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ESTIMATES OF ISR LUMINOSITIES WITH COOLED BEAMS

K. Hübner, D. Möhl, L. Thorndahl (CERN, Geneva) and P. Strolin (E.T.H., Zürich)

SUMMARY

A cooling ring of ~ 25 m radius using stochastic cooling at 4 GeV/c could serve the ISR and later on the SPS with dense beams of antiprotons and/or ions. In the ISR \bar{p} -p luminosities in the range of 10^{29} could be reached in the low β insertion using fast cooling in the ring and postcooling in the ISR. Only "spare" proton pulses from the PS (not used for injection to the SPS) are taken to produce antiprotons.

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1. INTRODUCTION

This note is to update antiproton cooling and accumulation schemes for the ISR ^{1,2} which were proposed in collaboration with the Novosibirsk Institute. We reconsider this work in view of the growing interest in antiproton physics at CERN ^{3,4} and in view of the recent progress in stochastic ⁵ and electron cooling ¹⁷.

Our present luminosity estimates are based on a scheme which is characterized by :

- Fast stochastic cooling of each \overline{p} burst in a 4 GeV/c, R = 25 m cooling ring (to profit from the fact that stochastic cooling can be fast at low intensity);
- Accumulation and postcooling in the ISR;
- Nonlinear momentum cooling (which uses the fact that damping rates can be faster as $\Delta p/p$ decreases).

The corresponding ISR luminosity with 24 h filling time is estimated at $L_{pp} \sim 10^{29}$ in a low β -insertion ⁹ compared to 10^{28} in previous schemes which were based on a more modest (R = 10-15 m) ring and less powerful cooling. As additional improvements we consider RF gymnastics to reduce the \overline{p} momentum spread prior to cooling and/or storage of protons for \overline{p} production in the free ISR ring ^{3,7}. These gymnastics could push the luminosity to $\sim 10^{30}$ cm⁻²sec⁻¹ which is the maximum obtainable in 24 h. using all antiprotons which the PS can produce within the acceptance of the cooler.

Apart from \overline{p} -p collisions in the ISR, antiprotons from the cooling ring could in principle be accelerated in the SPS for fixed target experiments yielding $10^8 \ \overline{p}$ /pulse at 400 GeV/c once every 45 s, or accumulated in a second ring for the \overline{p} -p collisions in the SPS proposed by Rubbia ³. Another interesting application of the same ring could be cooling of light ions to increase luminosity for ion physics with the ISR or the SPS. A location close to the PS-ISR-SPS transfer lines (TT2, TT10) would facilitate such a multiple use and permit to take profit of existing tunnels.

Although our luminosity estimates are based on stochastic cooling the limiting value of $\sim 10^{30}$ cm⁻²sec⁻¹quoted above remains true for electron cooling as well. No consideration will be given to this alternative in the rest of this paper although it may be that schemes based on this or a combination of stochastic and e-cooling may lead closer to the limit.

2. PREPARATION OF ANTIPROTONS BEAMS FOR THE ISR

2.1 Limiting performance

Let us first estimate the ISR luminosity obtainable if every available PS pulse is exploited for \overline{p} production. Assume 26 GeV/c protons are produced at a rate of 0.5 x 10¹³/s. Take a conversion efficiency $\overline{p}/p = 10^{-5}$ (Appendix 2) to obtain 0.5 x 10⁸ \overline{p}/s i.e. $\sim 4 \times 10^{12} \overline{p}/day$, yielding $L_{\overline{p}p} = 1.6 \times 10^{30}$ cm⁻² sec⁻¹ (see Appendix 1) with 24 h accumulation. This luminosity can only be reached if the cooling cycle (τ_c) can be made shorter than the PS cycle ($\tau_{PS} \sim 3$ s in a supercycle with SPS filling) or if a number $-c/\tau_{PS}$ of PS proton pulses is stored in the ISR (or elsewhere) for producing a new \overline{p} burst.

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2.2 The scheme considered

A special mode of running the PS is used to squeeze as many protons as possible into 5 bunches (Appendix 4). A 25 m radius "cooler" operating at 4 GeV/c accepts the corresponding antiprotons and damps them to a size suitable for injection in the ISR. Accumulation and postcooling take place at 4 GeV/c in one ISR ring.

Considerations that have led to the parameters (Table 1) of the cooler are explained in Appendix 2. The conversion efficiency is $\overline{p}/p \approx 1.3 \times 10^{-5}$ i.e. $1.3 \times 10^8 \ \overline{p}$ are accepted per PS pulse (25 Gev/c, 10^{13} protons). The damping system (Table 2) reduces $\Delta p/p$ by ~ 10 and the emittances by ~ 30 in 45 s. The number of pick-up electrodes, amplifiers and RF gaps for Δp cooling has been restricted to 16 each, to limit the complexity. In fact, if it were acceptable to increase e.g. the number of amplifiers and gaps by 10, then cooling times comparable to the PS cycle might be achieved even with $\Delta p/p = 2\%$.

The cooled beam is adiabatically bunched by a 9.5 MHz RF system in the cooler (\sim 10 kV RF voltage, \sim 10 ms bunching time) or by the ISR RF system and stacked in the usual ISR fashion (with 25 of the 30 ISR buckets suppressed). A stochastic damping system similar to the momentum precooling device decreases the size of the stack so that aperture is made continuously available for the next pulse. Accumulation of \sim 2000 pulses (24 h.) yields $N_{\overline{D}} \sim 2.5 \times 10^{11}$; $L_{\overline{DD}} \sim 10^{29}$ cm⁻² sec⁻¹.

After stacking, the beam is accelerated to the desired energy as in the previous schemes ^{1,2}. Acceleration and crossing of transition energy with a few 10^{11} p as well as stacking of μ Amp. beams need further study (theoretical and/or simulation with protons).

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2.3 Improvement possibilities

a) Debunching in the cooling ring

Suppose one uses an RF system in a "debuncher fashion" to turn \overline{p} -bunches by 90° in phase space. With a bunching factor of 1/6 or smaller we can aim at a reduction of $\Delta p/p$ by about 4. Note that with stochastic cooling the gain is twofold as cooling time can be decreased <u>and</u> the number of cooling times to reach a $\Delta p/p$ digestable in the ISR diminishes. In fact, cooling to $\Delta p/p = \pm 0.2\%$ will now only take ~ 5 s. However, to take full profit of this short cooling time, one has to make sure that :

- the RF structure of the fresh p burst disappears rapidly enough not to interfere with cooling;
- emittance cooling can be speeded up $(\tau \leq 1 s)$;
- postcooling can be made fast enough to digest new beam every, say, 5 s.

The RF voltage needed to rotate the $\Delta p/p = \pm 2\%$ beam is rather high. Taking

$$n = \frac{1}{\gamma_{r}^{2}} - \frac{1}{\gamma^{2}} - \frac{1}{2} - \frac{1}{2}$$

one needs $U_{RF} \approx 1 \text{ MV}^6$. The corresponding time ($\tau > 0.2 \text{ ms}$) and hence the duty cycle required are however small ¹⁸.

b) Proton buffering in the ISR 3,7

Assume storage of the PS proton pulses during each cooling cycle in the free ISR ring. Assume a special RF system to compress the stack into a \sim 145 m long bunch (bunch length = cooler circumference kicker gap) which is fast ejected (vertical ejection) onto the conversion target. Taking a momentum acceptance of \pm 1.5%, 60% stacking efficiency and a PS bunch area of 10 mrad, on : can stack up to 10 PS pulses. The overall gain is somewhat smaller (actor 6 with the cooling system of Table 2) since the $\Delta p/p$ damping tends to slow down with intensity. The drawback of this scheme is the large RF system required to bunch the beam. It must provide 2 MV. Furthermore an ejection system is needed.

3. COOLING OF IONS

Let us assume RF stacking and cooling of fully stripped ions (at 2 GeV/nucleon) in the cooler. For the same $\Delta p/p$, cooling will be longer than for protons because charge and mass are larger. However, it seems that the main bottleneck will be the current limit at 2 GeV/c in the ISR rather than the cooling time. Let us take this limit to be 3 Amp. (N = 6 x 10¹³/Z) which we obtain scaling optimistically I $\propto \gamma$ (in fact, transverse stability limit contains a U $\propto \frac{1}{\gamma^3}$ - term which starts to dominate the V $\propto \frac{1}{\gamma}$ at about $\gamma \sim 2-3$).

Luminosities at 8 GeV/c/nucleon (i.e. just below transition energy) are estimated :

L ion ion = $1.2 \ 10^{29}/Z^2 \ cm^{-2} \ sec^{-1}$ L ion proton = $4 \ 10^{30}/Z \ cm^{-2} \ sec^{-1}$

where the usual beam height (5 mm) was assumed at 2 GeV/c/nucleon and adiabatic shrinkage during acceleration to 8 GeV/c/nucleon. Compared to the source-limited situation without cooling ⁴ the gain is typically \sim 6 in ion-proton and \sim 20 in ion-ion luminosity for ions like Ne or Xe. Further transverse cooling of the current (transverse instabilities) limited beam at 2 GeV/c is not feasible because the beam would become unstable for dimensions \leq 5 mm.

4. ACCELERATION OF p-s FOR FIXED TARGET EXPERIMENTS AT THE SPS

The batch of $10^8 \ \overline{p}$ from the cooler could in principle be injected and accelerated in the SPS for fixed target experiments at, say, 400 GeV. Clearly developments would be needed at the SPS to work with :

- particles of negative charge,
- 4 GeV/c injection,
- intensities of the order of $10^8 \, \overline{p}/p$.

No attempt has been made to examine the feasibility in detail. From discussions ⁸ with experts we conclude that injection, magnetic field and RF could probably be adapted to these special requirements without major development. However, a new (or a modified) septum seems necessary to be able to work with reversed polarity. It is not clear whether a thin septum for slow extraction can be made with the required polarity. For fast extraction one may get away with a thicker magnetic septum. Change from proton to antiproton operation (reversal of polarities) would require a short shut-down.

Finally we want to mention that the cooler together with the ISR or probably in conjunction with a second "accumulation and postcooling" ring seems capable of preparing dense \overline{p} beams for \overline{p} -p collisions in the SPS.

CONCLUSION

A cooling ring of ~ 25 m radius using stochastic cooling at 4 GeV/c could serve the ISR and later on the SPS with dense beams of antiprotons and/or ions. In the ISR \overline{p} -p luminosities in the range of 10^{29} could be reached in the low β insertion using fast cooling in the ring and postcooling in the ISR. Only "spare" proton pulses from the PS (not used for injection to the SPS) are taken to produce antiprotons.

REFERENCES AND NOTES

- G.I. Budker, N. Dikansky, D. Möhl, A.N. Skrinsky, P. Strolin and
 L. Thorndahl Possibilities for Antiproton Beams at CERN using
 Cooling by Electrons, CERN/EP Internal Report 76-03 (1976).
- D. Möhl. L. Thorndahl, P. Strolin Stochastic Cooling of Antiprotons for ISR Physics, CERN EP Internal Report 76-05 (1976).
- 3. C. Rubbia Unconventional Use of the SPS and FNAL Rings...", Seminar given at CERN, Feb. 1976 and talk presented to this Workshop. See also: D. Cline, P. McIntyre and C. Rubbia, "Collecting Antiprotons in the Fermilab. Booster...", NAL preprint 1976.
- L. Bertocchi et al. Physics with Antiprotons, Deuterons and Light Ions, ISR Workshop October 1976, Study Group Report.
- 5. G. Carron, L. Faltin, W. Schnell, L. Thorndahl Stochastic Cooling of Betatron Oscillations and Momentum Spread, presented to All Union Accelerator Conference, Moscow, October 1976.
- 6. The RF voltage for debunching $U \stackrel{>}{\sim} \frac{\pi}{2} h \frac{|n|}{\gamma} \left(\frac{\Delta p}{p}\right)^2$ 938 MV could be reduced if it were possible to tune transition close to the working energy by use of lenses similar to the transition jump system of the PS. Clearly γ_{tr} has to be retuned prior to cooling in order to achieve randomisation $(m = |n| 2\pi R \frac{2\Delta p}{p} / \text{sample length} >> g)$.
- 7. W. Schnell, private communication
- 8. S. van der Meer, private communication.
 C. Zettler, private communication.
 W. Middlekoop, private communication.
- 9. L. Resegotti, communication at the Workshop on Future ISR Physics,Oct. 1976
 B. Austin and A. Verdier, Nouveau Scheme de Section Superconductrice
 à Faible β dans les ISR, ISR Internal Report CERN/ISR-MA/74-39 (1974).
- L. Thorndahl Stochastic Cooling of Momentum Spread and Betätron Oscillations for Low Intensity Stacks, CERN Internal Report ISR-RF 75-55 (1975).

- 11. Study Group for CPS Improvements The Second Stage CPS Improvement Study : 800 MeV Booster Synchrotron, CERN Internal Report MPS/Int. DL/B 67-19. See also C. Bovet - Further Thoughts on the Transfer Booster-PS, CERN Internal Report MPS-SI/Note DL/68-6 (1968).
- B.V. Chirikov, V.A. Tayursky, H.J. Möhring, J. Ranft, V. Schirmeister -Optimization of Antiproton Fluxes from Targets Using Hadron Cascade Calculations, K. Marx University preprint, Leipzig, KMU-HEP-7412 (1975).
- K. Hübner Luminosity with Beams of Different Height, ISR Performance Report ISR-TH/KH/14.11.1972.
- D. Dekkers et al. Experimental Study of Particle Production at Small Angles ..., CERN Internal Report NP/Int. 64-5 (1964).
- P.I. Kalmus et al. Low Momentum Antiproton Production, CERN Report 71-25.
- 16. It might be concluded that weak focusing $(\gamma_H \sim Q <<\gamma)$ should be chosen to ease mixing. However, since the conversion $\overline{p}/p \propto E \frac{\Delta p}{p}$, optimisation of N/ τ for a ring of given apertures favors large Q
- 17. G.I. Budker et al. New Experimental Results on Electron Cooling, Paper presented to the All Union High Energy Accelerator Conference, Moscow, October 1976 (Preprint Novosibirsk Physics Institute 76-32, translated at CERN by O. Barbalat - PS/DL/Note 76-25).
- 18. We are grateful to W. Pirkl for helpful discussions on the feasibility of such a high voltage low duty cycle RF system.

A P P E N D I X 1

Luminosity Estimates

We scale from ISR run 741 this year which gave

$$L/I_1I_2 = \frac{2.9 \times 10^{31}}{31 \times 33.5} \text{ cm}^{-2} \text{ sec}^{-1} \text{ Amp}^{-2}$$

We anticipate some further improvement and take a current of 40 A for the proton beam colliding with the antiprotons. We take cooled antiprotons with a beam size considerably smaller than the p beam. From ref. 13 we conclude that under such circumstances the luminosity gain is $\sim \sqrt{2}$. Hence we have

$$\frac{L}{I_{p}} = \frac{2.9 \times 10^{31}}{31 \times 33.5} 40 \times 1.41 \approx 1.5 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1} \text{ Amp}^{-1}$$

or

$$L/N_{\overline{p}} \approx 8 \times 10^{16} \text{ cm}^{-2} \text{ sec}^{-1}$$
 per antiproton.

Note that in our previous estimates¹,² which were scaled from earlier measurements we took

$$L/N_{\overline{p}} = 3.5 \times 10^{16} \text{ cm}^{-2} \text{ sec}^{-1} \text{ per } \overline{p}.$$

Finally we shall assume in the present work that the superconducting low β insertion⁹ will be installed and improve the luminosity by 5. Hence, we take $L/N_{\overline{p}} = 4 \times 10^{17} \text{ cm}^{-2} \text{ sec}^{-1}$ per \overline{p} in all luminosity estimates.

A P P E N D I X 2

Proton Antiproton Conversion and Parameters of the Cooling Ring

The conversion efficiency is
$$\overline{p}/p = \frac{d^2 N}{d\Omega dp} \Delta \Omega \Delta p \eta$$
 target

The differential cross-section has to be averaged over the angles accepted. With simple quadrupole focusing optimum overall efficiency $n_{target} \sim 1/3$ is obtained if the target lenth (l_t) is about one nuclear interaction length and if the primary proton beam spot (a_v, a_h) and the acceptances of the \overline{p} channel are related by

$$a_{h,v} \stackrel{\sim}{\sim} (0.7 + A_{h,v}/a)^{\frac{1}{2}}$$

It follows that

and

$$\Delta \Omega = (A_h A_v)^{\frac{1}{2}} / (0.7 \ell_t)$$

$$\overline{p}/p = \frac{d^2 N}{d\Omega dp} \frac{(A_v A_h)^{\frac{1}{2}}}{0.7 \ell_t} 2p \frac{\Delta p}{p} \times \frac{1}{3}$$

where $\Delta p/p$ is understood to be the half spread. In Fig. 1, conversion, accepted $\Delta\Omega$ and matched primary spot size $\sqrt{a_h} \times a_v$ for a 6 cm long target are plotted as a function of the acceptance $\sqrt{A_h} A_v$ and for $\Delta p/p = \pm 1\%$. The differential cross-section is extracted from the measurements by Dekkers et al. ¹⁴ and by Kalmus et al. ¹⁵ for 4 GeV/c p-s produced from 24 GeV/c protons. Note that we take $\frac{d^2 N}{d\Omega dp} = 0.015$ sterad⁻¹ (GeV/c)⁻¹ per interactive proton at 0° lab. angle. The results agree reasonably well with the more detailed calculations by Ranft et al.¹².

One notes that the conversion efficiency starts to run into "saturation" for emittances above, say, 200 π mm.mrad. This is due to the reduced yield at large angles. It is for this saturation that we have restricted $\sqrt{A_h A_v} \lesssim 250 \pi$ mm.mrad. The emittance ratio $A_h/A_v = 3/2$ fits the acceptance ratio of the ISR. Keeping in mind that vertical aperture is more expensive than horizontal one, such a choice seems adequately independent of the ISR. Finally the maximum $\Delta p/p$ is limited by the chromaticity of the magnetic lattice, by aperture considerations, by the RF voltage necessary for debunching, etc. On the other hand a large enough $\Delta p/p$ is desirable to have enough rerandomisation for stochastic cooling and high conversion efficiency. Fortunably the choice is not too critical since over a large range of parameters both yield and damping time increase linearly with Δp so that the rate dN/dt of \overline{p} production remains unchanged.

A P P E N D I X 3

Momentum Cooling

From ref. 10 the momentum cooling rate may be written as

$$\frac{1}{\tau} = \frac{2W}{N} \left[g - \frac{g^2}{2} \left(1 + \frac{\beta H^E H/\pi}{\alpha p^2 (\frac{\Delta p}{p})^2} + \frac{v^2}{(\Delta p/p)^2} \right) \right]$$

"heating terms

We propose to keep the first of the heating terms small by having the emittance cooling preceeding momentum cooling ($\Sigma_{\rm H}$ small during Δp cooling). However, even without heating at best g = 1 and hence the rate N/ τ of cooling is limited to N/ $\tau \leq W \sim 5 \times 10^8 \text{ sec}^{-1}$ or 4 x 10¹³ particles/day. In reality one is limited to values g < 1 by the broadband power available and by the need to have sufficient randomisation of the population of each beam sample (length $\ell_{\rm s} = \frac{c}{2W}$). This "mixing" requires

$$m/g = 2\pi R |n| 2 \frac{\Delta p}{p} / (\ell_s g) >> 1$$

and hence small g. Taking m/g = 1, $r_1 = 5 \times 10^{-2}$ and $p/p = \pm 0.27$ as given by the ISR inflector, we need $g < 9 \times 10^{-2}$.

Now from the analysis of ref. 10 it may be concluded that for the power limited amplifier one has 3 characteristic ranges (cf. Fig. 2) :

i) "low intensity" :
$$g \propto N^{1} (\Delta p/p)^{-1}$$

 $\tau \propto N^{0} \Delta p/p$
ii) "medium intensity" : $g \propto N^{\frac{1}{2}} (\Delta p/p)^{-1}$
 $\tau \propto N^{\frac{1}{2}} \Delta p/p$
iii) "high intensity" : $g = 1$
 $\tau \propto N$

One notes (Fig. 2 and Table 2) that the cooling ring works at the upper edge of the "low intensity range" and that at the end of cooling g is close to the "maximum possible" $g \gtrsim 1/m$.

Note that we attempt to increase $1/\tau$ as Δp decreases. To do so, we either have to mechanically move the pick-up electrodes or to provide for a special design which keeps the sensitivity constant as the beam shrinks.

APPENDIX 4

Injection Into the Cooler (an Example)

A "pseudo 5 bunch mode" is used in the PS as follows : 10 bunches are injected into the PS by super-imposing the booster beams 2 by 2 ¹¹. After acceleration to 26 GeV/c, the RF is turned down adiabatically to \approx 40 kV (2 cavities). The remaining cavities are then switched on nonadiabatically at half the usual RF (4.7 MHz) and the 9.5 MHz voltage is switched off. With proper phasing bunches rotate in phase space (Fig. 3) and after \sim 1/4 of a synchrotron oscillation, are superimposed 2 x 2. The 5 double bunches are then fast ejected and the corresponding \overline{p} bunches fill the cooler in two turns. The inflector has to be pulsed during the second turn as indicated in Fig. 4. The momentum spread of the double bunches ($\Delta p/p \leq \pm 4 \times 10^{-3}$) is reasonably close to the limit which can be safely handled by the PS fast ejection system.

TABLE 1

Tentative Cooling Ring Parameters

	and the second	
Momentum	4	GeV/c
Radius	25	m
Injected beam ($\Delta p/p$	<u>+</u> 2.2	%
E _H	300	".mm.mrad
	200	".mm.mrad
Ejected beam / △p/p	<u>+</u> 0.2	7
E _u	10	".mm.mrad
	7	π .mm.mrad
Typical momentum cooling time	15	sec
No. of cooling times (momentum)	3	
Matched primary spot on target (r_v)	2.4	mm
{r _H	2.7	mm
Accepted angle $\Delta\Omega$	18	msterad
Conversion p/p	1.3	x 10 ⁻⁵
No. of p per PS pulse	1.3	× 10 ⁸
No. of p per day	2.5	x 10 ¹¹
No. of pulses per day	1920	
Luminosity/day	1.1	x 10 ² cm ⁻² sec ⁻¹
Lattice parameters assumed :		
^β function	10	
Momentum compaction ^a p	2	m
Beam size at injection (a	<u>+</u> 45	
(a _H	<u>+</u> 75	
		1

TABLE 2

Momentum Spread Cooling

16 pairs of pick-up
16 gaps (50 Ω impedance)
16 amplifiers with W = 500 MHz ; P = 63 Watts
I noise = 200 nA
R = 25 m ; p = 4 GeV/c

N P	10	7	10	8	10	9	
∆p/p	<u>+</u> 2.2	0.22	2.2	0.2	2.2	0.2	~
I p	3.0		30	.6	30	6	µ Amp .
I sig	37		11	6	36	7	nAmp.
⁸ 0	0.03	2	0.2	52	0.7	71	
N	0.019	•	0.1	9	1.	9	10 ⁶
U sig	12	1.2	29.9	2.99	29	2.9	kV
g	0.0043	0.C43	0.0037	0.037	0.021	0.21	
m/g	1900	19	225	2.25	39	0.39	
τ	23	2.3	26.5	2.65	48	4.8	sec.

TABLE 3

Momentum Postcooling in ISR (4 GeV/c)

```
16 pairs of pick-up
16 gaps (50 Ω impedance)
16 amplifiers with W = 500 MHz
P = 63 Watt
Inoise = 200 nA
```

(same set of amplifiers as in the cooling ring).

N P	10	0 ⁸		10 ⁹	10	10	10 ¹	1	
p/p	0.22	0.02	2.2	0.2	2.2	0.2	2.2	0.2	7.
I p	0.00	51	0.05	1	0.5	1	5.	1	mA
I sig	0.04	7	0.15		0.4	7	1.	5	
go	0.05	3	0.36		0.8	5	0.	98	
^N s	0.03	2	0.32		3.2		32		10 ⁶
U sig	1.5	0.15	32	3.2	24	2.4	8.8	0.8	kV
8	0.007	0.07	0.0060	0.0 6	0.03	0.3	0.0 99	0.98	
m/g	95	0.95	1160	11.6	24	0.24	7.6	0.76	
τ	13.8	1.4	170	17	35 0	35	1020	202	sec
$\frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\Delta \mathbf{p}}{\mathbf{p}} \right)$	0.0	1	0.0	0 8	0.0	04	0.0	014	Zsec ⁻¹

Required $\frac{d}{dt} \left(\frac{\Delta p}{p}\right) = (0.27/45 \text{ sec}) 1/6 \frac{1/n}{\text{stack}} \approx 0.0014 \text{ Z sec}^{-1}$

cooler	"suppressed	stacking
performance	buckets"	efficiency



Fig. 1







