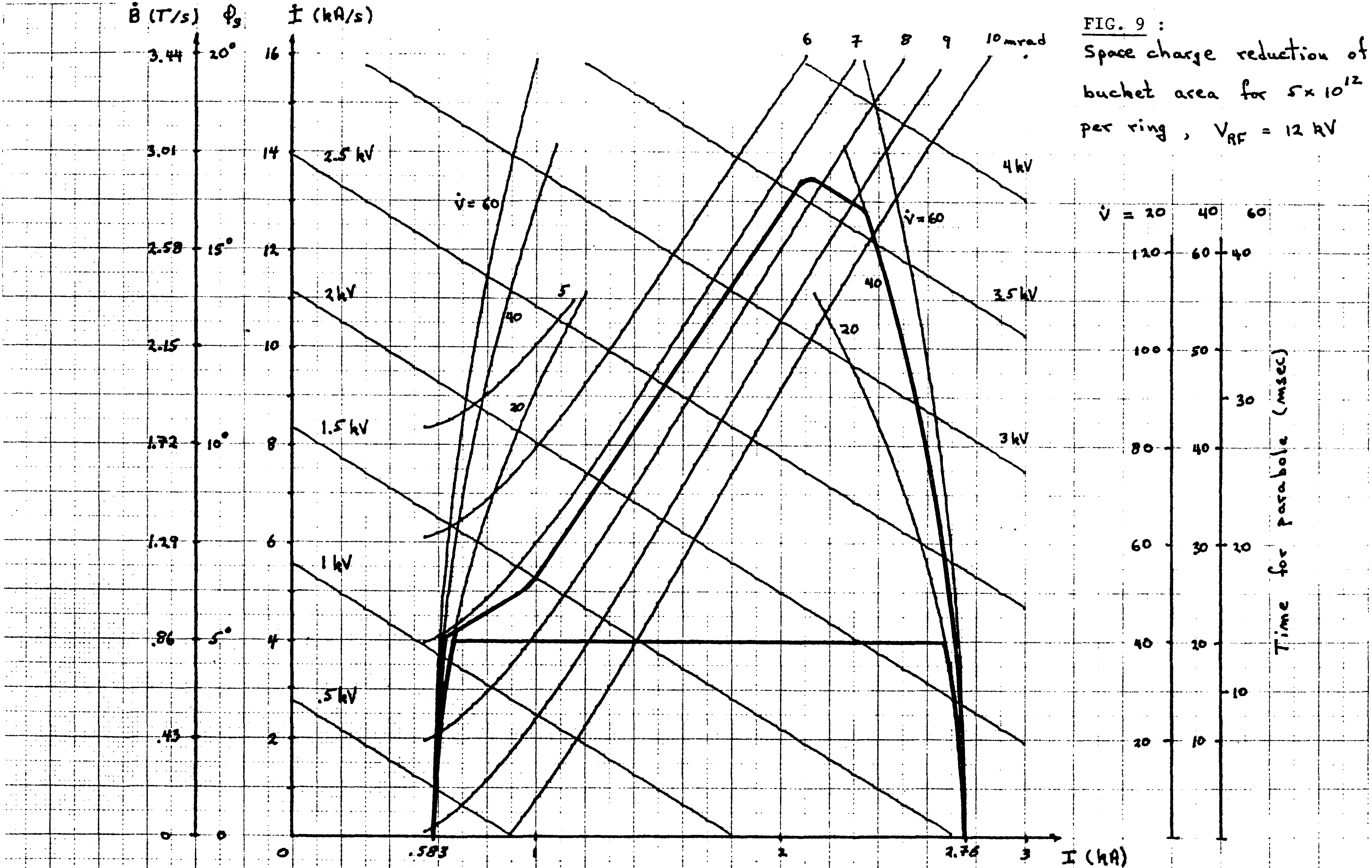


FIG. 9 :
Space charge reduction of bucket area for 5×10^{12} per ring, V_{RF} = 12 kV

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A possible faster cycle that maintains a bucket area of at least 7 mrad is shown in Fig. 9. The kink or knee in the curve assures that the bucket minimum occurs at low energy, to avoid losses along the cycle. The rise time from front porch to flat-top can be computed from

$$t_1 - t_2 = \frac{1}{b} \ln \dot{I}_1/\dot{I}_2 \qquad (4)$$

for the straight line segments $\dot{I} = a + bI$, plus the time for the parabolas :

21.0	parabola up
72.5	
130.6	
16.2	
64.0	parabola down
304.3	msec.

The bucket area along the cycle is shown in Fig. 10 for different intensities. For comparison, Fig. 11 shows the bucket area for the normal cycle. About 80 msec remain near the end of the proposed cycle for increasing the bucket area via Magnani Shaking[4)]. At least 10 mrad are required to pass transition in the PS at high intensity[5)].

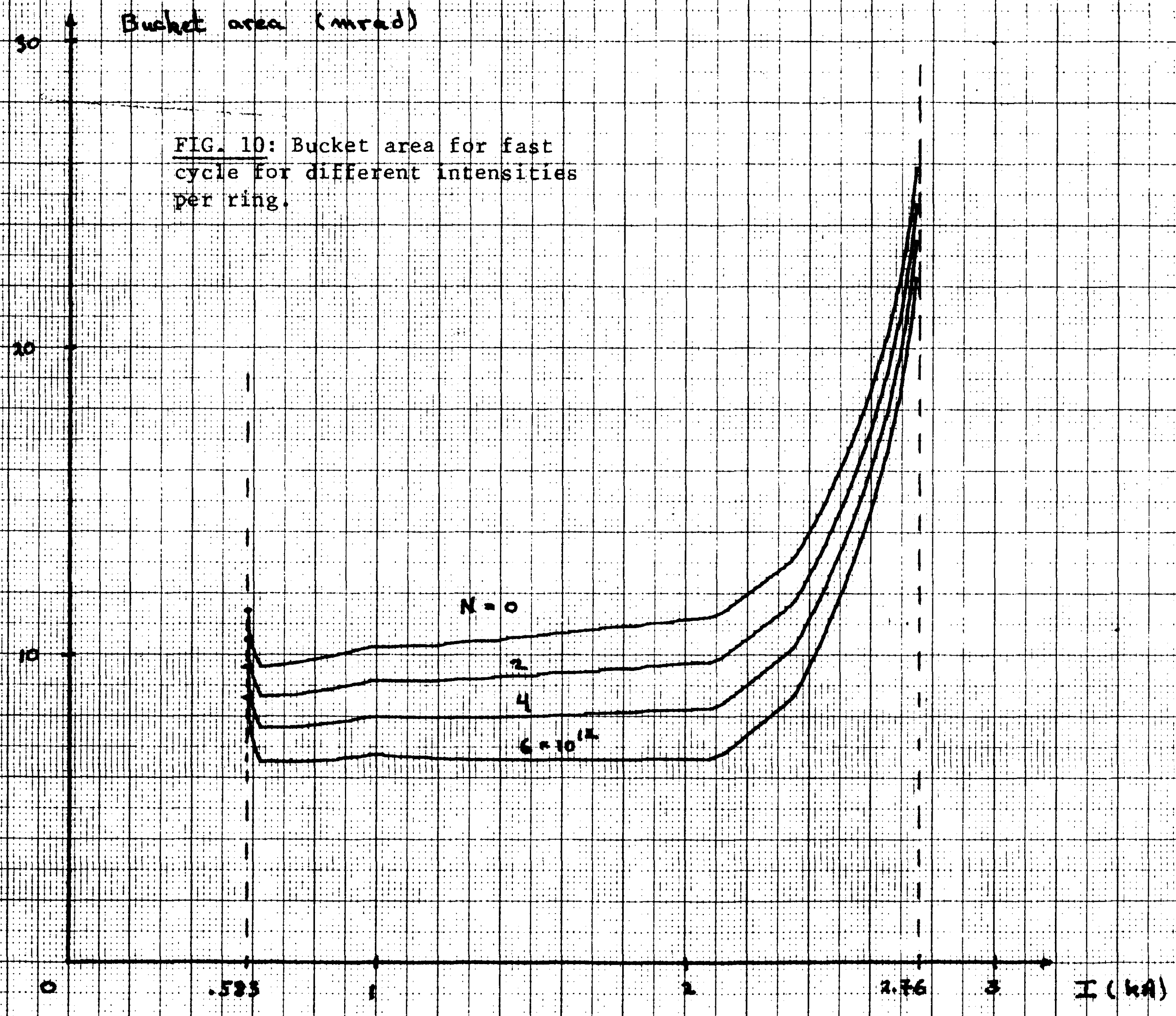

FIG. 10: Bucket area for fast cycle for different intensities per ring.

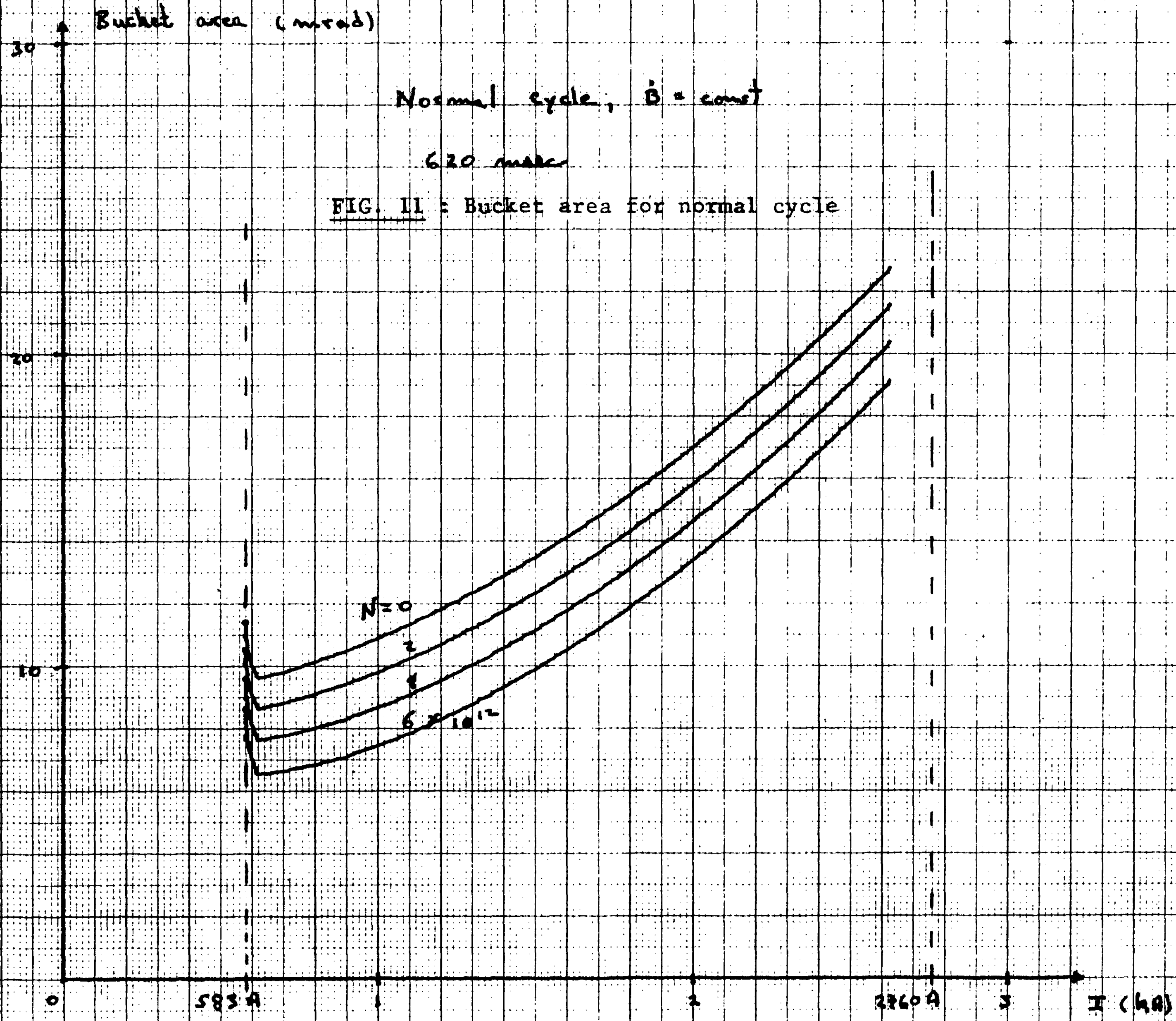

FIG. 11 : Bucket area for normal cycle

Transverse. Any decrease in the bunching factor ($B_F < 1$) due to the faster acceleration rate reduces the space-charge limit and causes further emittance blow-up[6]. Fortunately, for the small synchronous phase angles required in the Booster, the bunching factor is practically independent of the cycle chosen. A higher $\dot{B}$ reduces the bucket size, but on the other hand, more voltage is then required for acceleration and less remains to focus and squeeze the bunch. The two effects tend to cancel, leaving the bunch length nearly independent of $\dot{B}$.

A formula of W. Hardt (Appendix) has been used to compute the space-charge Q-shifts, and the results are shown in Fig. 12 for normal and fast cycles. Bunch lengthening due to longitudinal space-charge forces is included. We conclude that the transverse space-charge limits are the same for both cycles.

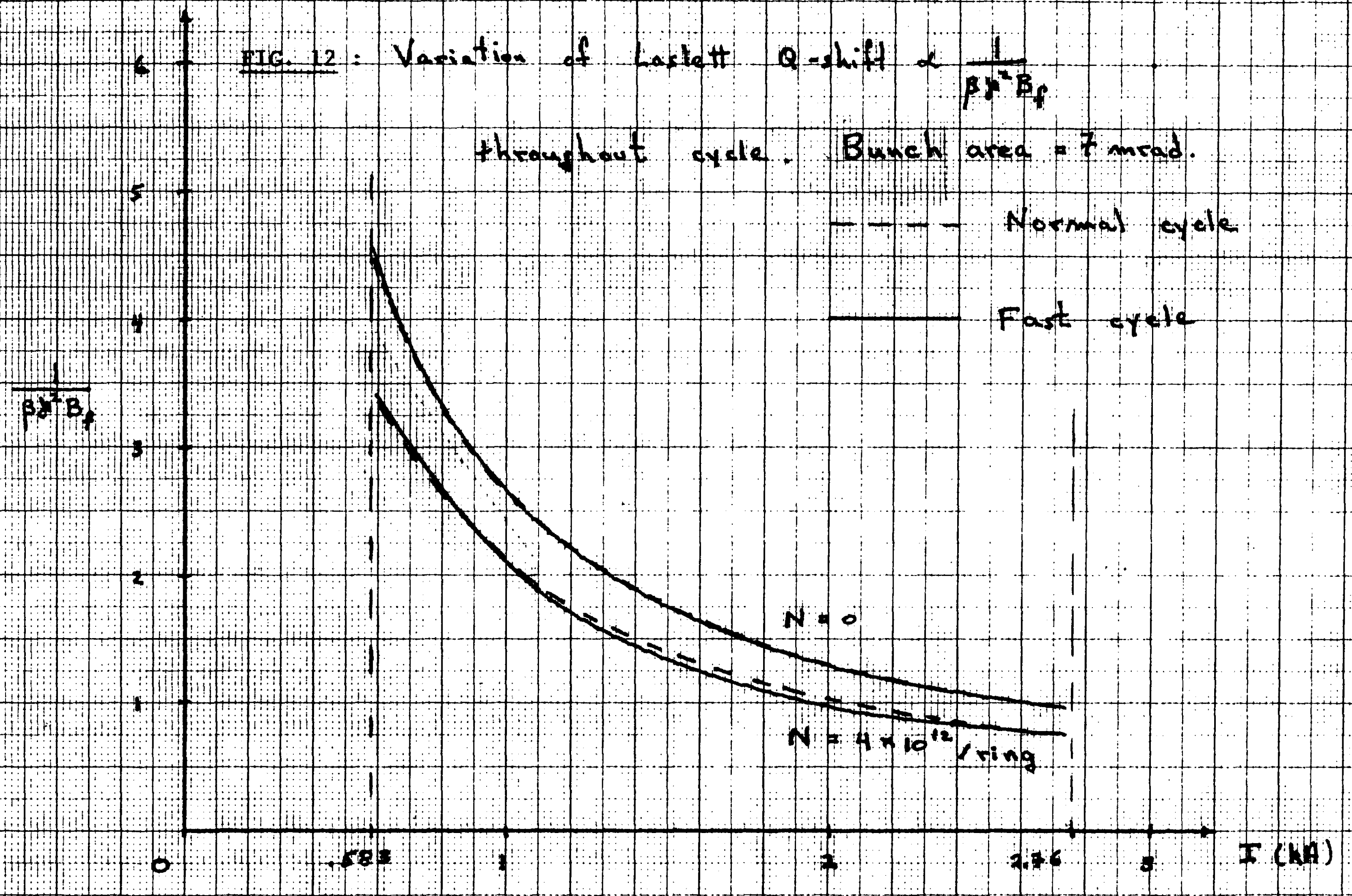

FIG. 12 : Variation of Laslett Q-shift $\propto \frac{1}{\beta\gamma^2 B_f}$ throughout cycle. Bunch area = 7 mrad.

- APPENDIX -

The bucket area is given by

$$A = \sqrt{\frac{\gamma}{|\eta|}}\,\alpha(\Gamma)A_0 \tag{A1}$$

where

$$A_0 = \frac{16}{\sqrt{2\pi h}}\sqrt{\frac{eV_{RF}}{E_0}} \tag{A2}$$

$$= 10.2 \times 10^{-3} \text{ rad}$$

for V_{RF} = 12 kV, E_0= 0.938 GeV, h = 5 (harmonic number).

The moving bucket factor α is a function of Γ,

$$\Gamma = \frac{2\pi R\rho}{V_{RF}}\dot{B} \tag{A3}$$

$$= 2.32 \times 10^{-5}\,\dot{I}$$

for R = 25 m, ρ = 8.239 m (bending radius). A good approximation for α is (Fig. 13)

$$\alpha = 3 + \Gamma - 2\sqrt{1 + 3\Gamma} \tag{A4}$$

or

$$\Gamma = 3 + \alpha - 2\sqrt{1 + 3\alpha}\,. \tag{A5}$$

The factors γ and η in (A1) are functions of magnet current I,

$$\gamma = \sqrt{1 + \beta^2\gamma^2} \tag{A6}$$

$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_T^2} \tag{A7}$$

where $\gamma_T = Q_H - 0.17 \simeq 4$ and $\beta\gamma = 0.566 \times 10^{-3}I$.

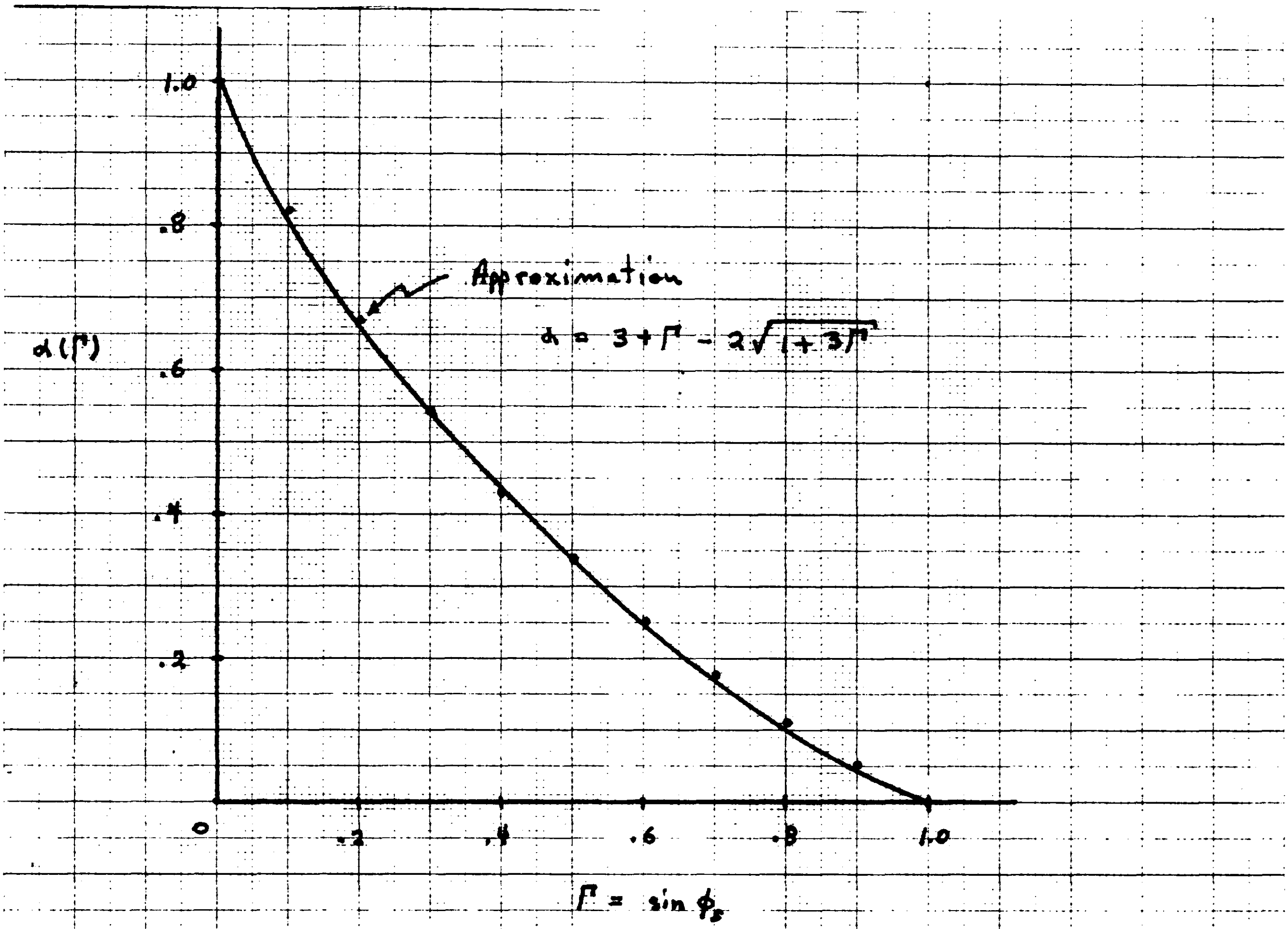

Fig. 13. Moving bucket factor

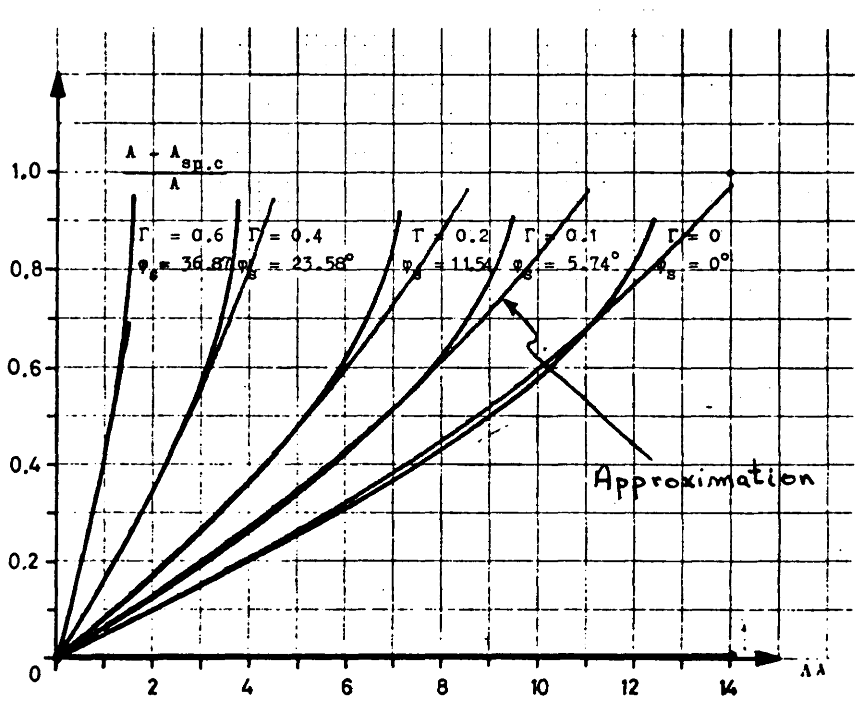

FIG. 14 : Approximation for space-charge reduction of bucket area.

Space-charge reduction of the bucket area has been computed in Ref. 7 for the water-bag distribution (constant particle density in phase space). The results are presented as graphs in Ref. 8. An approximate fit to the graphs is (Fig. 14)

$$A_{sc} = (1 - ax - bx^3)A \tag{A8}$$

where

$$a = 0.05(1 + 0.5\Gamma)/(1 - \Gamma)^2$$

$$b = 10^{-4}/(1 - \Gamma)^6$$

$$x = 4\pi \, g_0 \frac{E_0 r_p}{R \, \gamma^2 \, V_{RF}}$$

$$= 0.3 \, N \, g_0/\gamma^2$$

with

$$N = \text{particles per ring} \times 10^{12}$$

$$g_0 = 1 + 2 \ln (\text{vacuum chamber radius/beam radius})$$

$$= 2.9 + \ln \frac{I}{I_{inj}} \qquad (I_{inj} = 538 \text{ A})$$

$$r_p = 1.53 \times 10^{-18} \text{ m.}$$

W. Hardt (private communication) has derived an approximate formula accurate to a few percent relating bunch length to bunch area for partially filled buckets,

$$A = 16 f(\hat{\phi}) \sqrt{\frac{1}{2\pi h} \frac{\gamma}{|\eta|} \frac{U_T}{E_0}} \tag{A9}$$

where

$$f(\hat{\phi}) = \frac{\sin^2 \frac{\hat{\phi}}{2.255}}{\sin^2 \frac{\pi}{2.255}}$$

$2\hat{\phi}$ = bunch length in RF radians

$U_T = V_{RF}|\cos\phi_m| + V_{SC}$

$\sin\phi_m = \dfrac{\hat{\phi}}{\sin\hat{\phi}} \sin\phi_s$

$$V_{SC} = E_0 \text{ sign } \eta \frac{\pi h r_p g_0 N}{\gamma^2 R(\sin\hat{\phi} - \hat{\phi}\cos\hat{\phi})}$$

N = number of protons per ring.

Given the bunch area A, the bunch length $2\hat{\phi}$ is found by iteration. From an initial length $\hat{\phi}_i$ and deviation $\delta\hat{\phi}_i$, the area A_i and deviation δA_i are computed from (A9). The new length $\hat{\phi}_{i+1}$ is found from

$$\Delta A = \frac{\partial A}{\partial \phi} \Delta\phi \qquad \text{(A10)}$$

or

$$A - A_i = \frac{\delta A_i}{\delta \phi_i} (\hat{\phi}_{i+1} - \hat{\phi}_i).$$

About five iterations are required to achieve 1% accuracy.

Distribution :

MAC
Multipulsing Working Group
MST
BC

REFERENCES

1. D. Boussard et al., Interim Report of PS Multipulsing Working Group, PS/AE/Note 76-11 (1976).

2. R. Gailloud, Etude préliminaire en vue d'une diminution du temps de répétition du cycle de l'aimant principal du PS Booster, MPS/BR (1975).

3. F. Völker, Pulsing the PSB at 0.6 seconds repetiton period, PS/BR Note/76-8 (1976).

4. G. Gelato et al., 1975 Particle Accelerator Conference, Washington, p. 1334.

5. W. Hardt, 9th International Conference on High Energy Accelerators, Stanford (1974), p. 434.

6. J. Gareyte et al., 1975 Particle Accelerator Conference, Washington, p. 1855.

7. C. Nielsen and A. Sessler, Rev.Sci.Instr. 32, p. 85 (1959).

8. C. Bovet et al., A selection of formulae and data useful for the design of A.G. Synchrotrons, CERN/MPS-SI/Int. DL/70-4, p. 32 (1970).