

Evidence of a Near-Threshold Resonance in ^{11}B Relevant to the β -Delayed Proton Emission of ^{11}Be

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A narrow near-threshold proton-emitting resonance ($E_x = 11.4$ MeV, $J^\pi = 1/2^+$, and $\Gamma_p = 4.4$ keV) was directly observed in ^{11}B via proton resonance scattering. This resonance was previously inferred in the β -delayed proton emission of the neutron halo nucleus ^{11}Be . The good agreement between both experimental results serves as a ground to confirm the existence of such exotic decay and the particular behavior of weakly bound nuclei coupled to the continuum. R -matrix analysis shows a sizable partial decay width for both, proton and α ($\Gamma_\alpha = 11$ keV) emission channels.

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Loosely bound atomic nuclei can be understood as open quantum systems: a weakly bound ensemble of nucleons coupled to an external environment. The behavior and properties of such systems are deeply affected by the interplay with this external environment called continuum. Because of this interplay, these systems display generic properties that are common to all weakly bound systems, with near- or above-threshold excitation energies. This coupling to the continuum may manifest in a nuclear reaction that excites the system to a state near the particle emission threshold. Therefore, the study of these atomic quantum systems underlines the commonly contrived close link between reaction and structure. As the system becomes gradually less bound, many-nucleon correlations may manifest through the formation of particle clusters via narrow resonances in the vicinity of the particle emission threshold. Although the formation and emission of clusters with well-defined quantum states is ubiquitous in the nuclear physics domain, little is known about how their properties are defined.

Many examples of particle-emitting near-threshold narrow resonances of fundamental relevance for α clustering [1], proton radioactivity [2], and for reactions of astrophysical interest [3,4] can be found throughout the entire nuclear landscape. Such a correlation-driven nuclear binding gives rise to open quantum systems where the radial wave function of valence nucleons extends well beyond the bound core forming weakly bound nuclei known as halo [5]. Open quantum systems with particle-emitting states can be formed near the drip line where separation energies become negative [6], by β decay into unbound states [7], or by resonance scattering [8]. A very particular, near-threshold, narrow resonance was recently inferred from the β -delayed proton emission (βp) of the halo nucleus ^{11}Be , a counterintuitive decay in neutron-rich nuclei [9,10]. This exotic decay is possible, within a relatively small energy window, for systems with a low neutron separation energy, such as ^{11}Be (501.6(3) keV) [11]. One of the key questions is whether this type of decay proceeds via a two-step mechanism feeding unbound states on the daughter nucleus, or directly into the continuum. There exists clear experimental evidence supporting a direct decay in the β -delayed deuteron emission of ^6He [12] and of ^{11}Li [13]. Branching ratios for these decays amount to the order of 10^{-6} and 10^{-4} , respectively.

The βp decay of ^{11}Be was directly observed for the first time in the $^{11}\text{Be} \rightarrow ^{10}\text{Be} + \beta^- + p$ disintegration by our

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collaboration [9]. The experiment was performed by implanting a ^{11}Be beam in the prototype Active Target Time Projection Chamber [14]. A novel particle tracking algorithm was employed to distinguish between protons, α particles, and recoiling nuclei. The experiment yielded a branching ratio for the β^-p branch of $b_p = 1.3(3) \times 10^{-5}$. Moreover, from the energy distribution of the emitted protons, it was deduced that the β^-p decay was sequential. The disintegration proceeded via an intermediate near-threshold narrow resonance in the β^- decay product $^{11}\text{B}^*$ at an energy $E_R = 196(20)$ keV above the proton separation energy and a total width of $\Gamma_T = 12(5)$ keV and $J^\pi = (1/2^+, 3/2^+)$. No corresponding state had been observed in ^{11}B at the time. This result was contested by Riisager and collaborators, who employed an indirect approach to measure this same branching ratio [15–17]. They made use of a mass separator to measure the presence of ^{10}Be in a catcher where ^{11}Be had been implanted. Despite improving their experiment on at least three occasions, unexplained discrepancies between their results persisted. Because of these inconsistencies, an upper limit was adopted from the lowest branching ratio, $b_p < 2.2 \times 10^{-6}$ [17], in clear disagreement with the value reported in Ref. [9] and with their previous measurement [16].

These two experiments also linked ^{11}Be to the search for the decay of neutrons into dark matter, the so-called dark decay [18]. The dark decay, which involves physics beyond the standard model, tries to explain the long-standing neutron lifetime puzzle by hypothesizing that $\sim 1\%$ of free neutrons decay into an undetected dark-sector particle instead of a proton. In this model, weakly bound neutrons could also undergo dark decay, with the halo neutron in ^{11}Be being the most promising candidate [19]. The final product of the ^{11}Be dark decay would be a ^{10}Be nucleus plus an undetected dark particle. A precise measurement of the $^{11}\text{Be} \rightarrow ^{10}\text{Be}$ rate (similar to the attempts by Riisager and collaborators [15–17]) would measure a combination of the βp and dark decay branching ratios. It is, therefore, paramount to have a precise measurement of the βp mechanism and branching ratio in order to disentangle its contribution to the overall $^{11}\text{Be} \rightarrow ^{10}\text{Be}$ decay and thus extract any hypothetical dark decay branch.

There have been several attempts from theory to confirm the resonance in ^{11}B and to estimate the β^-p decay branching ratio in ^{11}Be . Before the experiments were conducted, Baye and Tursunov [20] deduced $b_p \sim 5 \times 10^{-9}$ employing a cluster model with no resonant intermediate state. More theoretical attempts were carried out after the publication of the first experimental β^-p results. Volya [21] performed shell model calculations and concluded that no suitable resonance existed in ^{11}B that could act as an intermediate state in order to enhance the β^-p decay branching ratio. These calculations also suggested that, if such a state existed, it would strongly favor

breaking into $^7\text{Li} + \alpha$ rather than emitting a proton. Okołowicz and collaborators [22] arrived at a rather different conclusion; using a shell model embedded in the continuum model, they were able to infer the presence of the intermediate resonance with $J^\pi = 1/2^+$ and a small contribution from the $^7\text{Li} + \alpha$ channel that highlights the orthogonality of both possible eigenstates. However, their model does not reconcile with the large b_p obtained in Ref. [9]. Their most recent calculations suggest that the b_p should be 40 times lower to harmonize with the Γ_p and the branching ratio for α decay (b_α) [23]. In their study, it is assumed that there exists a very close $3/2^+$ resonance (11.49 MeV) that decays predominantly by α emission. Such a resonance was indirectly deduced from an R -matrix fit but never observed before [24]. They also conclude that decay from the isobaric analog state, as suggested by Ref. [21], is ruled out. Lastly, Elkamhawy *et al.* [25] performed halo effective field theory with and without the intermediate resonance state in ^{11}B . Similarly to Ref. [20], for a direct decay (no resonance) the b_p obtained was orders of magnitude lower than the directly measured one [9]. On the other hand, when a resonance with parameters similar to those measured by this collaboration was introduced, the experimental b_p was reproduced.

It is clear, thus, that the exotic β^-p decay requires the presence of a near-threshold resonance to enhance it to the level observed in Ref. [9]. Since no suitable level has been observed in ^{11}B to date, a dedicated experiment employing the $^{10}\text{Be}(p, p)$ reaction was performed to clarify its existence and properties. The experiment was conducted at the ReA3 reaccelerator facility of the National Superconducting Cyclotron Laboratory using a pure 350A keV ^{10}Be radioactive beam with an intensity of about 10^3 pps. The ^{10}Be material was produced at Paul Scherrer Institut (Villigen, Switzerland) from proton-irradiated carbon [26]. The excitation function of the reaction was obtained by stopping the beam on a 9.6 μm thick CH_2 target foil (8.64 mg/cm 2). A very thin (tens of nm) aluminum layer was evaporated on the upstream side of the foil. Secondary electrons produced from the aluminum by the beam were deflected using permanent magnets into a microchannel plate detector in chevron mode manufactured by TECTRA. A 1 mm thick and 35 mm effective diameter single-sided silicon detector (Micron MSD035) was placed around 10 cm downstream of the target to measure forward scattered protons and α particles. A sketch of the experimental setup is shown in the upper panel of Fig. 1. Particle identification was performed using the time of flight (TOF, about few ns of resolution) and energy correlation, as shown in the lower panel of Fig. 1. The solid line refers to the calculated TOF as a function of the energy set between protons and α particles. As can be seen, the line clearly separates two regions in the identification matrix. Elastic scattered protons are located in the low energy region of the

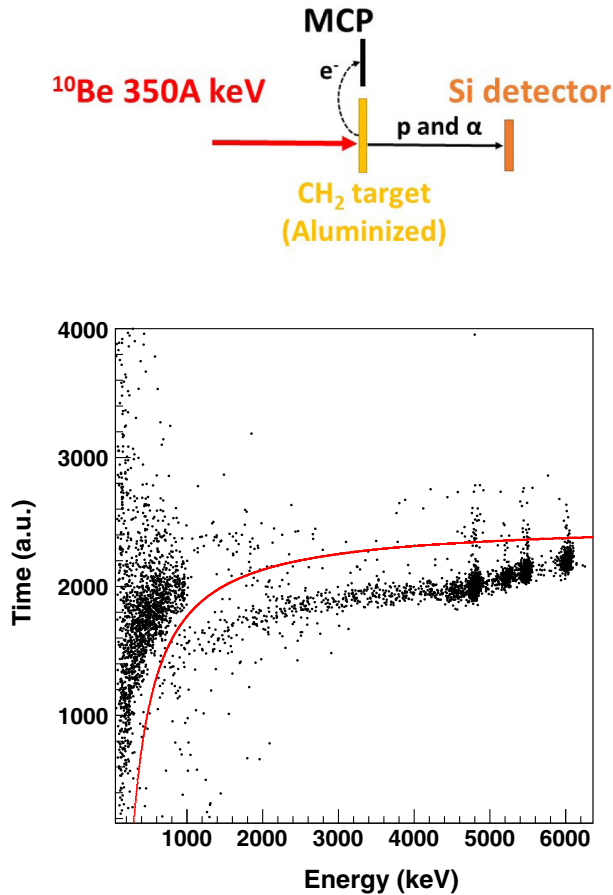


FIG. 1. Upper panel: Sketch of the experimental setup. Lower panel: Particle (proton and α) energy vs TOF. The peaks at high energy correspond to the ^{228}Th source α particles.

plot, below 1000 keV (corresponding to 350 keV in c.m.) and above the line. α particles, coming predominantly from a ^{228}Th calibration source, lie below the line. Within the TOF-E region of α particles, the ones coming from the decay of this particular resonance into ${}^7\text{Li} + {}^4\text{He}$ would have an energy of about 4000 keV, quite far away from the region of interest. It is also worth pointing out that reactions on carbon atoms in the target that could produce α particles are highly suppressed at these bombarding energies due to the penetrability [27]. The silicon detector was calibrated using a proton beam of different energies (down to 250 keV in the laboratory frame) and alpha particles from the ^{228}Th source. The reaction energy was corrected by the energy loss of the particles in the target. The detector resolution of 10 ± 1 keV (FWHM) in the c.m. was deduced taking into account the intrinsic resolution, straggling effects, and target thickness inhomogeneity.

Figure 2 shows the excitation function compared to an R -matrix calculation performed with the AZURE2 code [28]. These calculations were also compared to the ones yielded by the DSIGMAIV code [29], finding an excellent agreement between the two codes. A resonance effect interfering with Coulomb scattering can be clearly seen below 200 keV.

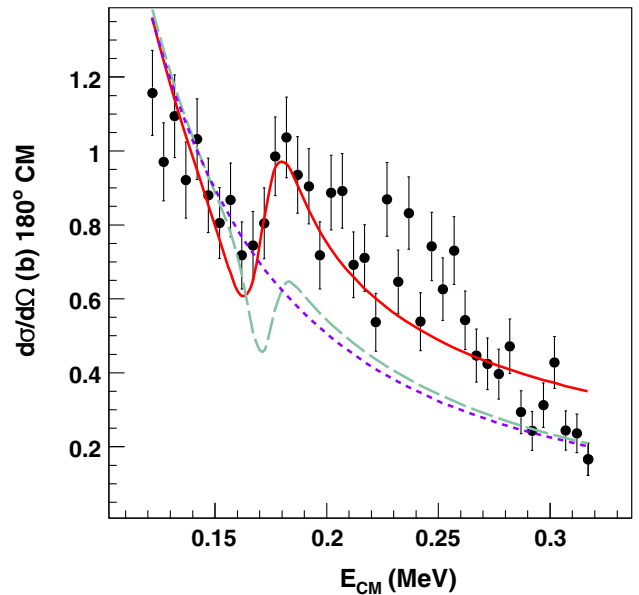


FIG. 2. Excitation function (c.m.) for the ${}^{10}\text{Be}(p, p)$ reaction (solid dots) and best R -matrix fits performed for $1/2^+$ (solid line) and for $1/2^-$ (dashed line). The dotted line refers to the Coulomb scattering cross section.

The resonance width (16 ± 3 keV) and energy $E_R = 171 \pm 20$ keV (11.40 ± 0.02 MeV excitation energy in ${}^{11}\text{B}$) inferred from the fit are in good agreement with the values reported in Ref. [9] from the ${}^{11}\text{Be}$ β -decay (12 ± 5 keV). Moreover, the best fit (solid line), with $\chi^2 = 2.7$, confirms the $J^\pi = 1/2^+$ assignment. The proton partial width amounts to only 4.5 ± 1.1 keV. In order to obtain a resonance effect, compatible with the experimental resolution, another decay branch has been assumed and attributed to the ${}^7\text{Li} + \alpha$ decay channel (11 ± 3 keV). The sharp energy cut at 350 keV in the c.m. is due to the maximum beam energy. As it is evident from the figure, where the Coulomb scattering is also presented (dotted line), the cross section does not converge to Rutherford after the resonance. With such a narrow width, it would be expected that the cross section converges to pure Rutherford scattering if the resonance has a simple Breit-Wigner form. The excitation function exhibits a clear departure from the Breit-Wigner shape. Such deviations from the Breit-Wigner shape are well-known [30–32] and are mostly due to the energy dependence of the partial widths. Here, the effect is enhanced by the interference effects with the (mostly Coulomb) background. An R -matrix fit for a resonance with $J^\pi = 1/2^-$ ($\chi^2 = 5.4$) is also presented in Fig. 2 (dashed line). In this case, for $l = 1$, the excitation function converges back to Coulomb scattering rapidly, in contrast to our data.

In order to corroborate that such a behavior is only due to specific properties inherent to the R -matrix formalism, such as the penetration factors, energy dependencies, resonance energy shifts, and phase shifts [33], we performed a search

of a potential resonance. For this, we employed the optical model code SPOMC [34]. The potential has been inferred from the optical model parametrization of the code and renormalized to produce an $l = 0, 2s$ resonance at the experimental energy. The imaginary potential was set to zero. The behavior obtained was very similar to the one with the R -matrix formalism, shown in Fig. 2, with the cross section remaining about a factor of 2 higher than the potential scattering, even far from the resonance. Another excellent example of such a threshold change of the cross section due to the combined effect of interference and of the fast change of penetrability, similar to the one observed here, is a $1/2^+$ low-lying resonance in ^{15}F [8].

This peculiar behavior of the resonance effect can be described for a given partial wave by an interference between a potential scattering and the resonance term using the collision matrix for elastic scattering within the one resonance level approximation [35]:

$$U_{cc'}^{BW} = \exp i(\phi_c + \phi_{c'}) \left[\delta_{cc'} + \frac{i\sqrt{\Gamma_c(E)\Gamma_{c'}(E)}}{E_R - E - i\Gamma(E)/2} \right], \quad (1)$$

where the partial width is $\Gamma_c = 2\gamma_{\text{obs},c}^2 P_c(E)$, with $\gamma_{\text{obs},c}^2$ and $P_c(E)$ being the observed reduced width and the penetrability, respectively. ϕ_c is the hard-sphere shift and $\delta_{cc'}$ is the Kronecker delta. In the case of a single channel, $\Gamma(E)$ corresponds to the width of the elastic channel. Below the Coulomb barrier, the penetrability may vary much faster than the term $(E_R - E)$. Then, the term $(E_R - E)$ can be neglected, and the expression will converge to $\delta_{cc'} - 2$. This very specific behavior of a near-threshold resonance no longer has a Breit-Wigner form, but resembles more a threshold behavior with a strong contribution to the scattering probability that remains almost constant after the resonance even at distances large as compared to the resonance width.

A measure of the single-proton content of the resonance is provided by the proton partial width, which can be extracted from the two-channel R -matrix fit of the excitation function, and it is proportional to the spectroscopic factor [33,35]. A Wigner limit of the single particle width [33,35] for a channel radius of 2.7 fm of 18 keV was obtained, yielding a spectroscopic factor of 0.25. This is in agreement with the conclusion of the previous Letter [9], suggesting that the resonance state contains a significant single-particle strength. Since the resonance is well below the Coulomb barrier, the experiment probes the asymptotic part of the corresponding state, and the elastic cross section is largely insensitive to the internal details of the wave function. The extracted spectroscopic factor is thus essentially proportional to the asymptotic normalization constant, which, as opposed to the spectroscopic factor, is an on-shell, well-defined, observable quantity. It is worth noting that in Ref. [9] the α decay could not be observed due to the very strong branching to other channels decaying

by α emission. Hence, the total width, as observed, contained the eventual contribution of this channel. In the present experiment, it was not possible to confirm directly the α decay, predicted to have around 5 times lower cross section, due to limited statistics. A direct measurement of the $^{10}\text{Be}(p, \alpha)$, with a complete determination of the branching to different excited states in ^7Li , is required to clarify the situation.

In conclusion, we have observed a near-threshold proton-emitting resonance in ^{11}B via the $^{10}\text{Be}(p, p)$ reaction at 350A keV. An R -matrix calculation was used to deduce the energy, spin parity, and resonance width, in good agreement with the values inferred in β -delayed proton emission of ^{11}Be . This is a strong indication that the exotic βp decay indeed proceeds via this intermediary state, explaining the relatively large branching ratio observed. The results also suggest that the resonance has a sizable decay width to the $\alpha + ^7\text{Li}$ channel. The characteristics of the resonance, a consequence of the interplay between the reaction mechanism and structure, reveal the open quantum system nature of such narrow resonances.

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Note added.—During production of our work, we became aware of Ref. [36].

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[1] K. Ikeda, N. Takigawa, and H. Horiuchi, *Prog. Theor. Phys. Suppl.* **E68**, 464 (1968).

- [2] V. Girard-Alcindor *et al.*, *Phys. Rev. C* **105**, L051301 (2022).
- [3] M. Wiescher and T. Ahn, *Nuclear Particle Correlations and Cluster Physics* (World Scientific, Singapore, 2017), <https://www.worldscientific.com/doi/pdf/10.1142/10429>.
- [4] M. Friedman, T. Budner, D. Pérez-Loureiro, E. Pollacco, C. Wrede, J. José, B. A. Brown, M. Cortesi, C. Fry, B. Glassman, J. Heideman, M. Janasik, M. Roosa, J. Stomps, J. Surbrook, and P. Tiwari, *Phys. Rev. C* **101**, 052802(R) (2020).
- [5] I. Tanihata, *J. Phys. G* **22**, 157 (1996).
- [6] R. J. Charity, L. G. Sobotka, T. B. Webb, and K. W. Brown, *Phys. Rev. C* **105**, 014314 (2022).
- [7] M. Munch *et al.*, *Phys. Rev. C* **93**, 065803 (2016).
- [8] F. de Grancey *et al.*, *Phys. Lett. B* **758**, 26 (2016).
- [9] Y. Ayyad *et al.*, *Phys. Rev. Lett.* **123**, 082501 (2019).
- [10] Y. Ayyad *et al.*, *Phys. Rev. Lett.* **124**, 129902(E) (2020).
- [11] B. Jonson and K. Riisager, *Nucl. Phys. A* **693**, 77 (2001).
- [12] R. Raabe, J. Büscher, J. Ponsaers, F. Aksouh, M. Huyse, O. Ivanov, S. R. Leshner, I. Mukha, D. Pauwels, M. Sawicka, D. Smirnov, I. Stefanescu, J. Van de Walle, P. Van Duppen, C. Angulo, J. Cabrera, N. de Séréville, I. Martel, A. M. Sánchez-Benítez, and C. A. Diget, *Phys. Rev. C* **80**, 054307 (2009).
- [13] R. Raabe *et al.*, *Phys. Rev. Lett.* **101**, 212501 (2008).
- [14] D. Suzuki, M. Ford, D. Bazin, W. Mittig, W. Lynch, T. Ahn, S. Aune, E. Galyaev, A. Fritsch, J. Gilbert, F. Montes, A. Shore, J. Yurkon, J. Kolata, J. Browne, A. Howard, A. Roberts, and X. Tang, *Nucl. Instrum. Methods Phys. Res., Sect. A* **691**, 39 (2012).
- [15] M. J. G. Borge, L. M. Fraile, H. O. U. Fynbo, B. Jonson, O. S. Kirsebom, T. Nilsson, G. Nyman, G. Possnert, K. Riisager, and O. Tengblad, *J. Phys. G* **40**, 035109 (2013).
- [16] K. Riisager *et al.*, *Phys. Lett. B* **732**, 305 (2014).
- [17] K. Riisager, M. Borge, J. A. Briz, M. Carmona-Gallardo, O. Forstner, L. Fraile, H. Fynbo, A. G. Camacho, J. Johansen, B. Jonson *et al.*, *Eur. Phys. J. A* **56**, 100 (2020).
- [18] B. Fornal and B. Grinstein, *Phys. Rev. Lett.* **120**, 191801 (2018).
- [19] M. Pfützner and K. Riisager, *Phys. Rev. C* **97**, 042501(R) (2018).
- [20] D. Baye and E. Tursunov, *Phys. Lett. B* **696**, 464 (2011).
- [21] A. Volya, *Europhys. Lett.* **130**, 12001 (2020).
- [22] J. Okołowicz, M. Płoszajczak, and W. Nazarewicz, *Phys. Rev. Lett.* **124**, 042502 (2020).
- [23] J. Okołowicz, M. Płoszajczak, and W. Nazarewicz, [arXiv:2112.05622](https://arxiv.org/abs/2112.05622).
- [24] J. Refsgaard, J. Büscher, A. Arokiaraj, H. O. U. Fynbo, R. Raabe, and K. Riisager, *Phys. Rev. C* **99**, 044316 (2019).
- [25] W. Elkamhawy, Z. Yang, H.-W. Hammer, and L. Platter, *Phys. Lett. B* **821**, 136610 (2021).
- [26] S. Heinritz, D. Kiselev, N. Kivel, and D. Schumann, *Appl. Radiat. Isot.* **130**, 260 (2017).
- [27] Q. Haider and F. B. Malik, *J. Phys. G* **12**, 537 (1986).
- [28] R. E. Azuma, E. Uberseder, E. C. Simpson, C. R. Brune, H. Costantini, R. J. de Boer, J. Görres, M. Heil, P. J. LeBlanc, C. Ugalde, and M. Wiescher, *Phys. Rev. C* **81**, 045805 (2010).
- [29] H. W. Wang, *High Energy Phys. Nucl. Phys.* **30**, 18 (2006), http://caod.oriprobe.com/articles/10219346/Study_of_Unbound_Nuclei_11N_Resonance_Energy_Level.htm.
- [30] C. E. Rolfs and W. S. Rodney, *Cauldrons in the Cosmos: Nuclear Astrophysics* (University of Chicago Press, Chicago, 1988).
- [31] F. de Oliveira Santos, *EPJ Web Conf.* **184**, 01006 (2018).
- [32] C. Angulo, G. Tabacaru, M. Couder, M. Gaelens, P. Leleux, A. Ninane, F. Vanderbist, T. Davinson, P. J. Woods, J. S. Schweitzer, N. L. Achouri, J. C. Angélique, E. Berthoumieux, F. de Oliveira Santos, P. Himpe, and P. Descouvemont, *Phys. Rev. C* **67**, 014308 (2003).
- [33] A. M. Lane and R. G. Thomas, *Rev. Mod. Phys.* **30**, 257 (1958).
- [34] L. C. Chamon, B. V. Carlson, L. R. Gasques, D. Pereira, C. De Conti, M. A. G. Alvarez, M. S. Hussein, M. A. Cândido Ribeiro, E. S. Rossi, and C. P. Silva, *Phys. Rev. C* **66**, 014610 (2002).
- [35] P. Descouvemont and D. Baye, *Rep. Prog. Phys.* **73**, 036301 (2010).
- [36] E. Lopez-Saavedra *et al.*, following Letter, Observation of a Near-Threshold Proton Resonance in ^{11}B , *Phys. Rev. Lett.* **128**, 012502 (2022).