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FAST PULSED DEFLECTING MAGNETS

by

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G E N E V E

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SUMMARY: A comparison is made between a single pair of current-sheets, fed by a matched line simulating voltage fed network, and a number of current sheets, each pair of which having its own discharging capacitor. In the latter case the voltage requirements are considerably less.

I. INTRODUCTION.

It may be desirable to dispose of a device which could deflect a stream of particles almost instantaneously over a given angle. In the CERN proton synchrotron the particles are located in 20 bunches which are 100 ns spaced in time and the distance between bunches is about 60 ns in time.

Now there exists a simple relation between the angle of deflection, the electric field strength and the rise time in a number of cases. Considering first the current sheets as a lumped inductance L, the applied voltage is given by

$$V = L \frac{di}{dt} \qquad (1)$$

The inductance of a one turn current sheet arrangement is given by

$$L = \mu_o \frac{d}{h} l \qquad (2)$$

in which d is the distance between the sheets, h the height and l the length. We assume here that $d/h \ll 1$ so that the magnetic flux density is mostly confined in the space between the conductors, or alternatively, that the flux lines are closed by a ferro-magnetic material.

The B 1 value of the deflector is a measure for the angle of deflection according to

$$B\,l = \theta\, B_o R_o \tag{3}$$

in which $B_o R_o$ is the magnetic rigidity of the particle and for a given energy a constant.

The flux density is related to the current according to

$$B = \frac{\mu_o i}{h} \tag{4}$$

Approximating $\frac{di}{dt} \approx \frac{\Delta i}{\Delta t} \approx \frac{i}{T}$ in which the rise time T is the time in which the current reaches from zero the desired value i, we find from the foregoing formulas

$$V = \frac{B\,l\,d}{T} \tag{5}$$

For instance if the required B 1 value is 0,2 Wb/m, corresponding with an angle of deflection for 25 GeV protons of 0,024 rad, and a rise time equal to the time between bunches i.e. 60 ns, the required voltage will be, with d = 0,03 m : $V = 10^5$ Volt.

The voltage is thus virtually independent of the length 1 of the current sheets. This remains so, even if the sheets can no longer be regarded as a lumped inductance.

In order to feed the coil from a line simulating voltage fed network, a useful scheme is to connect the coil as a four terminal network. On one side is connected the charged line, with the switch in series, and on the other side the load. The characteristic impedance of all sections should be equal, so as to avoid reflections due to mismatch.

Let the characteristic impedance be Z and the voltage on the charged network be 2 V, then upon switching the current will rise to a value such that :

$$V = iZ \tag{6}$$

Let the group velocity in the current sheet arrangement be v, the inductance per unit length L and distributed capacitance per unit length C, then the following relations hold :

$$Z = \sqrt{L/C} \quad \text{and} \quad v = 1/\sqrt{LC} \tag{7}$$

and therefore

$$V = i\ v\ L \tag{8}$$

The inductance per unit length is given by $L = \mu_o\ d/h$ so that with (4)

$$V = B\ v\ d \tag{9}$$

The delay time i.e. the time required for the pulse to propagate the length l of the current-sheets, is given by

$$T = l/v \tag{10}$$

and consequently one has obtained the same formula (5)

$$V = \frac{B\ l\ d}{T} \tag{5a}$$

One will appreciate that the delay time has the same effect as the rise-time, provided the group velocity is considerably less than the velocity of the protons, and this is of course borne out by inserting the appropriate values in (9).

Apart from the delay time, the pulse shape will suffer from imperfect switching and imperfect network components. The effect of imperfect switching may be evaluated in tens of ns and of imperfect network components in the time constant of individual sections. But ultimately the delay time is the dominant factor, with respect to the voltage requirements.

II. REDUCTION OF THE VOLTAGE.

In order to reduce the voltage, one may split the pair of current sheets into a number of sections and feed them in parallel. Only one charged pulse forming network is required as proposed by B. Kuiper and G. Plass. In this respect the 4 terminal arrangement of the current sheets is very convenient, as the proper connecting lengths may be easily obtained. Obviously the reduction in voltage is equal to the number of sections.

The rise-time of a pulse produced by a line simulating matched network is in the case of a Rayleigh network of the order of the delay time per section, or in general the pulse duration divided by two times the number of sections. This rise-time adds to the delay time of the current sheets with regards to the time in which the current in the deflector reaches the required level.

It seems plausible to divide the available time equally between rise-time of the pulse and the delay time of the current-sheets, i.e. the LC time of the individual sections is equal to the LC time of the current sheets. For instance in the case cited above the LC time will be 60 ns, 16 sections are required to produce a pulse duration of about 2 μ sec and at least one bunch of the circulating beam will be lost.

However, it is not at all certain that a 16 section network behaves as it should. This depends largely on the properties of the capacitors used. Associated with the construction of a capacitor is an internal inductance, which can be determined by measuring the ringing frequency i.e. the resonance frequency of the short circuited capacitor.

The analysis of the line simulating network requires therefore inductances to be added in the shunt branches. This is quite a lengthy calculation, but the following consideration gives a rough idea how small these inductances have to be, before they impair the shape of the pulse. When the pulse is being produced, the wave front is propagated with such velocity that the capacitors located near the wave front are losing half their charge in approximately the LC time of one section. We easily calculate, that a short circuited capacitor loses half its original charge in approximately the internal LC time of the capacitor.

If the LC time of one section were comparable with the internal LC time of the capacitor, one would draw more current from the capacitor than a perfect short possibly could give. Therefore the condition holds:

$$L_{internal} \ll L_{section} \qquad (11)$$

If one builds a network with more sections than relation (11) permits, the total pulse duration remains in first approximation the same, but the wave front will be spread out over several sections. The effect helps quadratically, since the section L's are in series and the internal L's are in parallel.

For a certain type of paper insulated capacitor, constructed under optimum conditions, the internal inductance seems to be proportional to the length and independent of the diameter. We assume here that the overlapping metal foils are would in cylindrical units; so many of them put in series as the voltage requires, and the return conductor closely surrounding the pile of units. The current density, considered on a macroscopic scale, must then be constant, apart from time dependency. This permits the calculation of the stored magnetic energy, from which the equivalent internal inductance is found :

$$L = \frac{\mu_o}{8\pi} l \qquad (12)$$

l is the total height of the pile.

The internal inductance is thus proportional to the design voltage; or even somewhat more, as for higher voltages the spacing between can and units become larger. Also the feed through of the insulator on top represents a considerable inductance. In view of the demand nowadays for capacitors with low internal inductance, one may hope for better designs for which (12) fortunately does not hold.

We consider now a practical design in which the deflector is divided in 4 parallel sections. The voltage of the pulse is therefore (5a) V = 25000 Volt, and the voltage on the network 50000 Volt.

The inductance per section of the network is easily found to be a sixteenth of the inductance of the undivided parts of current-sheets (2). If d=h and l = 1 meter, one obtains $L = 8.10^{-8}$ H/section. The internal inductance of a 50 kV capacitor is (12) probably of the order of $3 - 5 \cdot 10^{-8}$ H, so that the condition (11) is not fully satisfied. Considering in addition the effects of small mismatches of a fairly complicated system of parallel low impedance lines, one may conclude, barring better capacitors, that at least two bunches of the circulating beam will be lost.

III. SQUARE WAVE APPROACH.

We approach the problem anew by considering the properties of the line simulating network. In a conventional line type pulser a steep wave front goes together with a flat top. The latter feature is for a magnetic deflector not so stringent, because the particles in the beam have themselves already a fair amount of angular spread, say $6\ 10^{-4}$ radian. Therefore a fluctuation in pulse height of the same order appears to be acceptable. The design of such a network is much simpler; in fact the early designs, preceding Guillemin's theories on the subject, show exactly the properties which are acceptable in the present case.

The Fourier expansion of a square wave of unit amplitude is given by :

$$f\ (t) = \frac{4}{\pi}\left(\sin \alpha t + \frac{1}{3} \sin 3\alpha t + \frac{1}{5} \sin 5 \alpha t + \ldots\ldots\right) \qquad (13)$$

Curve 1 of Fig. 1 and 2 show, how the function looks for the first three, respectively the first four terms. (The factor $4/\pi$ is omitted). A two terminal network consisting only of shunt branches, each branch having an inductance and a capacitance in series, enables us to represent function (13) term by term

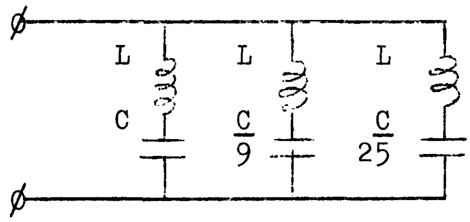

Fig. 3

When a unit step voltage is applied, the current in each branch is, with $\alpha = 1/\sqrt{LC}$:

$$i_r = \frac{1}{r} \sqrt{C/L} \sin r \alpha t \qquad r=1,3,5,\ldots.. \tag{14}$$

All those branch currents add up in the limit to the unit step current divided by the characteristic impedance Z

$$Z = \frac{4}{\pi} \sqrt{\frac{L}{C}} \tag{15}$$

Suppose that the lowest minimum of the pulse is high enough to deflect the particles a given minimum angle, i.e. point A on figure 1 and 2. The rise time is then found as the intersection of that level with the rising part of the pulse. From the figures one measures 8,3% for a three section network and 6,4% for a four section network. This is in both cases slightly more than one bunch lost. If this is acceptable, one may construct the network according to this principle and be fairly sure that the internal inductance of the capacitor does not spoil the pulse shape. In fact the ringing frequency should be higher than the resonance frequency of individual shunt-branches. For a 2 μ sec pulse the ground harmonic is 1/4 MC the next harmonic is 3/4 MC and so on.

IV. FURTHER IMPROVEMENTS.

Considerable advantages may be obtained by incorporating the deflector sections into the network of figure 3.

Firstly, the delay time of the deflector needs not to be added to the rise time of the pulse-forming network: both are identical. Secondly, it is no longer necessary to match the line with its characteristic impedance; the line may be short-circuited. For a short circuited line the voltage is half the voltage required for the matched line.

It is obvious that such a scheme is only warranted, if the network elements are small enough to be located in situ.

The network elements have to be modified, because in this case the Bl components have to add up to a square pulse.

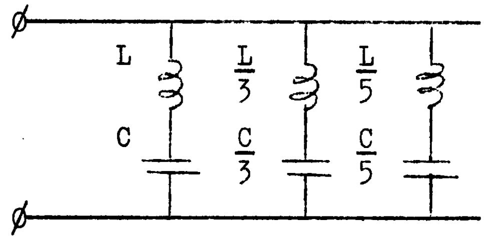

Fig. 4

Assuming the deflector elements, denoted by L, $\frac{L}{3}$, $\frac{L}{5}$ etc., all of the same basic structure and only differing in length, one appreciates, that in the above scheme the amplitude of all branch currents are equal to

$$\hat{i} = V \sqrt{C/L} \tag{16}$$

in which V is the voltage to which the capacitors are charged.

The amplitudes of the flux density in all deflector elements are thus also equal, and since the lengths of the deflector elements form the series 1, 1/3, 1/5 etc., the coefficients of the Bl Fourier expansion are in accordance with (13).

We show now that the voltage on the network is independent of the rise-time. Instead of this quantity we introduce the pulse duration P:

$$\alpha = \pi/P = 1/\sqrt{LC} \tag{17}$$

With (2), (4) and (16) one obtains then:

$$V = \frac{\pi}{P}\hat{B}ld \tag{18}$$

in which $\hat{B}$ is the amplitude of the flux density.

Relating $\hat{B}$ to the effective B one has in the limit

$$\hat{B} = 4B/\pi \tag{19}$$

so that

$$V = \frac{Bld}{P/4} \tag{5b}$$

comparing this result with (5) or 5 (a) one appreciates that under circumstances the voltage reduction is, say, a factor of eight.

The system has however limitations. According to (19) the amplitude of the flux density is higher than the effective by a factor of $4/\pi$. This factor has to be amended to about 1,4 if the amplitude is compared with the minimum pulse height A, as shown in figure 1 or 2. The total length of the deflector is about a factor 1,6 for a three element deflector and a factor 1,9 for a four element deflector higher than the effective length according to (5b). It will be shown in the next section that, in the case of ferroxcube clad current-sheets, built into a small field free section of the proton synchrotron, the Bl value is limited to about 0,2 Wb/m. This value is however sufficient to sweep the circulating beam in one go onto a thick target or onto the gap of a small deflecting magnet, which permits a further clearance to accomodate a large deflecting magnet.

The rise time for a practical network will be such, that certainly one bunch of the circulating beam, or perhaps two, will be deflected less than the minimum clearance angle. This is in itself not a disadvantage, since we have shown, that in all probability the results with conventional line type pulsers are not better. The switch has to conduct both ways. The current is given by (see 16).

$$i = V \sqrt{C/L} \, (\sin \alpha t + \sin 3 \alpha t + \sin 5 \alpha t + \ldots.) \qquad (20)$$

Probably a spark-gap gives the best results. The internal inductance of the spark-gap assembly is the coupling element between individual shunt-branches of the network. This inductance has therefore to be kept as low as possible.

The current will decay exponentially. The time constant for the decay in energy of a free vibrator is given by

$$t_o = Q/\alpha \qquad (21)$$

One would require for the first two sections, which determine the crude shape of the pulse : $Q > 10$.

The higher harmonics may eventually have lower Q values since their main purpose is to decrease the rise time.

The internal inductance of the capacitors lowers the coefficient of the Bl component of the deflector. This becomes more effective at the higher harmonics.

For comparison are given in Fig. 1 and 2 the pulse shape with the coefficients

$$b_r = \sin \pi ar/(r^2 \sin \pi a) \qquad r = 1,3,5\ldots\ldots$$

a = 0 —— curve 1
a = 0,05 —— curve 2
a = 0,1 —— curve 3

A similar modification may be necessary to bring capacitances to the nearest standard value.

V. PRACTICAL DESIGN.

The design is based on a beam diameter of 2 cm and a maximum deflector length of 1 meter. The current sheets are 3 cm in height and 3 cm spaced apart and surrounded by ferrox cube. This limits the amplitude of the flux density to about 3500 gauss. The length of the sections would be 60 cm, 20 cm, 12 cm, 8 cm and so on. The pulse duration P is chosen 10% higher than the revolution time of the protons, in order not to lose the last two bunches. Formula (18) provides the voltage to which the capacitors have to be charged

$$V = \frac{0{,}35\ 0{,}6\ 0{,}03}{2{,}2}\ 10^6 = 9000 \text{ Volt}$$

The peak current is found with (4)

$$\hat{i} = \frac{0{,}03\ 0{,}35}{4\pi}\ 10^7 = 8200 \text{ A}$$

The inductance of the main section is found with (2)

$$L = 4\pi 10^{-7}\ 0{,}6 = 0{,}75\ \mu \text{ H}$$

The capacitance in the main section is found with (11)

$$C = 0,75\ 10^{-6}\ (\frac{8200}{9000})^2 = 0,62\ \mu F$$

The minimum Bl value corresponding with point A is

$$Bl_{min} \approx 0,7\ \hat{B}l = 0,15\ Wb/m$$

and

$$Bl_{max} \approx 0,9\ \hat{B}l = 0,19\ Wb/m$$

The deflecting angles for 25 GeV protons are found to be

$$\Theta_{min} \approx 1,8\ 10^{-3}\ \text{radian}$$

$$\Theta_{max} \approx 2,3\ 10^{-3}\ \text{radian}$$

These angles appear to be sufficient to clear the kicked protons from the original beam envelope.

The deflector elements are located in such a manner that the smaller elements are grouped in the centre surrounded by the larger elements. All elements are identical except for the length.

The ferrox cube yoke consists of C shaped part in which the current sheets are located and forming an integral part with the capacitors and the sparkgap. This assembly is advanced from above to enclose the beam after it has contracted sufficiently. At the same time the ferrox cube closing slab is advanced from beneath.

The cans of the capacitors and one pole of the sparkgap are mounted on a baseplate which is at 9/kV potential.

The upper pole of the sparkgap and the common terminal of the deflector elements are also connected to a baseplate which is grounded and spaced from the life baseplate by the smallest amount. It may be necessary to back the ferrox cube slabs with a reinforcing material like alumina and to provide for an electric gap in the ferrox cube.

In figure 5 is shown a layout of a three section deflector.

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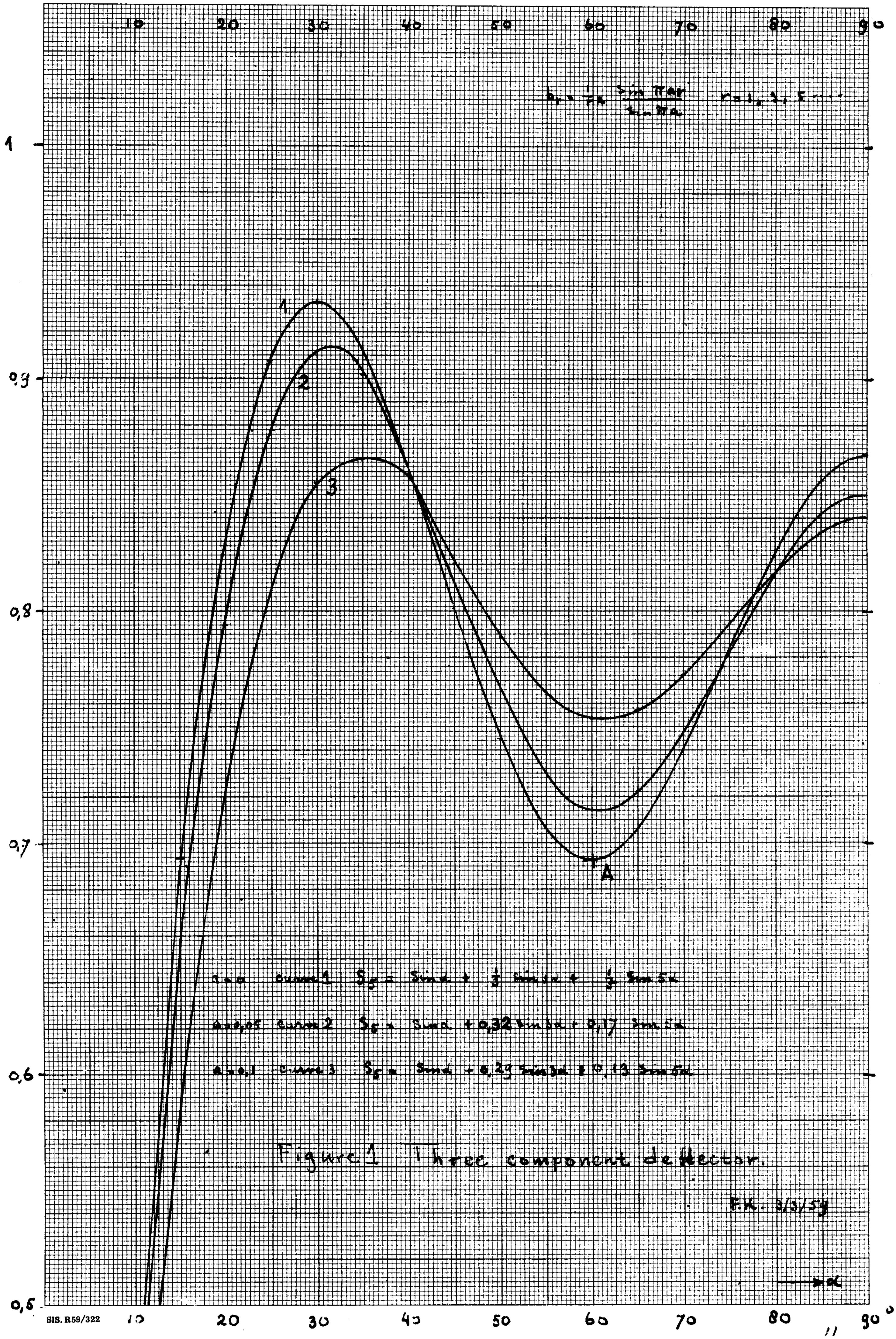

Figure 1 Three component deflector.

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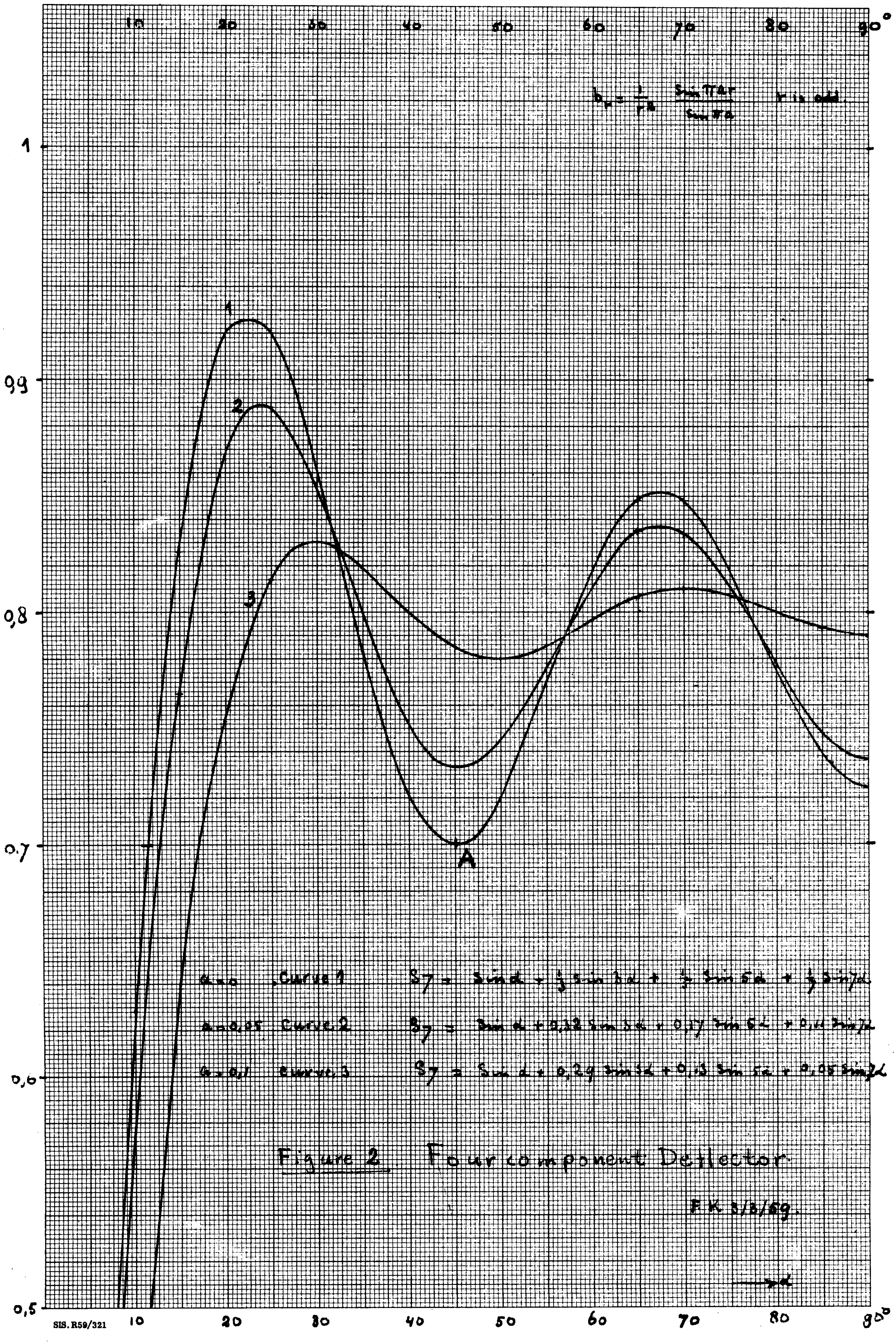

Figure 2 Four component Deflector

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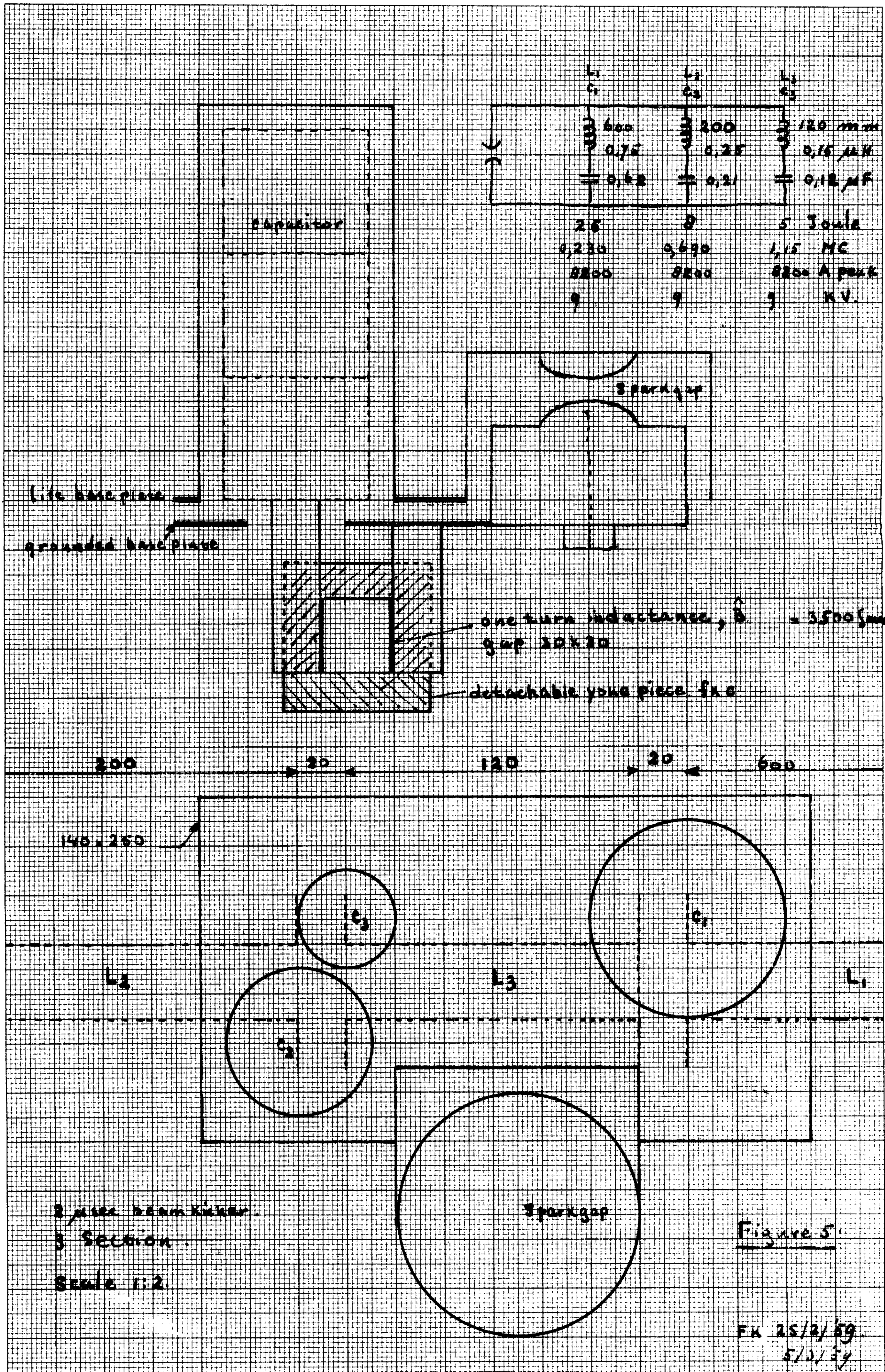
L1 C1
L2 C2
L3 C3
600
0,75
0,62
200
0,25
0,21
120 mm
0,16 μH
0,12 μF
25 8 5 Joule
0,230 0,690 1,15 MC
8200 8200 8200 A peak
9 9 9 KV.
capacitor
Sparkgap
Lite base plate
grounded base plate
one turn inductance, B̂ = 3500 gauss.
gap 20x20
detachable yoke piece fxc
200
20
120
20
600
140x260
C3
C1
L2
L3
L1
C2
Sparkgap
2 μsec beam kicker.
3 Section.
Scale 1:2.
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Figure 5