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SPECIAL LATTICE COMPUTATION FOR THE COMPACT LINEAR COLLIDER (CLIC)

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Abstract

In the study of a high-energy linear collider like CLIC, there are domains of activities where analytical developments are useful guides before doing fully numerical calculations or using standard accelerator design programs. The purpose of this report is to illustrate the interest of such an approach in designing rarely used magnetic insertions, such as isochronous modules or tunable achromats, which are required in several locations of the CLIC complex. Algorithmic codes have been written in this context. They give global information or guidance among a variety of approximated solutions and they help the user in selecting the most promising one before fully numerical calculations and eventually optimisations are carried out to obtain the solution consistent with the physical model.

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8 November 2000

1 Introduction

The main injector of CLIC has to be connected to the linear collider through transfer lines which have to be designed carefully to retain the quality of the beam. In particular, the bunch length growth due to the bending loops must be kept to a very low value which means that these lines should be as isochronous as possible, while not causing too large an emission of synchrotron radiation. Moreover, the driver complex which generates the RF power contains a few combiner rings and several transfer lines where the isochronicity or its tuning are of paramount importance. The standard lattice and insertion design programs, such as MAD [1], have difficulty to find stable solutions and need to be started from good approximations. These are usually obtained analytically by considering some or all the magnetic elements as thin magnetic lenses. However the analytical development generates a large variety of solutions, the choice between which is subject to many parameters. Thus a program based on this development should be able to help the designer in making the best choice by providing him with the needed clues, and by narrowing interactively the selection in this multi-parameter space.
The codes which had to be developed are written mainly in FORTRAN 77 for historical reasons but they are built in a modular fashion and run in a UNIX environment. Because they call upon large general purpose packages such as MAD, they also involve parts written at the script level.

2 Design of special lattices

The CLIC study [2] and the new CLIC test facility [3] involve magnetic lattices which are able to modify the length of the beam bunch (isochronicity tuning) or not (isochronicity) and to match them to each other or to a standard FODO line. The complexity of such lattices (rings or transfer lines) makes it very difficult to design them globally. Thus it has been proposed to build them from modules, each matched to the next one. This strategy (analogous to the standard breaking up of a complex computer program into many much simpler routines), proved to be very efficient and fast in treating special lattices. Three of these modules are described hereafter.

2.1 Isochronous achromat

It has been shown that the minimum number of deflecting magnets in an isochronous insertion is three. For reasons of simplicity the chosen insertion shown in Fig. 1 consists of three bending magnets geometrically and magnetically symmetric around the middle plane of the second magnet [4]. To

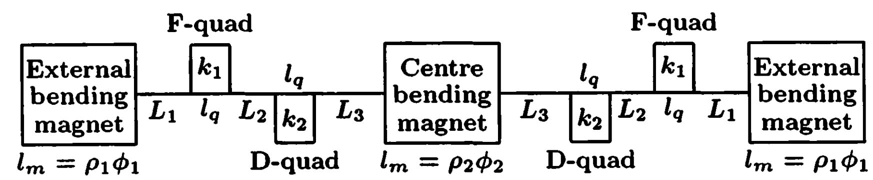

Figure 1: An isochronous insertion consisting of three bending magnets

simplify the algebra, these magnets are treated as sector magnets of equal length l_m but of different deflection angles ϕ_1 and ϕ_2 for the first and second dipole respectively. The space between the first two magnets is filled by a drift space of length L_1, by a focusing quadrupole of length l_q and normalised gradient k_1, by a second drift space of length L_2, by a defocusing quadrupole of length l_q and normalised gradient k_2 and finally by a third drift space of length L_3. On the assumption that the quadrupoles are perfectly centred, the parameter R_{56} is given by :

$$R_{56} = \int_{s_1}^{s_2} \frac{D_x}{\rho(s)} ds \tag{1}$$

where D_x is the horizontal dispersion, $\rho(s)$ the radius of curvature, and s_1,s_2 are the longitudinal coordinates of the beginning and end of the beamline. The R_{56} parameter is positive if high-momentum particles of the bunch travel longer paths. Assuming also that the dispersion and its derivative are zero at the entrance of the first dipole, the contributions of the first dipole and of half the second dipole to this integral are, respectively [4] :

$$\rho_1 (\phi_1 - \sin\phi_1)$$
$$D_j \sin(\phi_2/2) - \rho_2 D'_j [\cos(\phi_2/2) - 1] + \rho_2 [\phi_2/2 - \sin(\phi_2/2)]$$

where ρ_1 and ρ_2 are the trajectory radii in the first and of the second dipole respectively and D_j and D'_j are the dispersion and its derivative at the entrance of the second dipole. Thus the following equation is obtained :

$$\begin{aligned}\frac{R_{56}}{2} =& \rho_1 (\phi_1 - \sin\phi_1) + D_j \sin(\phi_2/2) - \\ & \rho_2 D'_j [\cos(\phi_2/2) - 1] + \rho_2 [\phi_2/2 - \sin(\phi_2/2)]\end{aligned}$$

In order to obtain a non dispersive module, the derivative of the dispersion at the point of symmetry must be zero, providing a second equation :

$$-\frac{\sin(\phi_2/2)}{\rho_2} D_j + D'_j \cos(\phi_2/2) + \sin(\phi_2/2) = 0 \tag{2}$$

From these two equations it is easy to obtain :

$$\begin{aligned} D'_j &= \frac{1}{\rho_2}\left[\frac{R_{56}}{2} - l_m\left(\frac{3}{2} - \frac{\sin\phi_1}{\phi_1}\right)\right] \\ D_j &= \rho_2 \left[1 + D'_j \cot(\phi_2/2)\right] \end{aligned} \tag{3}$$

It is possible to obtain the expressions of the lengths of the first two drift spaces as functions of k_1,k_2 and of L_3 :

$$\begin{aligned} L_1 &= a\frac{C_2 q_1}{C_1 q_2}\left(L_3 - \frac{D_j}{D'_j} + q_2\right) - l + q_1 \\ L_2 &= q_1 - q_2 + \frac{b}{L_3 - \frac{D_j}{D'_j} + q_2} \end{aligned} \tag{4}$$

where

$$\begin{aligned} l &= \rho_1 \tan(\phi_1/2) & a &= -\frac{D'_j}{\sin\phi_1} = -\frac{x}{\rho_2 \sin\phi_1} \\ b &= \frac{q_2}{C_2}\left(\frac{q_2}{C_2} + \frac{q_1}{aC_1}\right) & q_i &= \frac{C_i}{S_i\sqrt{k_i}} \\ C_1 &= \cos\left(l_q\sqrt{k_1}\right) & S_1 &= \sin\left(l_q\sqrt{k_1}\right) \\ C_2 &= \cosh\left(l_q\sqrt{k_2}\right) & S_2 &= \sinh\left(l_q\sqrt{k_2}\right) \end{aligned}$$

The lengths L_1 and L_2 depend on the parameter R_{56} through the quantities D_j and D'_j. The insertion is isochronous if $R_{56} = 0$. The expressions (4) generate a family of insertions according to the actual values of k_1,k_2 and L_3. Of course L_1 and L_2 must be positive and larger than a specified physical length δ. The algebra to find the valid ranges of k_1,k_2 and L_3 is tedious and can be found in [4]. The results are given in Table 1.

Table 1.

k_1	k_2	L_3
$k_1 \leq k_1^{(1)}$	$q_2 < q_1 - \delta$	$L_3 > L_3^{(0)}$
	$q_2 > q_1 - \delta$	$L_3^{(0)} < L_3 < L_3^{(1)}$
$k_1^{(3)} \leq k_1 < k_1^{(2)}$	$k_2 > k_2^{(1)}$	$L_3^{(2)} < L_3 < L_3^{(1)}$
$k_1^{(1)} < k_1 < Min\left\{k_1^{(2)}, k_1^{(3)}\right\}$ $l_q\sqrt{k^*} \leq 2$ or $l_q\sqrt{k^*} > 2$ and $u(x_m) \geq 0$	$k_2 > 0$	$L_3^{(2)} < L_3 < L_3^{(1)}$
$k_1^{(1)} < k_1 < Min\left\{k_1^{(2)}, k_1^{(3)}\right\}$ $l_q\sqrt{k^*} > 2$ and $u(x_m) < 0$	$0 < k_2 < k_2^{(2)}$ or $k_2 > k_2^{(2)}$	$L_3^{(2)} < L_3 < L_3^{(1)}$
$k_1^{(2)} < k_1 < k_1^{(3)}$	$k_2 < k_2^{(4)}$	$L_3^{(2)} < L_3 < L_3^{(1)}$
$k_1^{(1)} < k_1 < k_{max}$	$q_2 < q_1 - \delta$	$L_3 > L_3^{(2)}$

with the following definitions :

$$C^* = \frac{aq_1}{C_1\,(l - q_1 + \delta)} \quad , \quad k^* = \frac{1}{\left[\delta - q_1 - \dfrac{q_1^2}{C_1^2\,(l - q_1 + \delta)}\right]^2}$$

$$\frac{\tanh x_m}{x_m} = \frac{1}{l_q\sqrt{k^*} - 1}$$

$$u\,(x_m) = x_m \sinh\,(x_m) - l_q\sqrt{k^*}\,[\cosh\,(x_m) - C^*]$$

$$(l + \delta)\sqrt{k_1^{(1)}}\tan\left(l_q\sqrt{k_1^{(1)}}\right) - 1 = 0$$

$$\left[k_1^{(2)}\delta\,(l + \delta) - 1\right]\sin\left(l_q\sqrt{k_1^{(2)}}\right) - \sqrt{k_1^{(2)}}\,(l + 2\delta)\cos\left(l_q\sqrt{k_1^{(2)}}\right) = 0$$

$$(l + \delta)\sqrt{k_1^{(3)}}\sin\left(l_q\sqrt{k_1^{(3)}}\right) - \cos\left(l_q\sqrt{k_1^{(3)}}\right) - a = 0$$

$$\sqrt{k_2^{(1)}}\sinh\left(l_q\sqrt{k_2^{(1)}}\right) - \sqrt{k^*}\left[\cosh\left(l_q\sqrt{k_2^{(1)}}\right) - C^*\right] = 0$$

$\sqrt{k_2^{(2)}}$, $\sqrt{k_2^{(3)}}$ being the roots of the equation :

$$\sqrt{k_2}\sinh\left(l_q\sqrt{k_2}\right) - \sqrt{k^*}\left[\cosh\left(l_q\sqrt{k_2}\right) - C^*\right] = 0 \quad \text{with} \quad \sqrt{k^*} > 2/l_q \quad \text{and} \quad C^* > 1$$

$$\sqrt{k_2^{(4)}}\sinh\left(l_q\sqrt{k_2^{(4)}}\right) + \sqrt{k^*}\left[\cosh\left(l_q\sqrt{k_2^{(4)}}\right) - C^*\right] = 0$$

$$L_3^{(0)} = \frac{D_j}{D_j'} - q_2$$

$$L_3^{(1)} = L_3^{(0)} + \frac{b}{\delta + q_2 - q_1}$$

$$L_3^{(2)} = L_3^{(0)} + \frac{q_2 C_1}{aq_1 C_2}(l - q_1 + \delta)$$

An example of an isochronous arc is shown in the Fig.2 which displays the optics functions of the quarter part of a combiner ring in the preliminary design of the CLIC RF power source.

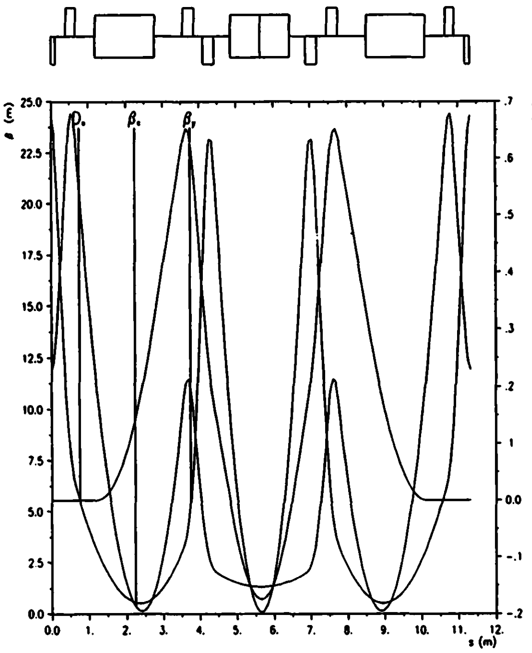

Figure 2: 90° isochronous arc for the CLIC RF power source.

2.2 Isochronicity-tunable achromat

It has been shown in [5] that it is possible to tune R_{56} between a minimum value (negative) $R_{56,min}$ and a maximum value (positive) $R_{56,max}$ without displacing the quadrupoles of this insertion. Thus the lengths of the first two drift spaces L_1 and L_2 are fixed and the normalized strengths k_1 and k_2 can be expressed as functions of R_{56}, which implies inverting the two equations (4). Unfortunately these are transcendental equations and no close-form can be obtained for k_1 and k_2. However it can be shown that it is possible in the thin lens approximation, that is for l_q sufficiently small to fulfill :

$$C_1 = C_2 = 1 \qquad S_1 = l_q\sqrt{k_1} \qquad S_2 = l_q\sqrt{k_2}$$

Then, the absolute values of the focal lengths $f_1 = l_q k_1$ and $f_2 = l_q k_2$ replace q_1 and q_2 respectively. In order to design an R_{56} tunable module it is necessary to find the ranges of L_1, L_2, L_3 such that f_1 and f_2 remain positive when R_{56} varies from $R_{56,min} < 0$ to $R_{56,max} > 0$.
Let x_{min} and x_{max} be defined by :

$$\begin{aligned} x_{min} &= \frac{R_{56,min}}{2} - l_m\left(\frac{3}{2} - \frac{\sin\phi_1}{\phi_1}\right) < 0 \\ x_{max} &= \frac{R_{56,max}}{2} - l_m\left(\frac{3}{2} - \frac{\sin\phi_1}{\phi_1}\right) \end{aligned} \tag{5}$$

The valid ranges of the lengths L_1, L_2 and L_3 can be determined by eight sets of conditions. The algebra to obtain them is tedious and can be found in [5]. Hereafter the results are summarised :

First set

$$\frac{\sin\phi_1}{\phi_1} + \frac{\tan(\phi_2/2)}{\phi_2} < \frac{3}{2}$$

$$x_{max} < -\rho_2\tan(\phi_2/2)$$

$$L_3 < \frac{\rho_2^2}{x_{max}} + \rho_2\cot(\phi_2/2)$$

$$-\mathcal{L}_3 - \frac{d\rho_2\sin\phi_1}{x_{min}} < L_2 < -\mathcal{L}_3$$

$$L_1 < -d - \frac{x_{min}(L_2 + \mathcal{L}_3)}{\rho_2\sin\phi_1}$$

Second set

$$\frac{\sin\phi_1}{\phi_1} + \frac{\tan(\phi_2/2)}{\phi_2} < \frac{3}{2}$$
$$x_{max} < -\rho_2 \tan(\phi_2/2)$$
$$L_3 < \frac{\rho_2^2}{x_{max}} + \rho_2 \cot(\phi_2/2)$$
$$L_2 > -\mathcal{L}_3$$
$$L_1 < -d - \frac{x_{max}(L_2 + \mathcal{L}_3)}{\rho_2 \sin\phi_1}$$

Third set

$$x_{max} < \rho_2 \sin\phi_1$$
$$\frac{\rho_2^2}{x_{min}} + \rho_2 \cot(\phi_2/2) < L_3 < \rho_2 \cot(\phi_2/2)$$
$$L_2 < -\mathcal{L}_3$$
$$L_1 > -d - \frac{x_{max}(L_2 + \mathcal{L}_3)}{\rho_2 \sin\phi_1}$$

Fourth set

$$x_{max} > \rho_2 \sin\phi_1$$
$$\frac{\rho_2^2}{x_{min}} + \rho_2 \cot(\phi_2/2) < L_3 < \rho_2 \cot(\phi_2/2)$$
$$L_2 < -\mathcal{L}_3 \frac{x_{max} - x_{min}}{\rho_2 \sin\phi_1 - x_{min}}$$
$$L_1 > -d - L_2 - \mathcal{L}_3 \frac{x_{max}}{\rho_2 \sin\phi_1}$$

Fifth set

$$x_{max} < \rho_2 \sin\phi_1$$
$$\frac{\rho_2^2}{x_{min}} + \rho_2 \cot(\phi_2/2) < L_3 < \rho_2 \cot(\phi_2/2)$$
$$L_2 > -\mathcal{L}_3$$
$$L_1 > -d - (L_2 + \mathcal{L}_3) \frac{x_{min}}{\rho_2 \sin\phi_1}$$

Sixth set

$$x_{max} > \rho_2 \sin\phi_1$$
$$\frac{\rho_2^2}{x_{min}} + \rho_2 \cot(\phi_2/2) < L_3 < \rho_2 \cot(\phi_2/2)$$
$$L_2 > -\mathcal{L}_3 \frac{x_{max} - x_{min}}{\rho_2 \sin\phi_1 - x_{min}}$$
$$L_1 > -d - (L_2 + \mathcal{L}_3) \frac{x_{min}}{\rho_2 \sin\phi_1}$$

Seventh set

$$x_{max} < 0$$
$$L_3 > \rho_2 \cot(\phi_2/2)$$
$$L_1 > -d - \frac{(L_2 + \mathcal{L}_3)\, x_{min}}{\rho_2 \sin\phi_1}$$

Eigth set

$$x_{max} > 0$$

$$\rho_2 \cot(\phi_2/2) < L_3 < \frac{\rho_2^2}{x_{max}} + \rho_2 \cot(\phi_2/2)$$

$$L_1 > -d - \frac{(L_2 + \mathcal{L}_3)\, x_{min}}{\rho_2 \sin \phi_1}$$

with d being defined by :

$$d = l - \frac{\rho_2}{\sin \phi_1}$$

This approach has been used to obtain a preliminary design of the transfer line between the Delay Loop and the Combiner Ring of the test facility to be built at CERN to validate the concept retained for the production of the 30 GHz RF power needed by the CLIC two beam scheme [3]. The transfer line should be able to increase or decrease the bunch length by 1.6 mm. Given the $\Delta p/p$ of the order of 1 %, the range of R_{56} is then between -0.16 m and 0.16 m. To accomodate this transfer line in a 'S' shape inside the available space, it is made of two insertions, one bending the beam by 75° and the other bending it back by $-75°$. A range of possible solutions has been identified without using numerical searches which are very unstable in this specific problem. Thus the insertion could be optimised to find a compromise between the overall length imposed by the building dimensions, and the optics (Twiss parameters). The most useful set of conditions in the design of this transfer line has been the third. The thick lens results have been easily obtained through the standard program MAD [1] by setting the quadrupole length at the nominal value of 0.2 m and by using the thin lens data as initial conditions. The three dipoles of the selected insertion have the same length (0.4 m) and generate the same beam deflection (25°). The drift lengths are $L_1 = 1.2$ m, $L_2 = 0.6$ m and $L_3 = 1.55$ m.

The Figures 3, 4 and 5 show the optical functions of the full insertion when the R_{56} parameter of half one single insertion is -0.04 m, 0 m, 0.04 m respectively.

For a beam energy of 400 MeV, the gradients of the first and second quadrupoles vary between 12.04 T/m and 7.81 T/m, and between 12.13 T/m and 1.29 T/m respectively. They are shown in Figure 6.

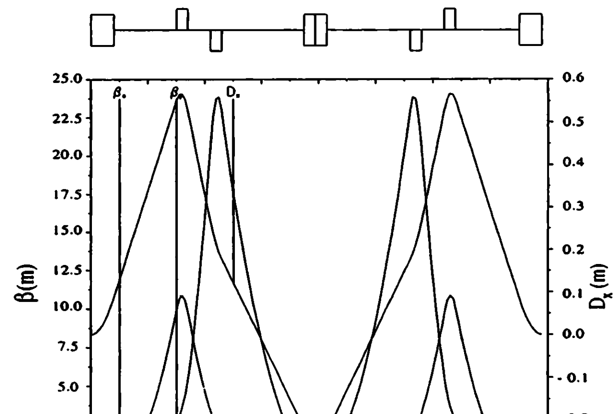

Figure 3: Optical functions for $R_{56} = -0.04$ m.

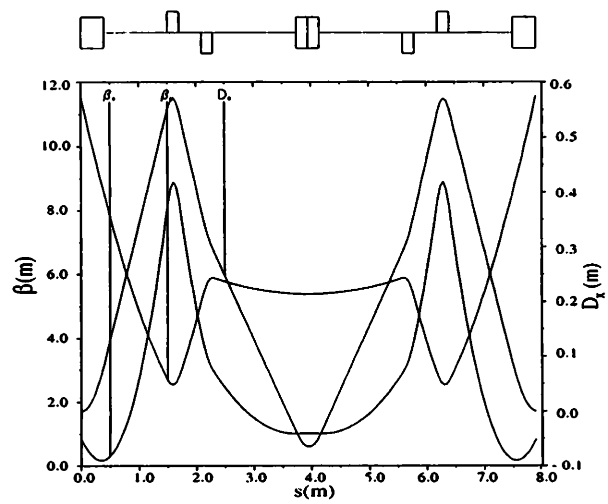

Figure 4: Optical functions for R_{56} = 0 m.

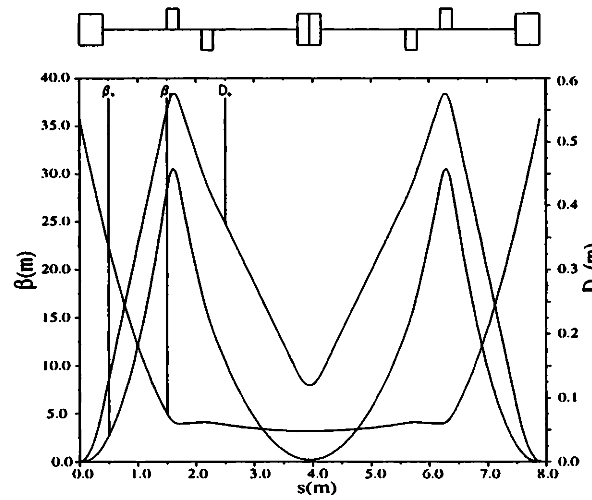

Figure 5: Optical functions for R_{56} = 0.04 m.

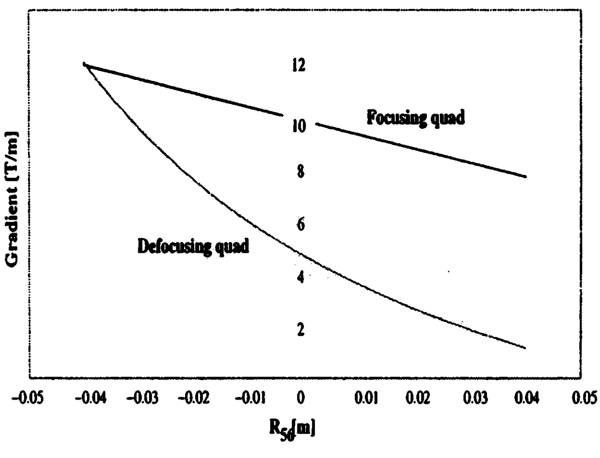

Figure 6: Quadrupole gradients at 400 MeV

2.3 Symmetric matching triplet

The special insertions presented above are usually part of transfer lines or rings. The connection between them is done by matching sections. To avoid large excursions of the betatron functions, the easiest way is to take advantage of the insertion symmetry and to ensure that the values of the Twiss parameters are the same at both ends. In order to reduce the contribution of magnetic errors and sextupole effects to a minimum , the phase advance over a small number of insertions should be as close as possible to an integer multiple of π in both planes. The symmetric triplet shown in Fig. 7 is the insertion with the smallest number of components which provides the required degrees of freedom.

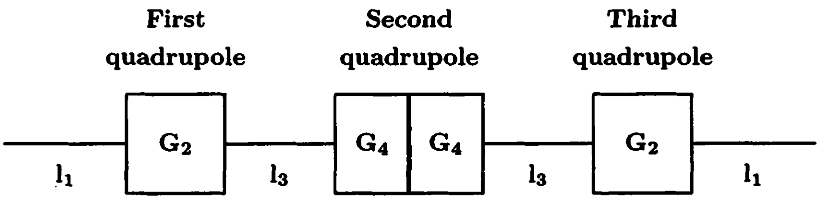

Figure 7: Schematic of a symmetric triplet

It has four free variables (the two drift lengths l_1, l_3 and the magnetic gradients G_2, G_4 respectively of the first, and half of the second quadrupole). It can easily be proved [6] that the diagonal elements of the transfer matrix for each plane are all the same. Thus the number of independent values which can be fixed by selecting the Twiss parameters at the entrance and at the exit of the triplet is also four. The number of solutions is finite and does not depend upon additional assumptions. Moreover, closed form solutions can be obtained in the thin lens approximation. The equality of the diagonal elements of the transfer matrix for each plane forces an important constraint on the betatron function in that plane at the entrance and at the exit of a symmetric triplet. Neglecting the special case where the phase advance is an odd multiple of π, we may distinguish two situations. The first one, which can be called a mirror-symmetry triplet, occurs when the betatron functions at both ends have equal values and their derivatives have opposite values, the phase advance being a free parameter. This is the case of interest for building up a ring with a chain of isochronous modules separated by such matching insertions. The second configuration occurs when the betatron function has different values at each end of the symmetric triplet with its derivatives being free parameters. But in this case the phase advance can no longer be freely chosen and is given by :

$$\cot\mu = \frac{\beta_2\alpha_1 + \beta_1\alpha_2}{\beta_1 - \beta_2}$$

where α_1, β_1 and α_2, β_2 are the Twiss functions at the entrance and exit of the triplet (see Fig. 7) respectively. Thus we have the choice only between two phase advances which differ by π. The more general case, which will be called a matching triplet, can be used for example in matching a FODO line to a special transfer line containing one of the achromats presented above.
In the thin lens approximation, the transfer matrices for the horizontal and vertical planes can be expressed as functions of the two drift lengths l_1, l_3 and the quadrupole strengths $g_2 = k_2 l_q, g_4 = k_4 l_q$ where l_q is the quadrupole length and k_2, k_4 are the normalized gradients. In this case the general solution is characterized by [6] :

$$\begin{aligned}
\ell_1 &= \frac{1}{c^2 - d^2}[(bc - ad)z - (bd - ac)] \\
g_2 &= \frac{c^2 - d^2}{(bc - ad)(1 - z^2)} \\
\ell_3 &= \frac{(bc - ad)(1 - z^2)}{z(c^2 - d^2)} \\
g_4 &= \frac{z^2(d - cz)}{1 - z^2}
\end{aligned}$$

where z is a solution of the cubic equation

$$z^3 - \frac{2d}{c} z^2 + z + \frac{d^2 - c^2}{c(bc - ad)} = 0 ,$$

and a, b, c, d are defined by:

$$a = (t_{h,11} + t_{v,11} - 2)/4 , \quad b = (t_{h,11} - t_{v,11})/4,$$

$$c = (t_{h,21} + t_{v,21})/4 , \quad d = (t_{h,21} - t_{v,21})/4 ,$$

$t_{h,nm}$ and $t_{v,nm}$ being the elements of the horizontal and vertical transfer matrices, respectively. It is assumed $bc - ad \neq 0$, and the trivial cases of $c = 0$ or $c = \pm d$ are neglected. Of course only real values of z leading to positive lengths of drift spaces are retained. The extension to thick lenses is easily obtained by solving for the normalized gradients k_2, k_4 from the two nonlinear equations [6] or by a standard numerical matching program such as MAD.

3 Concluding remarks

The aim of this paper is to show how important it is to develop analytical treatments not to only rely on numerically driven programs for insertion designs. Analytical approximations provide sure guidelines and a very useful insight of the main features of the problem. Moreover they give good starting points which not only speed up the numerical search but are often crucial for convergence, especially when the parameter hyper-space is not smooth in the neighbourhood of the required solution. The latter is frequently not unique, and the choice is based on additional constraints such as the maximum excursion of the Twiss parameters or geometrical conditions which are often mutually conflicting. Thus a compromise has to be taken which cannot be automated inside a program but necessitates the active intervention of the user, which requires interactive capabilities and an extensive array of tools to aid the designer, including graphics. Up to now, the programs available are implemented for historical reasons in FORTRAN 77 running in a UNIX environment. A consolidation is needed to obtain a fully portable product. At the same time the routines should be streamlined and standardized by moving to an object-oriented language.

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