LINAC QUADRUPOLE GRADIENTS AND MATCHING PARAMETERS
AT DIFFERENT BEAM INTENSITIES *

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ABSTRACT

With higher beam intensities the required strength of linac quadrupoles as well as the parameters of the matched beam differ more and more from the zero space charge values. A simple method of calculation is presented by which the quadrupole gradients, matching parameters and linac acceptances for any beam current can be obtained. The beam is considered as a uniformly charged ellipsoid and the whole treatment is linear.

A computer programme has been developed to determine the above parameters. It traces, in addition, the beam through the computed structure and displays its envelope on a graphical output. In this way it is possible to see how well the beam is matched.

The results obtained for ++-- and +- structures are presented.

Introduction

In modern protons linacs, the beam intensities are such as to invalidate the treatment of space charge effects as small perturbation phenomena. In the longitudinal phase plane the phase oscillations are affected and their adiabatic damping is no more proportional to $\beta^{-3/4}$ (in non-relativistic approximation). In the transverse phase planes the quadrupole gradients have to be increased (in absolute value) in order to prevent too big a drop in the linac acceptances. In connection with this, the parameters for a matched beam undergo some modifications too.

It is the purpose of this paper to present a method of calculation of the quadrupole gradients and matching parameters as function of the beam current. The calculations are carried out with a computer program.

Computer programs treating space charge effects can generally be classified into two groups :

    i) individual particle programs : the beam is represented by a certain number of particles (up to a few thousands) and the Coulomb forces among them are calculated by different methods;

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ii) beam programs : the beam is considered as an entity usually as a uniformly charged ellipsoid leading to linear space charge forces. The evolution of the beam envelope under the influence of external and internal forces is calculated by solving simultaneously a set of longitudinal and transverse envelope equations.

The programs of the first group give a more detailed insight into beam phenomena but are 10 ÷ 100 times more time consuming that the others.

The program ADAPT[1] presented in this paper belongs, to some extent, to the second group (linearised forces, beam treated as an entity), but it is more a "structure" program than a beam program. This is to say that space charge forces appear as external forces and are added to existing transverse and longitudinal forces of the linac structure. In this way the structure parameters under space charge conditions are calculated by the usual technique for periodic structures developed by Courant, Livingston and Snyder (CLS).

Hypothesis

In order to apply the CLS formalism several simplifications have to be made :

i) all the forces are considered to be linear (necessary for the application of the linear matrix theory);

ii) the defocusing action of the accelerating gaps is the same for synchronous and non-synchronous particles;

iii) the acceleration in gaps is the same for particles on and off axis;

iv) stray fields, image forces and forces from adjacent bunches are neglected.

Among these approximations, the most crude one is the linearisation of the longitudinal focusing forces in gaps. These forces are linear for small oscillation amplitudes, but become highly non-linear when particle excursions approach the separatrix. The calculation of the longitudinal linac acceptance and longitudinal matching parameters may therefore be in some error, but the transverse planes are practically not affected.

Principles

In this chapter we explain the principles on which the calculations are based. It will be seen how space charge forces are progressively introduced and added to the existing forces in the structure. For each beam intensity a new corresponding structure is defined and new structure calculations carried through.

Three phases can be distinguished in the calculations :

1. Focusing structure calculations :

Quadrupole gradients, linac acceptances and matching parameters are determined using the CLS formalism for periodic structures. The linac being only a quasi periodic structure, some special treatment is necessary in order to apply the CLS formalism. We adopted a method[2] consisting of a "cell by cell" analysis of the linac. This means when analysing the n-th cell, a complete structure period is formed by such cells, changing as necessary the sign of the quadrupole field. The results obtained apply only to the n-th cell and the calculation is repeated for each cell of the linac. (Another method using groups of cells was applied in the program ACCEPT[3]).

2. Beam Transfer :

A "zero charge" beam, matched in the six-dimensional phase space to the focusing structure is transfered through the linac by matrix calculations. The beam envelopes are determined at eight positions in each cell for two reasons :

i) to control how well the beam is matched all over the linac;

ii) to obtain beam dimensions necessary for space charge calculations.

3. Space Charge Calculations :

Space charge forces are calculated with the above beam dimensions and assuming first a small current intensity. To deal with linear space charge forces, the beam is represented (non self-consistently) as a uniformly charged ellipsoid. The potential at the inside of such an ellipsoid is given by [4] :

$$U_i = - Ax^2 - By^2 - Cz^2 + D$$

The coefficients A, B and C are of the form :

$$A \{B,C\} = \frac{abc \; \rho}{2\varepsilon_o} \int_o^\infty \frac{du}{(a^2\{b^2,c^2\} + u) \; \sqrt{(a^2+u)(b^2+u)(c^2+u)}}$$

where a, b and c are the ellipsoid half axes in the x,y and z direction, ρ is the charge density and u the integration variable. The integral is calculated by the Gauss integration method[5]. By performing some appropriate scaling in the integral (the value of A, B and C does not change when a, b and c are scaled by the same factor), a 10 point approximation is sufficient to give the desired accuracy. Introducing the beam current I instead of ρ , the space charge forces can be written as :

$$F_x \{F_y, F_z\} = \frac{3I \; T_{RF} \; A\{B,C\}}{8\pi \; \varepsilon_o \; abc} \; x \; \{y,z\}$$

where T_{RF} is the RF period. These forces are calculated at eight positions in each cell and then added to the focusing structure forces. The structure is now changed and new structure calculations have to be initiated. The results will apply to a beam with the assumed small current intensity.

Increasing progressively the intensity, the whole procedure described above is repeated until the desired current value is reached. At the end of the calculations, the evolution of the structure and matching parameters as function of the beam current is known.

Calculations : Criterion and Procedure

Criterion :

The focusing structure design can be based on different criteria. The one we adopted is an imposed phase advance μ per structure period. This phase advance is kept equal for low and high intensity beams in order to compare better the results. The μ itself varies along the linac $\propto \beta^{-\frac{1}{2}}$ (this law of μ variation is the one originally chosen for the CERN Linac) which corresponds to an icnrease in the mean beam envelope $\propto \beta^{\frac{1}{4}}$.

Computational procedure in the program ADAPT :

The organisation of the program, in its main lines, is shown on Fig. 1. The geometrical and electrical properties of the linac have to be supplied as data. The program begins with a zero space charge calculation, but the transfer matrices are already prepared for the inclusion of space charge terms. Structure calculations are performed on periods which have symmetries at mid F and mid D points ($\alpha = 0$) also in the presence of space charge. This is convenient for the determination of quadrupole gradients. "Ideal" beam envelopes correspond to these symmetries.

In the beam transfer calculations, the beam is matched to the linac input or to a structure symmetry point and then transferred through the linac. The envelopes thus obtained are "true" ones and do not have symmetries at mid F or mid D points.

Space charge calculations were usually carried through with a progressive current increase of $\sim$ 20 mA per step. Same results were obtained with a bigger current increase (50 - 100 mA), provided the calculation for a particular current was repeated 2-3 times.

Results

The geometrical and electrical data applied in the program ADAPT correspond to the CERN 3 MeV experimental linac. Two focusing structures were analysed, FFDD and FD for beam currents up to 200 mA.

For convenience, the results are presented in form of diagrams.

Fig. 2 shows the quadrupole gradients for beam currents of 0, 100 and 200 mA.

Fig. 3 gives the exponential law $G = G_0 \beta^{-n}$ (β = relativistic factor) of the gradient decrease along the linac.

Fig. 4 shows the evolution of the linac acceptances as function of the beam current. Note that the vertical and the horizontal acceptances are not equal (the vertical one is bigger due to the fact that the vertical amplitude function β_V has its first maximum farther down in the linac structure, where the drift tube bore radius is increased). In the calculations, the beam emittances filled up the acceptances.

Figs. 5 and 6 present the parameters of a matched beam at the input to the CERN 3 MeV Linac.

Figs. 7, 8, 9 and 10 show how well the beam is matched to the focusing structure; the crosses on the drawings indicate the calculated transverse envelopes while the sine-like full lines present the real envelopes obtained by transferring a matched beam through the linac. A 200 mA beam (Figs. 9, 10) is as well matched as a "zero current" one (Figs. 7,8).

It is worth mentioning that the overall matching along the linac depended on the "matching point" chosen. In fact, best results (those presented) were obtained by matching the beam to a β-minimum point (β = amplitude factor) rather than to the linac input.

Conclusion

The described computational method was found very useful in the analysis of the linac "front end" [6]. The main interest of the method lies in its capability to determine a functional relation between linac parameters and beam current intensity. The results can be introduced, as optimised data, into individual partical programs. The program ADAPT uses about 10 seconds of central processor time of the CDC 6600 for the complete analysis of a focusing structure (from 0.5 to 3 MeV) comprising the calculation of quadrupole gradients, linac acceptances, matching parameters and beam transfer with envelopes determination, all for 10 values of beam current in the region from 0 to 200 mA.

Acknowledgments

References

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4.   O.D. Kellogg. Foundations of Potential Theory. Dover, New York, 1953.

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Distribution (open)

Fig. 1

CALCULATION SEQUENCE IN ADAPT
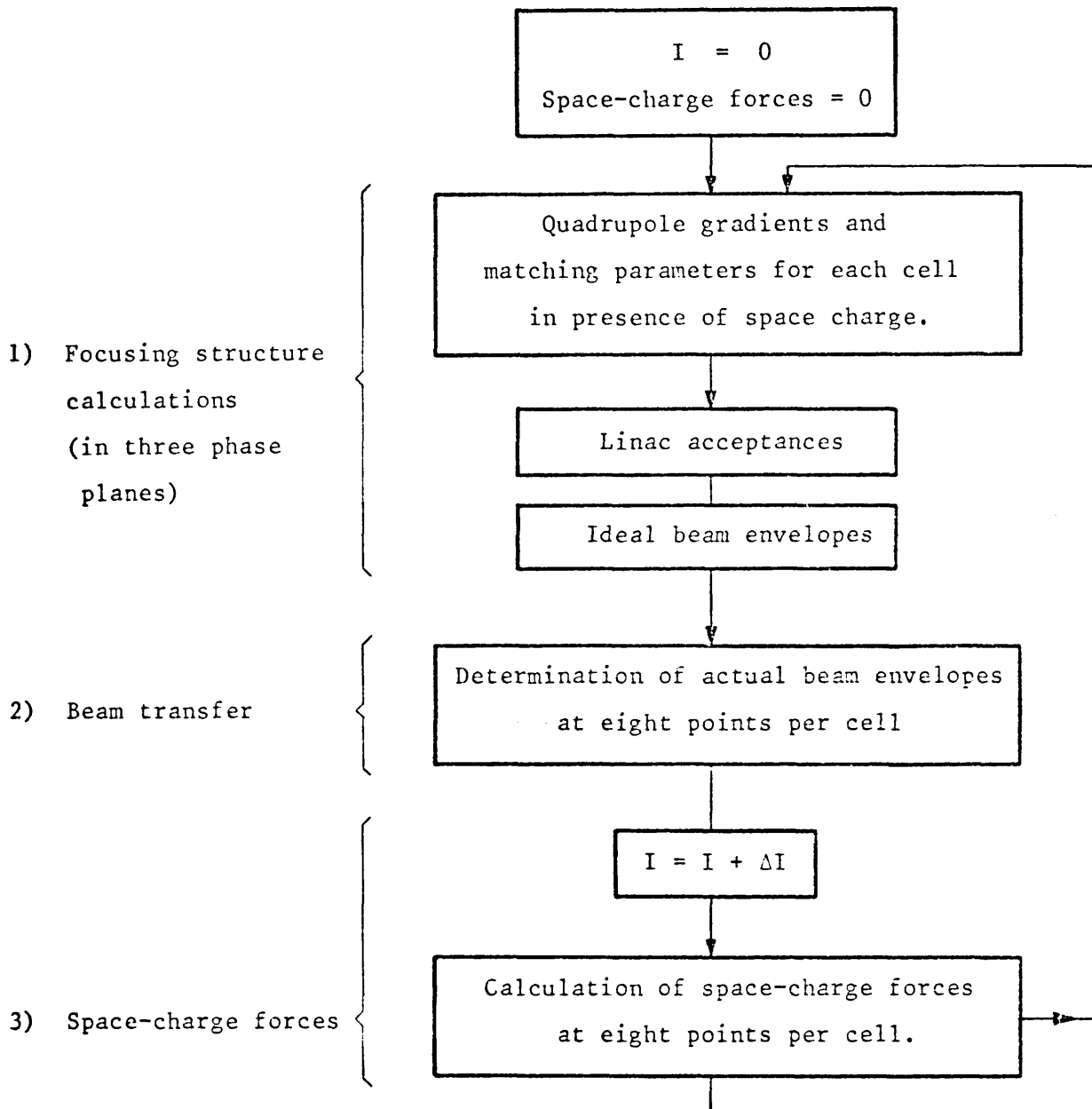
Fig. 2

QUADRUPOLE GRADIENTS FOR
FFDD AND FD FOCUSING STRUCTURES

GEOMETRICAL AND ELECTRICAL DATA
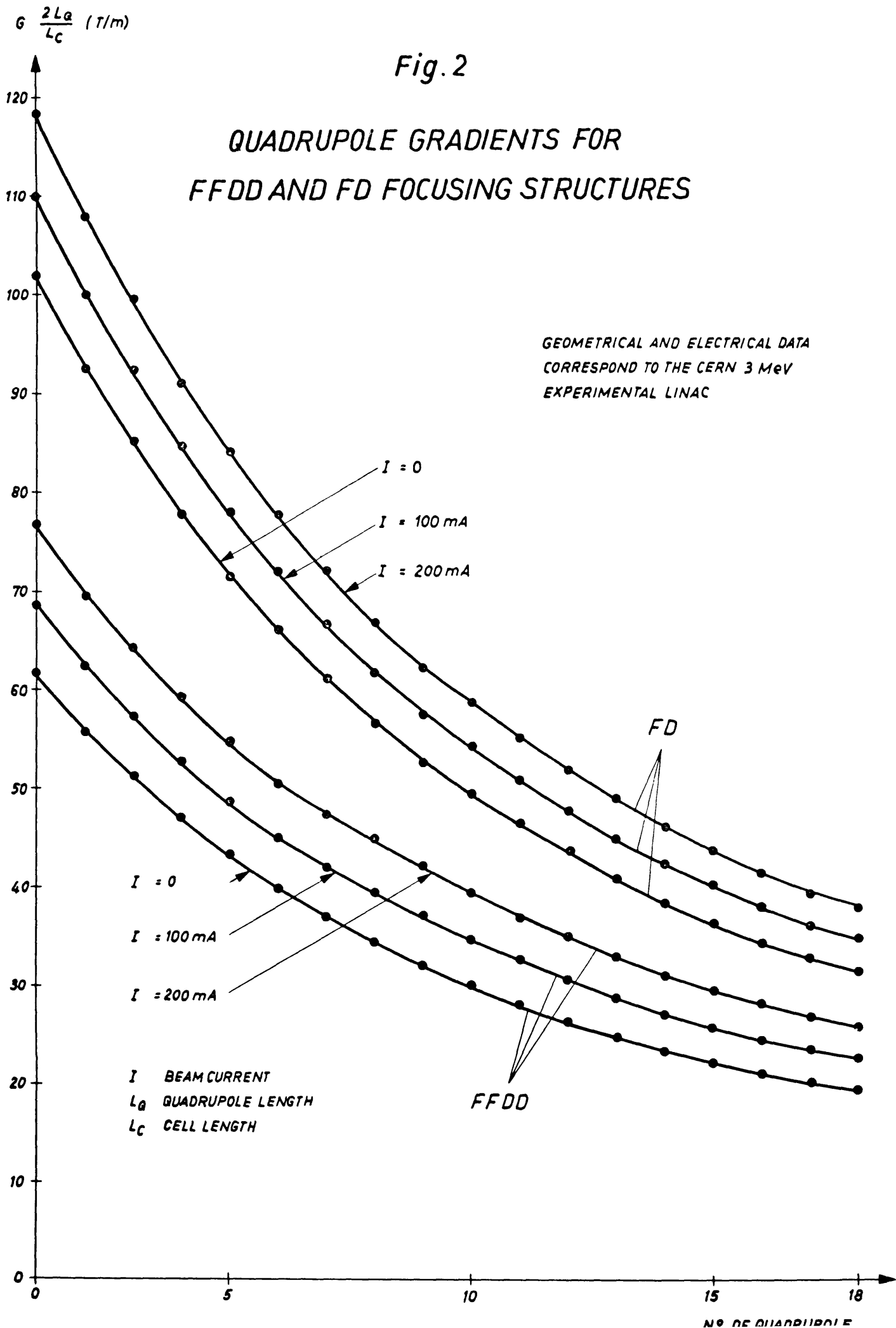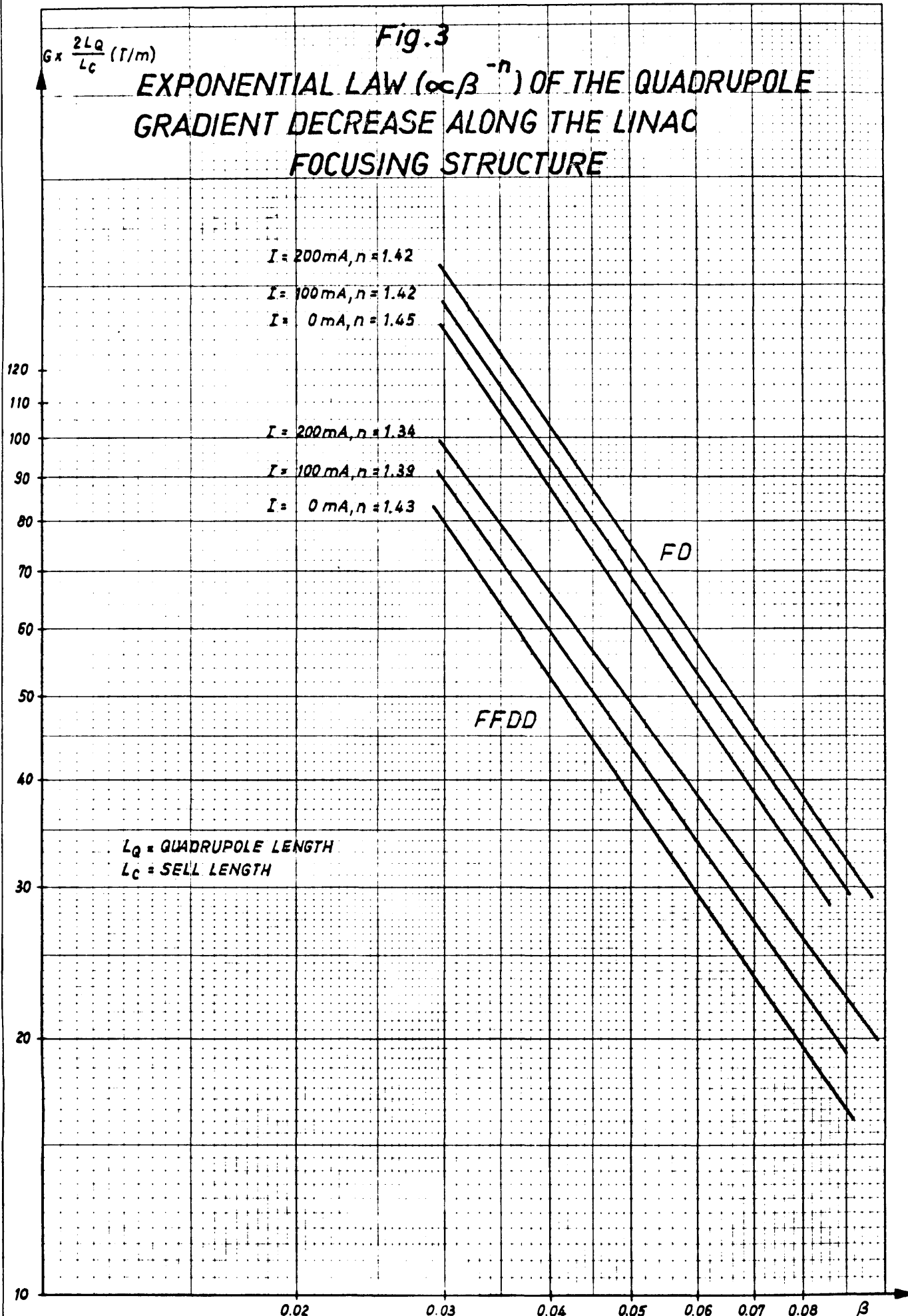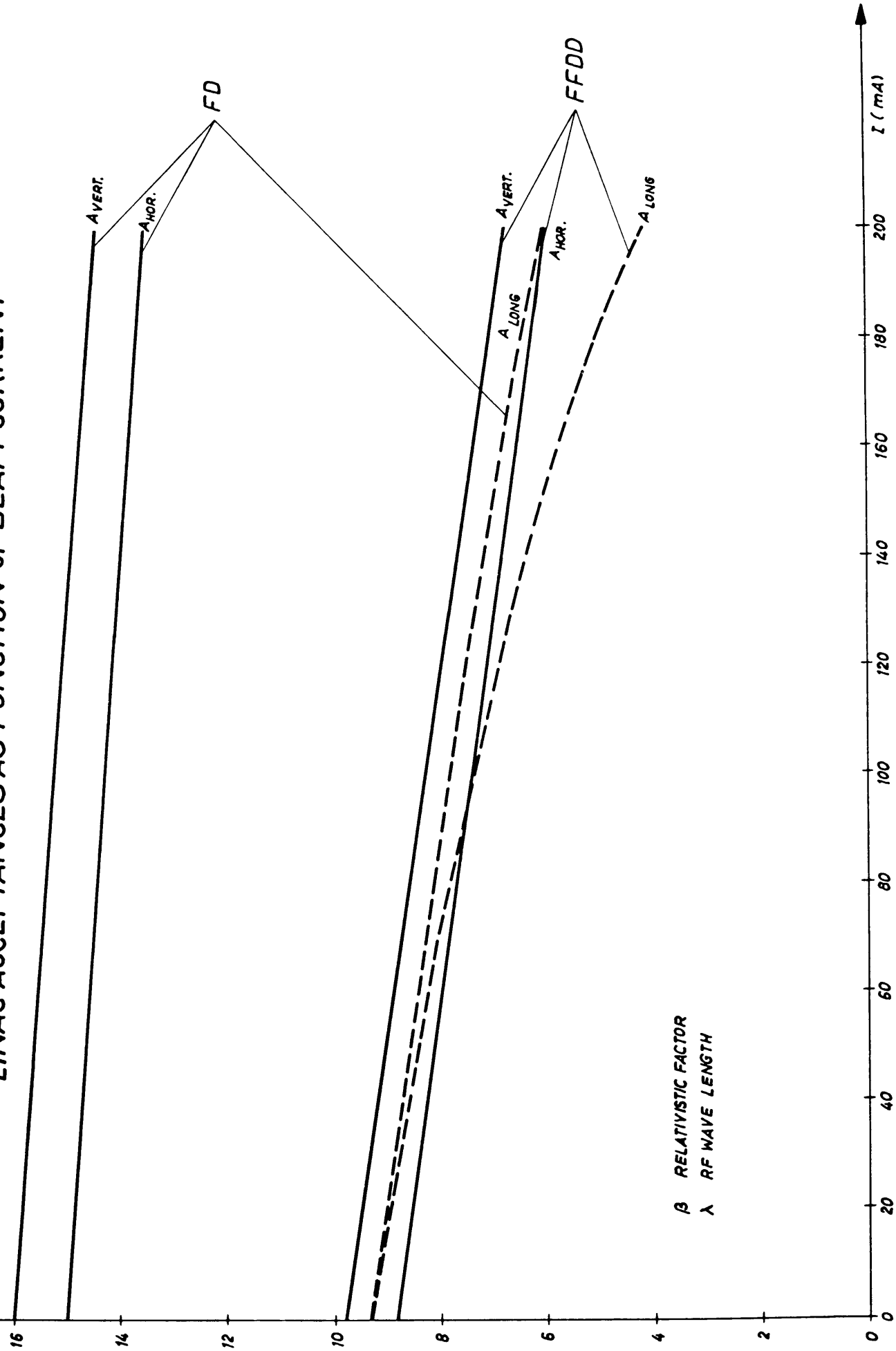CORRESPOND TO THE CERN 3 MeV
EXPERIMENTAL LINAC

Fig.3

EXPONENTIAL LAW ($\propto \beta^{-n}$) OF THE QUADRUPOLE GRADIENT DECREASE ALONG THE LINAC FOCUSING STRUCTURE

$G \times \frac{2 L_Q}{L_C}$ (T/m)

$I = 200\,mA, n = 1.42$
$I = 100\,mA, n = 1.42$
$I = \quad 0\,mA, n = 1.45$

$I = 200\,mA, n = 1.34$
$I = 100\,mA, n = 1.39$
$I = \quad 0\,mA, n = 1.43$

FD

FFDD

L_Q = QUADRUPOLE LENGTH
L_C = SELL LENGTH

Fig. 4

LINAC ACCEPTANCES AS FUNCTION OF BEAM CURRENT

$A.\beta.\lambda \; (mm^2 \times rad)$

$I \; (mA)$

FD

FFDD

$A_{VERT.}$

$A_{HOR.}$

$A_{VERT.}$

$A_{HOR.}$

A_{LONG}

A_{LONG}

β RELATIVISTIC FACTOR
λ RF WAVE LENGTH

Fig. 5

MATCHING PARAMETERS AT THE LINAC INPUT AS FUNCTION OF BEAM CURRENT

FOCUSING STRUCTURE : FFDD

α, β CLS PARAMETERS

$ENV_{H(V,L)}$ HORIZONTAL (VERTICAL, LONGITUDINAL) BEAM ENVELOPE

Fig. 6

MATCHING PARAMETERS AT THE LINAC INPUT AS FUNCTION OF BEAM CURRENT

FOCUSING STRUCTURE : FD

α, β CLS PARAMETERS

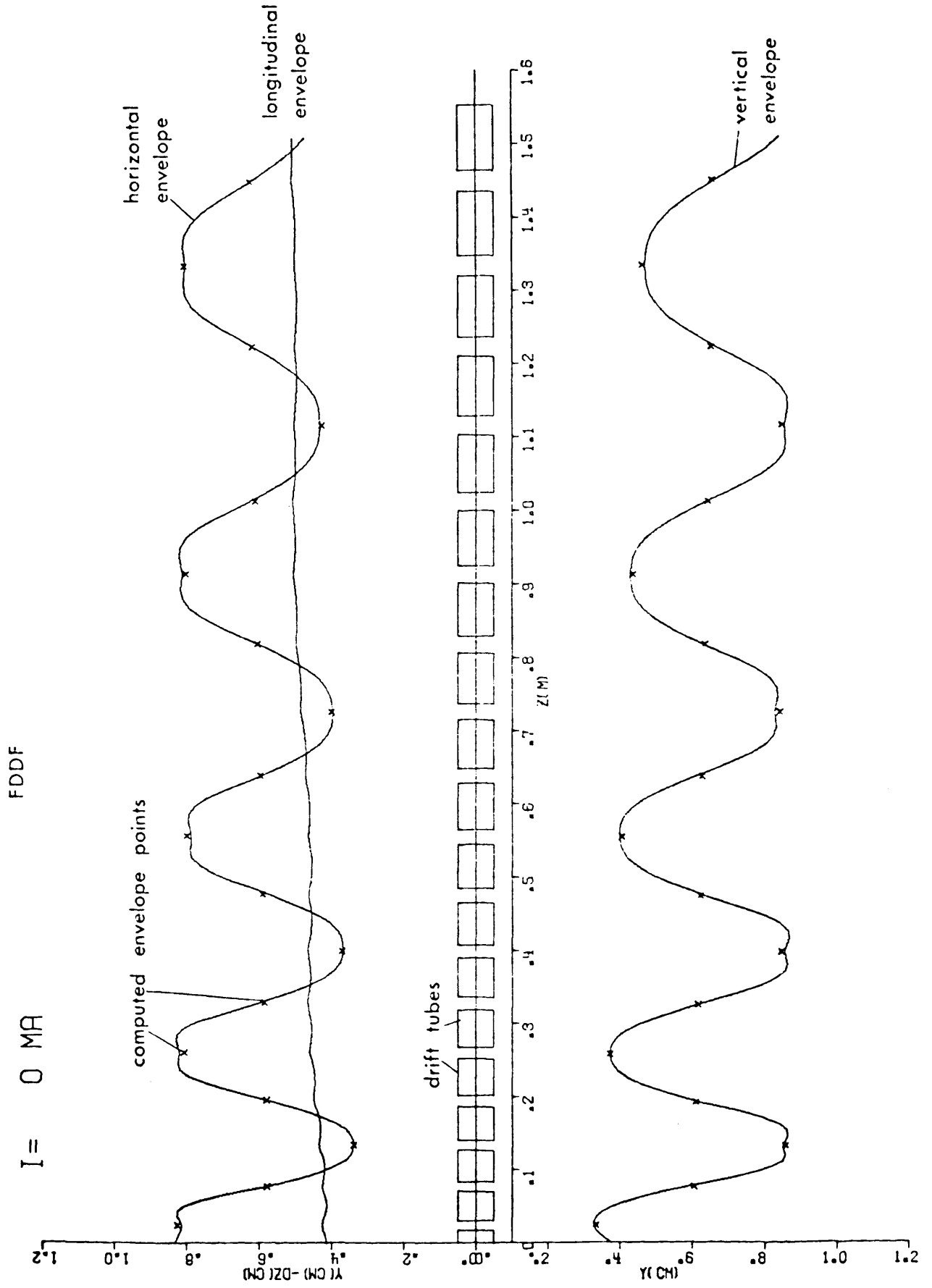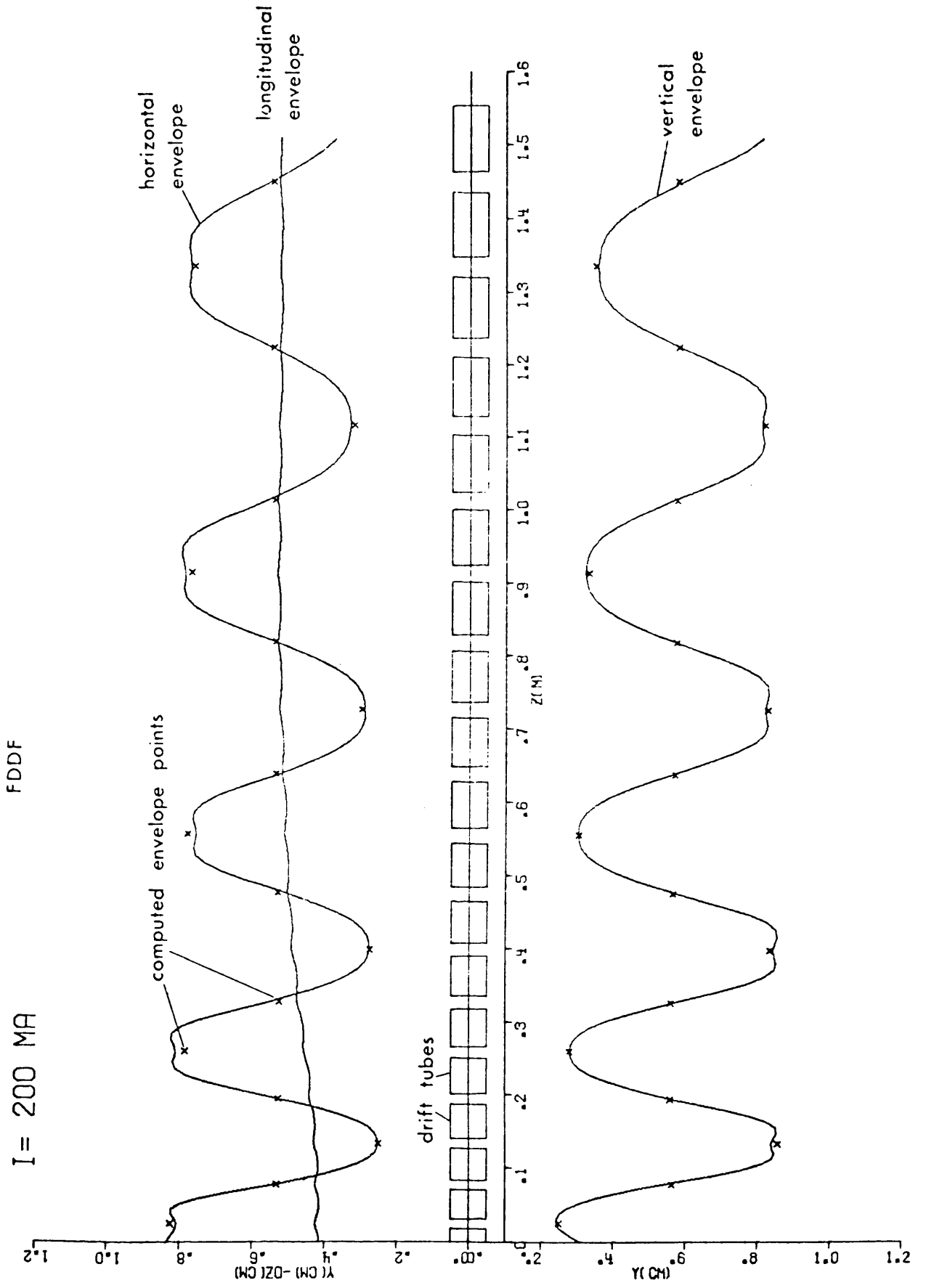$ENV_{H(V,L)}$ HORIZONTAL (VERTICAL, LONGITUDINAL) BEAM ENVELOPE
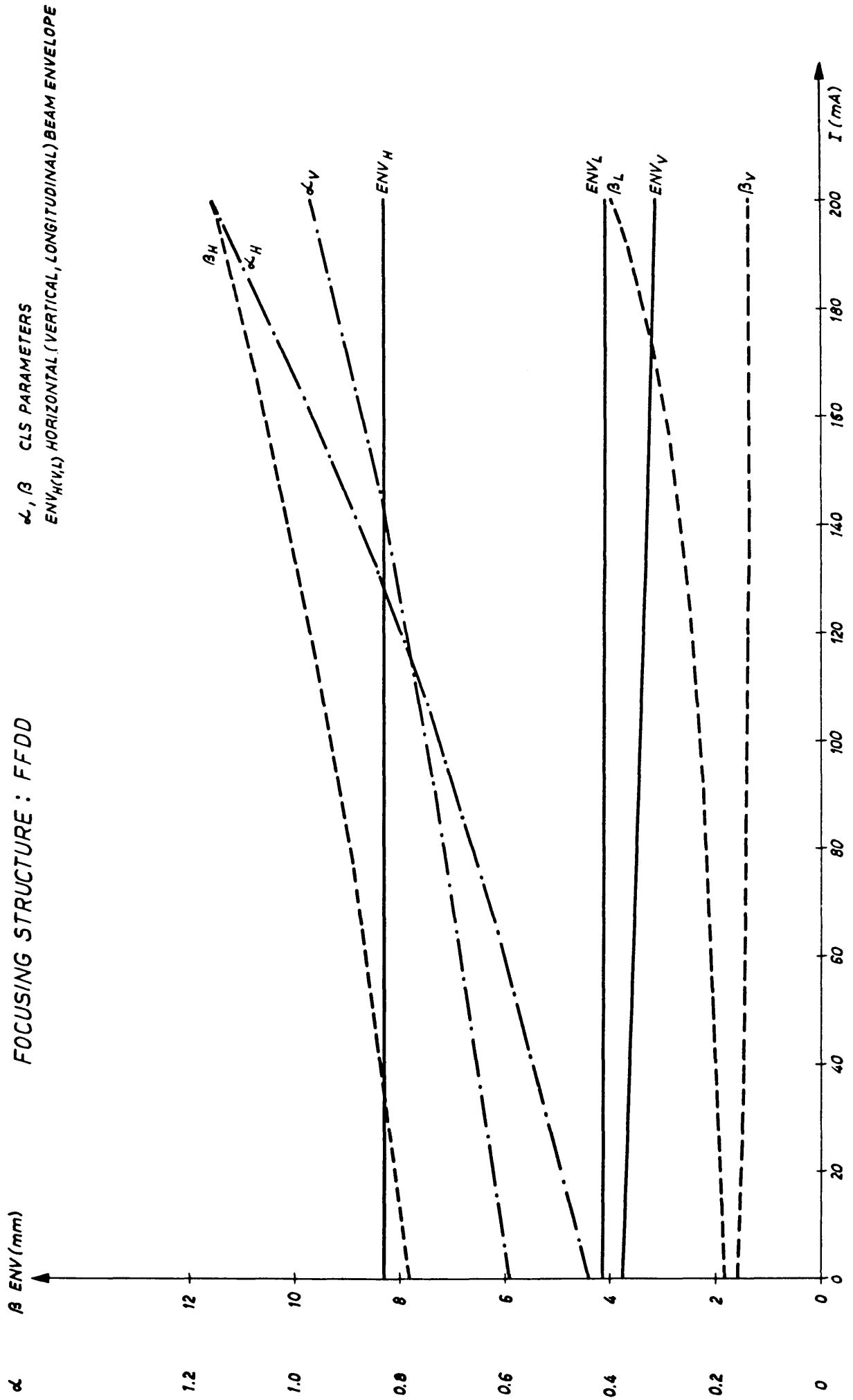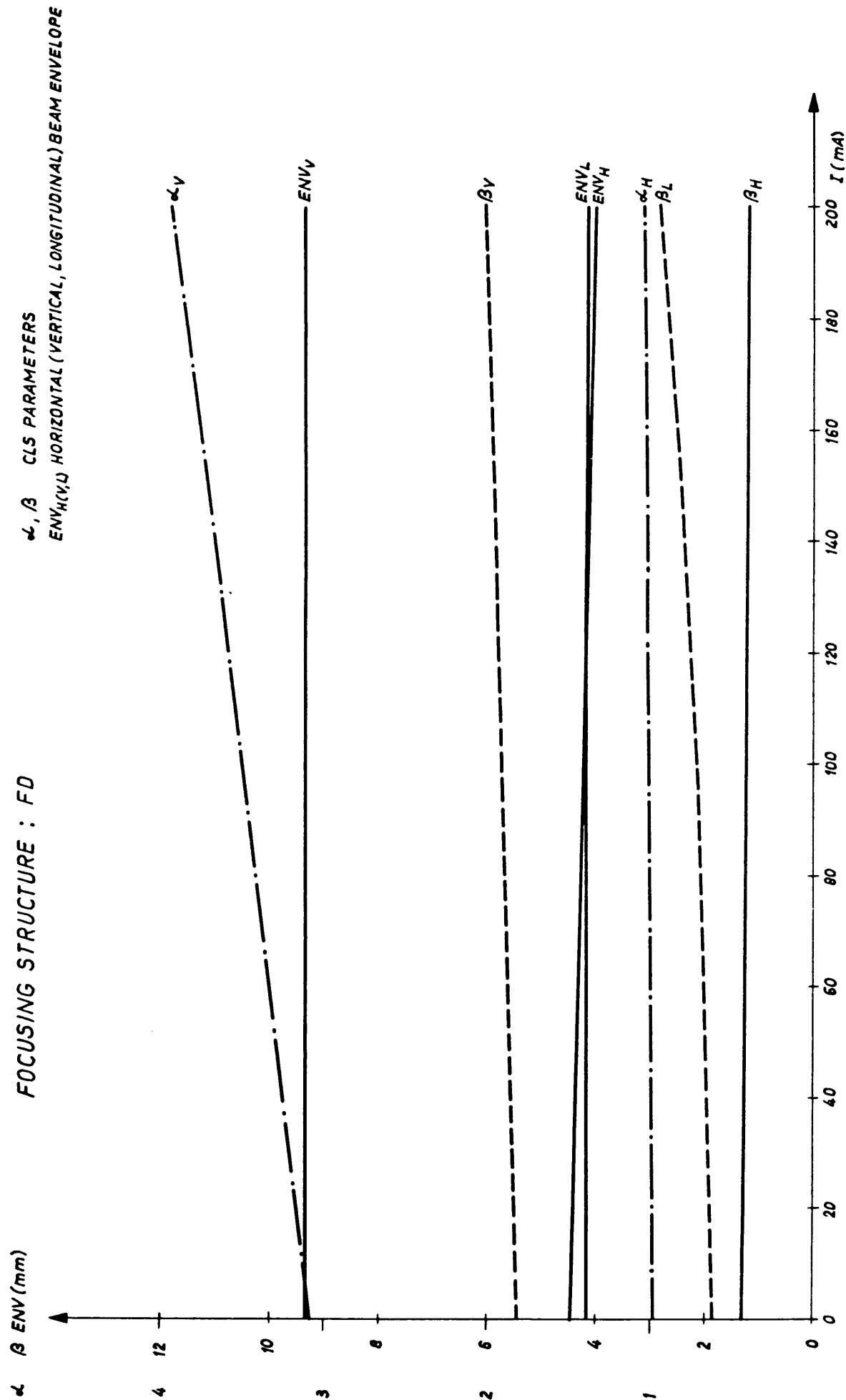
FIG.7

Envelopes of a transfered matched beam

FIG. 8

Envelopes of a transfered matched beam

FIG. 9

Envelopes of a transfered matched beam

I= 0 MA   FD

horizontal envelope

longitudinal envelope
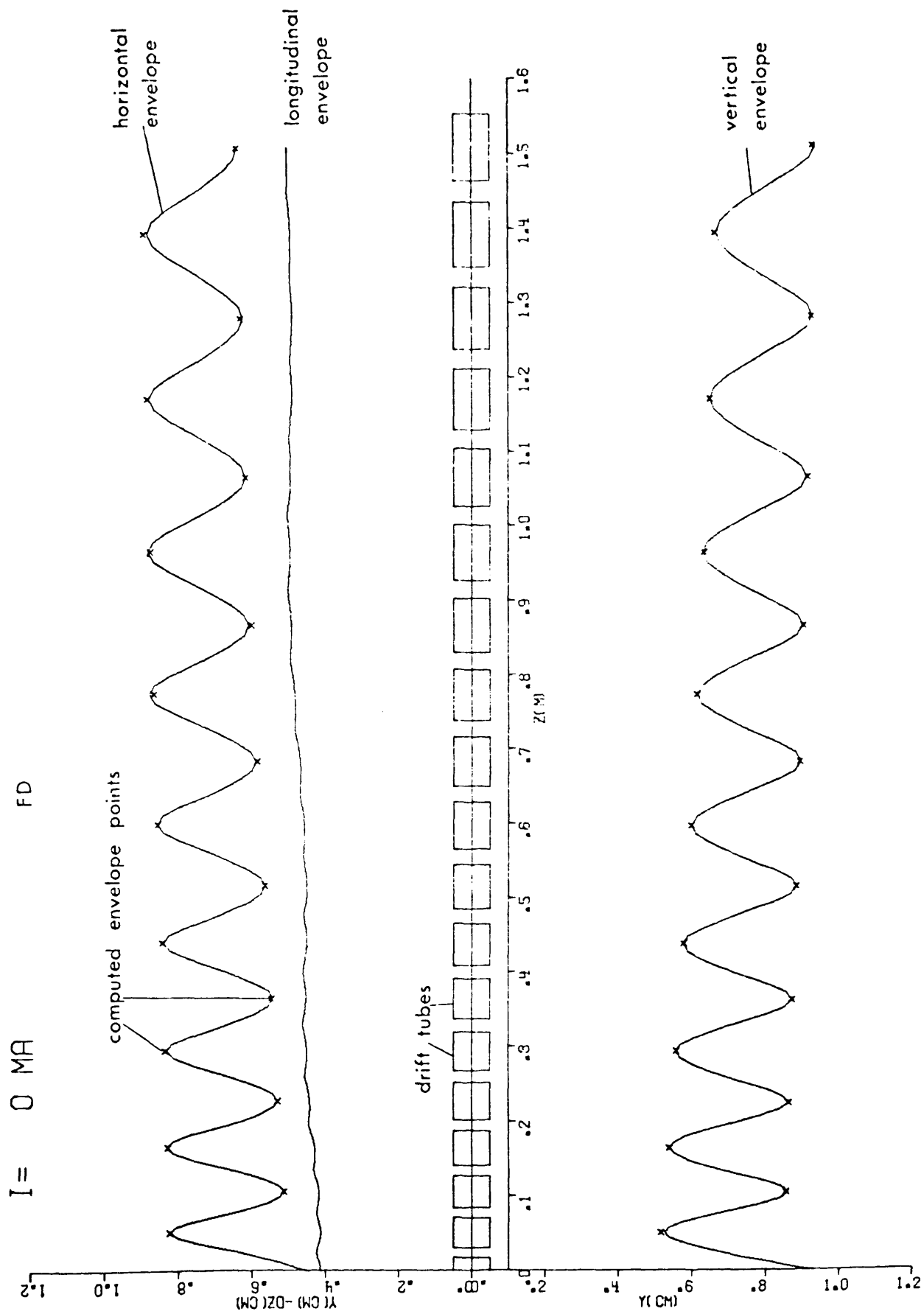
computed envelope points

drift tubes

vertical envelope

Z(M)

Y(CM) -0Z(CM) Y(CM)

FIG.10

Envelopes of a transfered matched beam