# Predicting today's cosmological constant via the Zel'dovich-Holographic connection

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This Letter proposes a solution of the Vacuum Energy and the Cosmological Constant (CC) paradox based on the Zel'dovich's ansatz, which states that the observable contribution to the vacuum energy density is given by the gravitational energy of virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state. The novelty of this work is the use of an ultraviolet cut-off length based on the Holographic Principle, which is shown to yield current values of the CC in good agreement with experimental observations.

### I. INTRODUCTION

The Cosmological Constant (CC) problem or Vacuum Catastrophe stands for the stark mismatch between the currently observed values of the vacuum energy density (the small value of the CC) and theoretical large value of zero-point energy suggested by quantum field theory. It is also associated with a possible explanation for the dark energy driving the Universe accelerated expansion. Its theoretical value should therefore match observations.

Unfortunately, due to about 120 orders of magnitude mismatch, it bears the reputation of "the worst prediction in the history of physics" [1], see also [2–6]. This note sets out to calculate and explain the current experimental values of the CC by revisiting an original idea proposed by Zel'dovich and combining it with the Holographic Principle.

## II. THE COSMOLOGICAL CONSTANT PARADOX

Despite being responsible for showing everyone that space-time is a dynamic entity, co-evolving with the matter that inhabits it, Einstein, for once, was not prepared for the idea that the entire Universe could be a dynamic entity as well. As a result, when faced with the irrefutable evidence that his equations did not admit a static universe as a solution, he resolved to add an "ad-hoc" term, the CC, for obtaining one. Shortly later, however, it was for experimental data to show that our Universe is actually expanding, at which point he famously termed the CC his "biggest blunder." However, to say with Joyce, "errors are the portal of discovery," and the CC has taken central stage in modern physics, mostly because of its potential connections with dark energy and the accelerated expansion of the Universe. Not without a huge riddle, though: the CC has dimensions of an inverse length squared and since its physical origin is generally attributed to spacetime fluctuations at the Planck scale, it is natural to assume that its value in Planck units should be of order 1, namely:

$$\Lambda L_{\rm P}^2 \sim 1. \tag{2.1}$$

By contrast, for the product  $\Lambda L_{\rm P}^2$  cosmological observations deliver a value of  $\sim 10^{-122}$ , namely 122 orders of magnitude smaller, making of (2.1), as mentioned before, "the worst prediction ever in the history of physics" [2–6]. Despite intensive efforts, the puzzle is still standing. Here, we begin by observing that  $10^{-122}$  is surprisingly close to the square of the ratio of the Planck length and the Universe radius  $10^{2(-35-27)}=10^{-124}$ , thereby providing a strong clue towards a theory where the "natural" Eq. (2.1) would be replaced by a much more accurate prediction:

$$\Lambda L_{\rm P}^2 = \left(\frac{L_{\rm P}}{L}\right)^2 \,, \tag{2.2}$$

where  $L \sim 10^{27} m$  is the current radius of the Universe. In the following, it is shown that the above relation

in the following, it is shown that the above relation is precisely what one obtains by a straightforward combination of a previous argument by Zel'dovich, with the Holographic Principle.

## III. REVISITING ZEL'DOVICH'S ANSATZ

Zel'dovich argued that since the bare zero-point energy is unobservable, the observable contribution to the vacuum energy density,  $e_v$ , is given by the gravitational energy of virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state [7, 8]. Therefore:

$$e_v(r) \sim \frac{Gm^2(r)}{r} \frac{1}{r^3} \,.$$
 (3.1)

In the expression above, also according to Zel'dovich, the vacuum contains particles with an effective density  $m(r)/r^3$ . Additionally, by considering the Compton's expression for the wavelength, the effective mass of the particles at scale r is taken as  $m(r) \sim \hbar/(cr)$ . Substituting this in Eq. (2.1), and defining a local CC as:

$$\Lambda(r) = \frac{Ge_v(r)}{c^4}, \qquad (3.2)$$

one readily obtains:

$$\Lambda L_{\rm P}^2 \sim \left(\frac{L_{\rm P}}{r}\right)^6 \,, \tag{3.3}$$

where  $L_{\rm P} = (\hbar G/c^3)^{1/2}$  is the Planck length. Next, we observe that the measured CC is likely to result from the average of the local CC over the full spectrum of active scales [9], ranging from a UV cutoff to an IR one, which we shall be taken here as the *current* radius of the Universe. It is worth emphasizing that in the present approach, such scales are not intended as regulatory devices to tame infinities but bear a physical meaning instead. They fix the boundaries of the spectrum of dynamically active scales arising from the collective motion of the nonlinearly interacting effective degrees of freedom[10]. As a mere analog, in fluid turbulence the IR cutoff is the macroscopic scale L of the problem, the molecular mean fee path  $L_{\mu}$  is the underlying microscale, and the Kolmogorov dissipative length  $L_d = L^{1/4} L_{\mu}^{3/4}$  represents the shortest dynamically active scale supporting coherent hydrodynamic motion. The effective UV cutoff of turbulence is therefore provided by  $L_d > L_\mu$  rather than  $L_{\mu}$ , consistently with the macroscopic (supramolecular), nature of fluid turbulence as a self-interacting classical vector field theory [11].

Further to be noted, the steep  $1/r^6$  dependence (intriguingly the same exponent of the attractive branch of molecular Lennard-Jones interactions [12]), implies that this average is largely dominated by the UV cutoff, namely:

$$\Lambda L_{\rm P}^2 \sim \left(\frac{L_{\rm P}}{L_{\rm UV}}\right)^6 \,, \tag{3.4}$$

where  $L_{\rm UV}$  denotes the (yet unspecified) UV cutoff length. To fix the latter we resort to the Holographic Principle, which states that the minimum observable length scale is not the Planck length itself but a much larger holographic scale, given by [13–15]:

$$L_{\rm H} = L^{1/3} L_{\rm P}^{2/3} \,.$$
 (3.5)

Hence, we stipulate

$$L_{\rm UV} = L_{\rm H} \,. \tag{3.6}$$

Inserting Eq. (3.6) in Eq. (3.3), and taking into account the expression (3.7), we finally obtain:

$$\Lambda L_{\rm P}^2 = \frac{L_{\rm P}^6}{L^2 L_{\rm P}^4} = \left(\frac{L_{\rm P}}{L}\right)^2,$$
(3.7)

which is exactly the sought relation (2.2). Such an expression shows that the current CCP (CC in Planck units) amounts to the second order term in a series of the cosmological smallness parameter  $L_{\rm P}/L$ . The fact that the second order term is entirely responsible for the value of the CCP reflects the  $r^{-6}$  Zel'dovich decay in space, along with the (2/3,1/3) UV-IR exponents structure of the holographic scale,  $L_{\rm H}$ , with no need of invoking any fine-tuning argument.

The present approach also provides a neat criterion to rule out other sources of vacuum energy. For instance, the energy density of Casimir fluctuations is given by  $e_{\text{Cas}}(r) \sim \hbar c/r^4$  [16, 17], yielding  $\Lambda L_{\text{P}}^2 = (\frac{L_{\text{P}}}{r})^4$ , namely, upon taking  $r \sim L_{\text{UV}} = L_{\text{H}}$  and recalling Eq. (3.4),  $\Lambda L_{\text{P}}^2 = (\frac{L_{\text{P}}}{L})^{4/3} \sim 10^{-83}$ , 40 orders of magnitude too large.

In full generality, the CCP associated with a vacuum energy scaling like  $1/r^n$ , is given by  $(L_P/L_{UV})^n$ , which recovers the desired value  $(L_P/L)^2$  under the condition

$$L_{\rm UV} = L_{\rm P}^{1-2/n} L^{2/n} \,.$$
 (3.8)

This is clearly satisfied by the Zel'dovic-Holographic combination (n = 6).

For the Casimir case, n=4, one computes  $L_{\rm UV}=L_{\rm P}^{1/2}L^{1/2}$  ( $L_{\rm UV}$ , i.e. the geometrical mean of  $L_{\rm P}$  and L), meaning that the cutoff should lie logarithmically midway between the Planck and the Universe scale, i.e.,  $L_{\rm UV}\sim 10^{-4}$  meters. We are not aware of any theoretical argument supporting such a UV cutoff based on Casimir physics. Finally, the "natural" choice  $L_{\rm UV}=L_{\rm P}$ , corresponds to  $n\to\infty$ , an infinitely steep decay, which does not look natural at all from the perspective discussed in this Letter.

## IV. TIME DEPENDENCE

All along this text, we have deliberately referred to the IR cut-off, L, as to the *current* radius of our Universe, in order to emphasize that the present analysis does not encompass Universe's entire expansion chronology. In other words, our explanation does not cover the value of the CC across full time span since the Big Bang until now, but it only addresses the value of the CC at the current time. It does so, though, by proposing an alternative and possibly more economic explanation (in terms of assumptions) as compared to previous ones [18, 19].

As it is well known, our Universe expansion chronology is parametrized by a dimensionless quantity, known as the cosmic scale factor a(t). Based on its time dependence, three characteristic eras can be distinguished: a radiation-dominated era encompassing the time scale from inflation until about 47,000 years after the Big Bang, where  $a(t) \sim t^{1/2}$ ; a matter-dominated era, between about 47,000 years and 9.8 billion years after the Big Bang, where  $a(t) \sim t^{2/3}$ ; and finally, the so called dark energy dominated era in which  $a(t) \sim e^{H_0 t}$ ,  $(H_0)$ 

being the actual value of the Hubble "constant") and where our Universe is currently undergoing an accelerated expansion as suggested by observations [20–24]. In the early universe, the mass-energy density effect was larger than the cosmological constant one, so the universal expansion was slowing down (note that any power-law expansion implies a 1/t decay of the Hubble parameter  $H = \frac{\dot{a}}{a}$ ). However, at around 6 billion years after the Big Bang, the mass-energy effect became so diluted that the cosmological constant one took over. As the universe evolved further, the mass-energy effect became less and less important as compared to the cosmological constant effect, as confirmed by experimental sources [21].

Finally, we note that the experimental evidence of a positive and small CC together with a potential eternal expansion of our Universe opens up the possibility that our Universe may asymptotically approach a De Sitter one [25]. Meaning a universe with no ordinary matter content but with a positive cosmological constant driving its expansion. In this context, our treatment might also offer a possible clue towards the explanation of the value of the CC in the mid-term and far-future regimes of our Universe.

### V. CONCLUSIONS

In this Letter, we have proposed a straightforward solution of the vacuum energy and the CC paradox, based on the Zel'dovich's ansatz combined with the Holographic Principle. The result is in nearly quantitative agreement with the experimental value, which is rather remarkable considering the simple nature of the supporting assumptions. This result suggests that, as originally proposed by Zel'dovich, the observable vacuum energy density today is given by the gravitational energy of virtual particle-antiparticle pairs, continually produced and annihilated in the vacuum state. Nevertheless, this argument alone does not suffice, as it requires a merger with the Holographic Principle, in order to select the appropriate UV cut-off length.

It should also be pointed out that Zel'dovich needed to consider the proton mass as the "typical" mass scale for producing a reasonably good order of magnitude result without a proper justification. Even then, his ansatz remained off the modern value by nine orders of magnitude. The approach suggested here does not necessitate any such restriction and provides a considerably better agreement with the experimental results. Such a simplicity might represent a potential indication of its plausibility "in the spirit of Occam's razor."

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