$|V_{cb}|$ from inclusive semileptonic B decays

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Abstract

We determine $|V_{cb}|$ from inclusive semileptonic B decays including resummation of supposedly large perturbative corrections, which originate from the running of the strong coupling. We argue that the low value of the BLM scale found previously for inclusive decays is a manifestation of the renormalon divergence of the perturbative series starting already in third order. A reliable determination of $|V_{cb}|$ from inclusive decays is still possible if one uses a short-distance b quark mass. We find that using the $\overline{\text{MS}}$ running mass significantly reduces the perturbative coefficients already in low orders. For a semileptonic branching ratio of 10.9% we obtain $|V_{cb}|(\tau_B/1.50\,\text{ps})^{1/2} = 0.041 \pm 0.002 \pm 0.002$. This work was done in collaboration with M. Beneke and V.M. Braun.

Since it was shown that the decays of heavy hadrons can be described within the framework of an operator product expansion (OPE) [1], the old idea of extracting $|V_{cb}|$ from inclusive semileptonic B decays has gained revived interest. A very welcome feature of the OPE is that it expresses the hadronic decay rate as that of the parton model plus non-perturbative corrections, which are suppressed by two powers of the heavy quark mass:

$$\Gamma(B \to X_c e \nu) = \Gamma(b \to c e \nu) \left(1 + \frac{\delta^{NP}}{m_b^2}\right). \tag{1}$$

The main efforts were first concentrated on determining δ^{NP}/m_b^2, which turned out to be small and of the order of 5%. Subsequently, however, it was shown [2, 3] that the pole quark mass, which was habitually used in all analyses although it was known to be an ill-defined quantity, suffers as manifestation of its ill-definedness from a renormalon induced uncertainty of order Λ_{QCD}, which would generate terms of order $1/m_b$ on the right-hand side of Eq. (1). As a next step these uncertainties were shown to cancel against corresponding ones in the perturbative corrections to $\Gamma(b \to c e \nu)$ [2, 4]. At the same time it turned out [5], that the introduction of a short-distance mass in (1), which *a priori* avoids all renormalon uncertainties, shifts the value of V_{cb} by $\sim 15\%$ at one-loop level with respect to analyses using pole masses [6]. Consequently, the question of higher order perturbative corrections to the quark level decay moved in the center of interest and in Ref. [7] the BLM prescription [8] was used to set the renormalization scale in the lowest order radiative correction, which corresponds to taking into account exactly the supposedly large correction in $\alpha_s^2 \beta_0$, β_0 being the lowest order coefficient of the QCD β-function. The resulting "optimum" scale of order 500 MeV is rather small and was interpreted as an indication for the possible breakdown of perturbation theory in heavy quark decays.

In Ref. [9], cf. also [10], we have generalized the "standard" BLM prescription to arbitrary order in α_s and applied it to semileptonic b decays in Ref. [11]. The full series reads

$$\Gamma(b \to c e \nu) = \Gamma_0 \left\{1 - C_F \frac{\alpha_s(m_b)}{\pi} g_0(a) \Big[1 + \sum_{n=1}^{\infty} \tilde{d}_n(a) \alpha_s^n(m_b)\Big]\right\}, \tag{2}$$

where a is the ratio of quark masses m_c/m_b and $g_0(a)$ is the one-loop correction first calculated in [12]. Writing

$$\tilde{d}_n(a) = \delta_n(a) + (-\beta_0)^n d_n(a) \tag{3}$$

with $\beta_0 = -1/(4\pi)\{11 - 2/3 N_f\}$, we can calculate with our method both the $d_n(a)$ and the sum

$$M_\infty^{b \to c}[a, -\beta_0 \alpha_s(m_b)] \equiv 1 + \sum_{n=1}^{\infty} (-\beta_0)^n d_n(a) \alpha_s^n(m_b), \tag{4}$$

$$\text{so that } \Gamma(b \to c e \nu) = \Gamma_0 \left\{1 - C_F \frac{\alpha_s(m_b)}{\pi} g_0(a) M_\infty^{b \to c}[a, -\beta_0 \alpha_s(m_b)]\right\} \tag{5}$$

in our approximation of neglecting the δ_n. In the BLM language the "optimum" scale is then $\mu_\infty^{b \to c}$, defined as $\alpha_s(\mu_\infty^{b \to c}) \equiv \alpha_s(m_b) M_\infty^{b \to c}[a, -\beta_0 \alpha_s(m_b)]$.

Diagramatically, the d_n can be obtained from the one-loop correction with the insertion of a chain of, say, i fermion loops into the gluon lines. These diagrams are proportional to N_f^i, N_f being the number of active quark flavours, and thus proportional to β_0^i. Remarkably enough, these contributions can be related to the one-loop correction $g_0(a, \lambda^2)$ calculated with a *finite* gluon mass λ, such that [9] $(g_0(a) \equiv g_0(a, \lambda^2 = 0))$

$$g_0(a)(-\beta_0\alpha_s)M_\infty[a, -\beta_0\alpha_s] = \int_0^\infty d\lambda^2 \, \Phi(\lambda^2) \, g_0'(a, \lambda^2) + [g_0(a, \lambda_L^2) - g_0(a)], \qquad (6)$$

where $\alpha_s = \alpha_s(\mu)$,

$$\Phi(\lambda^2) = -\frac{1}{\pi} \arctan\left[\frac{-\beta_0\alpha_s\pi}{1 - \beta_0\alpha_s \ln(\lambda^2/\mu^2 e^C)}\right] - \theta(-\lambda_L^2 - \lambda^2). \qquad (7)$$

Here $\lambda_L^2 = -\mu^2 \exp[1/(\beta_0\alpha_s) - C]$ is the position of the Landau pole in the strong coupling and C is a constant characterizing the renormalization-scheme, $C = -5/3$ for the $\overline{\text{MS}}$-scheme and $C = 0$ for the V-scheme. In this talk I cannot give a detailed discussion of the asumptions underlying Eq. (6), but refer the reader to the corresponding sections in Ref. [9]. Still, two short comments are appropriate.

First, note that the product $\alpha_s(\mu)M_\infty[a, -\beta_0\alpha_s(\mu)]$ is explicitly scale invariant, provided the coupling runs with leading-order accuracy. The result is also scheme-invariant, provided the couplings are consistently related in the same BLM approximation, that is by keeping only the terms with highest power in N_f. Secondly, notice that the second term in (6) involves the radiative correction analytically continued to a negative squared gluon mass, namely the position of the Landau pole, $\lambda_L^2 < 0$. The renormalon divergence of perturbation theory is reflected by non-analytic terms in the expansion of $g_0(a, \lambda^2)$ at small λ^2 and leads to an imaginary part in this continuation. The size of the imaginary part (divided by π), $\delta M_\infty \equiv 1/(\pi|\beta_0|\alpha_s) \, \text{Im} \, g_0(a, \lambda_L^2)$, yields an estimate of the ultimate accuracy of perturbation theory, beyond which it has to be complemented by non-perturbative corrections. The real part of (6) coincides with the sum of the perturbative series defined by the principal value of the Borel integral [9], and the imaginary part of $g_0(a, \lambda_L^2)$ coincides with the imaginary part of the Borel integral. Numerically, we find

$$\Gamma(b \to ue\nu) = \Gamma_0 \left\{ 1 - 2.41 \frac{\alpha_s(m_b)}{\pi} \left[1 + 0.75 + 0.67 + 0.70 + 0.87 + 1.27 + \ldots \right] \right\}$$

$$= \Gamma_0 \left\{ 1 - 2.41 \frac{\alpha_s(m_b)}{\pi} \left[2.31 \pm 0.62 \right] \right\} \qquad (8)$$

using a b quark pole mass and

$$\Gamma(b \to ue\nu) = \overline{\Gamma_0} \left\{ 1 + 4.25 \frac{\alpha_s(\overline{m}_b)}{\pi} \left[1 + 0.604 + 0.159 + 0.073 + 0.032 + \ldots \right] \right\}$$

$$= \overline{\Gamma_0} \left\{ 1 + 4.25 \frac{\alpha_s(\overline{m}_b)}{\pi} \left[1.92 \pm 0.01 \right] \right\} \qquad (9)$$
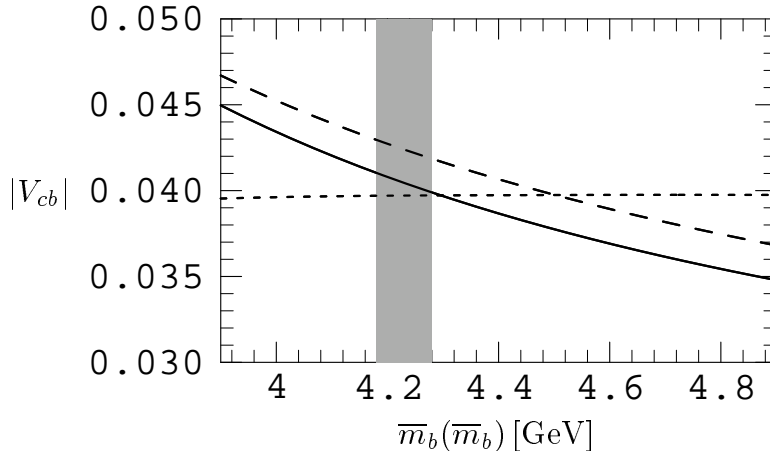
Figure 1: The value of $|V_{cb}|$ extracted from the inclusive B meson semileptonic decay rate after resumming $\beta_0^n \alpha_s^{n+1}$ radiative corrections, shown as a function of the $\overline{\text{MS}}$ b quark mass for fixed $\lambda_1 = -0.5 \, \text{GeV}^2$. The solid and long-dashed curves show the predictions obtained by using the $\overline{\text{MS}}$ and OS scheme, respectively. The central value coming from exclusive decays is shown by short dashes and the shaded area gives the interval of b quark mass values suggested by QCD sum rules. Experimental input: $\tau_B = 1.5 \, \text{ps}$, $B_{SL} = 10.9\%$, $\alpha_s(m_Z) = 0.117$.

using the $\overline{\text{MS}}$ mass $\overline{m}_b$. Obviously the introduction of the short-distance mass reduces the size of perturbative corrections considerably, whereas the gross divergence of the series in (8) is caused by the nearby $u = 1/2$ renormalon that is bound to cancel the corresponding one in the short-distance expansion of the pole mass.

Let us now turn to the determination of $|V_{cb}|$. We still need to fix m_b, m_c and the non-perturbative correction δ^{NP} that enters (1). We use the following value for the $\overline{\text{MS}}$ b quark mass suggested by QCD sum rules:[1] $\overline{m}_b(\overline{m}_b) = (4.23 \pm 0.05) \, \text{GeV}$. In order to fix the c quark mass, we make use of the fact that the *difference* between the pole masses of two heavy quarks is free from many ambiguities intrinsic to the mass parameters themselves and can be determined to a good accuracy from the expansion

$$m_b - m_c = m_B - m_D + \frac{1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)\left[\lambda_1 + 3\lambda_2\right] + O(\alpha_s/m, 1/m^2), \qquad (10)$$

where m_B and m_D are the B and D meson masses, respectively; λ_2 is given by $\lambda_2 \simeq 1/4\,(m_{B^*}^2 - m_B^2) \simeq 0.12 \, \text{GeV}^2$, and $-\lambda_1/(2m_b)$ is the kinetic energy of a heavy quark inside a B meson. For λ_1 an estimate is available from QCD sum rules [13], $\lambda_1 = -(0.6 \pm 0.1) \, \text{GeV}^2$. As for the non-perturbative corrections, we just mention that they partly depend on measurable quantities and partly on λ_1; in our analysis we use the value $\delta^{NP} = -(1.05 \pm 0.10) \, \text{GeV}^2$.

In Fig. 1 we show $|V_{cb}|$ as function of the b quark mass. The solid line shows the result

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[1]For references and a critical discussion see [11].

obtained using $\overline{\mathrm{MS}}$ masses, the long-dashed line is the result obtained for pole masses. The short dashes give $|V_{cb}|$ from exclusive decays. As compared to a corresponding analysis including only $O(\alpha_s)$ terms, we find the dependence on the definition of the quark masses to be considerably reduced. The final value we thus extract is

$$(\tau_B/1.5\,\mathrm{ps})^{1/2}\,|V_{cb}|_{incl} = 0.041 \pm 0.002 \pm 0.002, \tag{11}$$

where the first errors gives the theoretical uncertainty induced by the errors in the values of $\overline{m}_b(\overline{m}_b)$ and λ_1 and the second one comes from the uncertainty in the experimental branching ratio [14]. The full theoretical uncertainty inherent in our approach is larger and in particular constituted by the uncalculated part of the α_s^2 corrections. It remains to be hoped that semileptonic heavy quark decays do not behave different in that respect from other perturbative expansions, where *a posteriori* the $\alpha_s^2\beta_0$ term indeed turned out to be the dominant one, cf. the examples mentioned in Ref. [9].

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