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COMPUTER SIMULATION OF SPACE-CHARGE EFFECTS IN THE LINAC-BOOSTER TRANSFER LINE

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1. INTRODUCTION

The purpose of the program BEAM is to simulate, taking into account space-charge effects, the motion of a proton beam during its transfer from the Linac to the Booster. The transport line may consist of any required combinations of magnetic lenses, drift spaces and RF cavities.

The program traces a single bunch of protons represented here as a limited number of particles. The space charge forces are obtained by directly computing the interaction between all particles by an approximate method.

The main technical points of interest are as in the report [1] on the program BUNCH, from which BEAM is derived. Main points of difference are discussed in this report.

2. OUTLINE OF COMPUTATIONAL METHOD

The program simulates the dynamics of a proton beam as it passes through the various sections of a long beam line, the components of which are specified as data.

The initial dimensions of the simulated beam are deduced from measured data obtained at the end of the linac. Note that this point does not necessarily coincide with the origin of the computation.

The phase space coordinates of representative particles are assumed to belong to a 6-dimensional Gaussian distribution in $x, x' \left(= \frac{dx}{dz}\right), y, y' \left(= \frac{dy}{dz}\right), \varphi$ and W at the end of the linac, and the phase space is filled using this model.

The space charge forces are computed as follows :

a) A box is constructed which just fits the bunch in 3-dimensional real space, and this box is divided into a large number of small match-box shaped cells. The number of cells is fixed, but the dimensions of the box are variable. This arrangement is called the space charge 'cage'.

When doing a space charge computation, the longitudinal dimension of the bunch is reconstructed by interpreting the phases as longitudinal coordinates according to :

$$\Delta z = \frac{\beta\lambda}{2\pi} \Delta\varphi \quad .$$

This corresponds to switching back the independent variable from z to time (phase). The transverse coordinates are approximately taken as if all particles at the time when space charge forces are evaluated (i.e. when the centre of the bunch crosses the section under consideration) were on the same plane.

To check this approximation, the program BEAM was run against the program BUNCH, which does an exact time integration of the motion. The results in beam envelopes were identical, although small differences in scatter diagrams were noted.

The advantage of our method of movement is that the logic is much simpler, the limitation being that the transverse displacements must be small over a distance of the order of the bunch length.

3.3. METHOD OF DECIDING NEXT TRANSFER LINE COMPONENT

As the program has to have flexible logic, it was decided that the next component of the transfer line should be read in as data. Therefore, each data card contains the component code and essential parameters. The layout of the data card is explained in paragraph 3.9.

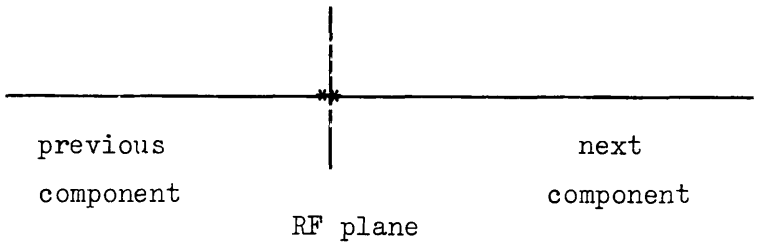
The logical sequence is :

a) The bunch centre is moved forward a distance dz at a time until it reaches the end of a component.

b) The main program then reads the next component as data and calls the subroutine which treats it. Each component can be divided into several steps dz, at each step a space charge calculation being carried out.

3.4. EQUATIONS OF MOTION IN AN RF CAVITY

The equations of motion are identical as those given in [1]. However, the cavity is treated as a plane at which impulsive corrections are made as follows :



a) The bunch is moved to the plane of the cavity,

b) Impulsive corrections to velocities are made according to the following FORTRAN type of replacement expressions *

$$X' = X' + \Delta X'$$
$$Y' = Y' + \Delta Y'$$
$$W = W + \Delta W$$

where $\Delta X'$, $\Delta Y'$ are given as in [1] and ΔW is function of the particle phase and of the buncher voltage.

c) The bunch then moves to the next component.

The particles are assumed to stay in position during this impulse and hence no recalculation of position and space charge acceleration is made at the cavity plane.

3.5. EQUATIONS OF MOTION IN A DRIFT SPACE

The formulae used are :

$$\Delta W_{long} = F_{lav.} \cdot dz$$

where F_{lav} is average longitudinal force over the step dz

$$F_{lav} = 0.5\ m\ (acc(z + dz) + acc(z))$$

and acc(z) is longitudinal space charge acceleration at z etc.

* In sects. 3.5. and 3.6. use is also often made of 'equations' which must be interpreted like FORTRAN replacement expressions. These are numbered F(n) to avoid confusion with genuinely algebraic expressions.

$$\Delta r' = \frac{dz}{mv^2} (F_r - r'F_z)$$

where r is x or y and v is the longitudinal velocity of each particle.

F(i) $VX = VX + 0.5 \frac{dZ}{VZ} (AX - AZ. \frac{VX}{VZ})$

F(ii) $VY = VY + 0.5 \frac{dZ}{VZ} (AX - AZ. \frac{VY}{VZ})$

F(iii) $VZ = (VZ^2 + AZ.DZ)/VZ$

F(iv) $X = X + \frac{dZ}{VZ} . VX$

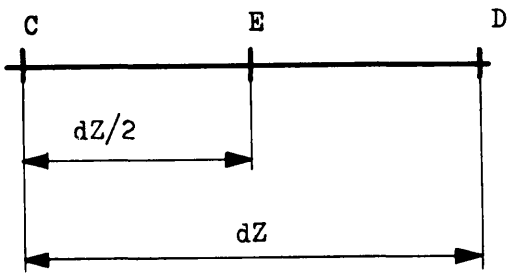
F(v) $Y = Y + \frac{dZ}{VZ} . VY$

F(vi) $\varphi = \varphi + d\varphi$

where $d\varphi = 2\pi\ fdz(1./VZ - 1./V)$
and V is velocity of bunch centre.

Symbols used : VX is x velocity, AX is x component of space charge. acceleratio and correspondingly for the other coordinates.

The computation is carried out as follows :



(1) The operations (i) to (iii) above are carried out once during which the velocities of all particles are calculated at E using the space charge forces at C.

(2) The coordinates X,Y,φ are transferred from C to D using the velocitie at E, assumed as average velocities for the step dZ.

(3) New space charge forces are calculated at D using the above new positions.

(4) The velocities are now transferred from E to D using the accelerations computed at D.

Using the above method both position and velocity are calculated to second order of accuracy.

3.6. EQUATIONS OF MOTION IN A QUADRUPOLE LENS

For a quadrupole lens, the formulae used in [1] apply, the independent variable being changed from t to Z.

The transfer of longitudinal coordinates is as in previous sections. Radially one has after a step dZ in a lens focussing in X and defocussing in Y :

$$F(vii) \qquad X = X.\cos kdZ + X' \frac{\sin kdZ}{k} + \alpha_x (1 - \cos kdZ)$$

$$F(viii) \qquad X' = - X.k \sin kdZ + X'.\cos kdZ + \alpha_x k.\sin kdZ$$

$$F(ix) \qquad Y = Y.\cosh kdZ + Y' \frac{\sinh kdZ}{k} + \alpha_y (\cosh kdZ - 1)$$

$$F(x) \qquad Y' = Y.k \sinh kdZ + Y' \cosh kdZ + \alpha_y \sinh kdZ$$

where $k^2 = qG/m_0\gamma v$, $\alpha_x = A_x/(kv)^2$, $\alpha_y = A_y/(kv)^2$

and q is elementary charge
G is lens strength
v is (average) longitudinal velocity
A_x is X space charge acceleration, assumed constant over the step dZ.
Similarly for A_y.

For the purpose of distinguishing between types of lenses in the program, a lens is considered positive if it focusses in x and defocusses in y and negative vice versa. In the latter case, the above equations hold with x and y interchanged. This sign convention agrees with TRANSPORT program [3].

4. FORMAT OF DATA CARDS

The basic input of data is as in [1]. The main differences are :

```
- 1 -        999  READ 1000, JOBNUM1, FREQ, INDEX, NOPT, ZREF
            1000  FORMAT (16, F10.0, 214, F10.3).
```

JOBNUM1 is a 3 digit number given by the user, only the most significant digit is important inside the program.

If JOBNUM1 = 0.. the program terminates.

" " = 6.. the program makes a dead start from the end of the linac and taped output is produced.

" " = 7.. the program makes a predetermined start using an input type. Tapes output is produced.

If JOBNUM1 is none of the above a normal run takes place with no tapes. The last data card must however have JOBNUM1 = 0.. to terminate correctly the program.

NOPT is the number of the output option the user wishes. These are :

1) Emittance tests for x,x',y,y' in XOUT1
2) Beam envelopes in x,x',y,y'
3) Histograms in φ, x space
4) Histograms and envelopes
5) Envelopes and statistic in XOUT2
6) Histograms and emittances
7) Statistics only
8) No output.

The option NOPT is fixed for the complete line.

INDEX is the number of components in the transfer line.

ZREF this is the starting position and is always zero except when JOBNUM1 is 7.. and a predetermined start is made from tape.

- 2 - READ 1005, ((C(I,K),I = 1.5), K = 1, INDEX)

1005 FORMAT (5F10.4)

This is the reading in of the transfer line parameters. Each card contains a complete set of data for each component. The fields are :

KODE	ZEND	RBORE	NSTEP	QMG ECAV

KODE is the code number of the element namely :

2 is drift

3 is buncher, debuncher

4 is lens

ZEND is the distance of the end of the component from the origin of the line in metres.

RBORE is the radius of the vacuum pipe of the component in metres.

NSTEP is the number of space charge computations wanted in that component.

QMG is the strength of the lens in Tesla/m.

ECAV is the voltage across the cavity gap in MeV.

For a lens focussing in x and defocussing in y, QMG is positive and negative vice versa. For a drift space this field is blank.

- 3 - READ 7715, (TITLE1(JI), JI = 1,36)

READ 7715, (TITLE2(JI), JI = 1.36)

7715 FORMAT (12A6)

These are headings for histograms in y and x respectively.

- 4 - READ 100, ((MU(I), V(I)), I = 1,6)

100 FORMAT (2F10.6)

These are the means and standard deviations for initial values of φ, W, x, x', y, y' respectively.

The units are :

coordinate	φ	W	x	x'	y	y'
Units	degrees	MeV	mm	mr	mm	mr

- 5 - READ 101, ((CORR(I,J), J = 1,6), I = 1,6)
101 FORMAT (6F10.6)

This is the reading of the initial correlations between the 6-variables. The order is as in - 4 - above.

- 6 - READ 106, ((D(I,J), J = 1,6), I = 1,6)
106 FORMAT (6F10.5)

Since the original filling of the phase space model may be made at different point from the experimental measurements, a matrix D is constructed which transfers the Gaussian 6-dimensional distribution to the starting point.

The matrix D has all main-diagonal entries equal to unity, all sub-diagonal and super-diagonal entries zero except for the x-x' and y-y' entries which are positive or negative depending on the position of the starting plane with respect to the measurement plane. Distances are given in mm.

5. FUTURE IMPROVEMENTS

The improvements which can be made to the program come into 2 classes :

i) Improvements in programming such as using assembler language routines, faster space charge routines, and a larger increase in the number of particles traced.

ii) Due to the flexibility of the program present structure, the user can easily add new subroutines. An obvious example is the inclusion of a routine which will simulate bending magnets.

ACKNOWLEDGEMENTS

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REFERENCES

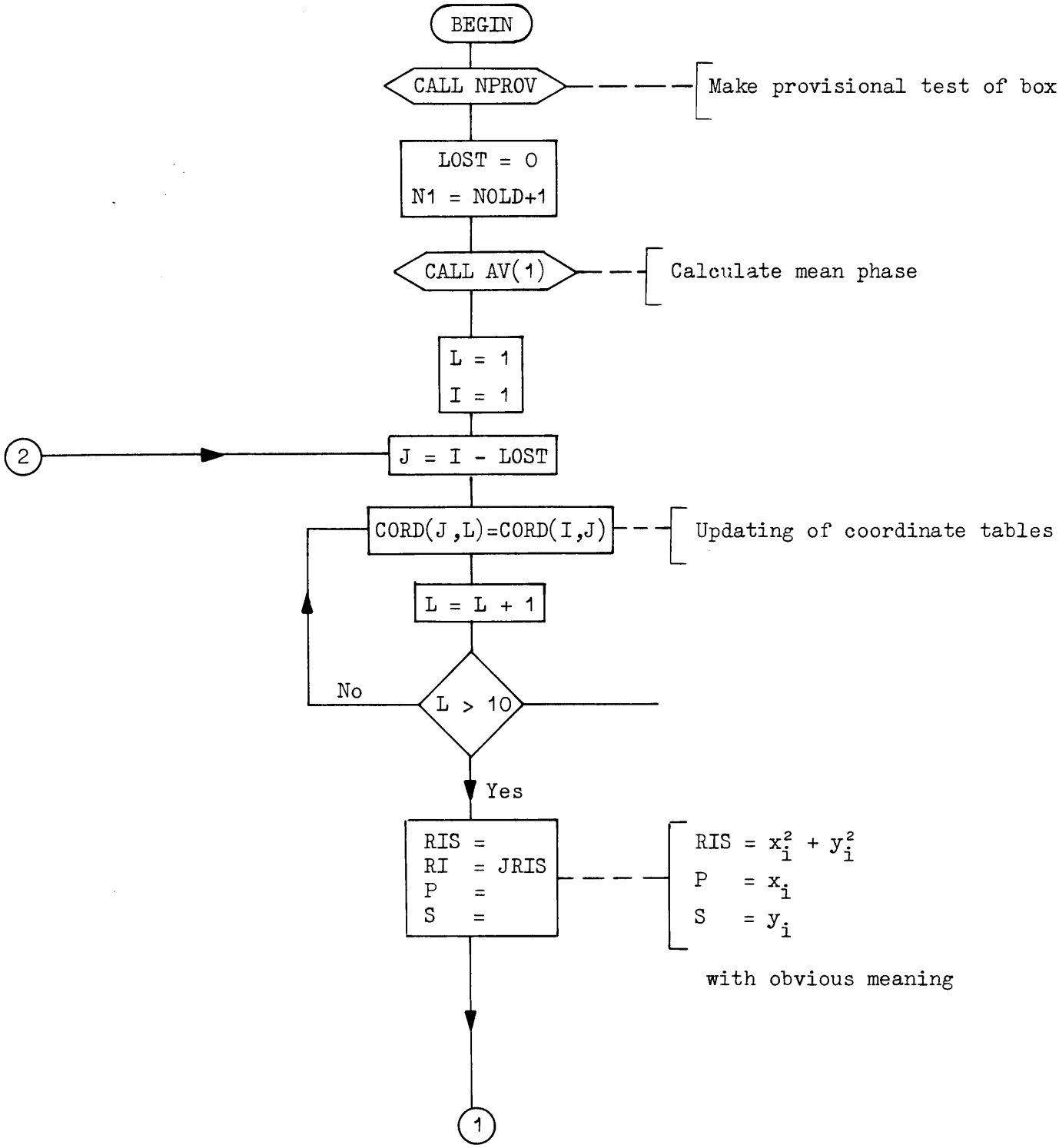
[1] F. Vermeulen, "Computer Simulation of Beam Bunching with the Inclusion of Space Charge Forces and Quadrupole Lenses", CERN, MPS/Int. LIN 67-5.

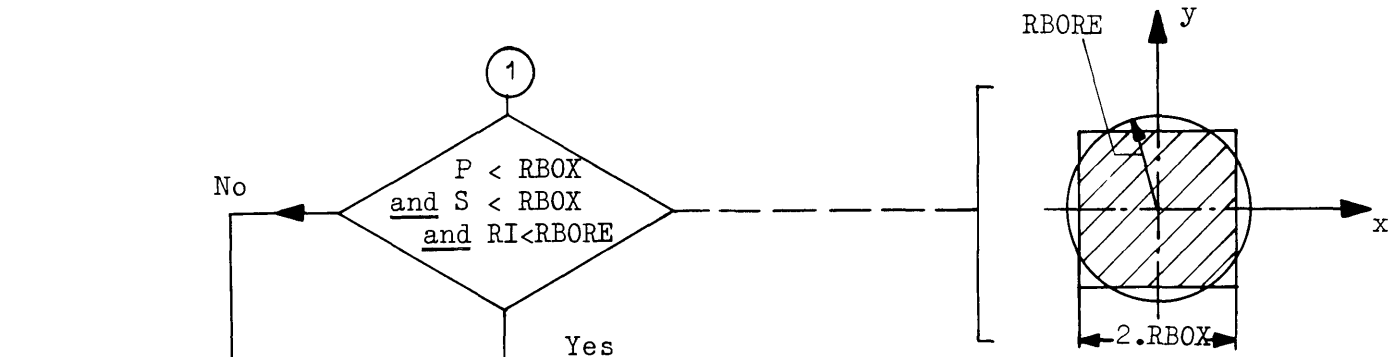
[2] W.T. Eadie, M. Martini, "Statistical Population of the Linac Beam Transverse Phase Space", CERN, MPS/Int. LIN 68-9.

[3] C.H. Moore, S.K. Howey, H.S. Butler, "A Computer Program for Designing Beam Transport Systems". To be published.

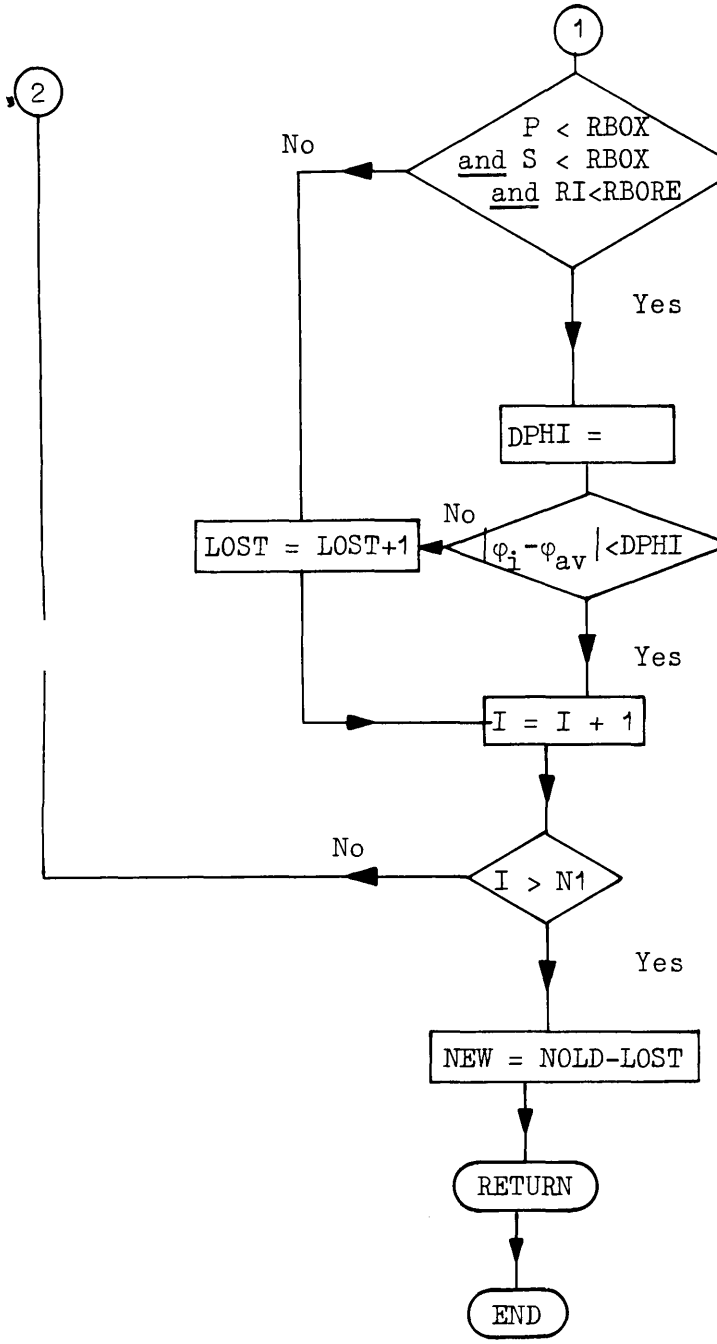
Distribution (open)

B) FLOW DIAGRAM FOR NEW





This checks particle is inside shaded area



This checks if particle will be outside box longitudinally at end of step.