SOLOPT: a Transport code with space charge in a solenoidal field

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Introduction

SOLOPT (SOLenoid OPTics) is a code that tracks particles in a transfer line where the focusing is performed by solenoids (cylindrical symmetry) in the presence of space charge. The magnetic focusing is introduced using Transport matrixes, the space charge by non linear kicks, calculated in the laminar approximation; this approach is well suited to study low energy electron beams, where the space charge overcomes the effect of thermal velocities.

The code has been written to design (and operate) the test line of the CTF − DC gun [1]; in this case it is used as a postprocessor of EGUN [2].

The magnetic field

The passage of a particle through a solenoid of length z is described by the linear transformation [3]:

$$\vec{w}(z) = M\vec{w}(0) = M_F R\vec{w}(0) \tag{1}$$

with:

$$R = \begin{pmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{pmatrix}, \quad M_F = \begin{pmatrix} C & S/\alpha & 0 & 0 \\ -S\alpha & C & 0 & 0 \\ 0 & 0 & C & S/\alpha \\ 0 & 0 & -S\alpha & C \end{pmatrix} \quad and \quad \vec{w} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$

and where $C = \cos(\alpha z), S = \sin(\alpha z), \alpha = \dfrac{B}{2(B\rho)}$, $(B\rho)$ is the magnetic rigidity and $'$ denotes the differentiation respect to the longitudinal coordinate z: there is not acceleration and in paraxial approximation it holds $z(t) = z_0 + \beta ct$. Equation (1) suppose that both in $z = 0$ and in z the magnetic field is null, while in between it is homogeneous and equal to B. The edge fields both at the beginning and at the end of the solenoid are then included (Fig.1); to stress this aspect equation (1) can be rewritten as:

$$\vec{w}(z^+) = M\vec{w}(0^-)$$

where 0^- means $0-\varepsilon$ when ε tends to 0, and z^+ indicates z + ε. To consider the case of a longitudinal field function of z SOLOPT divides the integration range in segments of length Δz and calculates the dynamics in each segment by the expression (1).

If the particles are generated in a point where $B \neq 0$, like a cathode or an electron − positron converter immersed in a magnetic field, this description has to be lightly modified; to satisfy the conditions of validity of equation (1) we have to calculate the coordinates that the particle would have outside the field. We have then to apply the inverse of the matrix that describes the edge field at the entrance of the solenoid:

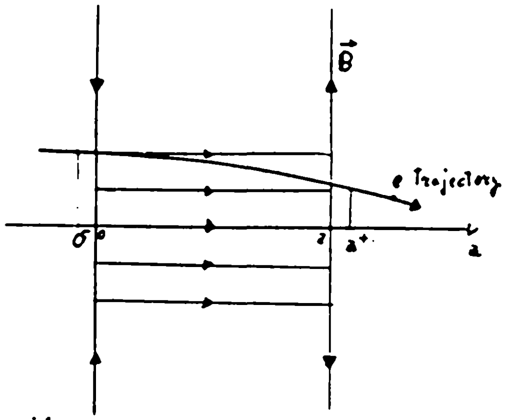


Figure 1: Field of a solenoid.

$$\vec{w}\,(z^+) = M M_{edge}^{-1}\,\vec{w}(0^+) \qquad with \qquad M_{edge} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 1 & 0 \\ -\alpha & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where 0^+ has been used to indicate that the particle is generated inside the field. In SOLOPT there is an option to consider this case.

The laminar approximation and the space charge

In SOLOPT the beam is considered to be laminar. By definition it means that the particle trajectories do not cross [4]: the beam distribution (with cylindrical symmetry) is then such that, at each z, r′ is a single valued function of r, r being the radius $\sqrt{x^2+y^2}$. This assumption is correct only if the thermal spread of velocities can be neglected.

As an example of motion laminar almost everywhere (i.e. except in a discrete set of points) let's consider a linear transfer line where $z=0$ is a point source; the dynamics is described by the transformation:

$$\begin{pmatrix} r(z) \\ r'(z) \end{pmatrix} = \begin{pmatrix} a(z) & b(z) \\ c(z) & d(z) \end{pmatrix} \begin{pmatrix} 0 \\ r'(0) \end{pmatrix} \tag{3}$$

so that at the generic z holds the relation:

$$r'(z) = \frac{d(z)}{b(z)} r(z). \tag{4}$$

The beam behavior is sketched in Fig. 2. At the image points of the source the laminarity is broken (and the denominator of (4) becomes 0). This very particular situation is of practical interest: for example a flat cathode can be considered in first approximation to be a point−like source located three gap lengths before the emitting surface [1]; the subsequent solenoid focusing determinates then within this approximation a laminar motion.

Consequence of the laminar motion is that the most external trajectory remains on the envelope, and its evolution describes the beam size at each z. The space charge effect can then be calculated using Gauss and Ampere theorems (Fig. 3):

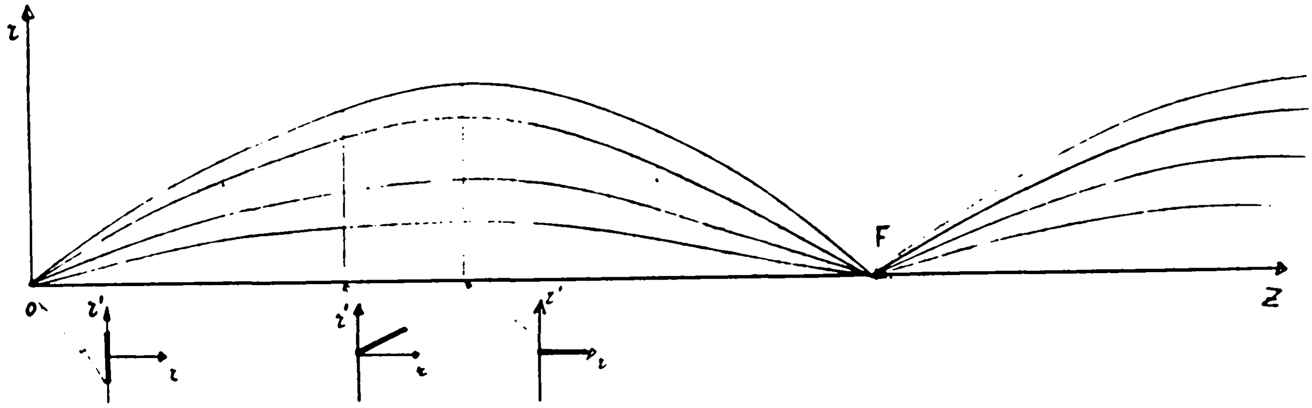


Figure 2: Sketch of a laminar beam.

$$E_r = \frac{I}{2\pi\varepsilon_0 \beta c r} \quad \text{and} \quad B_\phi = -\frac{\mu_0 I}{2\pi r}, \tag{5}$$

where I is the beam current (in the monoenergetic approximation), E_r is the radial electric and B_ϕ the transverse magnetic field. In paraxial approximation we have then:

$$m\gamma \frac{d^2 r}{dt^2} = F_r = e(E_r + \beta c B_\phi) = \frac{e}{2\pi\varepsilon_0 \beta c} \frac{I}{r\gamma^2}$$

where m is the particle rest mass, e is the elementary charge and F_r is the radial force. Taking now z as independent variable we get (MKS units):

$$r'' + \xi r = 0 \quad with \quad \xi = \frac{eI}{2\pi\varepsilon_0 m c^3 \gamma^3 \beta^3} = \frac{(2\times 10^{-7})I}{\gamma^2 \beta^2 (B\rho)} \tag{6}$$

In SOLOPT the effect of space charge in a segment of length Δz is introduced by a kick at z_0, performing the transformation:

$$\begin{aligned} x'(z_0^+) &= x'(z_0^-) + \xi \frac{x}{r^2} \Delta z \\ y'(z_0^+) &= y'(z_0^-) + \xi \frac{y}{r^2} \Delta z \end{aligned} \tag{7}$$

The code then alternatively applies at the phase space vector that represents the external trajectory the transfer matrix of the solenoidal field and the non linear kick due to the space charge.

How to run SOLOPT

SOLOPT can be run from any VXCERN account using the instruction:

$run DISK$QS:[PISENT.SOLOPT]MAIN

To prepare the input file FOR012.DAT we suggest to run:

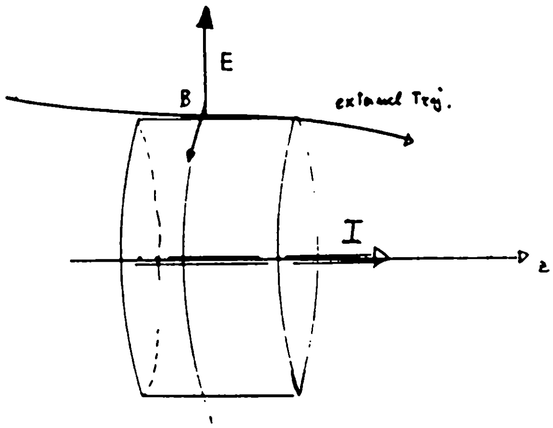


Figure 3: Calculation of the space charge effect.

$run DISK$QS:[PISENT.SOLOPT]INPUT

This program produce the input file asking questions for each data needed. We just mention that:

- deltaz is the interval between two points in the plot: the integration toperval Δz mentioned in the previous paragraphs is deltaz/10; the choice of zmax/deltaz = 10^3 is generally well suited.

- choosing the option 2 for the magnetic input one has to provide a file INB.DAT in free format with at each line B in Tesla calculated every mm.

Each run generates an output file SOLOPT.OUT and a GKS metafile SOLOPT.METAFILE.

An example: focusing by two ideal coils

This example can be run by the sequence:

$copy DISK$QS:[PISENT.SOLOPT]EXAMPLE.DAT FOR012.DAT

$run DISK$QS:[PISENT.SOLOPT]MAIN

In fig.4 are shown the input file and the ouput file relative to this run; in fig. 5 is plotted the metafile.

In this example a beam is focused by two ideal coils in such a way that in the three drift spaces is transmitted the maximum current allowed by space charge. The definition of this maximum current can be found in[1]; a characteristic feature of the maximum current condition is the ratio 0.43 between the maximum and the minimum radius.

The option 1 chosen for the graphics permits to have a longitudinal projection of motion, useful for considerations concerning the conservation of the Canonical Angular Momentum (Bush theorem [1] [4]).

Bibliography

1. Y. Baconnier, A. Pisent, 'CLIC Test Facility: the Photocathode Test Bench', Note PS/LP 89-16

2. W.B.Hermannsfeld, SLAC – 331, 1988

3. A.P. Banford 'The Transport of Charged Particles Beams', E.& F. N. Spon Limited (1966) pag. 129

4. J.D. Lawson, 'The Physics of Charged-Particles Beams', Clarendon Press, Oxford (1978) pag. 118

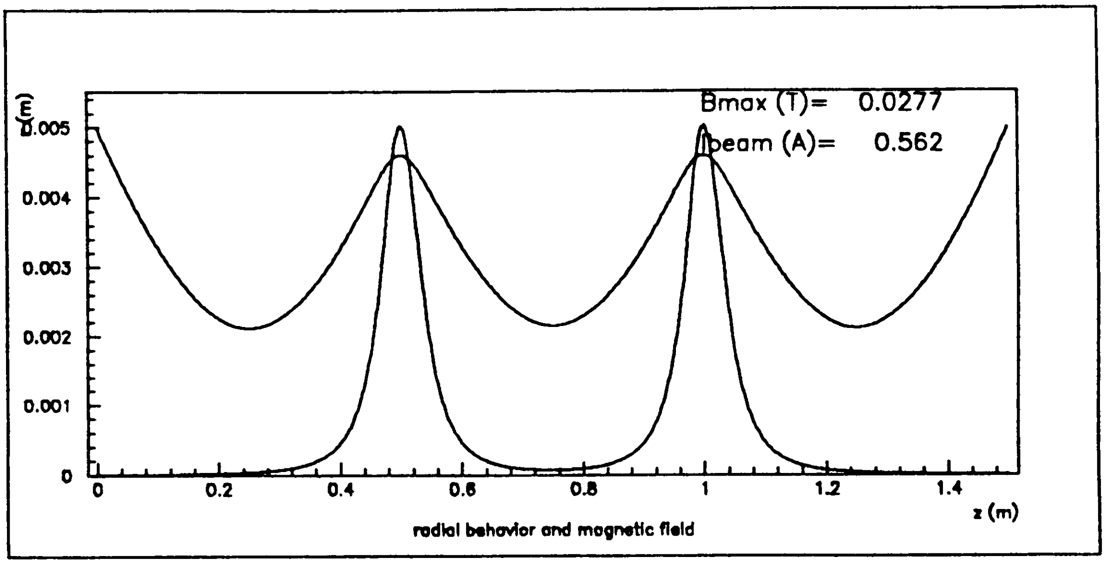
FOR012.DAT (as generated by INPUT.EXE)

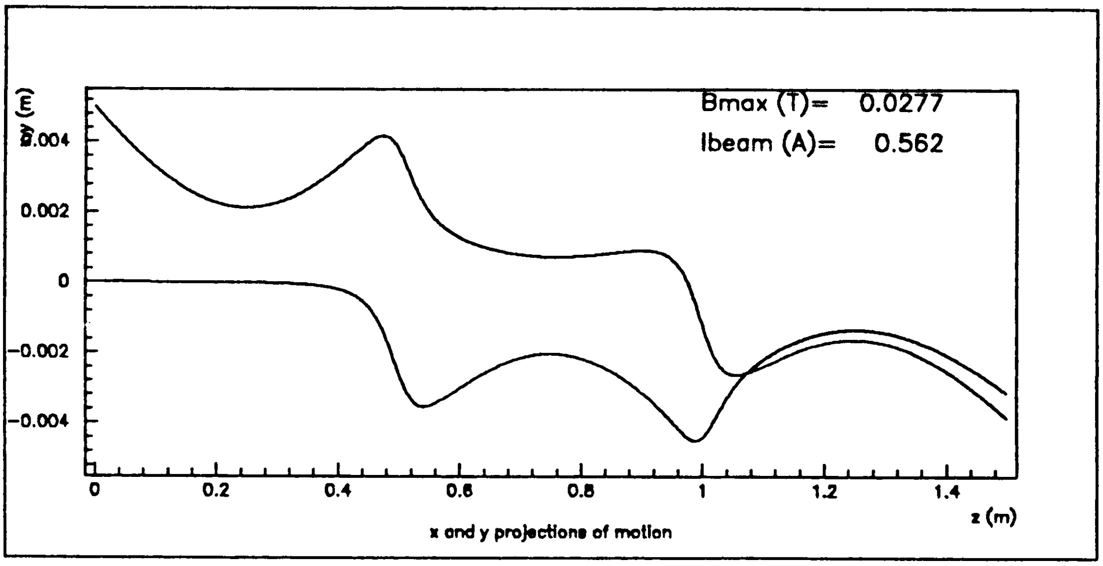
```
 1.5000000E-03   1.500000
 4.9999999E-03 -2.0000000E-02
 0.0000000E+00  0.0000000E+00
 0.5616000E+00  0.1000000
          1          1          0
          2
  2200.000      0.5000000      0.05
  2200.000       1.000000      0.05
```

SOLOPT.OUT (as generated by MAIN.EXE)

```
output of SOLOPT
________________
gamma,beta,Brho,csi
  1.195693      0.5482191      1.1172908E-03  3.5094093E-08
deltaz,zmax (m)
 1.5000000E-03   1.500000
x,dx/dz,y,dy/dz (m) (starting conditions)
 4.9999999E-03 -2.0000000E-02  0.0000000E+00  0.0000000E+00
beam current (A), particle energy (MeV)
 0.5616000      0.1000000
0 for constant field, 1 for ideal coils,2for external B
then 0 all graphics,1 no B,2 only one plot
then 1 for immersed source, 0 to close all field lines
          1          1          0
number of coils
          2
current(A),z(m),radius(m)
  2200.000      0.5000000      5.0000001E-02
  2200.000       1.000000      5.0000001E-02
bmax (T)
 2.7673244E-02
x,dx/dz,y,dy/dz (m) (final conditions)
-3.8834168E-03 -1.5518253E-02 -3.1595225E-03 -1.2647601E-02
r,dr/dz (m) (final)
 5.0063469E-03  2.0019434E-02
```

Figure 4: Input and output for the example described.





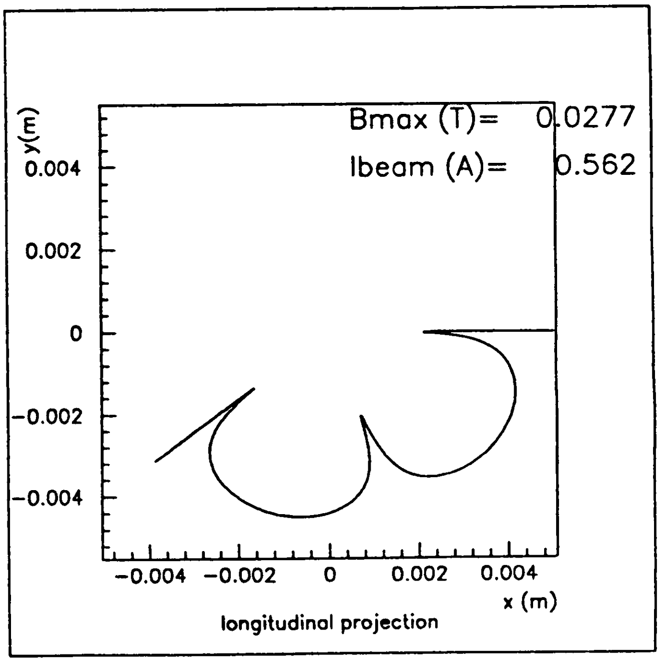


Figure 5: The graphical output.