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TOLERANCES FOR QUADRUPOLE FOCUSING IN THE LINAC

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SUMMARY

This note considers the importance of focusing mismatch in the Linac due to:

a) beam intensity fluctuations around the nominal value

b) two or more quadrupoles connected in series and excited to the same gradient.

Tolerances to be imposed on the components of the focusing system are derived accordingly.

1. FACTORS DETERMINING THE QUADRUPOLE GRADIENTS

In order to estimate the influence of various factors on the choice of quadrupole gradients we shall derive some approximate formulae.

For negligible beam intensities, the focusing in the linac is needed to

a) transfer a beam of finite emittance

b) compensate for RF defocusing.

In linear periodic systems (we shall work with linearized forces throughout), the mean oscillating frequency $\bar{\Omega}$ is related to the average force constant $\bar{K}$ by $\bar{\Omega}^2 = \bar{K}$. Taking + signs for focusing and - for defocusing we have:

$$\bar{\Omega}^2_{\beta} = \bar{\Omega}^2_{Q} - \bar{\Omega}^2_{RF} \qquad (1)$$

(the mean betatron frequency is determined by the difference of an average quadrupole focusing and RF defocusing).

But: $$\bar{\Omega}^2_{RF} = \frac{1}{2}\,\bar{\Omega}^2_{so}$$

(the transverse RF defocusing equals to one half of the longitudinal RF focusing, $\bar{\Omega}_{so}$ being the mean synchrotron frequency for I = 0).

$$\bar{\Omega}^2_{so} = \frac{1}{q^2}\,\bar{\Omega}^2_{\beta}$$

(q is the ratio of betatron to synchrotron frequencies).

Equation (1) becomes:

$$\bar{\Omega}^2_{\beta} = \bar{\Omega}^2_{Q} - \frac{1}{2q^2}\,\bar{\Omega}^2_{\beta} \rightarrow \underline{\bar{\Omega}^2_{Q} = \left(1 + \frac{1}{2q^2}\right)\bar{\Omega}^2_{\beta}} \qquad (2)$$

$$\text{or} \quad \underline{\bar{\Omega}^2_{Q} = \left(\tfrac{1}{2} + q^2\right)\bar{\Omega}^2_{so}} \qquad (3)$$

Formulae (2) and (3) show the amount of "additional" focusing required for the compensation of RF forces: for $q^2 = \frac{1}{2}$, the focusing is doubled, for $q^2 \ll \frac{1}{2}$ it is mostly determined by RF forces.

An FD focusing can be considered as a sequence of doublets; a strength of a doublet is approximately given by the product of strengths of individual quadrupoles, provided their respective gradients are roughly equal in absolute value. With this, the formulae for quadrupole gradients in the linac are derived from (2) and (3) as:

$$\underline{G \propto \bar{\Omega}_{\beta}\sqrt{1 + \frac{1}{2q^2}}} \qquad (4)$$

or

$$\underline{G \propto \bar{\Omega}_{so}\sqrt{\tfrac{1}{2} + q^2}} \qquad (5)$$

If RF defocusing is ignored, the quadrupole gradient $G' \propto \bar{\Omega}_{\beta}$. Thus

$$\frac{G}{G'} = \sqrt{1 + \frac{1}{2q^2}}$$

giving $\frac{G}{G'}$ = 2.254 and 1.374 for q = 0.35 and 0.75 respectively. These values agree quite well with computer results, compare Fig. 1.

In the presence of space charge, formula (1) becomes

$$\bar{\Omega}^2_{\beta} = \bar{\Omega}^2_{Q} - \bar{\Omega}^2_{RF} - \bar{\Omega}^2_{sc}, \qquad (6)$$

where $\bar{\Omega}^2_{sc}$ stands for an average space charge defocusing. The beam in the linac is bunched and considering the bunches as rotational ellipsoids we have:

$$\bar{\Omega}^2_{sc} = \frac{k_1}{b\,a^2}\, I \left[1 - f\left(\frac{b}{a}\right)\right] \qquad (7)$$

with $k_1 \cong 1{,}6 \cdot 10^{-7}$

I... beam current in A

$f(\frac{b}{a})$... dimensionless form-factor

b... half bunch length in m

a... mean beam radius in m

The transverse beam emittance E, mean beam radius a and $\bar{\Omega}_\beta$ are related through:

$$a^2 = \frac{E\beta\lambda}{\bar{\Omega}_\beta} \qquad (8)$$

(we work in a system where the independent variable is $\tau = \frac{c}{\lambda} t$).

Putting (8) into (7):

$$\bar{\Omega}^2_{sc} = \frac{k_1}{bE\beta\lambda}\, I \left[1 - f\left(\frac{b}{a}\right)\right] \bar{\Omega}_\beta = k(\beta,a,b)\, I\, \bar{\Omega}_\beta \qquad (9)$$

From (6), (9) and (2):

$$\bar{\Omega}^2_Q = \bar{\Omega}^2_\beta \left[1 + \frac{1}{2q^2} + \frac{k(\beta,a,b)I}{\bar{\Omega}_\beta}\right] \qquad (10)$$

From (10) we derive the formula for quadrupole gradients in the presence of space charge:

$$G \propto \sqrt{\bar{\Omega}_\beta \left[1 + \frac{1}{2q^2} + \frac{k(\beta,a,b)I}{\bar{\Omega}_\beta}\right]} \qquad (11)$$

To evaluate $k(\beta,a,b) = \frac{1.6 \cdot 10^{-7}}{bE\beta\lambda}\left[1 - f\left(\frac{b}{a}\right)\right]$, all values shall be expressed in mks units and Fig. 2 will be used to determine $f(\frac{b}{a})$*.

Example:

$E = 100 \cdot 10^{-6}$ m rad

$\beta\lambda = 5 \cdot 10^{-2}$ m

$b = 4.1 \cdot 10^{-3}$ m

$a = 3.7 \cdot 10^{-3}$

$I = 0.1$ A

from Fig.2 : $f(\frac{b}{a}) \cong f(1.10) \cong 0.355$

* Fig.2 is taken from CERN/AR/Int. SG/65-15 "Effets de la charge d'espace dans un accélérateur linéaire à protons" by P.M. Lapostolle.

$$\left.\begin{aligned} k\cdot I &= 0.50 \\ \bar{\Omega}_\beta &= \frac{E\beta\lambda}{a^2} = \frac{5\cdot 10^{-6}}{13.7\cdot 10^{-6}} = 0.365 \end{aligned}\right\} \quad \frac{k\cdot I}{\bar{\Omega}_\beta} \cong 1.37$$

For $a = 3.7\cdot 10^{-3}$ m $\rightarrow q \cong 0.75$ (from Fig. 3b**, valid for the present linac).

Putting the calculated values into (11), we get:

$$G \propto \bar{\Omega}_\beta \sqrt{1 + 0.89 + 1.37} \propto \bar{\Omega}_\beta \sqrt{3.26.}$$

The increase in G due to space charge is:

$$\frac{G}{G_o} = \frac{3.26}{1.89} \cong 31\%.$$

This is approximately what one finds with computer calculations, compare Fig. 3a**.

2. INFLUENCE OF BEAM INTENSITY FLUCTUATIONS

Misadjustments in quadrupole gradients due to statistical fluctuations in beam intensity I can be expressed as:

$$\frac{\Delta G}{G} = \frac{\frac{k\cdot I}{2\Omega_\beta}}{1 + \frac{1}{2q^2} + \frac{k\cdot I}{\bar{\Omega}_\beta}} \frac{\Delta I}{I} \tag{12}$$

Taking the same example as in section 1, one obtains:

$$\frac{\Delta G}{G} \cong 0.2 \frac{\Delta I}{I} \quad , \text{ for } I = 100 \text{ mA.} \tag{13}$$

Assuming $\frac{\Delta I}{I} \leqslant 5\%$, the equivalent gradient error is $\leqslant 1\%$.

For other operating conditions, e.g. $q = 0.35$, Fig. 3b yields $a \approx 5.5$ mm; after some calculations one obtains $\frac{\Delta G}{G} = 0.175 \frac{\Delta I}{I}$, which is pratically the same as eq. (13).

** Figure taken from CERN/LINP-Note 73-5 "Choice of the preaccelerator energy for the new Linac project" by M. Weiss.

The influence of gradient errors of -1% and -2.5% (errors of the same sign are more important than statistically distributed errors) on betatron matching is shown in Figs. 4 and 5 respectively. Fig. 6 shows a mismatch due to $\frac{\Delta I}{I} = 5\%$ (to be compared with Fig. 4 according to eq. (13)).

3. CONNECTION IN SERIES OF TWO OR MORE QUADRUPOLES

If the quadrupoles are pulsed, it is interesting to connect two or more in series in order to reduce the number of pulsers. In such a case the quadrupoles of one series would not be separately adjustable, and one would work with a "step" gradient law instead of a smooth one.

The quadrupoles are usually manufactured in batches; only those of the same batch are to be connected in series.

Various series connections, applicable to tank 2 and 3 of the present linac, have been analysed for two focusing conditions, q = 0.35 and 0.75 as:

a) 2 quadrupoles in series, Figs. 7 and 8

b) 2 quadrupoles in series, interlaced (+-+-+-+-), Figs. 9 and 10

c) 3 quadrupoles in series, Figs. 11 and 12

d) 4 quadrupoles in series, Figs. 13 and 14.

Best results are obtained with variant b), followed by c). Variants a) and d) bring about an unequal mean focusing in the two transverse phase planes, and are thus less satisfactory.

All the variants, except b), are sensitive to the choice of the q value; in general, a higher q value (stronger mean focusing) is preferable.

4. CONCLUSION: TOLERANCES ON QUADRUPOLE GRADIENTS

Fluctuations of the order of 5% in the nominal beam intensity of 100 mA can be "translated" into equivalent gradient errors of the order of 1%. Roughly of the same order are equivalent gradient errors in tank 1 due to the chromaticity in the beam: $\frac{\Delta p}{p} \cong (2.5 \div 3)\%$ at injection and $\cong (0.4 \div 0.5)\%$ at tank output.

In tanks 2 and 3 some gradient errors are generated if quadrupoles are connected in series.

Conclusion: the unavoidable fluctuations in the beam intensity of a few percent and the chromaticity in the beam cause equivalent gradient errors of the order of 1%. The tolerances imposed on the components of the focusing system should therefore be such as to limit additional gradient errors to only fractions of 1%.

ACKNOWLEDGMENTS

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Distribution

PS Scientific Staff

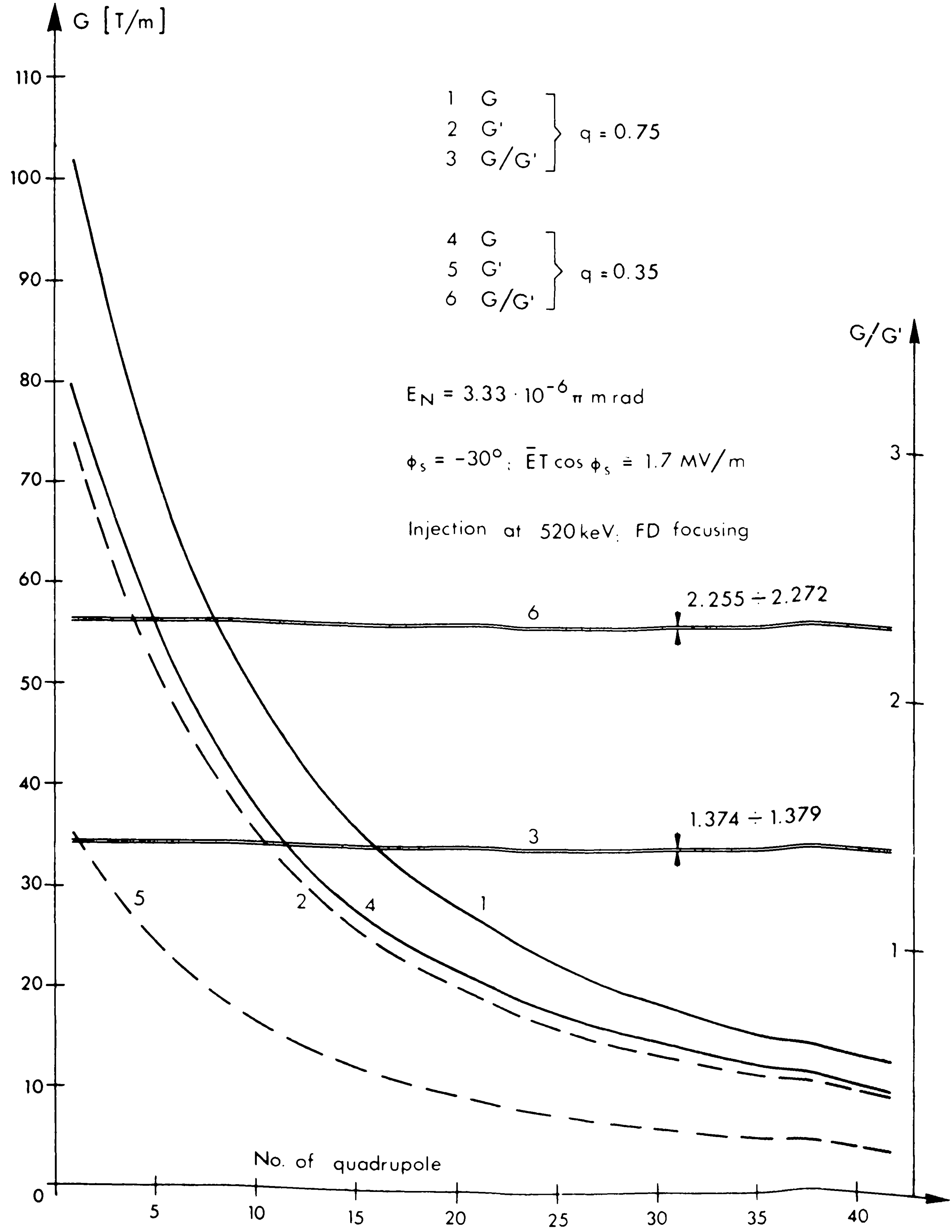

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FIG. 1: Quadrupole gradients in the first Linac tank for I = 0

(G' ... gradient in absence of RF defocusing)

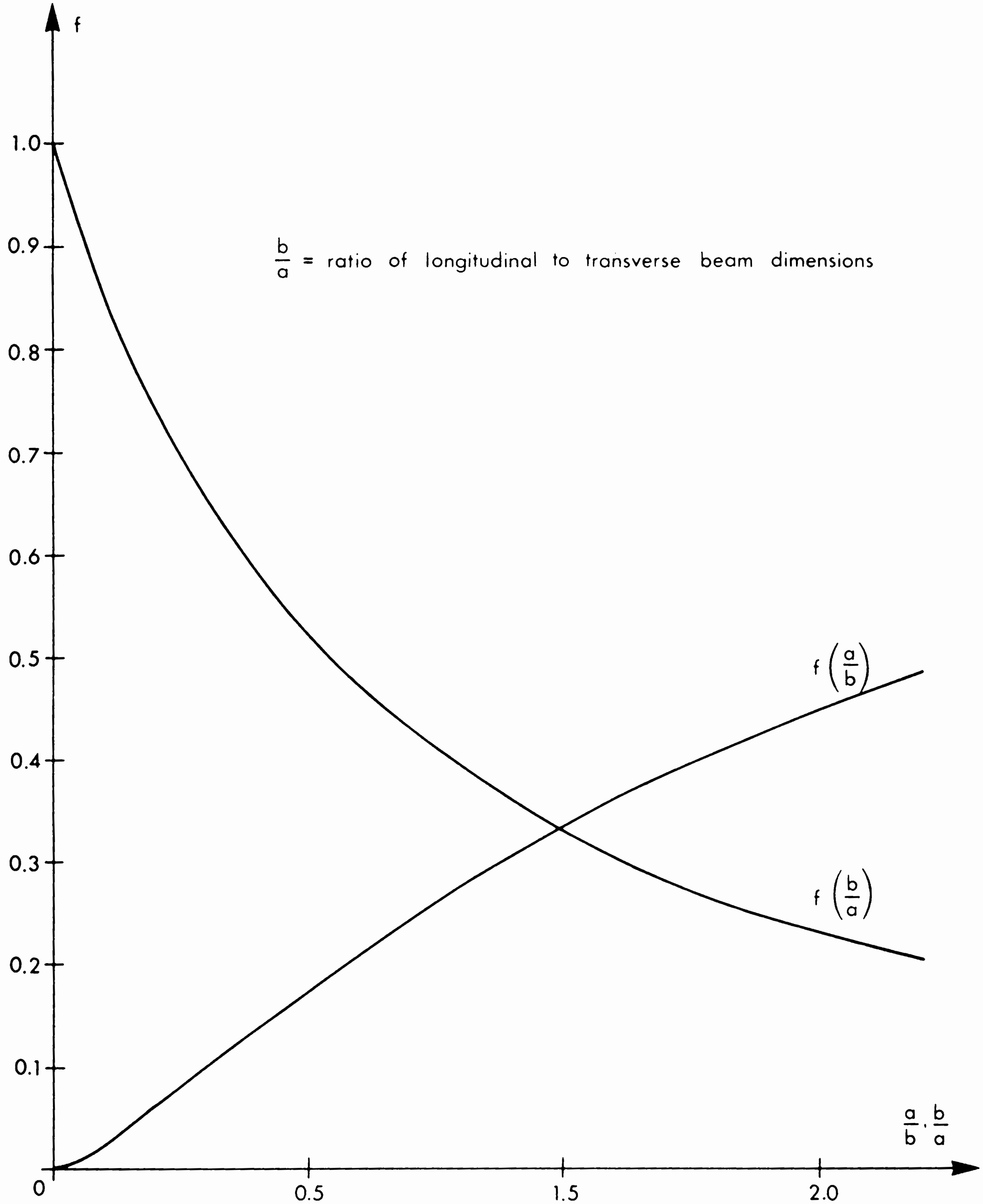

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FIG. 2: Dimensionless formfactors $f\left(\frac{b}{a}\right)$ and $f\left(\frac{a}{b}\right)$

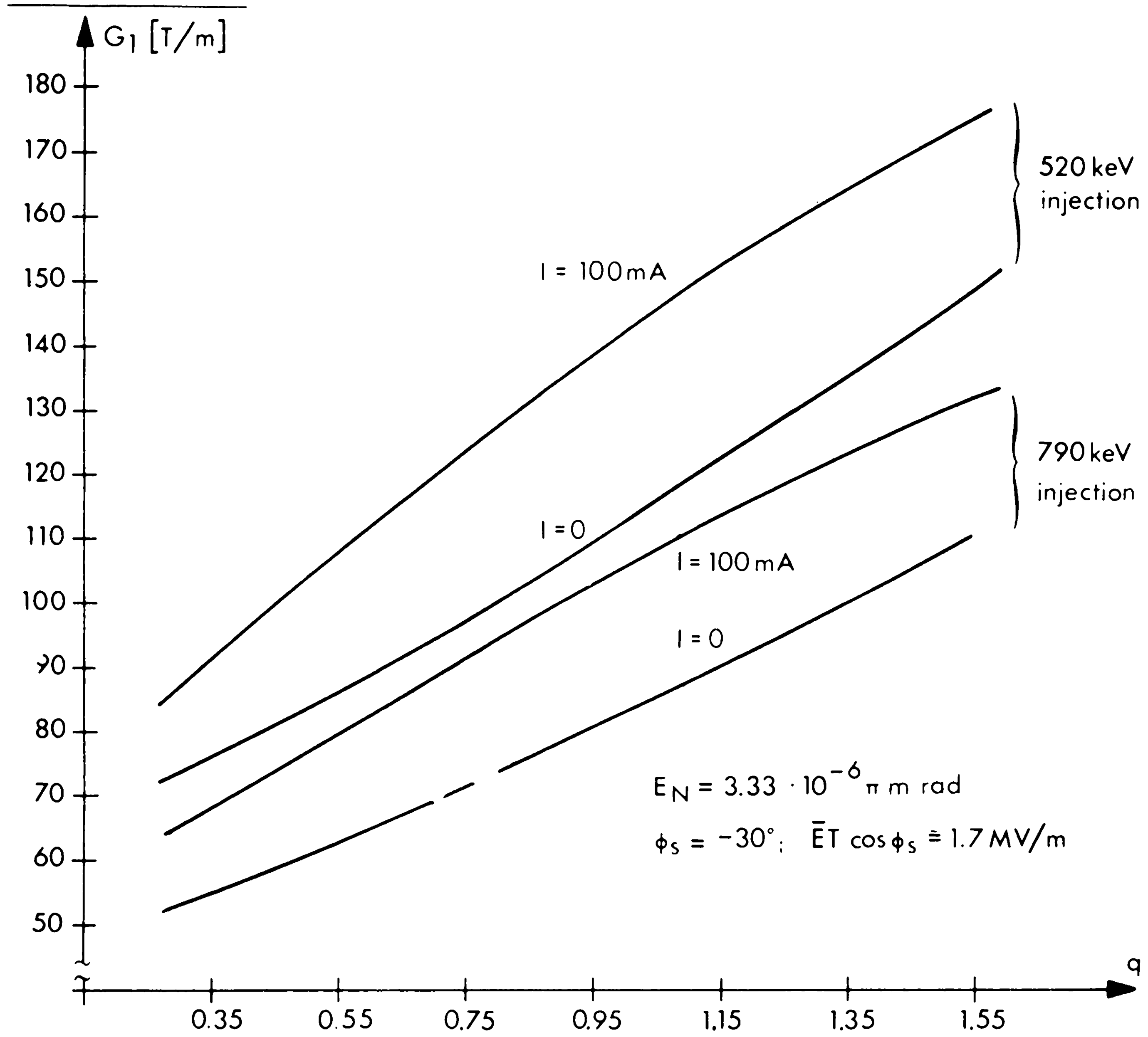

FIG.3a: Gradient of the first linac quadrupole as function of $q = \frac{\Omega_\beta}{\Omega_{s0}}$

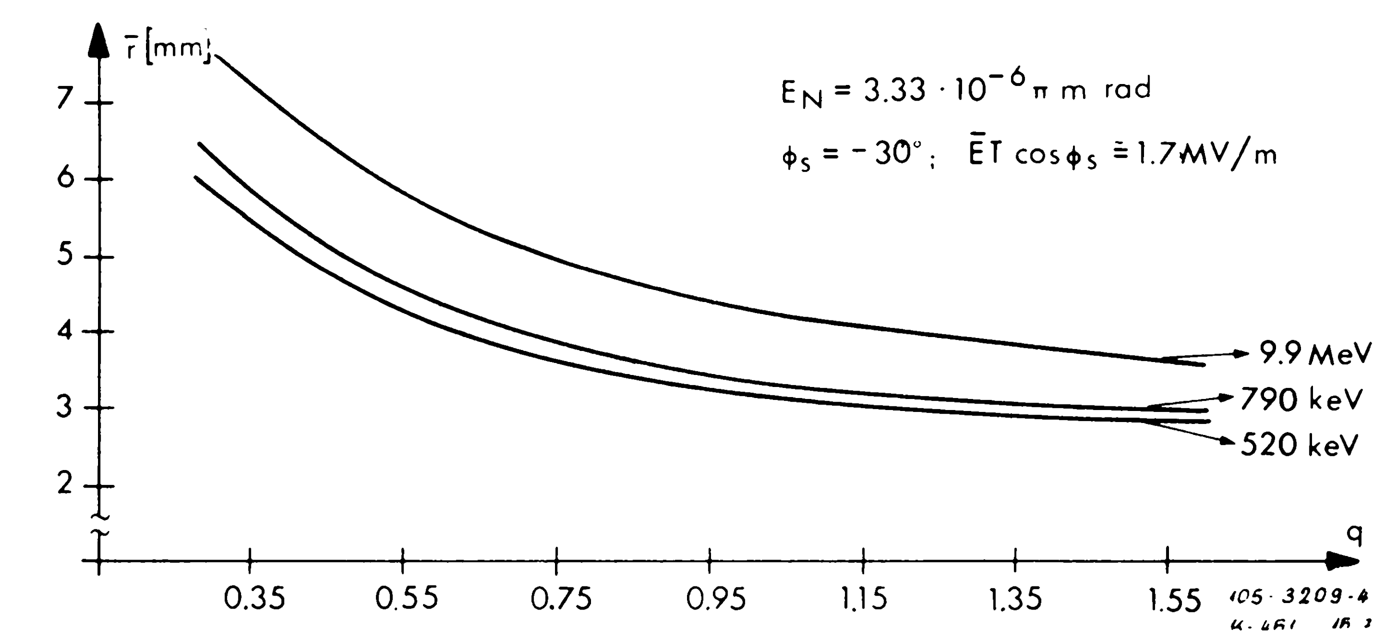

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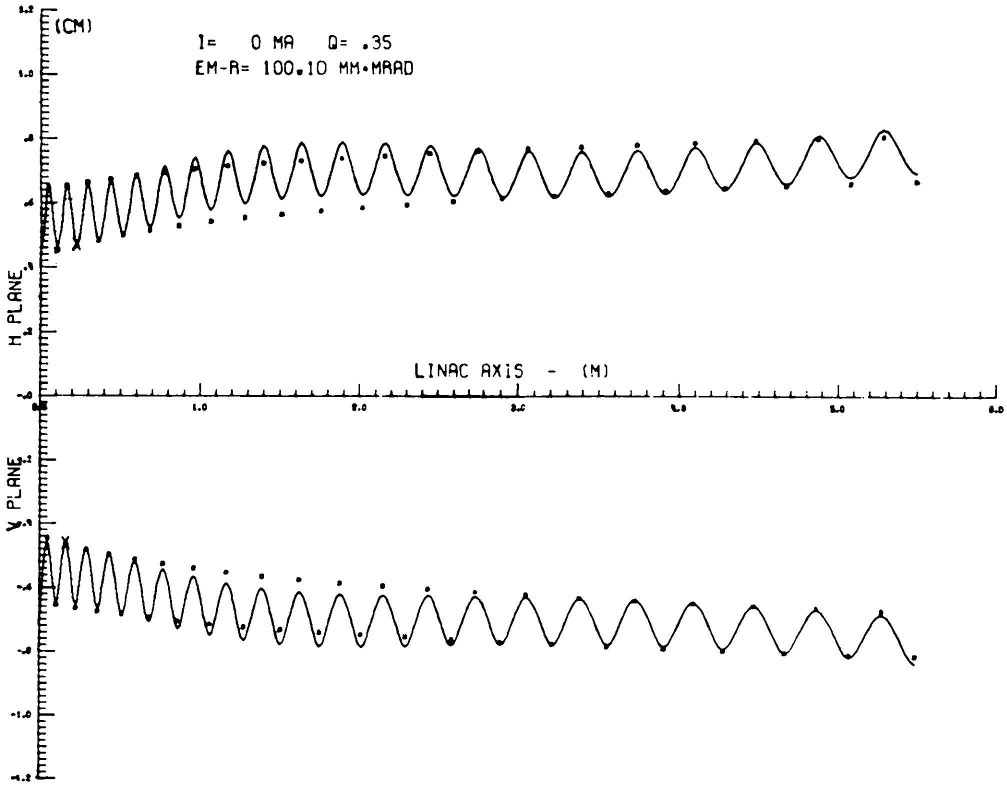
FIG.3b: Mean radius of matched beam at input and output of first tank



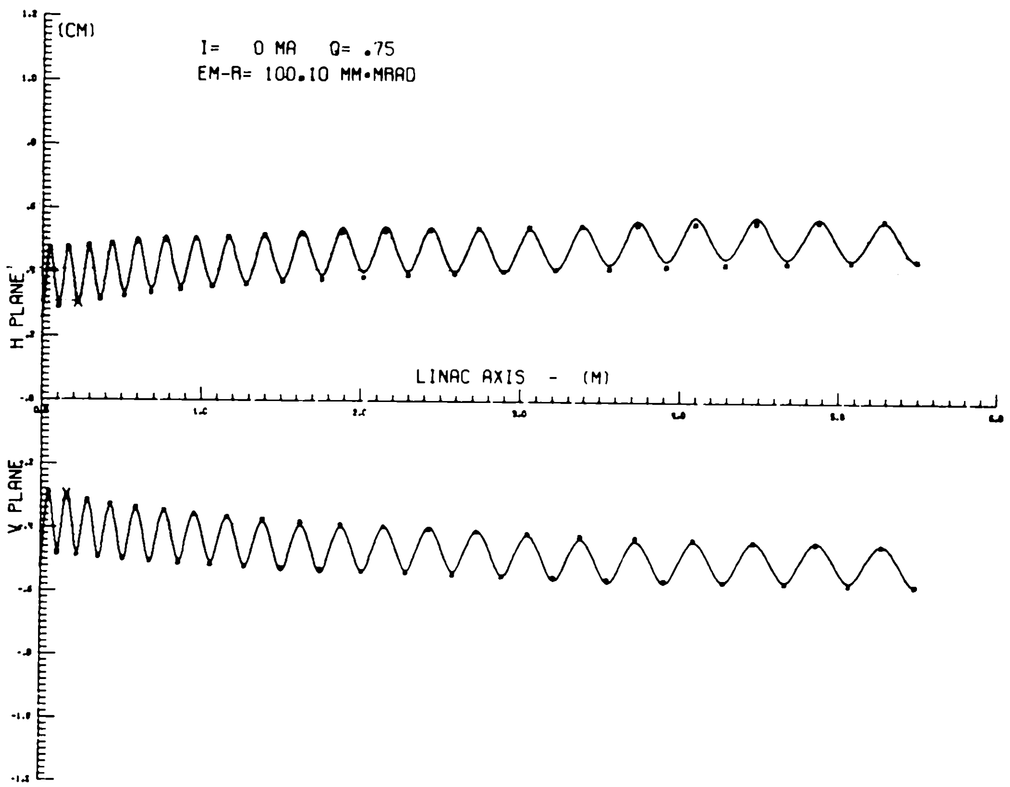
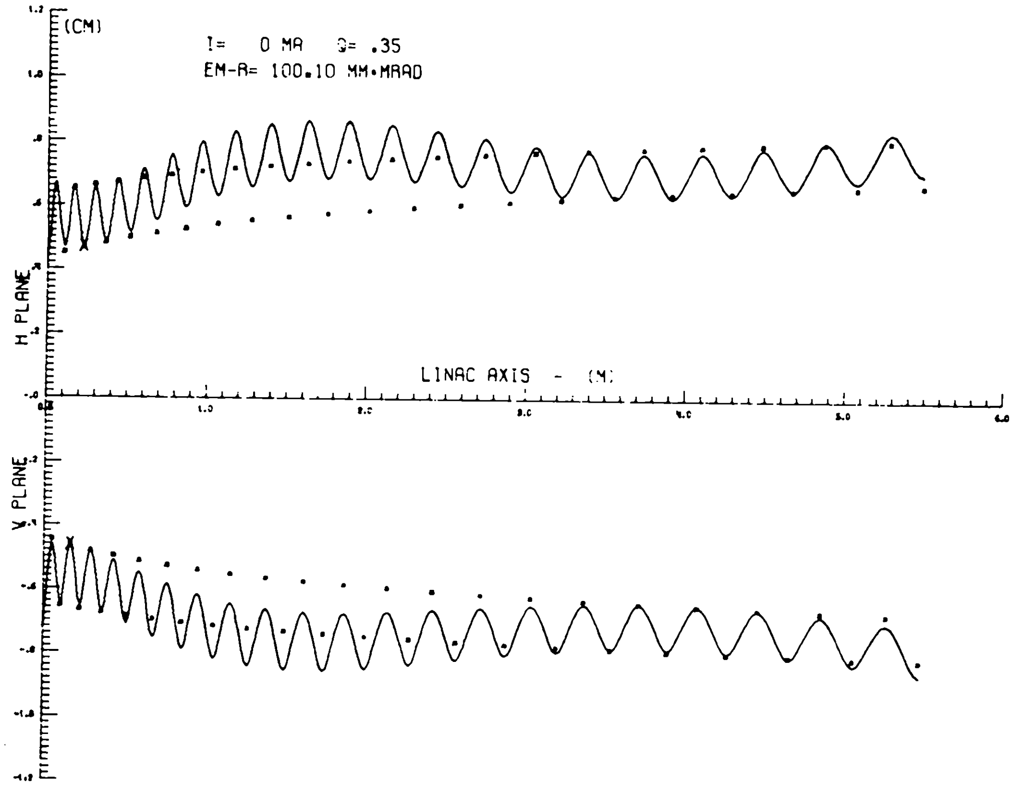


Fig. 4: Gradient errors of -1% - Tank I



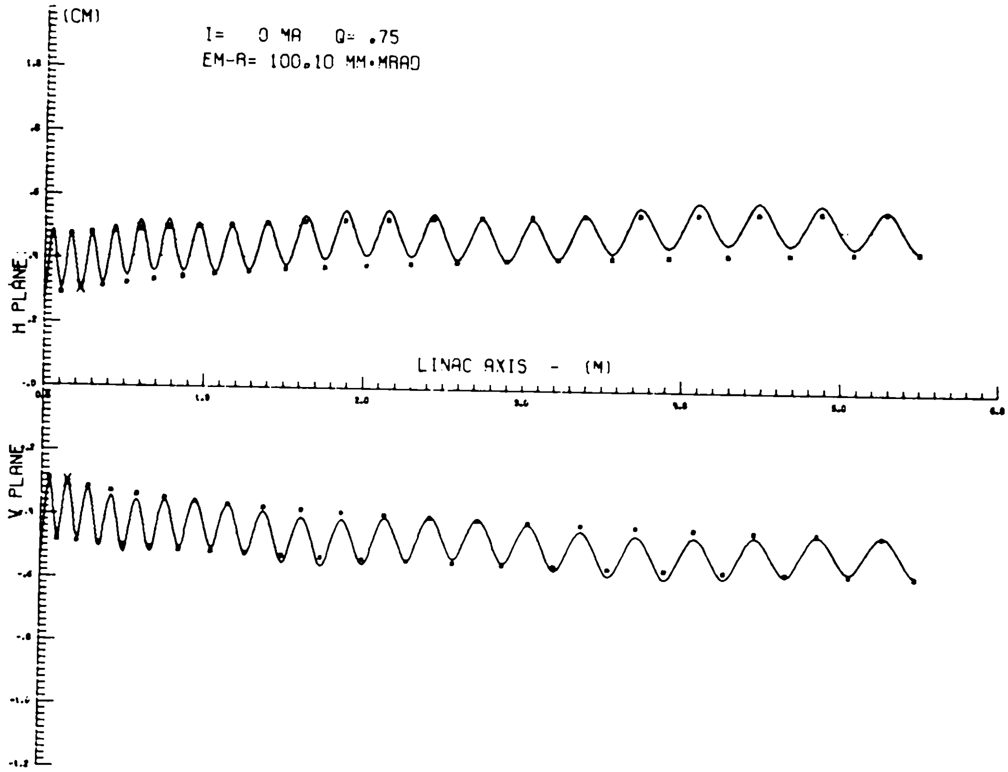
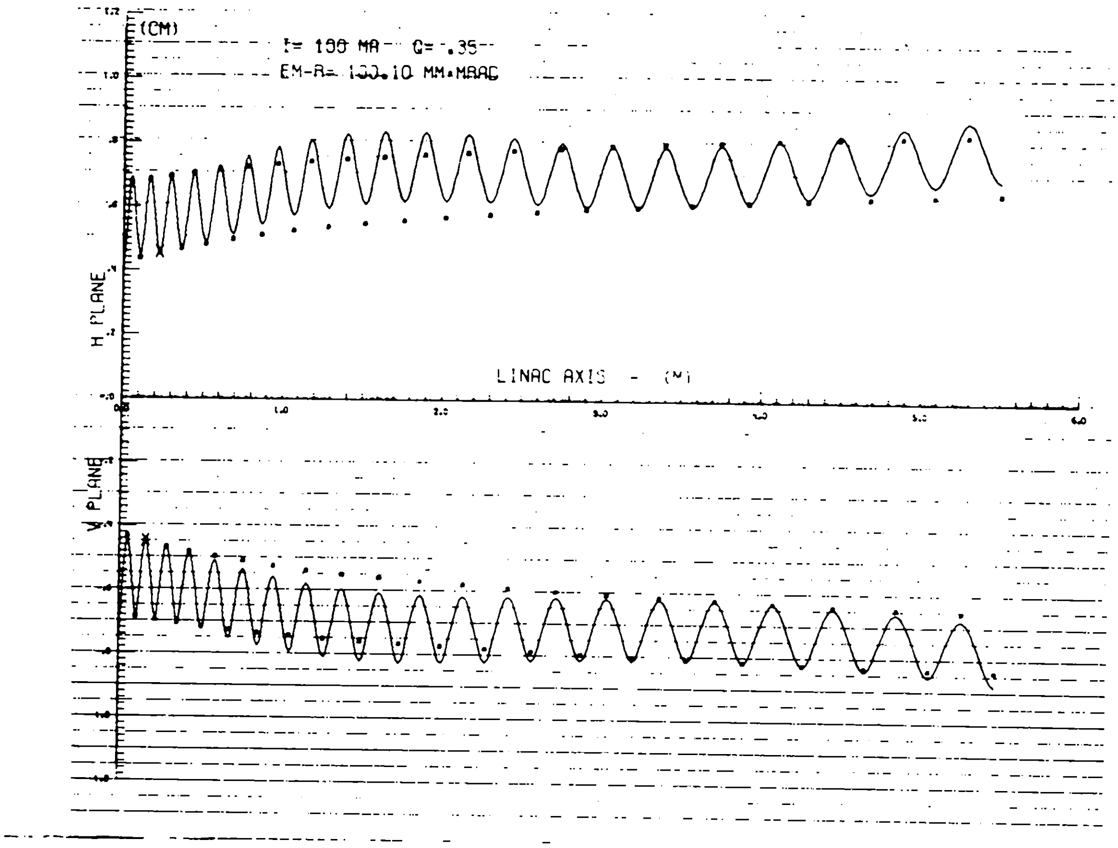


Fig. 5: Gradient errors of -2.5% - Tank I



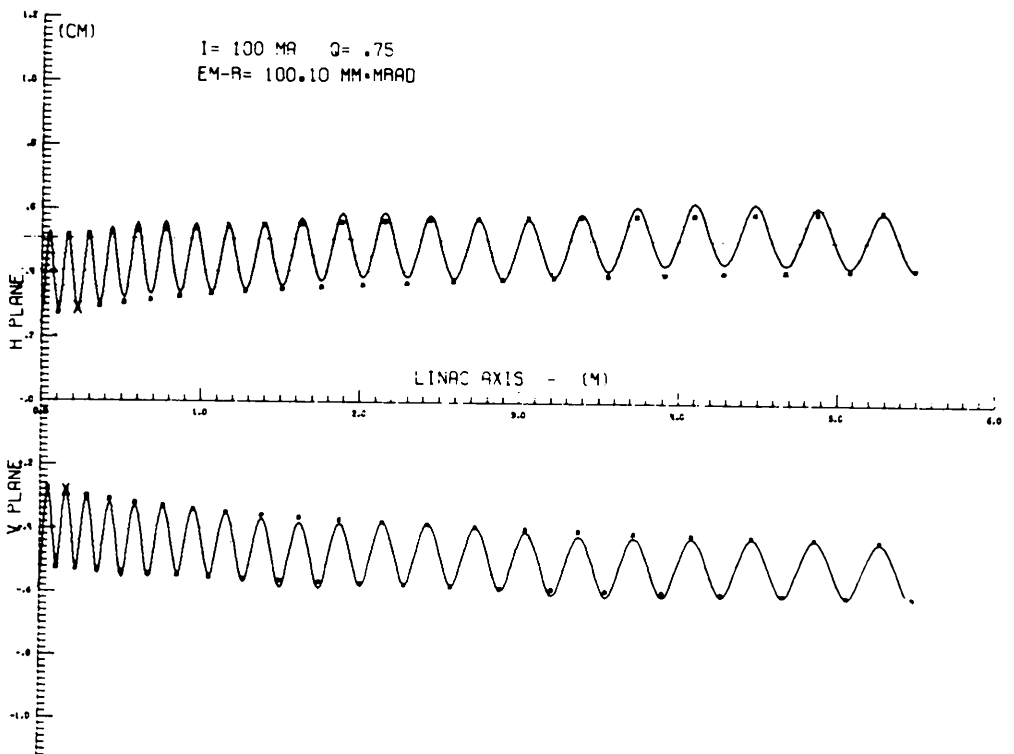


Fig. 6: Beam intensity fluctuation of +5% - Tank I

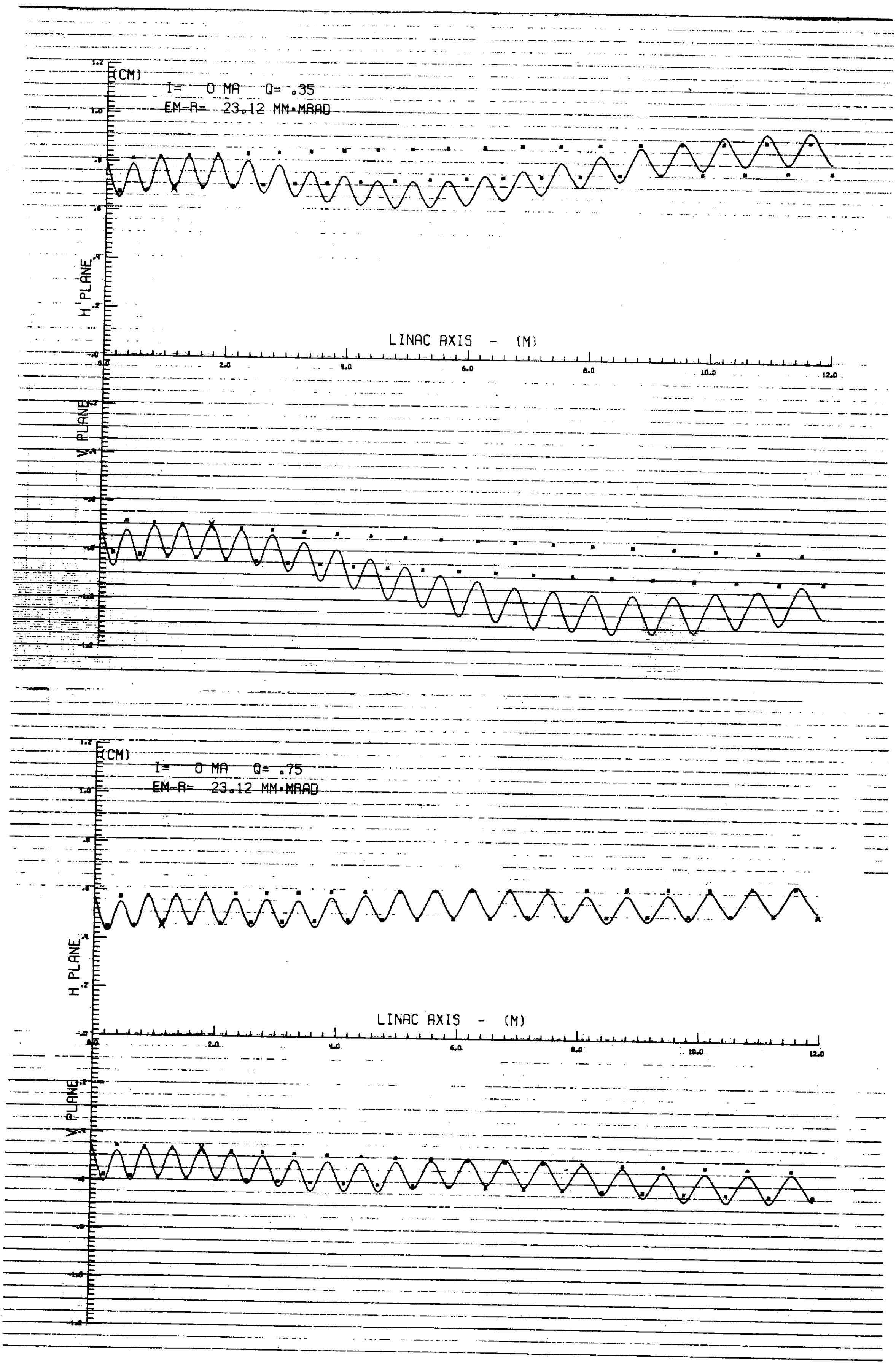
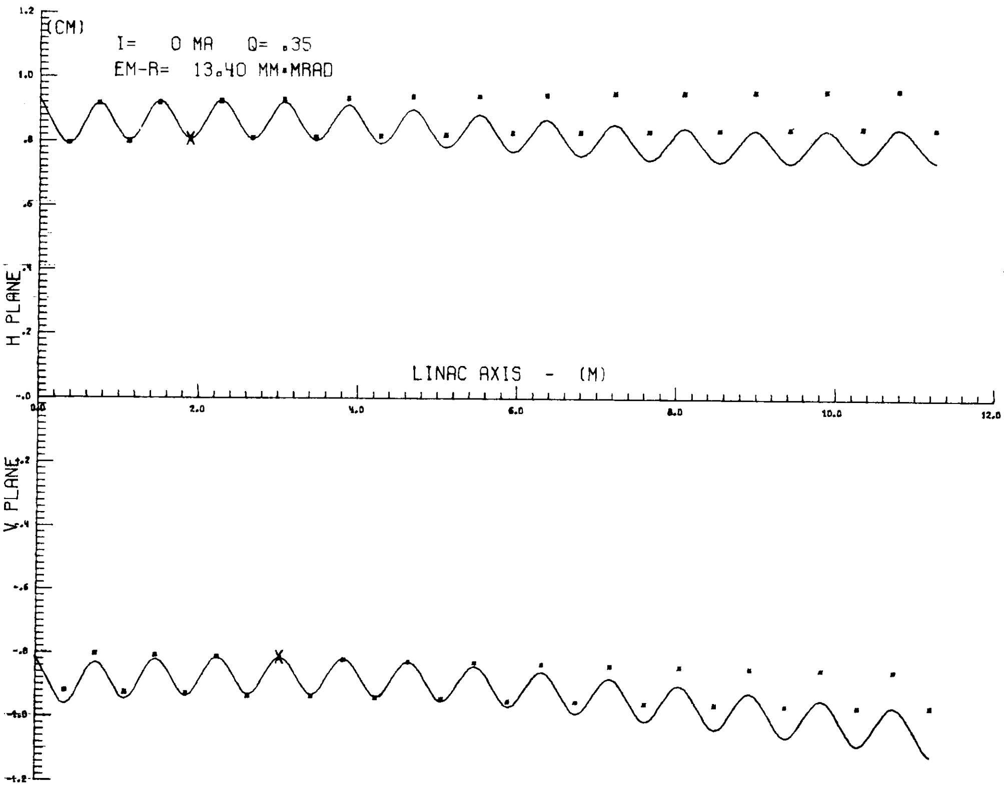


Fig. 7: Two quadrupoles in series - Tank II



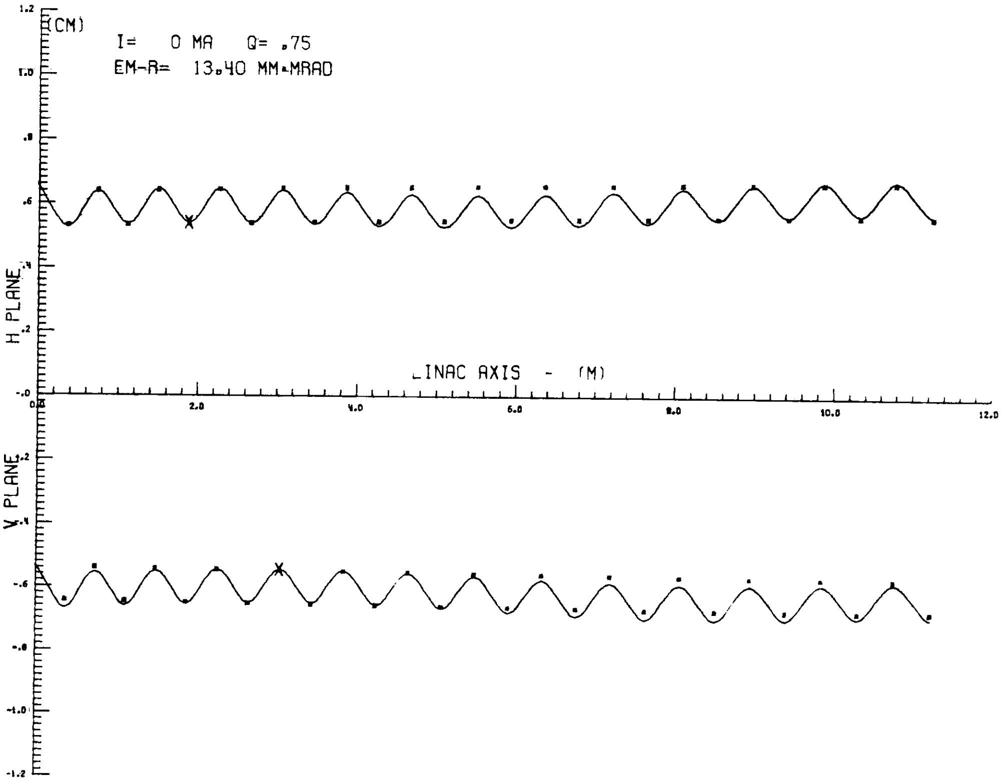
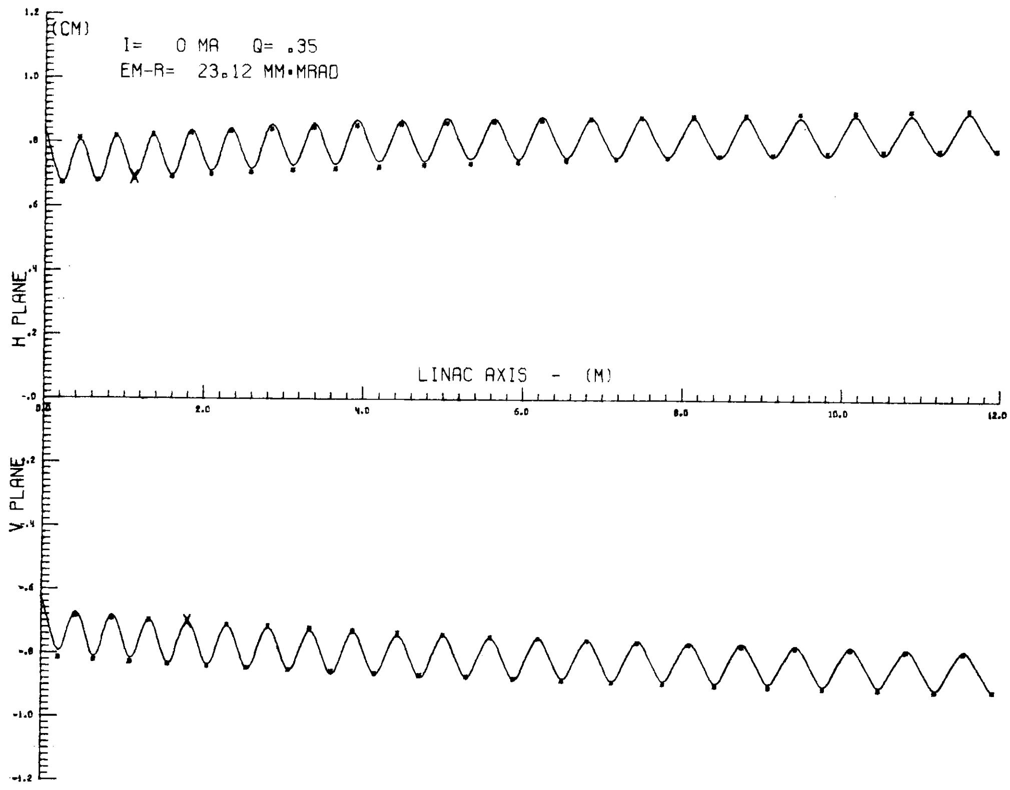


Fig. 8: Two quadrupoles in series - Tank III



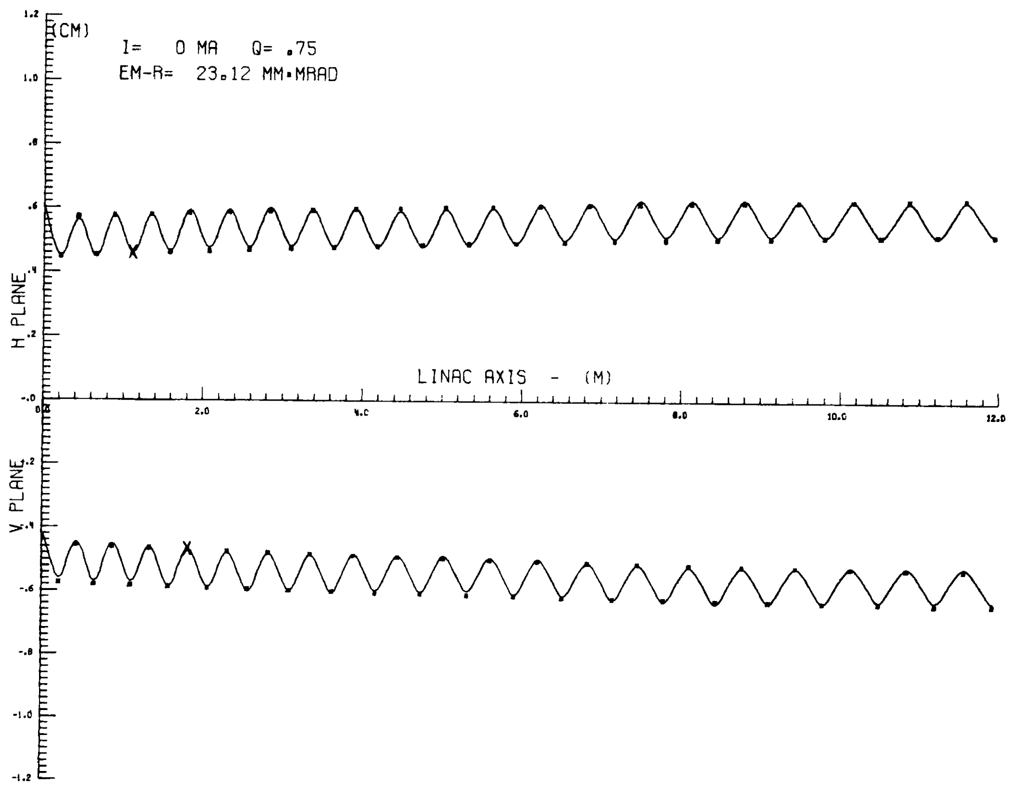
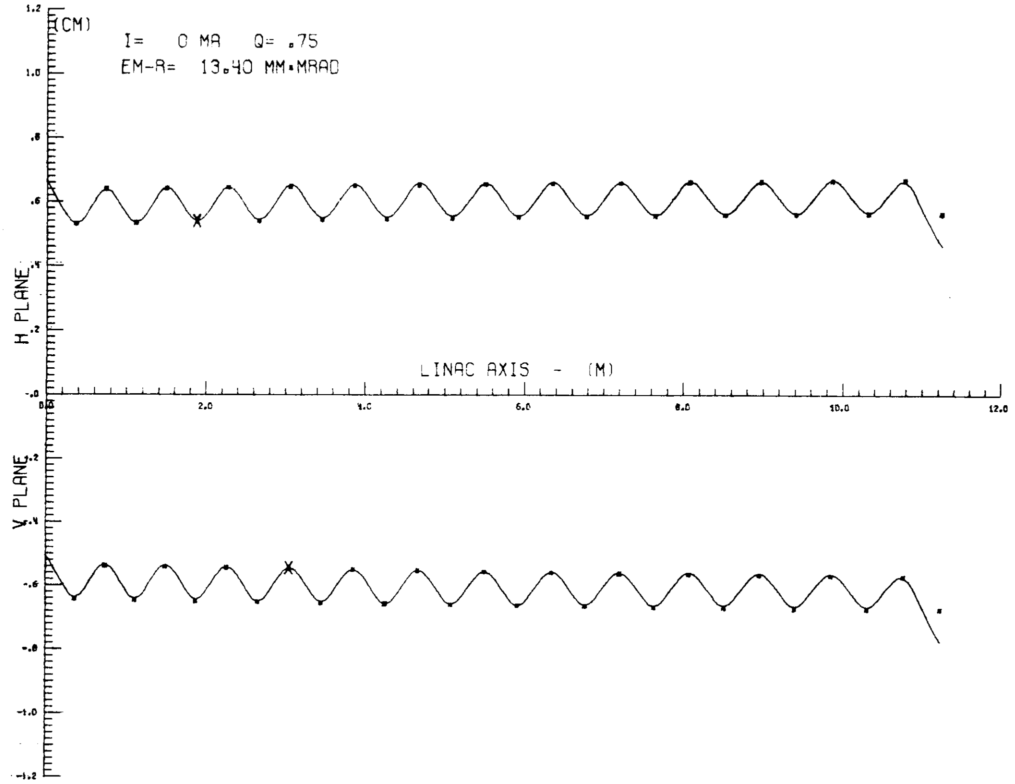


Fig. 9: Two quadrupoles in series, interlaced - Tank II



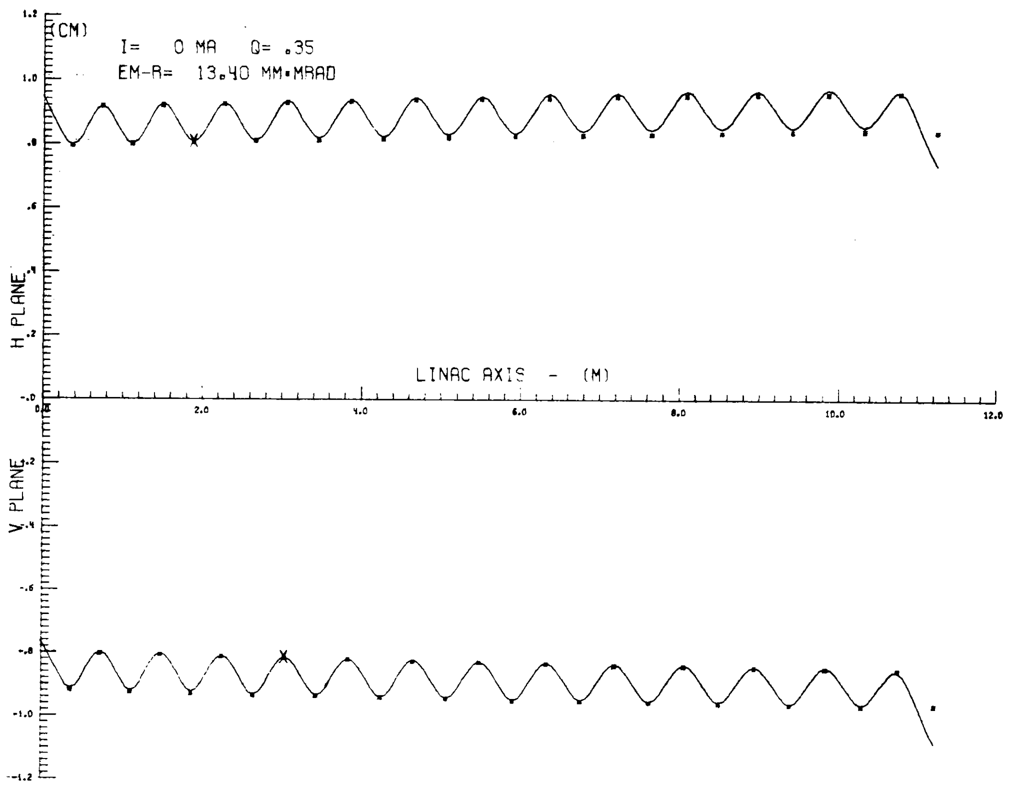


Fig. 10: Two quadrupoles in series, interlaced - Tank III

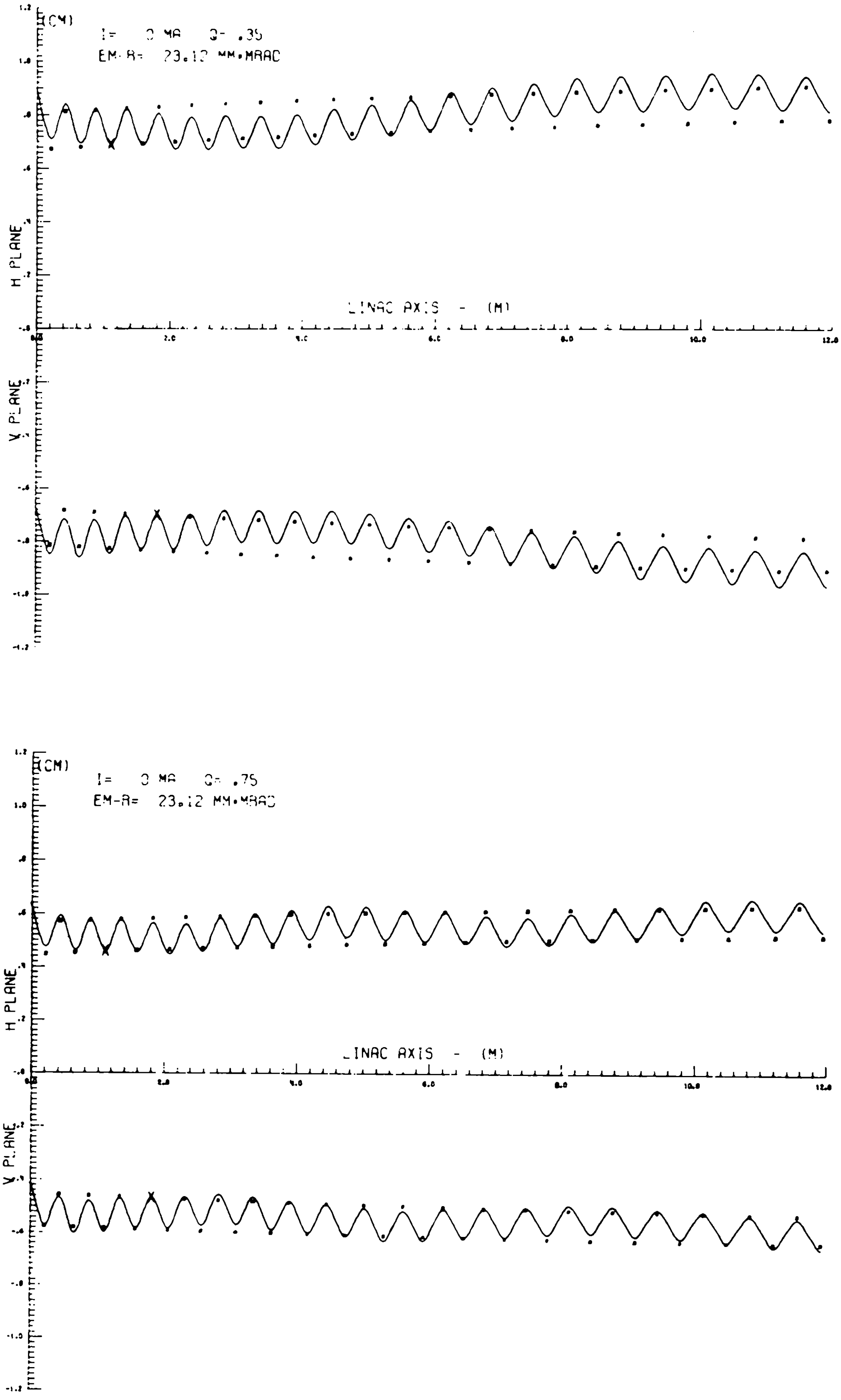


Fig. 11: Three quadrupoles in series - Tank II

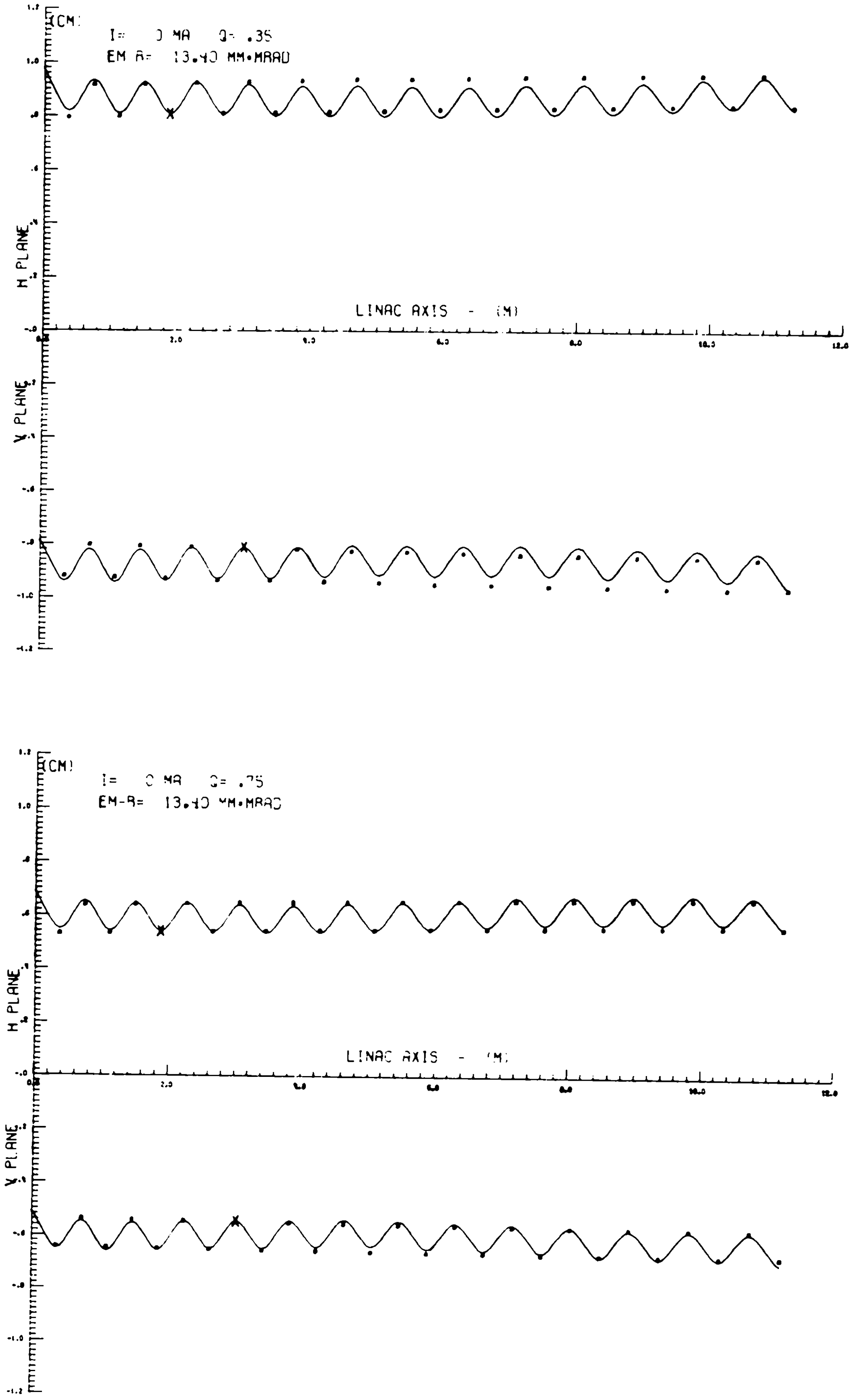
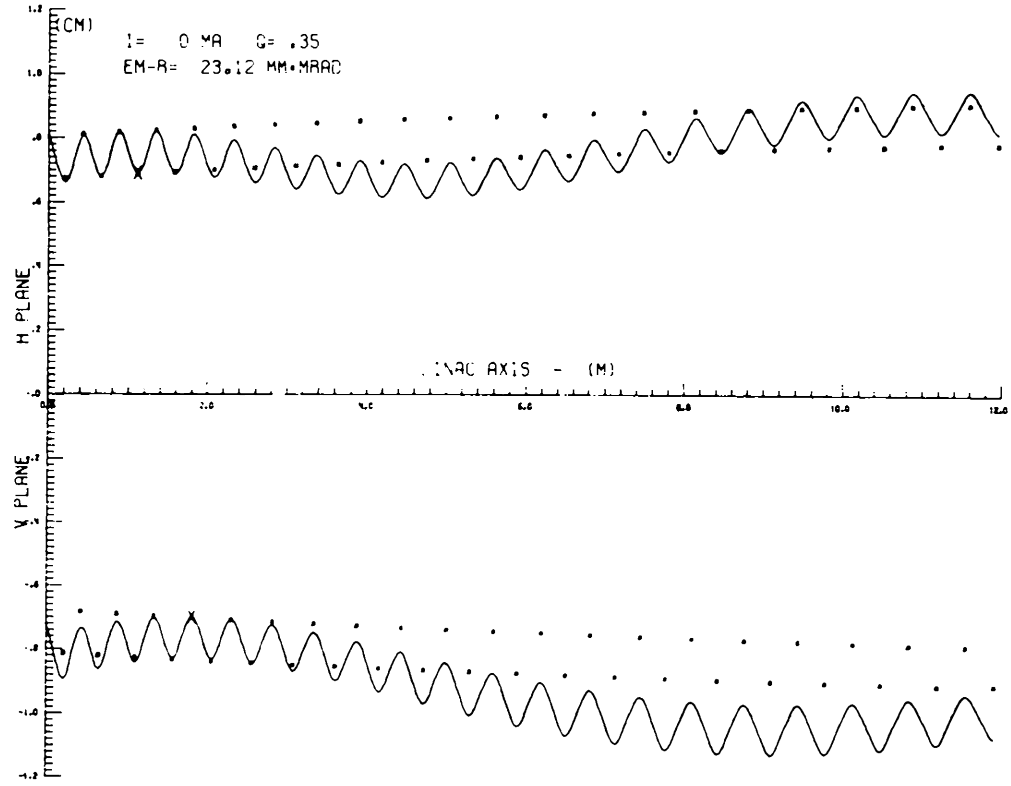


Fig. 12: Three quadrupoles in series - Tank III



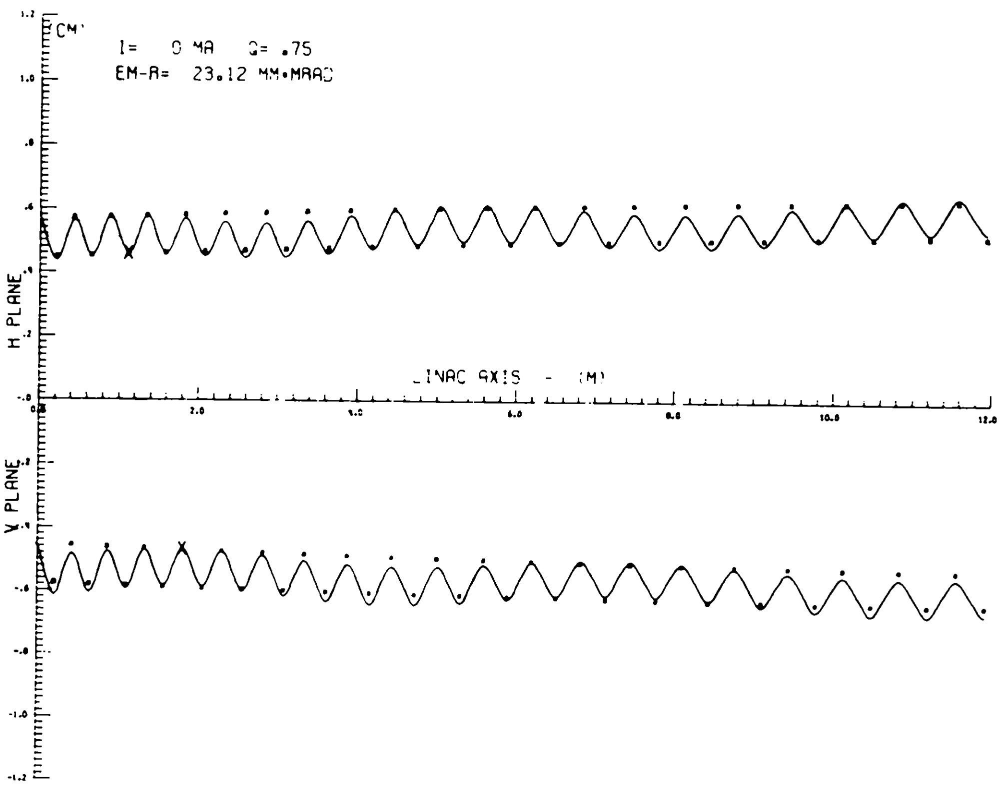


Fig. 13: Four quadrupoles in series - Tank II

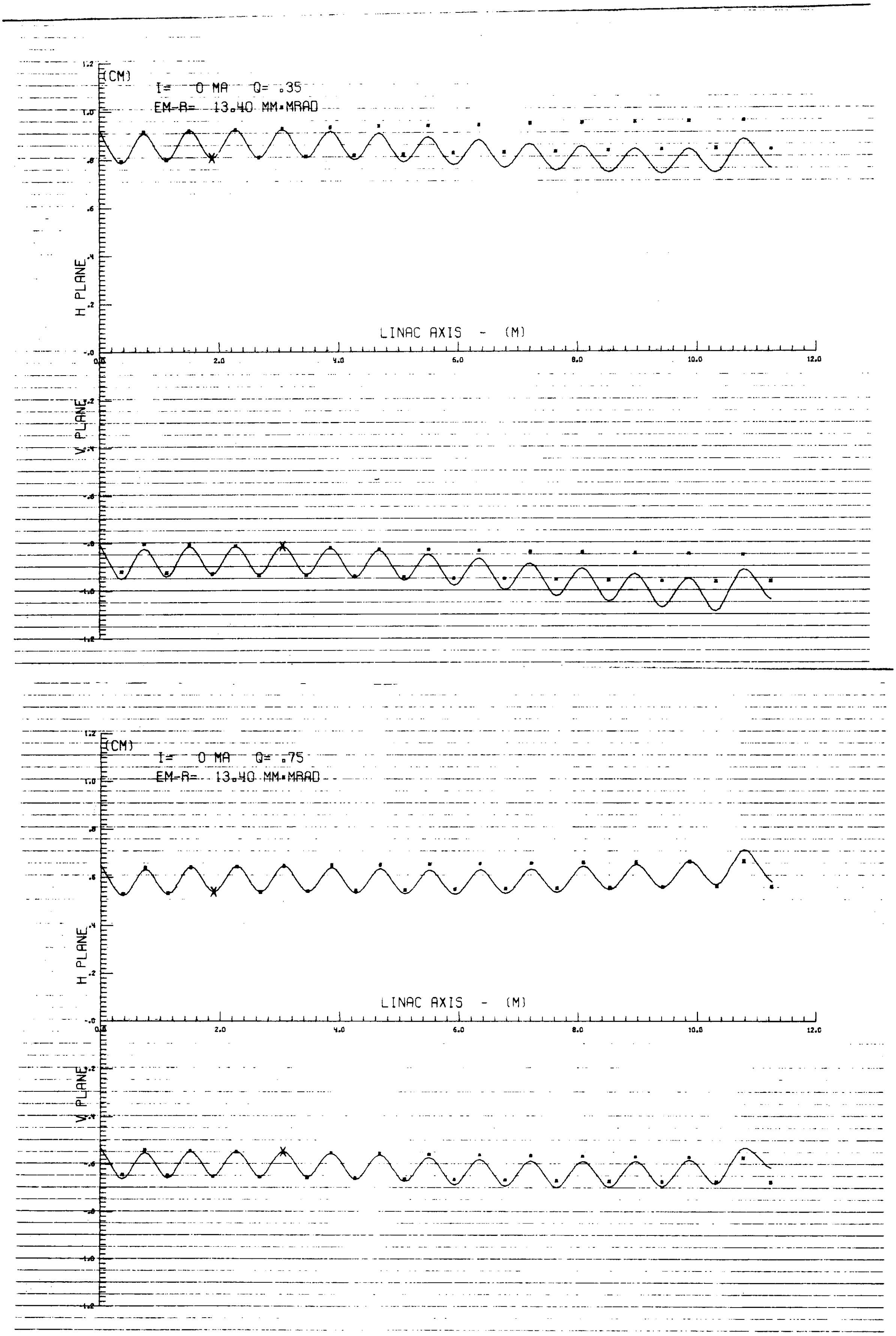


Fig. 14: Four quadrupoles in series - Tank III