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RADIATIVE CORRECTIONS TO THE NEUTRON DECAY RATE AT FINITE TEMPERATURE*

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ABSTRACT

We discuss the radiative corrections to the neutron decay rate at finite temperature within the Braaten–Pisarski resummation scheme. The result is relevant for the nucleosynthesis in the early universe. We find a positive but small contribution to the primordial helium abundance.

1. Introduction

Abundances of light elements in the universe as inferred today reflect the primordial ones, which provide one of the important tests for the cosmological big-bang model[1,2,3]. Furthermore, explanations of different problems of cosmology, e.g. the missing dark matter problem[4], require the knowledge of these abundances to high accuracy as well.

In the following we consider as the key quantity the fraction of neutrons as a function of time during the evolution of the universe.

In the big-bang scenario the period relevant for nucleosynthesis[3] starts at $t \simeq 10^{-4}$ sec ($T \simeq 100$ MeV), with a heat bath of nucleons, electrons, neutrinos and photons assumed to be in thermal (due to the strong and electromagnetic interactions) and chemical (due to the weak processes) equilibrium. These weak processes are (denoted by "thermal neutron decay" in the following)

$$\nu_e + n \leftrightarrow p + e^- \quad e^+ + n \leftrightarrow p + \bar{\nu}_e \quad n \leftrightarrow p + e^- + \bar{\nu}_e \quad , \tag{1}$$

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with any number of photons and e^+e^- pairs on both sides of the processes. The n/p ratio is determined by thermal distributions; neglecting the small neutron and proton momenta it is given by

$$n(T)/p(T) = \exp(-\Delta m/T) , \tag{2}$$

where the neutron-proton mass difference $\Delta m = m_n - m_p$ is introduced.

This ratio follows the cooling of the universe as long as the weak rate $\lambda_{weak}(n \to p)$ exceeds the expansion rate λ_{exp}. The freeze-out temperature is then determined by

$$\lambda_{weak}(T_o) = \lambda_{exp}(T_o), \tag{3}$$

or approximately

$$G_F^2 T_o^5 \simeq \left[\frac{8\pi\rho}{3\ m_{Planck}^2}\right]^{1/2} , \tag{4}$$

where ρ is the total energy density. The number of neutrons is essentially fixed at this temperature, $T_o \simeq 0.7$ MeV, since the processes (1) are no longer able to maintain chemical equilibrium. After a period of pure decay, almost all neutrons are finally captured into ${}^4He$ at $T \simeq 0.086$ MeV ($t \simeq 180$ sec) by reactions like

$$n + p \leftrightarrow D + \gamma \quad D + D \leftrightarrow T + p \quad T + D \leftrightarrow {}^4He + n \ . \tag{5}$$

The amount of ${}^4He$ is to a large extent determined by the Born approximation[5] to the processes (1). However, because the primordial abundances are now extremely well determined[4], it is crucial to understand all possible theoretical uncertainties that arise from different sources[6].

Important are the radiative QED corrections to (1). The first calculations[7,8] have been performed at fixed leading order in the fine-structure constant, i.e. at $O(\alpha)$. With the recent developments of field theory at finite temperature[9,10,11,12,13,14], especially using the methods for evaluating discontinuities of self-energy graphs[15,16], it became possible to show that collinear and infrared singularities[17,18,19,20,21] cancel in thermal rates. For the thermal neutron decay at $O(\alpha)$ this has been shown explicitely in ref.[22].

The now accepted Braaten-Pisarski[23] and Frenkel-Taylor[24] thermal resummation scheme resolves the problem of large contributions of higher order Feynman diagrams by resumming them in the hot thermal loop (HTL) approximation. The aim of the present discussion[25] is to see the effect of these higher order radiative correction on T_o and thus on the n/p ratio at this temperature.

In order to calculate the weak interaction rates (including their QED corrections) for the processes (1), where the energies and temperatures are small compared to the weak boson masses, we follow the accepted approximation by applying the effective $V - A$ theory of neutron decay with point-like nucleons[26]. We furthermore take the temperatures for the neutrinos, electrons and photons to be equal, and we neglect corrections of $O(T/m_{n,p})$ by computing the rates in the infinite neutron/proton mass limit ($m_{n,p} \to \infty$)[1,2,3].

We first summarize the finite temperature formalism[14] and review the relation between the interconverting rates and the neutron self-energy[15,16,22]. Next we analyse the relevant scales appearing in this problem and identify HTL – contributions which have to be resummed, and which are not included in the previous fixed order results in refs.[7,8]. Details of our calculation may be found in ref.[25]. We conclude with quantitative results.

2. Neutron-proton rates at finite temperature

The thermal non-equilibrium distribution for the neutron as a function of time, $n(t)$, is assumed to obey the kinetic "master" equation

$$\frac{dn}{dt} = -n\Gamma_d + (1-n)\Gamma_f \ , \tag{6}$$

where Γ_d and Γ_f denote the decay and formation rates for the neutron, respectively. When using equilibrium distributions in $\Gamma_{d,f}$, the detailed balance relation

$$\Gamma_f/\Gamma_d = \exp(-q_0/T) \tag{7}$$

holds, where q_0 is the neutron energy (the small chemical potential of the nucleons is neglected). According to the work by Weldon[15] and by Kobes and Semenoff[16,27] the rate $\Gamma \equiv \Gamma_d + \Gamma_f$ may be related to the imaginary part of the thermal neutron self-energy Σ by[22,28]

$$\Gamma = -\frac{1}{2q_0} tr[\ (\not{Q} + m_n)\ Im\Sigma(Q)] \ , \tag{8}$$

with $Q = (q_0, \vec{q})$ the Minkowski four-momentum of the neutron.

Since the processes (1) conserve the number of nucleons, i.e. $n(t)+p(t) =$ constant, where p denotes the proton distribution function, $\Gamma_{d,f}$ depend on p and thus on the neutron density n. Therefore (6) is a non-linear equation. However, in order to calculate the neutron-to-proton ratio the following approximations are usually applied (and accepted): (i) only the lowest order in the weak Fermi coupling G_F is taken into account, and (ii) the recoil of the nucleons is neglected in the infinite mass limit. This allows to factorize the rates into:

$$\Gamma_d \simeq (1-p)\ \lambda^{np} \quad , \quad \Gamma_f \simeq p\ \lambda^{pn} \quad , \tag{9}$$

where λ^{np} is the sum of all partial rates per neutron (including possible QED corrections),

$$\lambda^{np} \equiv \lambda(\nu_e + n \to p + e^-) + \lambda(e^+ + n \to p + \bar{\nu}_e) + \lambda(n \to p + e^- + \bar{\nu}_e) \quad , \tag{10}$$

and λ^{pn} is the sum of the rates per proton for the inverse processes.

Introducing the ratio X_n of neutrons to all nucleons, $X_n = n/(n+p)$, and approximating $(1-p) \simeq (1-n) \simeq 1$ (valid for $m_{n,p}/T \gg 1$), the "master" equation (6) becomes linear[1],

$$\frac{dX_n}{dt} = -\lambda^{np}\ X_n + \lambda^{pn}\ (1-X_n) \quad , \tag{11}$$

and the detailed balance relation (7) is replaced by

$$\lambda^{pn}/\lambda^{np} = \exp(-\Delta m/T) \ . \tag{12}$$

All the involved particles, including the nucleons, are assumed to be thermalized at one universal temperature.

For the actual calculation we use the real-time formalism for thermal Green functions[14], and evaluate $Im\Sigma$ of (8); we realize from (9) that[22]

$$\Gamma = \Gamma_d + \Gamma_f \simeq \Gamma_d \simeq \lambda^{np} \ , \tag{13}$$

since the proton distribution may be neglected, $p \ll 1$.

The neutron self-energy function is approximated by

$$\begin{aligned} Im\Sigma(Q) &= \frac{G_F^2}{2}\int \frac{d^4K}{(2\pi)^4}\, 2\pi\delta_+((Q-K)^2 - {m_p}^2)\epsilon(k_0) \\ &\cdot \ (1+n_B(k_0))[G_h^\mu(\not{Q} - \not{K} + m_p)G_h^\nu]\ Im\Pi_{\mu\nu}(K) \ , \end{aligned} \tag{14}$$

where ϵ is the sign function. The condition $\delta_+(P^2 - {m_p}^2) = \Theta(p_0)\delta(P^2 - {m_p}^2)$ stands for the final state proton. The Bose-Einstein factor is $n_B(k_0) = (\exp(k_0/T) - 1)^{-1}$, and the nucleon vertex function is denoted by $G_h^\mu = \gamma^\mu(g_V - g_A\gamma_5)$, according to the $V-A$ effective Lagrangian[26].

Having in mind that the weak interactions are mediated by (infinitely) heavy bosons, the imaginary part of the boson self-energy, $Im\Pi_{\mu\nu}(K)$, enters in (14). With the aim to include the HTL-improved electron propagator S_{HTL} we derive for a general propagator S

$$\begin{aligned} Im\Pi_{\mu\nu}(K) &= -\epsilon(k_0)(1 - \exp(-k_0/T)) \\ &\cdot \ \frac{1}{2}\int \frac{d^4P'}{(2\pi)^4}\, 2\pi\epsilon({q_0}')\delta({Q'}^2)n_F({q_0}')[G_\mu^l iS^+(P')G_\nu^l \not{Q}] \ , \end{aligned} \tag{15}$$

with $Q' = P' - K$, and the lepton vertex $G_\mu^l = \gamma_\mu(1-\gamma_5)$. The thermal factor for fermions is $n_F(q_0) = (\exp(q_0/T) + 1)^{-1}$.

In the Born approximation the free thermal electron propagator S_0 enters:

$$\begin{aligned} i{S_0}^+(P) &= (\not{P} + m_e)[\Theta(p_0) - n_F(|p_0|)]2\pi\delta(P^2 - {m_e}^2) \\ &= (\not{P} + m_e)\epsilon(p_0)(1 - n_F(p_0))2\pi\delta(P^2 - {m_e}^2) \ , \end{aligned} \tag{16}$$

which leads to the result (keeping the neutron at rest and neglecting the electron mass m_e)[22]:

$$\lambda^{np}_{Born} = A\int_{-\infty}^{\infty} dp_0'\ {p_0'}^2\ (p_0' - \Delta m)^2(1 - n_F(p_0'))(1 - n_F(\Delta m - p_0')) \ , \tag{17}$$

where $A = G_F^2\ (g_V^2 + 3g_A^2)/2\pi^3$.

3. QED corrections and hard thermal loops

In order to obtain a more precise estimate for λ^{np} the corrections have to be considered due to the coupling of the intense radiation background to the charged particles present in the thermal neutron decay processes (1). Although the total QED corrections at fixed $O(\alpha)$ to λ^{np} are found to be small[7,8], as expected, it may be observed (e.g. from Fig. 8 of ref.[8]) that individual processes, especially the exclusive neutron decay and its inverse, do indeed receive large corrections at high temperature; they even exceed the corresponding rate of the Born approximation! Such a behaviour, however, requires resummation of the fixed order perturbative expansion.

Hard loop contributions are present in self-energy corrections to the fermion lines as well as in the loop correction to the vertex functions. They are especially significant on the soft momentum scale, where they become of leading order. Therefore we have to analyse carefully the relevant scales entering the rates for the processes (1).

To get an idea of the de facto relevance of critical soft momenta we estimate the mean electron momentum in the exclusive neutron decay based on the Born approximation:

$$< p' >_{n \to pe^- \bar{\nu}_e} \simeq 0.65\text{MeV} , \tag{18}$$

which is indeed a soft momentum for temperatures $T \geq 2MeV$. However, it has to be noted that this exclusive channel gives only a small fraction to the total rate λ^{np}, (10), at high values of T. On the other hand the dominant scattering processes prefer average electron momenta of the $O(T)$, i.e. hard ones, but nevertheless the soft region cannot be neglected when the electron phasespace is integrated to calculate the rate (cf. (15)).

The proton propagation does not require HTL modifications, because the mass satifies $m_p \gg T$ such that the proton four-momentum stays hard in the interesting range of temperatures. By the same argument HTL corrections to the proton-electron vertex may be neglected: the corrections attached to the proton are therefore calculated consistently at fixed $O(\alpha)$ as in refs.[7,8].

4. HTL resummation

The rate λ^{np} receives contributions from hard as well as from soft electron momentum. In order to obtain the corrections due to thermal electron self-energy effects consistently in the HTL approximation we calculate the soft and hard parts separately, following a proposal of ref.[29], where we separate the soft and hard regime in the electron momentum by a hyperbolic boundary:

$$p'_0 = \sqrt{p'^2 - \mu^2} . \tag{19}$$

The cutoff parameter μ is chosen on an intermediate scale $\mu \sim \sqrt{e}T$. This kind of covariant boundary has been successfully applied to the case of the thermal production rate of hard photons[30], where the same answer is obtained using the momentum cutoff: $p^* = \sqrt{e}T$[31].

In order to include all leading contributions when the electron momentum is soft the hard thermal loops in the electron propagator have to be resummed. This can be accomplished by taking the electron to be described by the effective resummed fermion propagator S_{HTL}[32,33,34]:

$$iS^{+}_{HTL}(P) = [\Theta(p_0) - n_F(|p_0|)] \cdot \left[(\gamma_0 - \vec{\gamma}\hat{p})\, Im \frac{1}{D_+(p_0,p)} + (\gamma_0 + \vec{\gamma}\hat{p})\, Im \frac{1}{D_-(p_0,p)}\right], \quad (20)$$

where we neglect the electron mass. The properties of the functions $D_\pm$ are discussed in great detail in refs.[32-34], and they have been used e.g in refs.[30,31].

There are two distinct contributions to $Im\frac{1}{D_\pm}$, and therefore to the neutron rate. The first one is due to the poles of $1/D_\pm$ at $D_\pm = 0$, i.e. to the quasiparticle (QP) excitations, $\omega_\pm$. The second one arises from a logarithmic singularity in $D_\pm$ giving rise to an imaginary part for spacelike electron momenta. This is interpreted as Landau damping (LD) i.e. absorption of the spacelike electron into the medium as discussed in ref.[34].

For the hard electron contribution the electron's phasespace to the right of the boundary (19) in the $p_0 - p$ plane is relevant. Here HTL contributions are suppressed, and only the one-loop diagram for the electron propagator to order α has to be considered.

5. Results and conclusion

The final result is expressed by the sum of three partial rates, due to the quasiparticles, due to Landau damping and due to the hard contribution:

$$\lambda^{np}_{res} = \lambda^{np}_S|_{QP} + \lambda^{np}_S|_{LD} + \lambda^{np}_H \; . \quad (21)$$

As we are interested in radiative corrections we subtract the Born approximation (17) from the total result (21), $\Delta\lambda^{np} = \lambda^{np}_{res} - \lambda^{np}_{Born}$. This quantity is evaluated numerically as a function of temperature, keeping μ fixed at its "ideal" value $\mu \simeq \sqrt{e}T$. The relative corrections[25], normalized to the Born rate (17),

$$\Delta\lambda^{np}/\lambda^{np}_{Born} \simeq -0.001 \; , \quad (22)$$

turn out to stay approximately stable for temperatures above $1MeV$, supporting the validity of the HTL approximation. Since we neglect the mass of the electron, we do not investigate temperatures below $1MeV$: this could be done e.g. by taking into account the effective temperature dependent Lagrangian derived in ref.[35], in which $m_e \neq 0$ at all temperatures.

Next we may relate (22) to the mass-fraction Y of the ^4He abundance using the approximation given in ref.[3]:

$$\Delta Y \simeq -0.18\Delta\lambda^{np}/\lambda^{np}_{Born} \simeq 0.0002 \; , \quad (23)$$

indicating an increase of Y due to HTL contributions.

We note that (22) includes the fixed $O(\alpha)$ contribution, already evaluated in refs.[7,8], where the (total) finite-temperature radiative correction - including the one for the electron-proton vertex - is evaluated to $\Delta Y \simeq 0.0004$.

In order to be able to quote the "genuine" HTL contribution beyond the leading fixed order we have to subtract the $O(\alpha)$ part, which we estimate by expanding $\lambda_S^{np}|_{QP}$ and $\lambda_S^{np}|_{LD}$ of (21), respectively, in terms of α.

Quantitatively we find that in terms of Y the finite temperature radiative corrections of ref.[7] are reduced at high temperatures ($T > 10MeV$) by $\Delta Y_{HTL}/Y \simeq 0.0017$ when taking into account the HTL resummed electron propagator, whereas at lower values of T, i.e. $T \simeq 1MeV$, the fixed $O(\alpha)$ correction agrees with the HTL improved one.

We finally observe that the HTL correction to Y is smaller than the recently estimated effect of keeping the nucleon mass finite ($\Delta Y/Y \simeq 0.0057$), but it is as big as (even bigger than) the effect of neutrino heating ($\Delta Y/Y \simeq 0.0006$)[6].

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