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EMITTANCE ESTIMATES FOR THE FUTURE CIRCULAR COLLIDER

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Abstract

The alignment strategy of the FCC-ee could have a large impact on its luminosity. Looser alignment tolerances result in more coupling and a subsequently higher vertical emittance. At the same time, tighter alignment tolerances around the 100 km ring are a major cost driver. This paper applies analytical emittance estimate methods to the FCC-ee and validates their predictions with data from simulations for different alignment tolerances. These methods can be used to help understand the impact of misalignments of certain magnet groups and to develop an efficient alignment strategy.

THEORY AND BACKGROUND

In a high energy electron beam, the high levels of synchrotron radiation cause the beam emittances to damp rapidly, however, the discrete nature of photon emission changes the momentum of the particles and excites them with respect to new dispersive reference orbits, and, thereby, causes the particle distribution in the horizontal and longitudinal plane to reach some equilibrium with finite emittance values [1]. In an ideal ring, the vertical closed orbit is flat and does not depend on the momentum of the particles and discrete momentum changes should not excite the particles. However, in reality magnetic imperfections and alignment errors couple the vertical motion to the horizontal and longitudinal planes. This causes the vertical closed orbit also to be momentum dependant and subsequently the vertical emittance to be limited by quantum excitation. Moreover, the finite horizontal distribution of the particles, together with the transversely coupled motion, and quantum excitation in the presence of such coupling, additionally blow up the vertical particle distribution [2].

Consequently, a key challenge when designing an electron-positron collider is to predict the impact that magnet and alignment errors will have on the beam emittance and to develop strategies to minimise it. This can be studied is by simulating many machines with different error seeds and computing the emittance for each machine using matrix methods. The results can be used to then find the expected emittance distribution for a given error distribution. An alternative approach is an analytical treatment of the problem, where the expected vertical emittance for a given error distribution is derived. Such methods have been established, tested and are well described in the literature [3]. These methods quantify the average changes in dispersion and closed orbit in the vertical plane caused by errors and then consider how these changes affect the expected vertical emittance.

Our objective is to apply analytical methods to study the alignment sensitivity of the Future Circular electron-positron Collider (FCC-ee) optics [4]. This analytical approach is meant to complement the ongoing extremely detailed simulation studies, which consider various different sets of errors and correction schemes, and preliminary results of which have already been presented in the past [5]. Our goal here is to directly quantify the impact that different errors in different groups of magnets have on the vertical emittance. This information would be a useful instrument when assigning and budgeting alignment tolerances, in view of the possibility that the alignment over a 100 km circumference could be a major cost driver for the FCC-ee. The lattice used for this study is the FCC-ee 4IP $\mathrm{t\bar{t}}$ version 301 lattice. The results from the analytical expressions are compared against results obtained from simulations done using MADX [6], where random errors are applied using the internal random number generator, where the seed number can be specified.

QUADRUPOLE ROLL ERRORS

The first type of error tested is quadrupole roll errors. For these errors, the equation relating the expected vertical emittance, ϵ_y, to a normal distribution of quadrupole roll errors with variance, $< \theta^2_{\mathrm{quad}} >$, is given as [3]

$$\begin{aligned}\frac{\epsilon_y}{< \theta^2_{\mathrm{quad}} >} &= \frac{J_z \sigma_\delta^2}{\sin^2(\pi Q_y)} \sum_{\mathrm{quad}} \beta_y \eta_x^2 (k_1 L)^2 \\ &+ \frac{J_x[1-\cos(2\pi Q_x)\cos(2\pi Q_y)]\epsilon_x}{J_y[\cos(2\pi Q_x)-\cos(2\pi Q_y)]^2} \sum_{\mathrm{quad}} \beta_x \beta_y (k_1 L)^2,\end{aligned} \tag{1}$$

where J_x, J_y and J_z are the horizontal, vertical and longitudinal partition numbers respectively; Q_x and Q_y are the horizontal and vertical tunes; ϵ_x and σ_δ are the horizontal emittance and normalised momentum spread at equilibrium. The sums are computed over all quadrupoles with β_x, β_y and η_x being the horizontal and vertical beta functions and the horizontal dispersion in the misaligned quadrupoles and $k_1 L$ being the normalised integrated strength of the quadrupole.

To test the accuracy of this expression, nine sets of MADX simulations, were set up where roll errors were applied to all quadrupoles. In the first set of simulations the standard deviation of these roll errors was chosen as 10 µrad and 100 simulation with different seeds were performed. This was repeated eight times, each time increasing the standard deviation by 5 µrad and using the same 100 error seeds. For each set of simulations, the average emittance and its standard error was computed, this is shown by the black points in Fig. 1. Figure 1 also shows the emittance obtained using Eq. (1) over the same range of quadrupole errors.

Figure 1 shows that the analytical expression is very accurate at predicting the emittance and is always within the range of the standard error of the mean. After establishing that there is a good agreement when applying errors to all

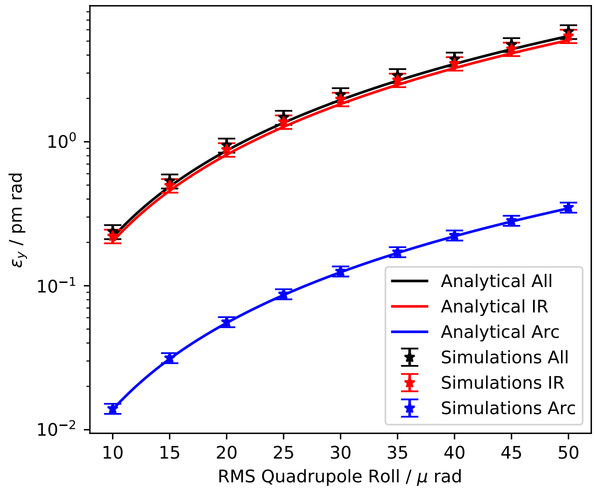

Figure 1: Vertical emittance due to quadrupole roll errors with increasing standard deviation, determined from analytical expression and MADX simulations.

quadrupoles, the next step was to establish whether Eq. (1) can also be used to predict the impact of subgroups of magnets. The most natural subgroups that one might sort the quadrupoles into could be arc and interaction region (IR) quadrupoles. Accordingly, the simulations and computations were repeated two more times, once only considering arc quadrupoles and the other time only IR quadrupoles, the results are also shown in Fig. 1 in blue and red respectively.

The results from the grouped quadrupoles shown in Fig. 1 not only indicate that Eq. (1) is accurate for subgroups of magnets, but also show that even though there are much fewer IR quadrupoles, roll errors in the IR quadrupoles have a much larger impact on the emittance than errors of comparable size in the arcs. A more detailed look at the accuracy of Eq. (1) and the impact that roll errors have on the emittance can be obtained by applying roll errors to individual quadrupoles and looking at the resulting vertical emittance. This was done by rolling the first quadrupole after the interaction point (IP) by 10 µrad whilst leaving all other quadrupoles aligned and computing the vertical emittance using MADX. This was repeated for every succeeding quadrupole until the second IP was reached, at which point the sequence repeats. At the same time Eq. (1) was evaluated for each individual quadrupole. The ratio between the analytical and MADX result is shown in Fig. 2, where the x-axis shows the location of the rolled quadrupole.

Figure 2 shows that Eq. (1) is able to predict the emittance due to roll errors in single quadrupoles to within 10% in most cases with few exceptions near the IPs. Figure 3 shows the vertical emittance produced by the rolls on the individual quadrupoles. The plot highlights that for equal roll errors, a small group of quadrupoles near the IP cause a vertical emittance that is several orders of magnitude larger than that caused by the majority of the other quadrupoles.

The final test was whether the emittance contributions for different quadrupoles add linearly, as indicated by the sums in Eq. (1). This was tested by choosing 100 groups of 300 random quadrupoles and performing 100 simulations for each group. In every group, random roll errors with a standard deviation of 10 µrad were assigned to the selected quadrupoles and the average emittance of the 100 simulations was computed. The expected emittance was also computed for each group by calculating the sum of the single emittance contributions shown in Fig. 3 over all quadrupoles in the group. For each group the ratio between the average simulated emittance and the analytical emittance estimate was computed and sorted into 20 bins, shown in Fig. 4.

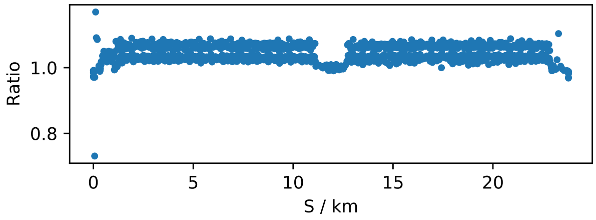

Figure 2: Ratio between vertical emittance from analytical expression and result from MADX for 10 µrad roll errors on quadrupoles around the FCC-ee.

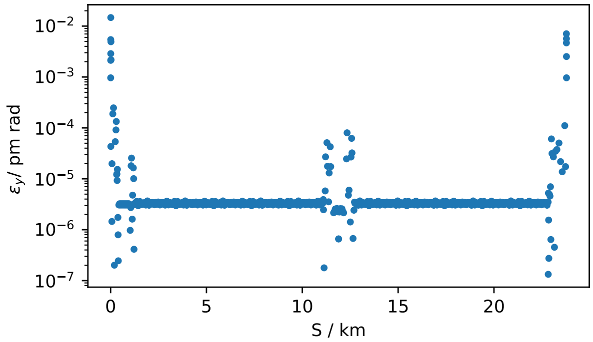

Figure 3: Vertical emittance from analytical expression for 10 µrad roll errors on quadrupoles around the FCC-ee.

The mean of this ratio for the 100 groups is 0.86 with a standard deviation of 0.12, indicating that for this case the analytical expression provides a slight underestimate for the emittance. Although it shows a slight underestimate,

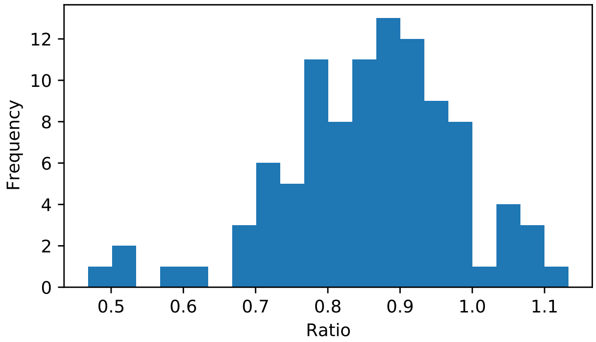

Figure 4: Ratio between vertical emittance from analytical expression and average result from MADX simulations when applying random errors to 100 groups of 300 magnets.

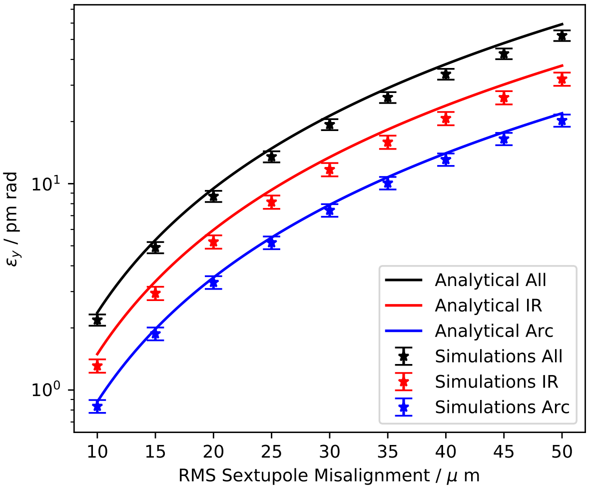

Figure 5: Vertical emittance due to sextupole vertical misalignment errors with increasing standard deviation, determined from analytical expression and MADX simulations.

the ratio largely resembles a normal distribution, indicating that there is no combination of magnets where Eq. (1) fails to provide an adequate order of magnitude estimate of the emittance. One cause for the underestimate might be that, as seen from Fig. 2, Eq. (1) yields a slight overestimate for the emittance due to arc quadrupoles, while it slightly underestimates the roll-error effect for some of the quadrupoles in the IRs and for all quadrupoles in the radiofrequency insertion (at about 11 km). At the same time, one can see from Fig. 3 that the quadrupoles in these regions tend to cause the vertical emittance to grow by orders of magnitude more than the quadrupoles in the arcs.

SEXTUPOLE MISALIGNMENTS

In the same way that a rolled quadrupole produces a skew quadrupole field with $k_s = k_1\theta$, a vertically misaligned sextupole produces a skew quadrupole field with $k_s = k_2 y/2$. Therefore, Eq. (1) can be adapted for sextupole misalignments by replacing k_1 with $k_2/2$ and $< \theta^2_{\text{quad}} >$ with the standard deviation of the vertical sextupole misalignment, $< y^2_{\text{sext}} >$ [7]. The accuracy of these formulas was again tested in a similar manner starting at 10 µm and increasing in steps of 5 µm. Again, this was done for all sextupoles but also for only the sextupoles in the arcs and only the sextupoles in the IRs. The results are plotted in Fig. 5.

Figure 5 shows that there is a good agreement between the analytical estimate and results from simulations with the analytical estimate slightly overestimating the emittance. Again, the contribution from the fewer IR magnets is larger than that of the arc magnets, however, for the sextupoles this difference is much smaller than for the quadrupoles. This can also be seen when misaligning single sextupoles by 10 µm and evaluating the vertical emittance, as shown in Fig. 6, where the difference between different locations is much smaller than for the quadrupoles, though there is large spread of sensitivities inside each arc. The qualitative difference between sextupoles and quadrupoles can be attributed to large differences in the relative strength of magnets in the arcs and in the IRs for these two elements. The arc quadrupoles — all of about the same strength — are much weaker than the final-focus quadrupoles, while strong arc sextupoles come relatively closer to the strength of the few sextupoles in the final focus. In addition, the strength of all $-I$ sextupole pairs in the arcs has been individually optimised, resulting in a large range of values, which explains the enormous sensitivity spread seen in Fig. 6. The ratio between the analytical and simulated emittance was also computed and is shown in Fig. 7.

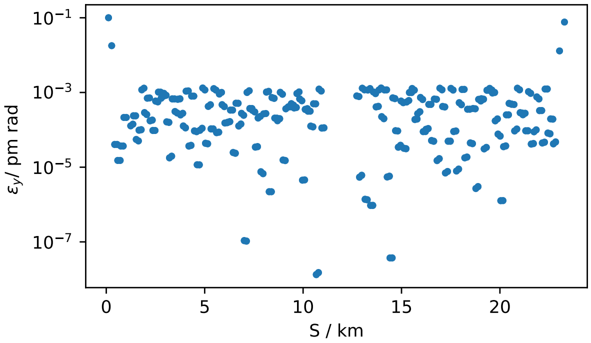

Figure 6: Vertical emittance from analytical expression for 10 µm alignment errors on sextupoles around the FCC-ee.

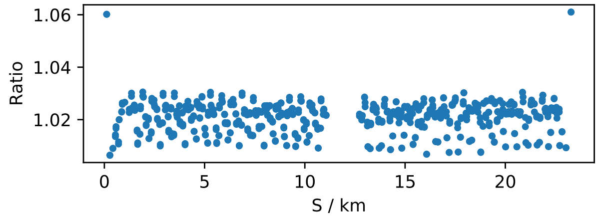

Figure 7: Ratio between vertical emittance from analytical expression and result from MADX for 10 µm alignment on sextupoles around the FCC-ee.

DISCUSSION AND OUTLOOK

The analytical expressions provide a good estimate of vertical emittance due to uncorrected quadrupole roll and sextupole misalignment errors. Whilst the estimate was occasionally off by a few percent, it captures the expected behaviour from increasing or decreasing the magnitude of the errors. The limits of linearity in the dependence of vertical emittance on the various alignment errors need to be further studied. In the future, we plan to also explore other types of errors, which change the vertical closed orbit and thus induce vertical dispersion and cause the vertical emittance to blow up. We could then also study the combined effect of multiple types of errors applied simultaneously.

Ultimately, we intend to construct a tolerance budget for various different types of errors, separating between arcs, IRs, and other insertion straights, in order to appropriately balance tighter tolerances in the IRs and looser tolerances in the collider arcs.

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