EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/MPS-DL/70-10
CERN/ISR-TH/70-53

DAMPING OF TRANSVERSE COHERENT OSCILLATIONS BY AN ELECTRODE STRUCTURE

by

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Geneva, 30th October 1970

1. INTRODUCTION

The Novosibirsk Storage Ring Group has investigated a method of passive damping of coherent transverse bunch oscillations[1,2,3]. The principle is to put an electrode structure (plat into the vacuum chamber, which together with the chamber walls acts as a transmission line. This electrode extends over part of the machine circumference. It is terminated by its characteristic impedance at either end. The original analysis of this device[1,2] as well as the experimental results obtained on VEPP-2 apply to the situation of a beam consisting of one short bunch or equivalently many short bunches oscillating independently of each other. From a more recent paper[3] it appears that the same damping method can be applied to a coasting beam as well.

The present note presents a different analysis of this device. For simplicity we only regard the limiting cases of "very long" and "very short" plates. This analysis seems to reveal that the terminated line method is ineffective in damping coherent betatron oscillations of an unbunched beam. One might still think of using this method to damp coherent betatron oscillations after injection into the ISR, wher the beam is still bunched. However, we find that the damping times obtainable are too long compared to the coherence time of betatron oscillations to avoid emittance increase. Similar results hold for single bunch modes in the PS.

2. COASTING BEAM

2.1 Method of calculation

In the commonly accepted notation coherent stability may be discussed in terms of the quantities U and V , as defined in Ref. 4), except that we use U instead of U + V for the real part of the frequency depression.

The "fast damping method" discussed in Ref. 3) may be described as the introduction of a V-term of appropriate sign and large enough to produce stability of otherwise unstable modes. In the following subsections we will give two independent derivations which both seem to reveal that the terminated plates are ineffective for fast damping of coasting beam instabilities.

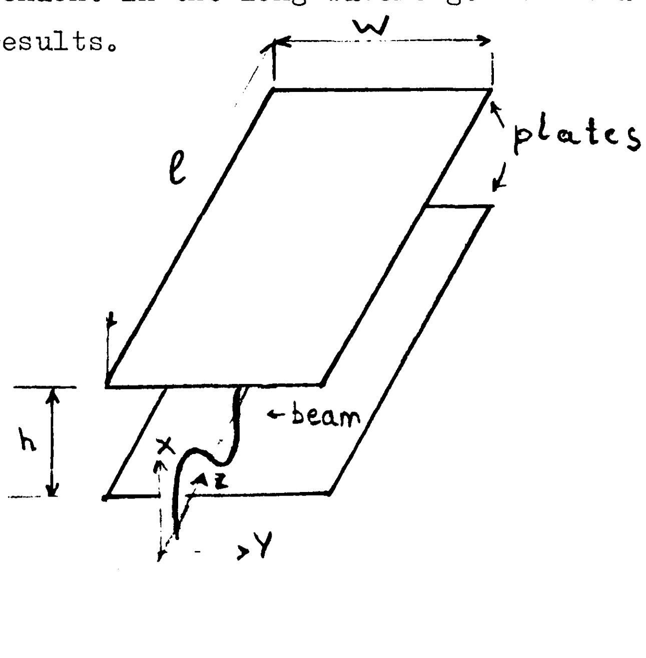
The first approach starts with the self-field of a point charge oscillating transversely between the electrodes of an infinite transmission line. The field of a coasting beam is obtained by integration over the corresponding sources. We find that the terminated plates do not change the V-term, at least for wavelengths long compared to the transverse dimensions of the plate.

The situation is different for a short bunch. Here the plates may introduce a large change of the V-term because the bunch spectrum contains wave numbers $k \approx k_o$, where ik_o is the propagation constant of the line ($k_o = \omega/c$ for an unloaded line).

The second approach which yields the same principal result, is an extension of Laslett's transmission line method[5]. For simplicity, in this second approach calculations are only performed for the limiting case, where the wavelength is short compared both with the longitudinal and with the transverse extent of the plates.

2.2 The point charge method

This subsection closely follows the derivation given in Ref. 2) for bunched beams. The calculations are performed in a coordinate system moving with the particle velocity $\beta_p c$. They are valid for the geometry sketched below. Different beam and electrode geometries can be accounted for by introducing a geometry factor which is frequency independent in the long wavelength limit and does not alter the principal results.



Consider a point charge q at azimuthal position $z = z_o$ which oscillates vertically with $x_o = \xi\, e^{-i\omega t}$. In addition to the pure image fields a wave is emitted along the line. In the long wavelength limit this wave (TEM wave) can be described by a potential difference u between the plates. Following Ref. 2) we define the potential to be u on the top plate, and 0 on the bottom plate; Z_o is the impedance between top and bottom plate. In subsection 2.3 we follow Laslett's convention and put $V = \pm u$ on the top and bottom plate respectively and take Z_o for the impedance between either of the plates and ground ($v = o$).
From ref. (2) we conclude that

$$u(z,t) = -\frac{1}{2}\, q\, i\, \omega\, Z_o\, \frac{x_o}{h}\, e^{-i\, k_o |z - z_o|}\ , \tag{1}$$

where Z_o is wave impedance of the line and ik_o is the line propagation constant. The minus sign in (1) and the factor $e^{-i\, k_o |z - z_o|}$ are justified by energy conservation (we assume $x \propto e^{-i\omega t}$) and by symmetry considerations.

Now we consider a coasting beam oscillating transversely. For simplicity we assume its diameter $d \ll h$. Let us approximate the physical situation by assuming an infinite beam between plates extending from $z = -\infty$ to $z = \infty$. This model (beam and plates from $z = -\infty$ to $z = +\infty$) is a good model for plates which have very low losses and cover the whole circumference of the machine without any break and without any termination.

Transverse oscillation modes are described by a beam displacement:

$$x_o(z,t) = \xi\, e^{i(k_z - \omega t)} \tag{2}$$

where k is related to the mode number n of the instability: $k = n/R$ [4].

As we are working in a system moving with the particles, the plates are moving with velocity-$\beta_p c$ in the z-direction. However, for an infinite line free of losses, the beam does not notice this movement.

We obtain the self-field of the coasting beam by replacing $q \rightarrow \lambda dz_o$ in (1), introducing x_o (z_o, t) from (2) and integrating over the sources:

$$u(z,t) = -\frac{\lambda}{2} i\omega z_o \frac{\xi}{h} e^{-i\omega t} \int_{-\infty}^{\infty} e^{-i(k_o|z-z_o|-kz_o)} dz_o \quad (3)$$

This integral is meaningful if we assume that the plates are long enough, and have small losses(k_o complex) so that $e^{-ik_o \ell} \rightarrow 0$ for $\ell \rightarrow \infty$. Further we assume that $k_o \neq k$. For an unloaded line the latter of these assumptions is equivalent to requiring $Q/n \neq \pm (1 \pm 1/\beta_p)$, because in the lab system $k_{o\ lab} = \frac{\beta_p n}{R} \pm \frac{Q\omega_o}{c}$ and $k_{lab} = \frac{n}{R}$. With these assumptions the integration (3) yields:

$$u(z,t) = \lambda\omega Z_o \frac{\xi}{h} e^{i(k_z - \omega t)} \frac{k_o}{k^2 - k_o^2} \quad (4)$$

i.e.

$$u(z,t) = \lambda\omega Z_o \frac{x}{h} \frac{k_o}{k^2 - k_o^2} \,. \quad (5)$$

In the coordinate system moving with the beam, the transverse component of the Lorentz force is

$$eE_x = -\frac{e\ u(xt)}{h} = -\frac{e\lambda\omega}{h^2} Z_o \frac{k_o}{k^2 - k_o^2} \quad x \quad (6)$$

We have thus found that the self-force is in phase with the beam displacement x. Such force components only cause a real frequency shift, but do not change the V-term. This property is unchanged, when we transform to the lab-system.

Let us now consider the case of a short δ-function bunch, to elucidate the different behaviour of bunched beams. We may of course immediately use the result (1) and find that the self-force

$$eE_x = \frac{i}{2} q \omega \frac{z_o}{h^2} x$$

is imaginary and reduces the growth rate of modes with $x \propto e^{-i\omega t}$ ("fast damping"). However, to see the difference from the coasting beam case it is instructive to expand the bunch spectrum into modes of type (2). This is readily done by Fourier transformation of the δ-function. In analogy to (2) we write

$$\lambda . x(z_o,t) = q \xi e^{-i\omega t} \delta(z_o) = q\xi e^{-i\omega t} \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ikz_o} dk \qquad (2a)$$

Then using the same procedure as above (eqs. 3 - 5) we obtain:

$$u(z,t) = q\omega Z_o \frac{\xi}{h} e^{-i\omega t} \frac{k_o}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{ikz}}{k^2 - k_o^2} dk \qquad (1a)$$

This expression is of course fully equivalent to (1). However, from (1a) we conclude that mainly the pole ($k = k_o$) contributes to make $u(z,t)$ imaginary.

In other words, the fact that the bunch spectrum (2a) contains components with a wave number k close to the modulus k_o of the line propagation constant ($i\ k_o$) introduces fast damping. The condition $k = k_o$ is a necessary but not a sufficient condition for fast damping. In Appendix 1 we find that plates with $k = k_o$ tend to increase the growth rates of the unstable coasting beam modes.

Following ref. 2) 3) we have assumed an infinite structure rather than a circular machine in this subsection.

The strong damping effect resulting from the pole of the integrand in (1a) is in general different in a circular structure where the integral in (3) has to be replaced by a sum (see Appendix 1).

For infinite structures(1) or equivalently (1a) may be used as "Greens functions" of an oscillating charge. However, (1a) is more difficult to use (pole in the integrand). To summarize: the infinite line is a weak model as it neglects end effects, as well as the periodicity of the circular machine. But even from this model we do not find fast damping of coasting beam modes if we use (1) rather than (1a) as Greens function.

2.3 The Transmission Line Method

The transmission line method permits one to take into account the effect of the end of the plates. In the case of plates having a single termination(*), the transmission line method[5][7] suggests that:

(i) In the "long wavelength limit" (i.e. when the wavelengths both of the perturbation and of the TEM wave which propagates on the plate are infinitely long compared to the longitudinal extent of the plate ($k\ell = k_o\ell = 0$) the response of the plate to transverse oscillations of a coasting beam is purely inductive, namely $U > 0$ and $V = 0$. Moreover, this response is independent of the position and of the impedance of the termination(*).

(ii) For wavelengths close to this limit ($k\ell << 1$ and $k_o\ell << 1$), one has an additional V-term which has a positive sign, is proportional to the frequency, to the square of the length of the plate and to the resistance of the termination and is independent of the position of the termination(*).

(*) In this paper we consider only the case where the termination impedance has a finite value in comparison with the characterisitc impedance of the plate. This condition assures that the plates are effectively terminated and not electrically floating.

In this sub-section we apply the transmission line method to plates terminated at both ends by their characteristic impedance. We find:

(i) In this particular case one can confirm what point (i) above seems to suggest, namely that in the long wavelength limit the response is independent of the way of terminating the plate and is characterised by

$$V = 0 \tag{7a}$$

Further on in this subsection, using simple physical arguments, we show that this statement has a general validity also for more elaborate ways of terminating the plate.

(ii) The V-term which arises when we depart from the long-wavelength limit, is qualitatively the same as in the case of a single termination. In the range $k\ell \ll 1$ and $k\ell \ll 1$, for ultrarelativistic particles one finds a V-term

$$V \simeq \frac{1}{(2\pi)^2} \frac{N}{\gamma_p} (Z_o c) \frac{r_o}{R} \left(\frac{\ell}{h}\right)^2 \frac{\omega}{Q} \tag{7b}$$

This result does not indicate the presence of any fast damping mechanism.

Laslett's equations for the scalar and vector potential of the transmission line mode propagating on the plate are(5):

$$\begin{aligned} \frac{\partial V_1}{c\partial t} + \frac{\partial A_1}{\partial z} &= - ickZ_o(\beta_p - \beta_w)\, \lambda_I \\ \frac{\partial V_1}{\partial z} + \frac{\partial A_1}{c\partial t} &= 0 \end{aligned} \tag{8}$$

Here $\lambda_I = -(p/h) \exp [i(kz - \omega t)]$ is the charge per unit length induced on the upper plate (the opposite sign should be taken for the lower plate) and p is the electric dipole per unit length introduced by the transverse perturbation of the beam.(*).

(*) We take for Laslett's coupling factor K its approximate value $K = w/2h$.

Solutions of these equations have the form:

$$\binom{V_1}{A_1} = \frac{pZ_oc}{h}\left[ae^{i\frac{\omega}{c}(z-z_1)} \pm be^{-i\frac{\omega}{c}(z-z_1)} + \binom{\beta_w}{1}\frac{\beta_p-\beta_w}{1-\beta_w^2}\,e^{ikz}\right]e^{-i\omega t} \qquad (9)$$

The coefficients a and b are determined by the boundary conditions at the two ends of the plate (termination impedance $Z_T = Z_o$) :

$$\left.\begin{array}{ll} A_1 + Z_oI_I + V_1 = 0 & \text{at the upstream end } z = z_1 \\ A_1 + Z_oI_I - V_1 = 0 & \text{at the downstream end } z = z_1+\ell \end{array}\right\} \qquad (10)$$

These equations are the Kirchoff's node equation for the ends of the plate. They relate the current $I_1 = A_1/Z_o$ of the transmission line mode which propagates on the plate to the current V_1/Z_T drawn by the termination impedance. One obtains

$$a = \frac{\beta_w}{2}\,\frac{1-\beta_p}{1-\beta_w}\;e^{ikz_1}$$

$$b = -\frac{\beta_w}{2}\;\frac{1+\beta_p}{1+\beta_w}\;e^{i(k+k_o)\ell}\;e^{ikz_1} \qquad (11)$$

We introduce (11) into (9), evaluate the n-th harmonics V_n and A_n of the potentials. We evaluate the additional transverse force per unit charge acting on the centre of the beam due to the presence of the plate. We consider the case where the plate has the same distance from the beam centre as the smooth vacuum chamber in the rest of the machine circumference.

$$\left.\begin{array}{l} \langle F\rangle = -\dfrac{\langle V_n-\beta_pA_n\rangle}{h/2} \\ \text{since}^{(4)}:\ (U+iV) = \dfrac{Nr_oc}{4\pi Q\beta_p\gamma_p}\,\dfrac{\langle F\rangle}{p} \end{array}\right\} \qquad (12)$$

We find

$$(U+iV) = -\frac{N}{\beta_p\gamma_p}\;\frac{r_oc^2}{2\pi Qh^2}\;Z_o\left(\frac{\ell}{2\pi R}\right)P\,(\omega,n) \qquad (13)$$

where N is the total number of particles, r_o is the classical particle radius, $\gamma_p^{-2} = (1-\beta_p^2)$ and Q is the number of betatron oscillations per revolution. The P-factor represents the electrical response of the plates

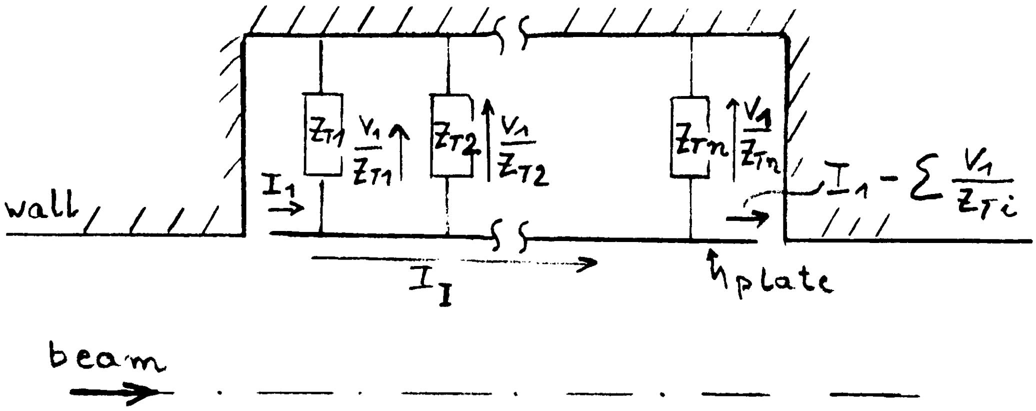
$$P(\omega,n) = \frac{i\beta_w}{2k\ell} \left\{ \left(\frac{1-\beta_p}{1-\beta_w}\right)^2 \left[e^{i(k_o-k)\ell}-1\right] + \left(\frac{1+\beta_p}{1+\beta_w}\right)^2 \left[e^{i(k_o+k)\ell}-1\right] - \frac{(\beta_p-\beta_w)^2}{1-\beta_w^2} \right. \qquad (14a)$$

In the case $k\ell \ll 1$, $k_o\ell \ll 1$ one obtains:

$$P(\omega,n) \simeq -\beta_p^2 - ik_o\ell \ \frac{1+\beta_p^2}{2} \qquad (14b)$$

It emerges from (13) and (14a,b) that in the long wavelength range the response is predominantly inductive (U>0) and that the V-term is as anticipated by (7).

We now examine the case of plates having any number, impedance and position of terminations (see sketch below). We restrict ourselves to the long wavelength limit (as defined at the beginning of this section). This implies that the induced current I_I is uniform all along the plate and that the TEM current $I_1 = A_1/Z_o$ and the TEM voltage V_1 are uniform along sections of plate in between terminations or free ends. Let us set I_1 and A_1 in such a way that the boundary conditions are satisfied at one end of the plate say the upstream end (see the sketch below). Let us move towards the other end, supposing $V_1 \neq 0$. Since V_1 is constant, terminations draw currents all of the same sign. To account for these currents in the node equation at the terminations one has always to decrease (or increase) I_1, I_I being obviously constant all along the plate. Then at the other end of the plate one faces an imbalance between I_1 and I_I, so that it is impossible to satisfy the boundary conditions unless $V_1 = 0$, independently of the way of terminating the plate. This is indeed the result which one obtains from (9) and (11) in the long-wavelength limit.



In fact, as discussed in ref. (7), the effect of finite i.e. $Z_T/Z_o \ll 1/k\ell$) termination impedances is only to shunt the electric fields (hence $V_1 = 0$ and $V = 0$) between the plates and the wall, so that the γ-cancellation of the e.m. forces is removed in the region between the plate and the wall. The wall currents travel further away from the beam than in the smooth pipe case. The factor $(1-\beta_p^2)$ in the expression of these forces is then transformed into the factor $(-\beta_p^2)$ which appears at the r.h.s.of eq. (11a) and represents a purely inductive effect. On the other hand, it has been shown in ref.(7)that the opposite limiting case of an electrically floating plate $(Z_T/Z_o \gg 1/k\ell)$ does not remove the γ-cancellation of the e.m. forces, so that its response still contains the factor $(1-\beta_p^2)$. We conclude that in the long-wavelength limit the beam does not discern how a plate is terminated, but only sees if it is terminated or electrically floating.

Let us now consider a plate with matched terminations at the two ends. One might argue that, since there are no reflections, the fields are the same as on an infinite waveguide 3). Using this approach one replaces the two terminations by two semi-infinite pieces of line. Then one misses the boundary conditions at the two ends (or "end effects") represented by eq. (10) and consequently one does not find the result $V_1 = 0$. This approach seems therefore to be incorrect for plates or finite length. Moreover one has to consider that the sources at the perturbation pass the end of the line. This is another reason why end effects should be included.

3. Fast Damping of Bunched Beams in the ISR

One might think of using the fast damping method to eliminate coherent oscillations due to injection errors in the ISR. We will use the following expression for the e-folding time τ_o, to estimate possible damping rates[2]

$$\frac{1}{\tau_o} = \frac{N_B}{4\gamma_p}\left(\frac{r_o}{h}\right)\left(\frac{\ell}{2\pi R}\right)\left(\frac{c^2 Z_o}{h}\right) \text{ ; cgs units} \tag{15}$$

or

$$\frac{1}{\tau_o} = \frac{\pi}{\mu_o}\frac{N_B}{\gamma_p}\left(\frac{r_o}{h}\right)\left(\frac{\ell}{2\pi R}\right)\left(\frac{Z_o}{h}\right) \text{ ; mks units.}$$

r_o: classical particle radius

For single bunch modes N_B is the number of particles per bunch.

Let us use as upper limit for the relevant ISR-parameters

$N_B = 10^{12}$	number of protons
$(\ell/2\pi R) = 0.01$	circumference factor of the plates
$Z_o = 50$ ohm	wave impedance of the plate-chamber system
$h = 3$ cm	distance between the plates

We obtain:

$$r_o^{ISR} \approx 15 \text{ msec.} \tag{16}$$

Typical coherence times in the ISR are 0.5 msec. ref. (9). Therefore the damping rate (16) is too small to prevent emittance dilution due to injection errors in the ISR. In conclusion the fast damping method seems not feasible for the ISR.

4. Fast Damping in the PS

Vertical instability is observed in the PS above transition energy[6]. This instability seems to be a single bunch effect, the growth time is of the order of 50 msec. at 1.5×10^{12} p/p. From the experimental observation it appears to be unlikely that the effect is a pure dipole oscillation. Therefore it is not clear whether the Novosibirsk method would damp these oscillations. Let us nevertheless use (15) for an estimate of possible damping times in the present PS. We insert

$N_B = 7.5 \times 10^{10}$ (protons per bunch)

$h = 0.07$ m

$\frac{\ell}{2\pi R} = 1.6 \times 10^{-3}$ (circumference factor for plate of $\approx$ 1m length)

$Z_o = 50\ \Omega$ (wave impedance of plate chamber system)

$\gamma = 6.5$

with these numbers we find from (15)

$$\tau_o^{PS} \approx 200 \text{ msec } \frac{\gamma}{\ell \text{ (metres)}} \simeq 1.4 \text{ sec.}$$

This damping time is much longer than the observed growth rate. In fact to have a damping time less than 50 msec. one would need about 30 metres of plate (assuming the above parameters, especially $Z_o = 50\ \Omega$). It is concluded that rather long plates would be required for damping of single bunch modes in the PS.

The situation is different for coherent bunch modes. Damping times may be shorter by a factor $1/_{20}$. However, it appears that part of these modes would be excited rather than damped by the plates.

Acknowledgement

It is a pleasure to thank H.G. Hereward for many very valuable comments and suggestions.

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APPENDIX 1

Circular Machine vs. Infinite Structure

Section 2.2 does not include the case $k = k_o$. We want to include this case for a coasting beam. We work in the lab system. We consider a circular machine rather than an infinite structure.

In the lab system any coasting beam oscillation may be represented by a Fourier expansion

$$\text{(A1)} \qquad x_{(z,t)} = \sum_{n=\infty}^{\infty} \xi_n e^{i\left(\frac{n}{R} z - (Q+n)\omega_o t\right)}$$

which is a super position of normal modes. The frequency of mode n seen by an observer in the lab system may be written as

$$\text{(A2)} \qquad \omega_n = (Q \pm |n| \omega_o = \left(\frac{Q}{R} \pm |k|\right) \beta c$$

The - sign is due to the fact that (A1) includes negative n and therefore also negative $k = \frac{n}{R}$. It is verified that the result (4) of section (2) is valid for a circular machine where lossless plates cover the whole circumference (no termination, no break).

We include loss assuming in the usual way a series resistance R' and a shunt resistance 1/G' per unit length of the line. We have to replace in (4)

$$Z_o k_o \rightarrow Z_o k_o \left(1 + i \frac{R'}{\omega L'}\right)$$

$$\text{(A3)} \qquad k_o \rightarrow k_o^2 \left(1 + i \frac{R'}{\omega L'}, + i \frac{G'}{\omega C'},\right)$$

where L', C' are the capacitance and inductance per unit length of the line and $k_o = \omega \sqrt{LC} = \frac{\omega}{v_e}$. $Z_o = \sqrt{\frac{L'}{C'}}$. We find instead of (4)

$$\text{(A4)} \quad u_{(z,t)} = \lambda\omega \frac{\xi}{h} e^{i(kz-\omega t)} \frac{Z_o k_o (k^2-k_o) + i[R'k^2 + \frac{G'}{Z_o^2} k_o^2]}{(k^2-k_o^2)^2 + k_o^4 (\frac{R'}{k_o Z_o} + \frac{G'Z_o}{k_o})^2}$$

The electric field is $E_x \approx -\frac{u}{h}$, the magnetic field may be obtained from $-\dot{B} = \text{curl } E$ which yields in the present case $B_y = -\frac{1}{h}\frac{k}{\omega}u$. Therefore the imaginary part of the Lorentz force $f_x = e(E_x + v_z . B_y)$ is (A5)

$$I_m(f) = \frac{e\lambda\omega x}{h^2} \frac{R'k^2 + G'k_o^2}{(k^2-k_o^2)^2 + k_o^4 \left(\frac{R'}{k_o Z_o} + \frac{G'Z_o}{k} \right)^2} \; x \; (1+\beta \frac{c}{v_e} \frac{k}{k_o})$$

For "forward waves" ($n > 0$ in [A1]) ω, k, k_o are positive and the force (A3) yields damping. For "backward waves" ($n < 0$) (A5) may be positive or negative (stable or unstable), however, for the "slow waves" ($n < -Q$) (A5) is always negative and will lead to anti-damping. Note that this result remains true for the special case $k_o = k$.

We have thus found that the line will always anti-damp "slow wave" modes. However, these are just the unstable coasting beam modes which one would like to damp.

As pointed out, the above analysis is valid for continuous plates. It neglects the effect of the terminations, a weakness that it shares with the models used in ref. (2) and (3).

However, we have inserted the periodicity of the machine. This has the consequence that (1) or (1a) are no longer the "Green's function" of the system but rather

$$\text{(A6)} \qquad u_{(z,t)} = q\omega Z_o \frac{x}{h} \frac{k_o}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{e^{i\frac{n}{R}z}}{(\frac{n}{R})^2 - k_o^2}$$

Note that (A6) is the same as (1a), section 2.2, except that the Fourier integral is replaced by a sum.

This will, in general, remove the pole in (1a) which is responsible for the fast damping of a single bunch in an infinite structure.

Therefore, the treatment of ref. (2) and (3) is questionable as it neglects the periodicity of the machine. In addition for plates of finite length end effects are important and it seems incorrect to replace finite plates by infinite ones weighted with a circumference factor.