Inflationary gravitational leptogenesis

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We consider the generation of the baryon asymmetry in models with right-handed neutrinos produced through gravitational scattering of the inflaton during reheating. The right-handed neutrinos later decay and generate a lepton asymmetry, which is partially converted to a baryon asymmetry by Standard Model sphaleron processes. We find that a sufficient asymmetry can be generated for a wide range of right-handed neutrino masses and reheating temperatures. We also show that the same type of gravitational scattering produces Standard Model Higgs bosons, which can achieve inflationary reheating consistent with the production of a baryon asymmetry.

I. INTRODUCTION

One of the most elegant mechanisms for generating the baryon asymmetry of the Universe is leptogenesis [1]. In its simplest version, the lepton number violating out-of-equilibrium decay of a heavy right-handed neutrino produces a lepton asymmetry if C and CP are violated in the decays. Then the baryon number B and lepton number L violating (but $B - L$ conserving) sphaleron processes [2] distribute the asymmetry between leptons and baryons. As long as the effective lepton number violating operators remain out of equilibrium, the baryon (and lepton) asymmetry will be preserved [3–5].

The obvious attractiveness in models of leptogenesis is the fact that the only element beyond the Standard Model required is a massive right-handed neutrino, often introduced to generate light Standard Model neutrino masses via the seesaw mechanism [6]. Differences among models of leptogenesis often relate to the means by which right-handed neutrinos populate the Universe or their embedding in a UV completion of the Standard Model [7,8]. For example, a common assumption is that the right-handed neutrinos are in thermal equilibrium in the radiation bath newly created after inflation [9]. This requires that

the reheating temperature T_{RH} exceeds the right-handed neutrino mass m_N. However, this requirement is not necessary, as right-handed neutrinos produced in the decay of the inflaton, provide a direct source of nonthermal right-handed neutrinos [10], and require only $m_N < m_\phi/2$, where m_ϕ is the mass of the inflaton ϕ.

Most models of nonthermal leptogenesis carry some form of additional model dependence, namely, how one couples the inflaton to the right-handed neutrino. In some cases, one might associate the inflaton with the supersymmetric partner of the right-handed neutrino [11], or in supergravity models, there maybe a gravitational coupling induced by the chosen forms of Kähler potential and superpotential [12,13].

In this paper, we consider a model-independent formulation for the production of right-handed neutrinos leading to leptogenesis. That is, once the inflaton potential is specified, we make no assumptions about how the inflaton couples to the right-handed neutrino sector. Its production from the inflaton condensate is purely gravitational [14–18].[1] We consider as an example, the class of inflationary models called T models [20] and show that for all such models for which the equation of state parameter during the period of reheating $w \geq 0.5$, the proper baryon asymmetry may be generated for reasonable choice of the right-handed neutrino mass and reheating temperature.

In what follows, we first compute the number density of right-handed neutrinos produced gravitationally from inflaton oscillations. We then apply this result to obtain

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[1]Note that production from the inflaton condensate almost always dominates over the gravitational production of matter from the thermal bath [19].

the resulting baryon/lepton asymmetries from the out-of-equilibrium decay of the right-handed neutrinos. We also show that even if the Higgs-inflaton coupling is only gravitational (minimal or nonminimal), it may be possible to *simultaneously* produce the lepton asymmetry as well as the entropy of the universe without the need to consider a specific coupling of the inflaton to matter.

II. GRAVITATIONAL PRODUCTION RATES

The simplest process for producing a lepton asymmetry from the out-of-equilibrium decay of a right-handed neutrino is a direct decay of the inflaton to N. If such a coupling exists and m_N exceeds the maximum temperature after inflation, then N will be produced out-of-equilibrium thus realizing the original leptogenesis scenario [1]. However, even in the absence of a direct coupling between the inflaton and the right-handed neutrino, N can be produced from scattering within the thermal bath or directly from the inflaton condensate. The former is the common mechanism leading to thermal leptogenesis [9]. The latter, on the other hand, is inevitable for processes mediated by gravity [15,18].

Gravitational interactions are described by the Lagrangian (see e.g., Ref. [21]),

$$\sqrt{-g}\mathcal{L}_{\text{int}} = -\frac{1}{M_P}h_{\mu\nu}(T^{\mu\nu}_{\text{SM}} + T^{\mu\nu}_{\phi} + T^{\mu\nu}_{N}). \quad (1)$$

Here, SM represents Standard Model fields, ϕ is the inflaton, and N is the right-handed neutrino. We assume the standard form of the stress-energy tensor $T^{\mu\nu}_i$, which depends on the spin of the field, $i = 0, 1/2, 1$. In Fig. 1, we show the s-channel exchange of a graviton obtained from the Lagrangian (1) for the production of right-handed neutrinos from the inflaton condensate. In addition, a similar diagram exists for the production of Standard Model fields during the reheating process. The Planck suppression due to graviton exchange is partially compensated by the energy available in the inflaton condensate at the end of inflation.
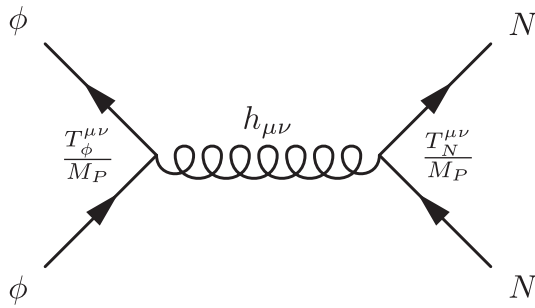

FIG. 1. Feynman diagram for the production of right-handed neutrinos N through the gravitational scattering of the inflaton condensate ϕ.

The rate for producing right-handed neutrinos can be found in Refs. [18,22] where it has been calculated in the context of a fermionic dark matter candidate. Note that the results of Ref. [22] do not agree with the results presented here, since their choices of parameters are in conflict with the tensor-to-scalar ratio bound from Planck. Because the production mechanism depends on the time-dependent oscillations of the inflaton subsequent to the period of exponential expansion, we must specify examples of the inflaton potential we consider.

As a specific example, we consider the α-attractor T model [20],

$$V(\phi) = \lambda M_P^4 \left| \sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right|^k, \quad (2)$$

which can in fact arise from a simple superpotential [23],

$$W = 2^{\frac{k}{4}+1}\sqrt{\lambda}M_P^3\left(\frac{(\phi/M_P)^{\frac{k}{2}+1}}{k+2} - \frac{(\phi/M_P)^{\frac{k}{2}+3}}{3(k+6)}\right), \quad (3)$$

in the context of no-scale supergravity. Equation (2) can be expanded about the origin,

$$V(\phi) = \lambda\frac{\phi^k}{M_P^{k-4}}, \quad \text{for } \phi \ll M_P, \quad (4)$$

and we will use this generic form hereafter. Therefore, it should be noted that the remaining discussion is general and not limited to T models of inflation. Phenomenological aspects of T models were recently considered in [24].

At the end of inflation, the time-dependent oscillating inflaton field can be parametrized as

$$\phi(t) = \phi_0(t) \cdot \mathcal{P}(t), \quad (5)$$

where $\phi_0(t)$ is the amplitude and

$$\mathcal{P}(t) = \sum_{n=-\infty}^{n=+\infty} \mathcal{P}_n e^{-in\omega t} \quad (6)$$

describes the periodicity of the oscillations, with frequency ω given by [25]

$$\omega = m_\phi\sqrt{\frac{\pi k}{2(k-1)}}\frac{\Gamma(\frac{1}{2}+\frac{1}{k})}{\Gamma(\frac{1}{k})}, \quad (7)$$

where $m_\phi^2 = V''(\phi_0)$. The rate (per unit time and volume) for the production of right-handed neutrinos can be expressed as [18]

$$R_N^{\phi^k} = \frac{2 \times (1/2) \times \rho_\phi^2}{4\pi M_P^4}\frac{m_N^2}{m_\phi^2}\Sigma_{1/2}^k, \quad (8)$$

TABLE I. Coefficients $\Sigma_{1/2}^k$ relevant for the gravitational production rate of right-handed neutrinos.

$\Sigma_{1/2}^6 = 0.101$	$\Sigma_{1/2}^8 = 0.133$	$\Sigma_{1/2}^{10} = 0.157$	$\Sigma_{1/2}^{12} = 0.177$
$\Sigma_{1/2}^{14} = 0.192$	$\Sigma_{1/2}^{16} = 0.205$	$\Sigma_{1/2}^{18} = 0.216$	$\Sigma_{1/2}^{20} = 0.225$

where during oscillations $\rho_\phi = V(\phi_0)$ is the energy density of the inflaton and the explicit factor of $2 \times (1/2)$ accounts for two particles produced in the final state and for the Majorana nature of N. Considering the potential (4), the effective inflaton mass squared is given by $m_\phi^2 = \lambda k(k-1)\phi_0^{k-2}/M_P^{k-4}$ and

$$\Sigma_{1/2}^k = \sum_{n=1}^{+\infty} |\mathcal{P}_n^k|^2 \frac{m_\phi^2}{E_n^2} \left[1 - \frac{4m_N^2}{E_n^2}\right]^{3/2}, \quad (9)$$

where the Fourier modes $\mathcal{P}_n^k$ are obtained from [25–27]

$$V(\phi) = V(\phi_0) \sum_{n=-\infty}^{\infty} \mathcal{P}_n^k e^{-in\omega t} = \rho_\phi \sum_{n=-\infty}^{\infty} \mathcal{P}_n^k e^{-in\omega t}, \quad (10)$$

$E_n = n\omega$ being the energy of the nth inflaton oscillation mode. The values of $\Sigma_{1/2}^k$ are given in Table I for $k \leq 20$. In this table and the remainder of the paper, the kinematic suppression in Eq. (9) is assumed to be negligible. Since this may have an effect only if m_N is close to the inflaton mass evaluated when the dominant production occurs, this depends on the precise shape of the inflaton potential. We leave for future work the detailed study between the exact inflationary models and gravitational production of the right-handed neutrinos.

The number density of right-handed neutrinos, n_N, is obtained by solving the Boltzmann equation, which can be expressed as

$$\frac{d(n_N a^3)}{da} = \frac{a^2 R_N^{\phi^k}(a)}{H}, \quad (11)$$

where a is the cosmological scale factor. The a dependence in the rate comes from the evolution of ρ_ϕ, which is given by [23,25]

$$\rho_\phi(a) = \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a}\right)^{\frac{6k}{k+2}}, \quad (12)$$

where a_{end} is the scale factor when inflation ends (defined when the slow-roll parameter equals unity). The equation of state for ϕ is $w = (k-2)/(k+2)$.

Equation (11) for the density of right-handed neutrinos can be solved analytically [18],

$$n_N(a_{\text{RH}}) \simeq \frac{m_N^2 \sqrt{3}(k+2)\rho_{\text{RH}}^{\frac{1}{2}+\frac{2}{k}}}{24\pi k(k-1)\lambda^{\frac{2}{k}} M_P^{1+\frac{8}{k}}} \left(\frac{\rho_{\text{end}}}{\rho_{\text{RH}}}\right)^{\frac{1}{k}} \Sigma_{1/2}^k, \quad (13)$$

evaluated at the time of reheating, which for now we assume is a result of the decay of the inflaton to Standard Model particles. We define ρ_{RH} as the energy density in radiation when it becomes equal to the inflaton energy density and $\rho_{\text{RH}} = (\pi^2 g_*(T_{\text{RH}})/30)T_{\text{RH}}^4$, with $g_*(T_{\text{RH}})$ the number of relativistic degrees of freedom[2] at T_{RH}.

III. LEPTOGENESIS

Once produced, the right-handed neutrinos decay rapidly,

$$N \to L_\alpha + H$$
$$N \to \bar{L}_\alpha + \bar{H}, \quad (14)$$

where L and H are the left-handed lepton and Higgs electroweak doublets, respectively. If CP is violated in the decay of N, then a lepton asymmetry,

$$Y_L \equiv \frac{n_L}{s} = \epsilon \frac{n_N}{s} \quad (15)$$

is produced. Here, $s = (2\pi^2 g_*(T_{\text{RH}})/45)T_{\text{RH}}^3$ is the entropy density. The CP violation is encapsulated in [28,29]

$$\epsilon \equiv \frac{\Gamma_{N \to L_\alpha H} - \Gamma_{N \to \bar{L}_\alpha \bar{H}}}{\Gamma_{N \to L_\alpha H} + \Gamma_{N \to \bar{L}_\alpha \bar{H}}}. \quad (16)$$

A nonzero value for ϵ requires at least two right-handed neutrinos. We assume the existence of three right-handed neutrinos and denote the lightest of these as N with mass m_N. The remaining two will be denoted as $N_{2,3}$ with masses $m_{2,3}$, and we assume $m_N \lesssim m_\phi \ll m_{2,3}$. Furthermore, we assume that the light and mostly left-handed neutrino masses are determined by the seesaw mechanism [6] so that

$$m_{\nu_i} \simeq \frac{|y_i|^2 v^2}{m_i}, \quad (17)$$

where y_i is a Yukawa coupling, $v \approx 174$ GeV is the Standard Model Higgs expectation value, and that the neutrino mass matrix is nearly block diagonal.

We note that this assumes a type-I seesaw with a singlet fermion N. There also exist type II and type III seesaw mechanisms, where new particles are $SU(2)$-triplet scalars and $SU(2)$-triplet fermions, respectively (see Ref. [8] for a review). Additionally, the neutrino mass hierarchy can have normal ordering or inverted ordering. In the following, we will continue to assume type-I with normal ordering. That is, we assume that y_3 is the largest neutrino Yukawa coupling so that m_{ν_3} is the largest neutrino mass despite a larger suppression from m_3.

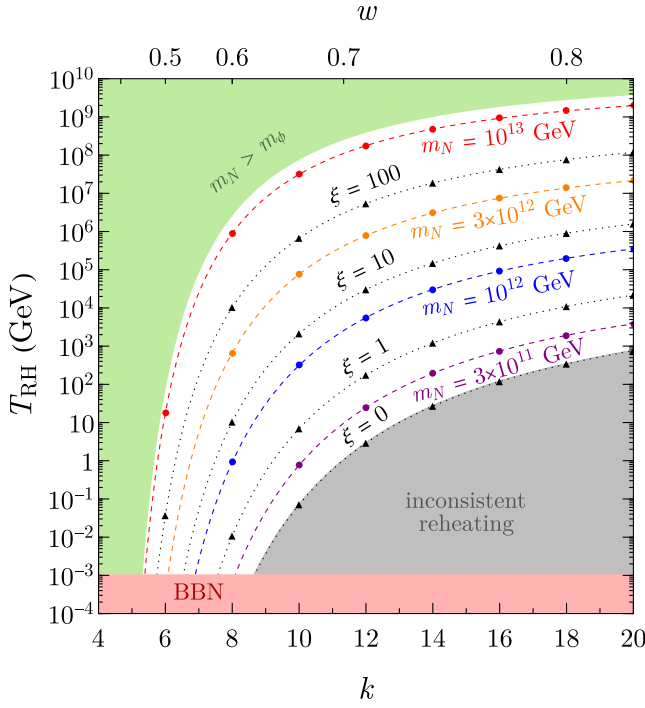[2]$g_* = 427/4$ for the full Standard Model particle content.

FIG. 2. The colored dashed curves show values of the reheating temperature $T_{\rm RH}$ required to explain the observed baryon asymmetry as a function of k, the exponent of the potential defined in Eq. (4), for different choices of the right-handed neutrino mass m_N. The black dotted lines show the reheating temperature obtained from purely gravitational reheating for different choices of ξ, the nonminimal gravitational coupling constant defined in Eq. (20). A minimal gravitational coupling ($\xi = 0$) gives a minimum $T_{\rm RH}$, excluding the gray region. The upper x axis labels the equation of state $w = (k-2)/(k+2)$ of the inflaton, during reheating.

Using the seesaw expression, we can write [13]

$$\epsilon \simeq -\frac{3\delta_{\rm eff}|y_3|^2}{16\pi} \cdot \frac{m_N}{m_3} \simeq -\frac{3\delta_{\rm eff}}{16\pi} \cdot \frac{m_{\nu_3} m_N}{v^2}, \qquad (18)$$

where $\delta_{\rm eff}$ is the effective CP violating phase in the neutrino mass matrix and $0 \leq \delta_{\rm eff} \leq 1$.

Finally, this lepton asymmetry is converted into the baryon asymmetry via the electroweak sphaleron processes that freeze out at the electroweak phase transition, giving $Y_B = \frac{28}{79} Y_L$ [4,13,30] and

$$Y_B \simeq 3.5 \times 10^{-4} \delta_{\rm eff} \frac{n_N}{s} \left(\frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \left(\frac{m_N}{10^{13} \text{ GeV}} \right), \qquad (19)$$

while the observed value is $Y_B \simeq 8.7 \times 10^{-11}$ [31].

The required reheating temperature $T_{\rm RH}$ for specific choices of the right-handed neutrino mass m_N as a function of the equation of state parameter w is displayed in Fig. 2. Here, we take $\rho_{\rm end} = (5 \times 10^{15} \text{ GeV})^4$ and $\lambda = 18\pi^2 A_{S*}/(6^{k/2} N_*^2)$ with A_{S*} being the amplitude of the curvature power spectrum measured to be
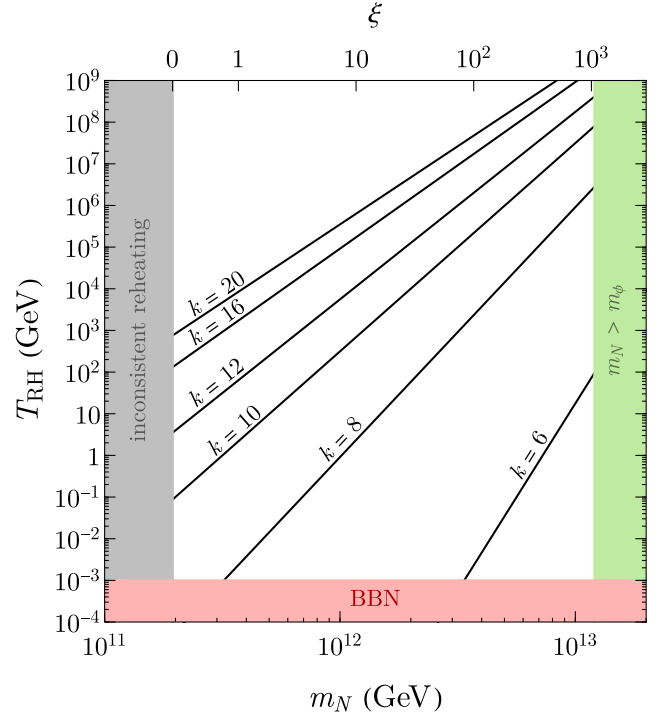

FIG. 3. Values of the reheating temperature $T_{\rm RH}$ required to explain the observed baryon asymmetry as a function of m_N for different choices of k. The upper x axis labels the approximate values of ξ necessary to achieve the corresponding $T_{\rm RH}$ through purely gravitational reheating.

$\ln(10^{10} A_{S*}) = 3.044$ [31,32] and the number of e-folds $N_* = 55$ for the Planck pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$.[3] This choice of parameters leads to an inflaton mass, $m_\phi \simeq 1.2 \times 10^{13} \text{ GeV}$, at the end of inflation that places an upper bound on m_N due to kinematics. However, as mentioned earlier, we do not include the kinematic factor when computing $\Sigma_{1/2}^k$ to remain inflationary model independent as much as possible. The circles along each curve correspond to even values of k in the inflaton potential. For $k = 2$, from Eq. (13), $n_N/s \propto m_N^2 T_{\rm RH}$ and $Y_B \simeq 6 \times 10^{-70} m_N^3 T_{\rm RH}/\text{GeV}^4$ is far too small to produce the required asymmetry. For $k = 4$, Y_B is independent of $T_{\rm RH}$ but requires a large $m_N \simeq 3 \times 10^{14} \text{ GeV}$, which exceeds the inflaton mass. However, for $k \geq 6$, for reasonable $T_{\rm RH}$ and a sufficiently large m_N, the proper asymmetry can be generated. This is also demonstrated in Fig. 3, where the required $T_{\rm RH}$ as a function of m_N is shown for different values of k. For $k = 6$, the correct asymmetry is obtained for $T_R \simeq 20 \text{ GeV}(m_N/10^{13} \text{ GeV})^9$. More generically, $T_R \propto m_N^{3k/(k-4)}$ for any $k \neq 4$. Lastly, the green regions are not feasible because the gravitational production of N is kinematically forbidden.

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[3]The variables $\rho_{\rm end}$, λ, and N_* are in principle functions of k and $T_{\rm RH}$ [25] in order to explain the CMB observations, but fixing to the aforementioned values gives an excellent approximation.

IV. GRAVITATIONAL REHEATING

The previous calculation of the lepton asymmetry was based solely on gravity for the production of right-handed neutrinos and n_L. However, we assumed that the entropy (and hence, T_{RH}) was produced by the decay of the inflaton to radiation. For example, a coupling $y\phi\bar{f}f$ would produce a reheating temperature proportional to $y^{k/2}$ for $k < 7$ and to $y^{3k/(2k-8)}$ for $k > 7$ [25] and would allow temperatures in the range shown in Fig. 2 for $y < 1$. We now ask whether both the numerator and denominator in $Y_L = n_L/s$ can be produced purely by gravity. It was recently shown that a gravitational coupling could be sufficient to ensure a complete reheating of the Universe [18]. However, to ensure a sufficiently large reheating temperature ($T_{RH} \gtrsim 1$ MeV) to avoid conflict with big bang nucleosynthesis, which excludes the red regions in Figs. 2 and 3, one needs to consider $w \gtrsim 0.65$ [33] or $k = \frac{2+2w}{1-w} > 9$. This lower bound comes from the fact that, for higher k, the inflaton energy density redshifts faster [see Eq. (12)] so the transition to radiation domination is achieved sooner. The requirement for large w can be relaxed if one considers nonminimal couplings of the Higgs to the gravitational sector of the type [34],

$$\mathcal{L}_\xi = -\frac{\xi}{2}|H|^2\mathcal{R}, \qquad (20)$$

where $\mathcal{R}$ is the Ricci scalar. This generates effective couplings between the inflaton and the Higgs boson,

$$\mathcal{L}_\xi^{\phi H} = \frac{\xi}{M_P^2}\left[2V(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]|H|^2. \qquad (21)$$

We can then write the Boltzmann equation for the radiation,

$$\frac{\partial\rho_R}{\partial t} + 4H\rho_R = R_k, \qquad (22)$$

where R_k the amount of energy transferred per unit time and per unit volume. To compute R_k, one needs to add the standard gravitational contribution corresponding to the exchange of graviton [18] and the nonminimal contribution from the coupling (20).

The rate can then be written $R_k = R_k^0 + R_k^\xi$ with

$$R_k^0 = N_s\frac{\rho_\phi^2}{16\pi M_P^4}\sum_{n=1}^{\infty}2n\omega|\mathcal{P}_{2n}^k|^2,$$

$$R_k^\xi = N_s\frac{2\times\xi^2}{8\pi M_P^4}\sum_{n=1}^{\infty}2n\omega\left|2\mathcal{P}_{2n}^k\rho_\phi + \frac{(n\omega)^2}{2}\phi_0^2|\mathcal{P}_n|^2\right|^2, \quad (23)$$

where $N_s = 4$ is the number of real scalars in the Standard Model and we neglect the Higgs mass. We considered even values of k, each mode n transferring an energy $2n\omega$ per scattering to the bath. For the case $k = 2$, only the mode $n = 1$ contributes, and we have ($\mathcal{P}_1 = \frac{1}{2}$, $\mathcal{P}_2^2 = \frac{1}{4}$), which gives for $N_s = 4$,

TABLE II. Coefficients α_k relevant for the rate of gravitational reheating.

$\alpha_6 = 0.000193 + 0.00766\xi^2$	$\alpha_8 = 0.000529 + 0.0205\xi^2$
$\alpha_{10} = 0.000966 + 0.0367\xi^2$	$\alpha_{12} = 0.00144 + 0.0537\xi^2$
$\alpha_{14} = 0.00192 + 0.0702\xi^2$	$\alpha_{16} = 0.00238 + 0.0855\xi^2$
$\alpha_{18} = 0.00281 + 0.0993\xi^2$	$\alpha_{20} = 0.00319 + 0.112\xi^2$

$$R_2 = R_2^0 + R_2^\xi = \frac{m_\phi\rho_\phi^2}{32\pi M_P^4}(1 + 36\xi^2), \qquad (24)$$

in agreement with [18,34]. Taking into account the ϕ_0 dependence in ω in Eq. (7), we can define

$$R_k \equiv \alpha_k M_P^5\left(\frac{\rho_\phi}{M_P^4}\right)^{\frac{5k-2}{2k}}. \qquad (25)$$

The values of α_k are given in Table II for $k \leq 20$.

Finally, we can solve Eq. (22) to obtain

$$\rho_R(a) \simeq \alpha_k\frac{k+2}{8k-14}\sqrt{3}M_P^4\left(\frac{\rho_{end}}{M_P^4}\right)^{\frac{2k-1}{k}}\left(\frac{a_{end}}{a}\right)^4, \qquad (26)$$

$$\rho_{RH} = M_P^4\left(\frac{\rho_{end}}{M_P^4}\right)^{\frac{4k-7}{k-4}}\left(\frac{\alpha_k\sqrt{3}(k+2)}{8k-14}\right)^{\frac{3k}{k-4}}$$

$$= \frac{g_*(T_{RH})\pi^2}{30}T_{RH}^4, \qquad (27)$$

which predicts the reheating temperature shown by the black dotted curves in Fig. 2 for different values of ξ, where the triangles along the curves correspond to even values of k. As can be seen, the black and colored curves are nearly parallel, and this correlation gives the upper x axis of Fig. 3 based on the value of m_N. The gray regions in Figs. 2 and 3 are inconsistent because even minimal gravitational interactions ($\xi = 0$) cannot achieve such low T_{RH}. That is, gravitational interactions alone provide a lower limit to the reheating temperature which is k dependent. Furthermore, in much of the parameter space, gravitational interactions can provide sufficient reheating without the need of additional inflaton couplings.

V. SUMMARY

In this paper, we have demonstrated that purely gravitational interactions of the inflaton ϕ can produce a sufficient abundance of right-handed neutrinos, which later decay and generate the observed baryon asymmetry of the Universe via leptogenesis. This mechanism, which we call inflationary gravitational leptogenesis, can explain the baryon asymmetry for a wide range of right-handed neutrino masses m_N and reheating temperatures T_{RH}. This mechanism works for a class of inflationary models as long as the equation of state of ϕ is $w \geq 0.5$ during

reheating, which is the case when the potential takes the form ϕ^k with $k \geq 6$ near the origin. Inflationary reheating can also successfully complete through gravitational interactions with the Higgs boson. This paradigm requires only gravitational interactions, so the results shown in Figs. 2 and 3 are robust and general. Indeed, these results provide a lower limit to the reheating temperature as gravitational interactions are necessarily present. This paves the way for new opportunities in inflationary model building and baryogenesis.

ACKNOWLEDGMENTS

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