5 Bright muon beams and muon colliders

5.1 Executive summary

High-energy lepton colliders can serve as facilities for precision and discovery physics. The decrease of s-channel cross sections as $1/s$ requires that luminosity increases with energy, ideally proportional to s, the square of the centre-of-mass energy. The only mature technology to reach high-energy, high-luminosity lepton collisions is linear electron-positron colliders; the highest energy for which a conceptual design exists is Compact Linear Collider (CLIC) at 3 TeV.

Muon collider (MC) technology must overcome several significant challenges to reach a similar level of maturity. An increased level of R&D effort is justified at the current time, because the muon collider promises an alternative path toward high-energy, high-luminosity lepton collisions that extends beyond the expected reach of linear colliders. The strong suppression of synchrotron radiation compared to electrons allows beam acceleration in rings making efficient use of the RF systems for acceleration. The overall power consumption of a 10 TeV MC is expected to be lower than that of CLIC at 3 TeV. Additionally the beam can repeatedly produce luminosity in two detectors in the collider ring. The ratio of luminosity to beam power is expected to improve with collision energy, a unique feature of the MC. The compactness of the collider makes it plausible that a cost effective design might be achieved; however this must be verified with more detailed estimates. If the technical challenges can be overcome, the MC offers a potential route to long-term sustainability of collider physics.

Past work has demonstrated several key MC technologies and concepts, and gives confidence that the concept is viable. Component designs have been developed that can cool the initially diffuse beam and accelerate it to multi-TeV energy on a time scale compatible with the muon lifetime. However, a fully integrated design has yet to be developed and further development and demonstration of technology are required. In order to enable the next European Strategy for Particle Physics Update (ESPPU) to judge the scientific justification of a full conceptual design report (CDR) and demonstration programme, the design and potential performance of the facility must be developed in the next few years.

We propose a programme of work that will allow the assessment of realistic luminosity targets, detector backgrounds, power consumption and cost scale, as well as whether one can consider implementing a MC at CERN or elsewhere. Mitigation strategies for the key technical risks and a demonstration programme for the CDR phase will also be addressed.

The work programme will develop a muon collider concept at 10 TeV and explore a 3 TeV staging to mitigate technology and operational challenges. The 3 TeV option might cost around half as much as the 10 TeV option, and can be upgraded to 10 TeV or beyond by adding an accelerator ring and building a new collider ring. Only the 4.5 km-long 3 TeV collider ring would not be reused in this case. The use of existing infrastructure, such as existing proton facilities and the LHC tunnel, will also be considered.

If the next ESPPU recommends further investment, a CDR phase could then develop the technologies needed to mitigate identified project risks and demonstrate that the community can execute a successful MC project. No cost estimate for the CDR phase exists but experience indicates that typically 5–10% of the final project cost would need to be invested. A muon cooling demonstrator facility would be expected to be the largest single component of the CDR programme, with the potential to provide direct scientific output in its own right.

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The resources available to the MC programme over the next five years will depend on decisions made in both Europe and the US; the strategy process in the latter case will conclude in 2023. Currently, CERN plans a budget of 2 MCHF per year and several person-years have already been committed at INFN, allowing the work to start. Two resource scenarios in the time span before the next ESPPU have been developed, with strong support from the community. Both require resources beyond those currently committed. An aspirational MC development scenario is consistent with achieving all the above goals by the time of the next ESPPU. A minimal scenario has a significantly reduced scope and lacks most preparations for the demonstration programme.

The muon collider programme is synergistic with other Roadmap efforts, and will directly benefit from progress in the high-field magnet, RF and energy recovery linac programmes. In case of the high-field magnet programme this will require efforts towards the development of very high field solenoid magnets, in addition to the focus on high-field dipoles for proton facilities.

Bright muon beams are also important for neutrino physics facilities such as NuSTORM or ENUBET. These have a physics case of their own right. A muon cooling demonstrator facility could potentially share a large part of the infrastructure with these facilities, from the proton source to the target.

5.2 Introduction

5.2.1 Past work

Muon colliders offer enormous potential for exploration of the particle physics frontier. Muons, like electrons, are fundamental particles, so the full energy of the particle is available when they collide, whereas protons are composites of quarks and gluons so only a fraction of the energy is available. Unlike electrons, the high mass of the muon leads to suppression of synchrotron radiation so that muons can be accelerated to high energy in rings. This results in a facility footprint that can be small compared to other proposed future energy-frontier facilities, while yielding comparable results. Studies indicate that the luminosity per beam power increases linearly with energy, making it a plausible route to 10 TeV scale collisions.

Unlike proton and electron machines, muon accelerators have received relatively little attention from the accelerator physics community owing to the challenges in producing and capturing muons, and their short lifetime. Muons are typically created by firing protons onto a target, yielding pions which then decay to muons. The resultant muon beam can have a large current but is diffuse in physical and phase space. Conventional techniques to increase beam brightness such as stochastic cooling cannot be applied to muons due to the 2.2 µs muon rest-frame lifetime.

Existing muon sources overcome this obstacle simply by collimating the muon beam, resulting in a muon rate that is low when compared to equivalent proton or electron beams. Applications for such muon sources have mostly been limited to rare decay searches, studies of the muon fundamental properties such as the anomalous magnetic moment, and material physics studies employing polarised muons. These types of sources, when used in a collider, could not achieve the required luminosity.

Over the past two decades, a dedicated effort has been undertaken in Europe, America and Asia to explore techniques to achieve higher muon brightness and accelerate muons. Two high-energy applications have been studied: the production of neutrinos for the study of neutrino oscillations in a neutrino factory; and the collision of muons in a collider. Concepts for muon-electron and muon-ion colliders have also been proposed. These studies have yielded key results.

- The principle of ionisation cooling, proposed to increase the beam brightness, has been demonstrated. RF component tests and ionisation cooling simulations have indicated that there exists a viable path to yield a beam with brightness suitable for a muon collider.
- Studies of the collider rings have yielded potential techniques for management of radiation arising from muon-decay neutrinos with TeV energies.

- Studies of the collider interaction region have demonstrated the possibility to optimise the design to shield detectors from the majority of beam induced background arising from decay electrons being lost in the neighbourhood of the detector. Using detectors with precise timing sensitivity and granularity it seems possible to achieve the required sensitivities for physics measurements.

5.2.2 Baseline concept

The current muon collider baseline concept was developed by the Muon Accelerator Program (MAP) collaboration [1], which conducted a focused program of technology R&D to evaluate its feasibility. Since the end of the MAP study seminal measurements have been performed by the Muon Ionization Cooling Experiment (MICE) collaboration, which demonstrated the principle of ionisation cooling that is required to reach sufficient luminosity for a muon collider [2]. The MAP scheme is based on the use of a proton beam to generate muons from pion decay and is the baseline for the collider concept being developed by the new international collaboration. An alternative approach Low Emittance Muon Accelerator (LEMMA), which uses positrons to produce muon pairs at threshold, has been explored at INFN [3].

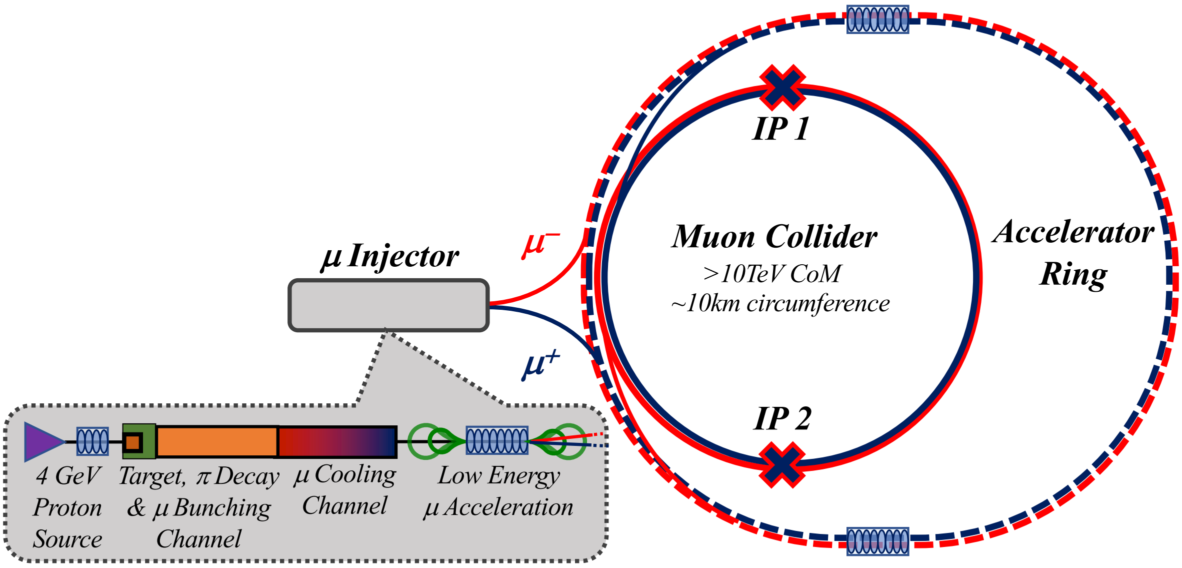

Fig. 5.1: A conceptual scheme for the muon collider.

MAP developed the concept shown in Fig. 5.1. The proton complex produces a short, high-intensity proton pulse that hits a target and produces pions. The decay channel guides the pions and collects the muons produced in their decay into a buncher and phase rotator system to form a muon beam. Several cooling stages then reduce the longitudinal and transverse emittance of the beam using a sequence of absorbers and RF cavities in a high magnetic field. A linac and two recirculating linacs accelerate the beams to 60 GeV. One or more rings accelerate the beams to the final energy. As the beam is accelerated, the lifetime in the lab frame increases due to relativistic time dilation so later stage accelerators have proportionally more time for acceleration, so that fast-pulsed synchrotrons can be used. Fixed-field alternating-gradient (FFA) accelerators are an interesting alternative. Finally the two single-bunch beams are injected at full energy into the collider ring to produce collisions at two interaction points.

The MAP study demonstrated feasibility of key components, but several important elements were not studied. The highest collision energy studied by MAP was 6 TeV. Technical limitations such as beam-induced backgrounds have not been studied in detail at higher energies. Individual elements of the muon source were studied, but integrated system design and optimisation was not performed. Cooling studies assumed limits in practical solenoid and RF fields that now appear to be too conservative; an updated performance estimate would likely yield a better assessment of the ultimate luminosity of the facility. MAP studies considered gallium, graphite and mercury target options, which should be progressed and studied in more detail to assess fully the performance and technical limitations of the system.

5.3 Motivation

5.3.1 Physics motivation

A muon collider with 3 TeV centre-of-mass energy would have similar or greater physics potential as an electron-positron collider such as CLIC, the physics reach of which is well established and documented [4]. A muon collider with a centre-of-mass energy of 10 TeV or more would open entirely new opportunities for the exploration of fundamental physics [5]. On one hand, it would feature a mass-reach for the direct discovery of new particles that surpasses in most cases the HL-LHC exclusion potential and in some cases is superior to future hadron collider projects. The MC could exploit a large production cross-section in case of high-mass states and it would benefit from favorable signal-to-background rates for low-mass states [6]. On the other hand, it would enable precision measurements through which new physics could be discovered indirectly, or the validity of the SM confirmed at a currently unexplored energy scale. The growing interest of the theory community in muon colliders has recently been expressed in the context of the ongoing Snowmass21 initiative [6, 7]. Several sensitivity projection studies have been completed during the last two years, and summarised at three workshops [8–10] and at regular meetings on the muon collider physics potential [11]. Detector studies indicate that the potential of the muon collider can be exploited with present state-of-the-art detector technologies at 3 TeV, but requires further R&D for a 10 TeV facility, as discussed in detail in the Detector R&D Roadmap.

5.3.2 Cost and power efficiency

Compared to other frontier particle accelerators and colliders, the Muon Collider shows advantages in terms of sustainability. The most obvious aspect is the relatively moderate land use thanks to the compactness of the accelerator complex. The collider ring is expected to have a circumference around 4.5 km and 10 km at 3 TeV and 10 TeV, respectively. The acceleration complex will be longer; the length depends on available technology and design choices. Superconducting fast-ramping magnets might allow a more compact design than normal-conducting magnets and it is possible to integrate more than one acceleration stage in the same tunnel. Also the use of existing tunnels can be considered. It has been suggested, see Ref. [12], that a muon beam can be accelerated from 0.45 TeV to 7 TeV in the LHC tunnel, using either one stage of superconducting or two of normal-conducting fast-ramping magnets. More R&D effort is required to evaluate the technologies and to make the design choices.

A second, decisive advantage concerns the energy efficiency, and more precisely the beam power, and hence the specific electrical power consumption per unit of luminosity. To maintain similar rates of s-channel events, the luminosity has to increase in proportion to s. Relevant targets for a lepton collider are 1, 10 and 20 ab^{-1} for centre-of-mass energies of 3, 10 and 14 TeV respectively. The luminosity that can be achieved per MW or wall-plug power is shown in Fig. 5.2, comparing the MAP muon collider and CLIC. The CLIC luminosity is limited by the beam size at the collision point; the current machine parameters are the fruit of a decade-long, intense development programme.

Assuming that the required technologies are available, the main parameters affecting the luminosity in a muon collider are summarised in the following scaling formula:

$$L \propto \gamma B P_{\text{beam}} \frac{N\sigma_\delta}{\varepsilon_n \varepsilon_l}. \tag{5.1}$$

P_{beam} denotes the beam power, N the particles per bunch, σ_δ the relative energy spread, ε_n the normalised transverse beam emittance, ε_l the normalised longitudinal beam emittance, and B the average dipole field in the collider ring.

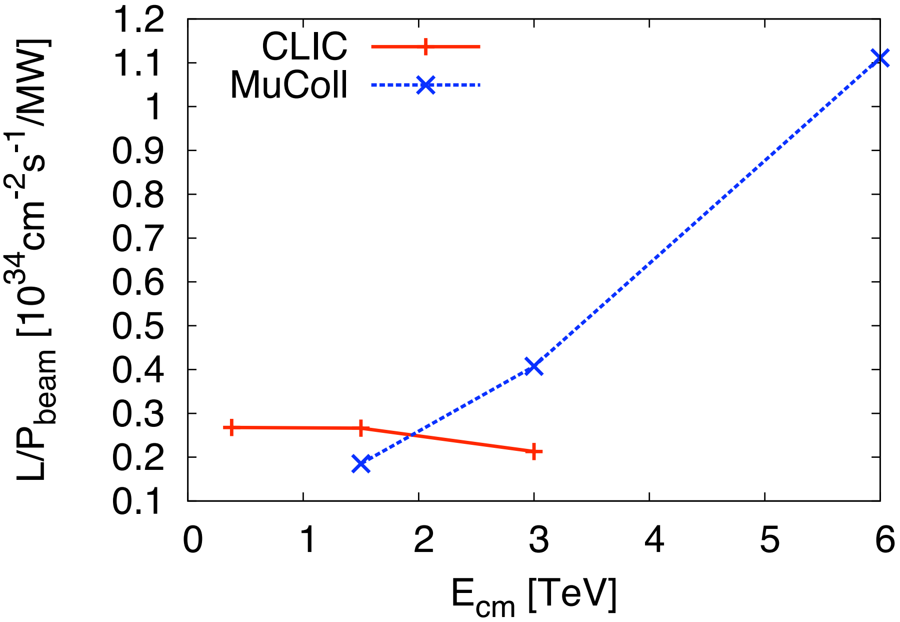

Fig. 5.2: Luminosity of the muon collider per MW of beam power, compared with CLIC, as a function of collision energy. For CLIC the full luminosity is given; due to beamstrahlung at 3 TeV about one-third of this value is above 99% of the nominal centre-of-mass energy. The muon collider luminosity per power is expected to increase linearly with energy beyond 6 TeV if the technology can be developed accordingly.

The advantageous scaling of efficiency with energy is evident. Table 5.1 shows that the beam power in a 10 TeV MC is expected to be half of the beam power in CLIC at 3 TeV. It is expected that the wall-plug power consumption of the 10 TeV MC is below that of a 3 TeV CLIC. However, the absolute value of the power consumption for a given collision energy has not been studied or optimised in detail. In particular, the energy-efficient design of rapid cycling synchrotrons with recovery of the magnetic field energy from cycle-to-cycle, and the reduction of large unrecoverable losses from eddy currents, are important topics for optimisation. Other aspects include minimising beam-induced heat load at cryogenic temperatures and efficient RF acceleration systems. The proposed programme will address these key questions and allow a quantitative assessment.

Finally, the modularity of the MC complex will allow synergy with other accelerator projects through reuse of subsystems: for instance, the high-intensity proton driver which could also serve a neutrino factory.

5.3.3 Timescale

A muon collider with a centre-of-mass energy around 3 TeV could be delivered on a time scale compatible with the end of operation of the HL-LHC. A technically limited time line is shown in Fig. 5.3. To deliver a muon collider on such a timescale, essential technical work to determine cost scale and feasibility must begin now in order for a fully informed recommendations to be made at the next ESPPU. The outcomes of the immediate R&D programme will of course allow a refinement of the long-term plan, and permit an increased level of certainty.

A programme of dedicated hardware prototyping could begin in around five years time to support the conceptual design study. Prototypes would include rapid-cycling synchrotron magnets and power supplies, high-field solenoids, high-power and high-gradient RF cavities, high-power targets, and essen-

Table 5.1: Tentative parameters for a muon collider at different energies, based on the MAP design with modifications. These values are only to give a first, rough indication. The study will develop coherent parameter sets of its own. For comparison, the CLIC parameters at 3 TeV are also given. Due to beamstrahlung only 1/3 of the CLIC luminosity is delivered above 99% of the nominal centre-of-mass energy ($\mathcal{L}_{0.01}$). The CLIC emittances are at the end of the linac and the beam size is given for both the horizontal and vertical planes.

Parameter	Symbol	Unit	Target value			CLIC
Centre-of-mass energy	E_{cm}	TeV	3	10	14	3
Luminosity	$\mathcal{L}$	$10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.8	20	40	5.9
Luminosity above $0.99 \times \sqrt{s}$	$\mathcal{L}_{0.01}$	$10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.8	20	40	2
Collider circumference	C_{coll}	km	4.5	10	14	—
Muons/bunch	N	10^{12}	2.2	1.8	1.8	0.0037
Repetition rate	f_r	Hz	5	5	5	50
Beam power	P_{coll}	MW	5.3	14.4	20	28
Longitudinal emittance	ϵ_L	MeVm	7.5	7.5	7.5	0.2
Transverse emittance	ϵ	μm	25	25	25	660/20
Number of bunches	n_b		1	1	1	312
Number of IPs	n_{IP}		2	2	2	1
IP relative energy spread	δ_E	%	0.1	0.1	0.1	0.35
IP bunch length	σ_z	mm	5	1.5	1.07	0.044
IP beta-function	β	mm	5	1.5	1.07	
IP beam size	σ	μm	3	0.9	0.63	0.04/0.001

tial proton driver components such as high-current ion sources. Additionally, a beam demonstration is necessary to show the efficacy of ionisation cooling in both transverse and longitudinal phase space and at low emittance compared to previous R&D. Such a programme would require a significant ramp-up of resources. In order to permit such an investment, the collaboration must establish within the next five years the scientific and technical justification.

Resource scenarios for the immediate programme fall within a broad range. In a minimal programme, the collaboration will study the key challenges and design drivers in order to make fundamental design choices and provide realistic targets for functional specifications of key components. This programme would provide supporting studies to show that key beam performance goals can be met, identify the key risks and provide a rough cost estimate. This will allow the decisions at the next ESPPU in the light of a better understanding of the challenges and technologies inherent in the muon collider.

A full programme would address additional key challenges, develop technologies unique to the muon collider and prepare the demonstration programme. In particular, this would enable the collaboration to provide start-to-end studies of the accelerator performance and improve the maturity of key technologies. Alternative technologies will be investigated that may enable reduction in cost and risk. This would build a higher level of confidence that the technical risks can be successfully addressed, and allow a rapid move towards the CDR phase. The full programme would therefore provide a baseline concept, well-evidenced performance expectations, and an assessment of the associated risks and cost and power-consumption drivers. It will also identify the R&D path to a full conceptual design for the collider and its experiments. This will allow fully informed decisions to be made at the next ESPPU and support similar strategy processes in other regions.

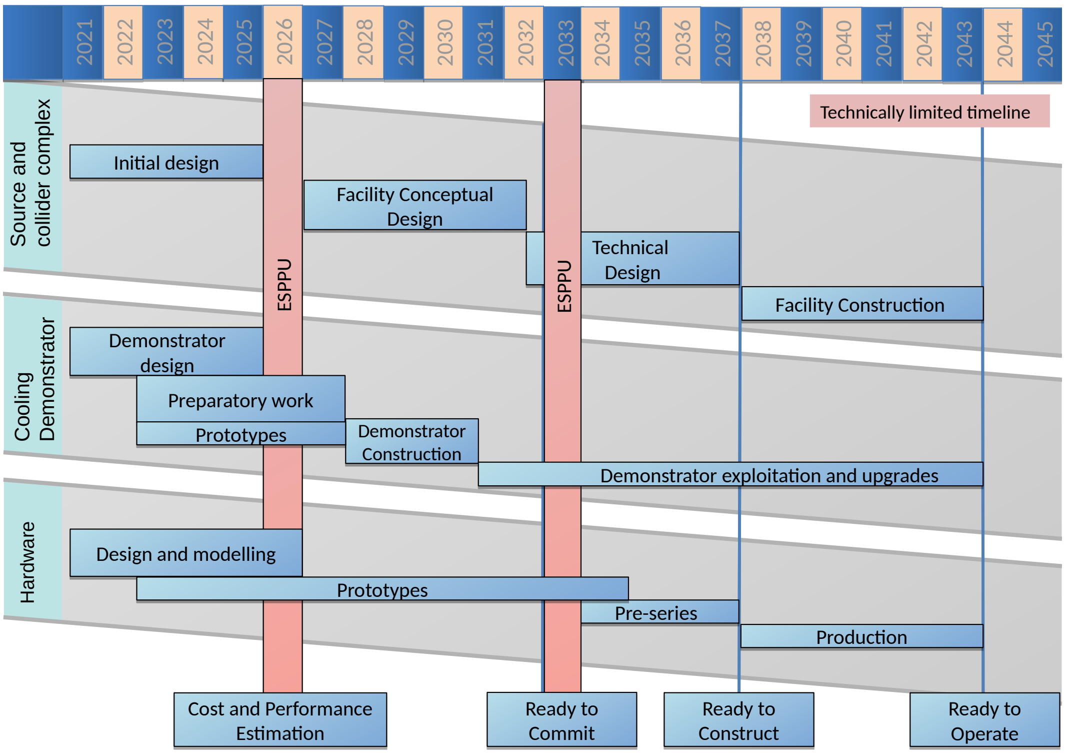

Fig. 5.3: A technically limited timeline for the muon collider R&D programme.

5.4 Muon beam panel activities

The panel employed three main routes to develop the input for the Roadmap:

- closed, fortnightly meetings of the panel to organise the work and to use the expertise of the members;
- meetings of the muon collider collaboration, which address the R&D planning;
- dedicated community meetings and workshops that draw on the world-wide expertise.

Four community meetings were held in 2021.

- A workshop held from 24–25 March to assess the testing opportunities for the muon collider, helped to arrive at a first definition of the scope of the demonstrator.
- A community meeting held from 20–21 May with nine working groups. These working groups, coordinated by an international group of conveners, identified the key R&D challenges across the project.
- A community meeting held from 12–14 July completed the formulation of the list of R&D challenges and prepared a set of proposals to address the key challenges that must be addressed before the next ESPPU.
- A community meeting in October discussed the proposed roadmap and provided feedback to the panel during the preparation of the final report.

This approach combined the expertise of the panel members, the participants in the new MC collaboration, and the participants in the earlier efforts. Contributions from the US community were extremely valuable, but necessarily limited pending the outcome of the ongoing US strategy process.

5.5 State of the art

5.5.1 *Previous developments*

Muon colliders were first proposed in 1969 by Budker [13] and later developed by Skrinsky *et al.* [14]. The concept was taken up in the late 1990s, principally in the U.S. Around the turn of the century the discovery of neutrino oscillations led to enthusiasm for a muon-based neutrino source, which could be compatible with the initial stages of a muon collider. Studies were taken up by the US Neutrino Factory and Muon Collider Collaboration. In Europe design concepts were advanced for a neutrino factory sited at CERN [15] or Rutherford Appleton Laboratory culminating in the EuroNu study [16]. The decision was made to focus on development of a neutrino source as it was viewed as a less demanding stepping stone to a muon collider.

Neutrino factory studies yielded significant advancements in the concepts required to deliver a muon collider. Concepts were developed to deliver a multi-MW proton beam based on European, American and Japanese siting options. In particular, it was realised that a very demanding short proton pulse would be required to maintain a short pion and muon bunch. Conceptual designs for appropriate bunch compression schemes were developed.

The first pulsed proton accelerators capable of delivering MW-scale proton beams were being commissioned around this time, using graphite and liquid mercury targets capable of withstanding high beam powers. Significant effort was invested in theoretical studies to understand how such target designs could be developed to be capable of withstanding several MW of beam power. Moving targets received particular attention, with liquid mercury the initially favoured option in US studies. Later studies considered alternatives such as gallium and graphite and found good performance. In Europe options were considered including tungsten powder fluidised by helium gas, multiple fixed graphite targets exposed successively to the beam to reduce the average power and beds of metal beads designed to absorb instantaneous shock. Experiments were undertaken on liquid mercury [17] and fluidised tungsten powder targets in beams [18].

Studies were also undertaken to develop solutions for capturing both positive and negative pions. Conventional neutrino targets employ horn optics, which acts as a focusing element for one pion charge and a defocusing element for the opposite charge. In order to focus both positive and negative pions, a solenoid was proposed [19]. To capture a large phase space volume, solenoids with fields in the range 15 to 20 T were considered. Consideration of shielding led to designs having large aperture in order to accommodate sufficient material to absorb the radiation from the target without causing radiation damage to the superconductor and impractical heat deposition in the liquid helium.

Following the target, the design of the solenoid field employed a taper to lower values in order to contain the pions as they decayed to muons and transport the beam through to later parts of the accelerator [20]. Despite the short proton beam, the resultant pion beam still occupied a large longitudinal phase space with a huge energy spread. Initial studies dealt with this using low frequency RF in the 50 to 100 MHz range. Owing to the low frequency, the RF cavities were large, some 2 m in diameter, with significant challenges concerning practicality of construction and integration with transport solenoids.

A novel scheme was developed employing more manageable RF cavities with frequencies in the range 325 to 650 MHz. In order to accommodate a beam that was much longer than the RF frequency, cavities near the target had a low voltage while cavities downstream had a higher voltage to adiabatically introduce a train of microbunches into the beam. An energy-time correlation developed before the bunches were properly captured, which evolved during the capture process. This was managed by employing higher-frequency cavities near the target and lower-frequency cavities downstream. After capturing the bunches, the design employed cavities that had a slightly different frequency to the microbunch spacing so that the early bunches experienced a decelerating field and the late bunches experienced an accelerating field. In the end, simulations yielded a bunch train that was flat in energy and captured within a RF bucket corresponding to conventional accelerator frequencies.

It was observed that significant beam impurities were transported by the muon front end. While this might be dealt with by a dipole-type chicane in more conventional machines, in the muon transport lines the beam had a large emittance that was challenging to transport through a regular chicane. A pure solenoid dogleg chicane was designed, drawing from experience from early stellarator designs. Solenoids in this arrangement induce a vertical dispersion, so high momentum particles scraped off on collimators in the roof or floor of the chicane, depending on the sign. Simulations indicated a very sharp cut-off momentum, below which even high emittance particles were transported with very little emittance growth for momenta very close to the cut-off. At the end of the chicane low-momentum protons, which lose much more energy than pions and muons, were stopped in a plug of beryllium.

By the end of the EuroNu design study, the concept for the muon capture system was considered mature, although further iterations were made to optimise RF frequency and solenoid field strength. Challenges remained especially in the target region, where practical experience of graphite and liquid mercury targets in high-power environments yielded new insights and detailed engineering of the challenging magnet was not performed.

Downstream of the muon capture region, the neutrino factory studies focused on reducing the transverse emittance of the beam so that it was suitable for acceleration. Conventional cooling techniques such as stochastic or electron cooling are not capable of cooling the beam on a sufficiently short time scale. Ionisation-cooling schemes were proposed as an alternative. In ionisation cooling, beams are passed through material, which absorbs transverse and longitudinal momentum due to ionisation. The longitudinal momentum is restored by RF cavities, resulting in a beam having lower transverse emittance. Multiple Coulomb scattering off atomic nuclei degrades the cooling effect. In order to decrease the contribution from scattering, energy absorbers having low atomic number are considered along with tight focusing. Nonetheless, systems have a minimum equilibrium emittance where the ionisation cooling and scattering effects cancel.

Initial cooling studies employed an elaborate system capable of yielding relatively low transverse emittance beams with no longitudinal cooling. Most designs for acceleration employed relatively high-gradient RF to promote rapid acceleration with a large RF bucket, so longitudinal emittance was considered manageable. It was later found that a more practical and simpler cooling system would yield a sufficiently low-emittance muon beam while saving on cost and complexity [21,22].

The muon cooling system was felt to be sufficiently complex and novel that an experiment was required. The Muon Ionisation Cooling Experiment was initiated in this period by a collaboration drawn from Europe, North America and Asia. An entirely new muon beam line was constructed at Rutherford Appleton Laboratory, together with bespoke beam instrumentation capable of measuring individual muons as they traverse the apparatus and a tightly focusing arrangement of solenoids. This led to the demonstration of the transverse muon ionisation cooling concept for the first time [2].

For the neutrino factory, energies in the 5 to 50 GeV range were considered, in order to generate the desired neutrino energy spectrum. Acceleration used a linac at lower energies where the beam is not fully relativistic and geometric emittances are higher. At higher energies combinations of recirculating linear accelerators (RLA) and fixed field alternating gradient accelerators (FFA) were considered. The RLAs are conceptually similar to the acceleration phase of a multi-pass ERL. Muons decay so the energy cannot be recaptured [23].

A beam test was carried out for FFAs using a scaled model based on electrons, the so-called Electron Model with Many Applications (EMMA) [24]. This showed that rapid acceleration was possible, with large acceptance despite the beam passing many resonances, and acceleration using fixed-frequency RF despite the time-of-flight of the beam changing as the beam increased in energy.

While the focus in this period was on neutrino production, development was ongoing in muon colliders. The development of techniques to reduce longitudinal as well as transverse emittance, 6D cooling, and the discovery of the Higgs boson meant the muon collider became topical. The Muon

Accelerator Programme collaboration was formed in the US to develop the muon collider concept while maintaining the possibility to develop an intense neutrino beam as a first stage.

Initial ideas for 6D cooling involved rings. Dispersion, when combined with wedge-shaped absorbers, would enable transfer of emittance from longitudinal phase space to transverse phase space. However, practical issues surrounding injection and extraction proved very challenging. Instead, linear systems having solenoids superimposed with dipole fields were found to yield sufficient dispersion to enable significant longitudinal cooling. Such systems had the advantage that, as lower emittances were reached, tighter focusing systems could be employed. Typically such cooling systems had a smaller minimum equilibrium emittance, but at the expense of reduced dynamic aperture, so tapering of the cooling lattice was envisioned which is not possible in a ring.

The ultimate limit of the 6D cooling was determined assuming that High Temperature Superconductors would not be available in such an arrangement. The system assumed closely packed coils with adjacent coils having opposite polarity. Preliminary force calculations indicated that the simulated lattices were feasible. In principle improved performance could be achieved by using higher fields and closer packing of the coils; the ultimate technical limit was not studied.

Other novel systems were considered. An alternative cooling system using a helical dipole-solenoid arrangement appeared capable of rapid cooling, but the scheme did not reach the same emittance and transmission as the rectilinear cooling scheme outlined above. At higher emittance a 6D cooling lattice was investigated capable of cooling both positive and negative muon species simultaneously. This would yield a much lower emittance, making the separation of positive and negative muons easier. A system for merging the bunch train produced by the front end was also designed. This was an important component of the system, as a single merged bunch would yield a significantly higher luminosity than the bunch train.

Studies undertaken as part of EuroNu indicated that the size of the RF bucket was a crucial parameter and high real-estate gradients were important not just to keep the cooling channel short but also to prevent beam losses in the presence of energy straggling. Magnetic fields were well-known to induce breakdown well below the normal limit. A dedicated hardware R&D programme yielded two solutions: either using hard cavity materials less prone to damage by electrons such as beryllium; or insulation of the cavities with high-pressure gas to absorb multipacting electrons.

Initial studies yielded lattice simulations indicating several-orders-of-magnitude reduction in longitudinal and transverse emittance, yielding luminosities suitable for a Higgs factory. In order to reach luminosity suitable for collision at a multi-TeV muon collider, additional transverse emittance reduction was required. In order to reach extremely small transverse emittance, very strong solenoids were considered, operating at low momentum to get the strongest possible focusing. Solenoids up to 30 T with aperture of a few cm, beyond the state of the art at the time, and momentum below 100 MeV/c were considered. In this energy range, well below minimum-ionising energy, muons with low energy lose more energy than muons with high energy. This results in increased energy spread and longitudinal emittance growth, but the transverse cooling more than compensated and the final designs appeared capable of reaching high luminosity. Preliminary studies were performed that indicated that further significant luminosity improvements could be achieved using higher field solenoids, but the prospect of higher field solenoids seemed unrealistic at the time.

Parametric Resonance Ionisation Cooling was also investigated for final cooling, using tight focussing available in near-resonance conditions. Progress was made in maintaining sufficient dynamic aperture, but further studies are required to demonstrate competitive performance.

Studies for acceleration considered a staged scheme, that could at first yield a neutrino source, and subsequently yield a collider at the Higgs resonance, with less detailed consideration of acceleration to multi-TeV energies. Acceleration for a neutrino source would use a linac or a recirculating linear accelerator (RLA). Acceleration to 63 GeV for a Higgs factory would be achieved by adding another RLA, with

Rapid Cycling Synchrotron stages added to reach TeV energies. Very rapid cycling times are required in order to accelerate on a time scale compatible with the muon lifetime, making significant demands on dipole magnets and power supplies. Studies were made considering combined fixed superconducting magnets with rapid cycling normal conducting magnets.

Collider ring studies investigated the possible luminosity that could be achieved. In order to make the largest number of bunch crossings before muons decay, the ring should have the lowest circumference possible. The luminosity is therefore proportional to the magnetic field.

In order to avoid the hour-glass effect, short bunches were required meaning low longitudinal emittance and low momentum-compaction factors. Studies showed a tight final focus could be achieved yielding a high luminosity, albeit with challenging magnet parameters.

Particular attention was given to the effect of neutrino radiation originating from muon decay. Decay neutrinos may interact with material near to the surface, and a long way from the their production point, giving rise to a small but significant shower. Mitigations for this weak off-site radiation were considered, for example the use of wide aperture magnets with the beam path slowly moved to spread the radiation so that it is not concentrated in a particular area. Muon decays also yielded an issue for the detector, where they would induce a significant electron background. Shielding of the detector, together with background rejection techniques such as timing cuts, appeared to be capable of reducing the effects of the beam-induced background to a manageable level.

Other developments have taken place since the end of the MAP programme that are important to muon collider development. Promising experiments have been performed at PSI on frictional cooling. Frictional cooling works in a similar fashion to ionisation cooling, but at muon energy below 1 MeV. In this energy regime lower energy muons lose less energy than higher energy muons owing to different energy loss contributions of inner electrons, so there is natural longitudinal cooling. However, the energy acceptance of the system is naturally lower and a full evaluation of such a system's suitability for a muon collider is required.

An alternative scheme, LEMMA, to produce a muon beam using positrons impinging on a target very near to the muon production threshold has been considered at INFN. An injector complex produces an extremely high-current positron beam. The positrons impact a target with an energy of 45 GeV, sufficient to produce muon pairs by annihilating with the electrons of the target. This scheme can produce small emittance muon beams. However, it is difficult to achieve a high muon beam current and hence competitive luminosity. Novel ideas are required to overcome this limitation.

An energy recovery linac for electrons, CBETA, has been demonstrated in the US employing FFA arcs. Such a machine would be similar to the FFAs that were considered for early stage muon acceleration, although energy recovery of muons is not possible owing to muon decay.

5.5.2 Current status of the feasibility R&D

Significant investment into muon accelerator R&D was made as part of EuroNu: the MICE collaboration completed a detailed measurement of the ionisation cooling process [2]; rapid acceleration of muons in a fixed field accelerator was demonstrated by EMMA; and schemes for high power targetry using liquid metal [17] and fluidised powder jets [18] were demonstrated.

By the beginning of the MAP study, designs for several components of the muon collider existed. The MAP Collaboration initiated its study with an evaluation of the feasibility of the key sub-systems required to deliver an energy-frontier collider [25]. Several issues were identified as part of the MAP feasibility assessment that had the greatest potential to prevent the realisation of a viable muon collider concept. These included:

- operation of RF cavities in high magnetic fields in the front end and cooling channel;
- development of a 6D cooling lattice design consistent with realistic magnet, absorber, and RF

cavity specifications;

- a direct demonstration and measurement of the ionisation-cooling process;
- development of very-high-field solenoids to achieve the emittance goals of the final cooling system;
- demonstration of fast-ramping magnets to enable RCS capability for acceleration to the TeV scale.

While other machine design and engineering conceptual efforts were pursued to develop the overall definition of a muon collider facility, research in the above feasibility areas received the greatest attention as part of the MAP effort.

An important outcome of MAP was that progress in each of the above areas was sufficient to suggest that there exists a viable path forward. The test program at Fermilab's MuCool test area demonstrated operation of gas-filled and vacuum pillbox cavities with up to 50 MV/m gradients in strong magnetic fields [26, 27]; a 6D cooling lattice was designed that incorporated reasonable physical assumptions [28]; a final cooling channel design, which implemented the constraint of a 30 T maximum solenoid field, came within a factor of two of meeting the transverse emittance goal for a high energy collider [29] and current development efforts appear poised to deliver another factor of $\sim$1.5 improvement; and while further R&D is required, fast-ramping magnet concepts [30] do exist that could deliver TeV muon beams.

Since the end of the MAP studies a number of technologies have developed, which make the muon collider a promising avenue of study. In particular, new studies are required to leverage the now-increased limits of solenoids and RF cavities, which theory suggests should give an improved cooling channel performance.

5.6 R&D objectives

Based on the MAP design, tentative target parameter sets have been defined for the collider as a starting point, shown in Table 5.1 above. If all design goals are met, these parameters would deliver the desired integrated luminosities within about five years of the end of commissioning. These design goals serve to clarify the critical design issues and, once detailed studies are available, the parameters will be revised and operational budgets that account for sources of beam quality degradation will be added. This might increase the time needed to achieve the integrated luminosity target.

The parameter sets have a luminosity-to-beam-power ratio that increases with energy. They are based on using the same muon source for all energies and a limited degradation of transverse and longitudinal emittance with energy. This allows the bunch in the collider to be shorter at higher collision energy and smaller beta-functions used. The design of the technical components to achieve this goal are a key element of the study.

A 10 TeV lepton collider is uncharted territory and poses a number of key challenges.

- The collider can potentially produce a high neutrino flux that might lead to increased levels of radiation far from the collider. This must be mitigated and is a prime concern for the high-energy option.
- The machine detector interface (MDI) might limit the physics reach due to beam-induced background, and the detector and machine need to be simultaneously optimised.
- The collider ring and the acceleration system that follows the muon cooling can limit the energy reach. These systems have not been studied for 10 TeV or higher energy. The collider ring design impacts the neutrino flux and MDI.
- The production of a high-quality muon beam is required to achieve the desired luminosity. Optimisation and improved integration are required to achieve the performance goal, while maintaining low power consumption and cost. The source performance also impacts the high-energy design.

Integrated accelerator design of the key systems is essential to evaluate the expected performance, to validate and refine the performance specifications for the components and to ensure beam stability and quality. Tables 5.2 and 5.3 describe the key technology challenges and their relation to the state-of-the-art.

Table 5.2: Description of principal technical challenges for series hardware items, where a significant number of each item will be required.

Proton driver bunch compression: Similar proton beams have been used at facilities such as SNS, however none with the short bunch that is required to achieve a good quality muon beam. Simulations performed for a neutrino factory at Fermilab, RAL or based on the proposed CERN SPL indicate that such a bunch compression is achievable but need to be matched to the specific conditions proposed here.
Muon cooling design: The muon cooling design has been worked through in simulation of individual components to yield the low emittance beams assumed in this document. Simulation of the final cooling system indicates 55 µm transverse emittance and 75 mm longitudinal emittance could be achieved. This document assumes that an improvement to 25 µm transverse emittance could be achieved. No start-to-end simulation has been performed and performance may be improved in the light of new magnet and RF technologies. If low emittance cannot be reached, higher power on target would be required or luminosity would be reduced.
Muon cooling rectilinear magnets: The MAP baseline design assumed rectilinear cooling channel solenoids with fields up to 13.6 T in a closely packed configuration with adjacent magnets having opposite polarity. Mechanical analysis shows satisfactory performance but indicates the lattice needs to be adjusted to enable a suitable support structure in proximity to the RF cavities.
Muon cooling rectilinear RF: The RF cavities in the MAP design, simulated with up to 28 MV/m on-axis at 650 MHz, sit very close to the magnets, which can induce breakdown. Tests that have been performed using a single cavity filled with high pressure hydrogen gas showed operation with 65 MV/m on-axis at 805 MHz while immersed in a 3 T field. Additional tests have been performed using a single beryllium-walled cavity that operated with 56 MV/m on-axis at 805 MHz, also in a 3 T field.
Muon acceleration RF: Beam loading of the RF cavities is a principal concern during acceleration. High gradients may be available using 1.3 GHz RF, for example operating at the ≈35 MV/m demonstrated for ILC, but the smaller cavities are more sensitive to beam loading, and optimisation of the frequency must be performed to understand the appropriate parameters.
Muon acceleration magnets: The muon collider requires fast-ramping synchrotron magnets. Ramps from −2 T to 2 T on a time scale of 2 ms have been considered. Normal-conducting magnets capable of ramping at 2.5 T/ms with peak field of 1.81 T have been demonstrated. HTS superconducting magnets have been demonstrated operating with faster ramp speeds, 12 T/ms, but a lower peak field of 0.24 T, have also been demonstrated. As several km of magnets are required, the cost and efficiency of the power supplies is a critical parameter.
Collider dipoles: The collider ring demands a small bending radius to attain the highest number of bunch crossings before decay. Dipole fields have been assumed of 11 T with a bore aperture of 15 cm for the 3 TeV collider and 16 to 20 T for the 10 TeV collider. A similar magnet has been demonstrated operating at 14.6 T with a bore aperture of 10 cm.

Table 5.3: Description of principal technical issues for unique hardware items, where only one or a few of each item will be required.

Muon collider target: The muon collider target will operate around 5 GeV and with a 5 Hz repetition rate with beam power around 2 MW, depending on the performance of the muon beam cooling system. This is equivalent to the state of the art; T2K receives 750 kW on target at considerably higher energy, while SNS operates with a liquid mercury target. Care must be taken to ensure the survival of the target under such conditions.

Muon collider target magnet: The muon collider baseline is to employ a very high field solenoid to capture pions. This will require an extremely large bore in order to accommodate radiation shielding. The highest proposed proton beam power for such a target is for rare muon decay experiments where targets are proposed up to around 100 kW with fields in the range of a few T. The fallback is to use horn-type focusing, which efficiently captures only a single sign of muon.

Muon final cooling solenoids: The MAP scheme had final cooling solenoids operating with fields up to 29 T and yielded a transverse emittance that was a factor two higher than outlined in this document. Commercial MRI magnets are now available with fields of 28 T and the highest field pure-superconducting magnets are in use with fields of 32 T, with bores similar to those required for the muon collider.

Final focus quadrupoles: Designs for a 3 TeV collider employed a final focus gradient of 250 T/m and 0.08 m aperture. This can be compared for example with the HL-LHC final focus quadrupoles, with a gradient of 132.2 T/m and 0.15 m aperture.

5.6.1 Neutrino radiation

Muon decay produces a large flux of high-energy neutrinos in a very forward direction. This can lead to a high local flux of neutrinos in the plane of a collider ring, which has a small likelihood of producing showers when exiting the ground at a distance from the facility. The insertions produce a very localised flux in a limited area; the arcs in contrast produce a ring of flux around the collider.

Minimising the flux in public areas is a prime goal of the study; this implies staying well below the legal limit for off-site radiation, for example at a level comparable to that arising from LHC operation. Using formulae from Ref. [31], one finds that, even in a 200 m deep tunnel, decays in the arcs of a 10 TeV collider approach the legal limit for the neutrino flux.

The proposed solution is a system of movers to deform the beamline periodically in the vertical plane so that narrow flux cones are avoided. Flux from insertions can be further minimised by acquiring the affected surface land and by using a large divergence in the focusing triplets. This solution improves on a previous, less performant, proposal to move the beam within the magnet apertures [32]. This system could achieve radiation levels similar to the LHC. The development of a robust system is the key to siting the collider in a populated area. Impact on the ring performance must be minimised. Proper consideration for vacuum connections and cryogenics systems must be made. Management of neutrino flux is a critical issue for the muon collider.

5.6.2 Machine-detector interface

Detector design at a muon collider has to be performed together with the machine-detector interface due to the substantial flux of secondary and tertiary particles coming from muon beam decay. Integrated studies of the detector and the collider are needed to ensure a properly optimised performance. Beam-induced background, arising both from muon decays and incoherent e^+e^- pair production, is a serious concern for the detector performance. The current solution to mitigate the background arriving at the detector consists of two tungsten cone-shaped shields (nozzles) in proximity to the interaction point, dimensionally accurate and optimised for a specific beam energy. A framework based on FLUKA has been developed to optimise the design at different energies [33].Studies performed so far demonstrate that, given reasonable assumptions of detector performance, it will be possible to perform the most challenging physics measurements [34]. Optimisations, for example using improved pixel timing in the tracker detector and novel trigger algorithms, are in progress and may yield further improved performance. This requires additional studies at higher energies. Combined design of the interaction region, detector shielding and detector should be performed to confirm physics performance at 3 TeV and 10 TeV.

5.6.3 Proton complex

Based on MAP calculations, the average proton beam power required in the target is in the range of 2 MW, but this needs to be fully validated by an end-to-end design of the facility. The proton beam energy should be in the range of 5 to 15 GeV. The power appears feasible; spallation neutron sources like SNS and J-PARC already operate in the MW regime and others like ESS and PIP-II are under construction. The Superconducting Proton Linac (SPL), an alternative injector complex considered for the LHC, would have provided 4 MW of 5 GeV protons. The collector and compressor system merges the beam into 2 ns-long pulses with a repetition rate of 5 Hz. Alternatively, the use of an FFA or fast-pulsed synchrotron could be considered, profiting from synergies with the next generation of spallation neutron sources in the UK and experience in Japan. In this case, the optics, magnet design and collective effects needs to be studied. The challenge of generating a high-intensity, short bunch at low repetition rate should be investigated. In particular, designs for an accumulator and compressor system should be developed, taking into account existing H^- ion sources and capability of H^--stripping systems for injection into the ring.

5.6.4 Muon production and cooling

Muons are produced via tertiary production ($p \to \pi \to \mu$) by delivering a multi-MW proton beam onto a target. The baseline design concept in MAP assumed a 6.75 GeV H^- linac with accumulator and buncher rings to properly format the proton beam, with a final combiner system to bring multiple proton bunches simultaneously onto the target for pion production. The proton energy was chosen in order to facilitate a neutrino factory, but in the 5 to 15 GeV proton energy range the muon production rate is proportional to the beam power and exhibits only a weak dependence on the beam energy, so other energies in this range are suitable [35].

The front-end systems begin with a multi-MW target enclosed in a high-field, large-bore solenoid magnet to enable simultaneous capture of both positive and negative species [19]. A tapered solenoid section matches into a decay channel where the pions produced at the target decay into muons. RF cavities capture the muons into a bunch train and then apply a time-dependent acceleration to decrease the energy spread of the muons [36].

The bunched muons from the front end must be rapidly cooled to achieve the required emittances for a collider before the unstable muons can decay. In the MAP scheme, an initial cooling channel [37], capable of cooling both species of muons simultaneously, reduces the 6D phase space of the beam by a factor of 50. The two muon species are subsequently separated [38] into parallel 6D cooling channels to continue reducing the beam emittance to the levels required for luminosity production in a collider. This emittance reduction for the individual species occurs in four distinct steps:

1. 6D cooling of the bunch train that is delivered from the Charge Separator;
2. a Bunch Merge stage to combine the bunch trains into a single bunch of each species [39];
3. a second 6D Cooling section to reduce the emittance of the individual bunches;
4. a Final Cooling section that trades the longitudinal emittance for improved transverse emittance of the beam.

Table 5.4: Parameters for a selection of proposed and operational pion and neutron production targets.

Facility	Average power on target [kW]	Beam energy [GeV]	Repetition rate [Hz]	Target material	Secondary particle species	Focusing type
T2K	750	30	0.5	Graphite	Pion	Horn
LBNF (proposed)	1200	60-120	1	Beryllium	Pion	Horn
Mu2E (Under Construction)	8	8	0.75	Aluminium	Pion	Solenoid
COMET Phase I	3	8	0.4	Aluminium	Pion	Solenoid
ISIS	200	0.8	50	Tungsten	Neutron	None
ESS (Under Construction)	5000	3	15	Tungsten	Neutron	None
SNS	1400	1	60	Mercury	Neutron	None
JPARC	500–1000	3	25	Mercury	Neutron	None

In the MAP studies, the best 6D cooling performance achieved was based on the so-called rectilinear cooling channel [28] while the performance of the baseline final cooling channel [29] was limited by the maximum achievable B-field for the solenoid magnets in the design.

A solid target might be able to handle 2 MW beam power, but evaluations of the stress and heating must be performed. The short proton bunch length and 5 Hz operation result in a large instantaneous power. Preliminary studies indicate target lifetime in these circumstances may be compromised and target heating will be an issue. A liquid metal [40] or a fluidised tungsten target [41] are alternative solutions in case a solid target cannot withstand the 2 MW or start-to-end studies indicate that the muon survival is insufficient and higher production rates, and hence beam power, are required. Tabel 5.4 shows the parameters of a selection of high-power target designs for current or near-future facilities.

The system of high-field solenoids with tapered fields around the target and downstream is challenging. At the target the field of a 15 T superconducting solenoid is boosted to 20 T with an inner copper solenoid. An alternative 15 T solution has also been explored by the MAP collaboration and may have sufficient performance [19]. The large 1.2 m aperture of the superconducting solenoid provides space for shielding from the target debris to avoid quench and radiation damage. The magnet design, with associated proton dump, and the radiation environment are key for overall machine performance. A preliminary engineering study of the target magnet should be performed, including consideration of radiation arising from beam interaction with the target. Studies of stress and heat load on the target should be performed. Alternative solutions, for example using liquid metal, should be considered to manage the large instantaneous power.

While subsystem designs exist that indicate the required cooling performance for the target luminosity, they have not been integrated, and further optimisation is expected to yield significant performance improvements.

The accelerating cavities are key to cooling efficiently and with limited loss of muons. Large real-estate gradients are required to ensure sufficient longitudinal acceptance so that the beam is well-contained. The lattice is very compact to yield very tight focusing so cavities sit in significant magnetic fields. Magnetic fields are known to compromise the available RF gradient. Two approaches were considered in MAP: either using high-pressure hydrogen-filled cavities, or beryllium end-caps, both of which are unconventional technology. The two approaches were each demonstrated on single test cavities but never incorporated into a cooling cell. The accelerating cavities should be developed experimentally so that they can be properly integrated into a cooling demonstrator. New solutions to the high gradient problem could also be investigated.

The baseline final cooling uses high-field solenoids to minimise the beam emittance. Pushing their field beyond the current state of the art, around ∼30 T, would improve the collider performance and appears feasible given the rate of progress in magnet R&D. The luminosity increases roughly linearly with the field and the high-energy systems could potentially have smaller apertures, which can simplify their design. The current and expected availability of high-field solenoids should be examined and appropriate magnet options should be incorporated into the muon collider design.

The overall design has to be optimised to improve the transverse emittance by a factor two and achieve the target performance. Further improvements would facilitate the machine design in the high energy complex. Alternative options have been proposed and need to be evaluated. In addition, the collective effects and beam-matter interactions should be explored further to validate the overall emittance performance. Integration of the muon production subsystem designs should be performed. Optimisation should be performed, paying particular attention to those areas that can significantly improve facility performance.

5.6.5 High-energy complex

Cooled muons finallly traverse a sequence of accelerators. The MAP scheme envisioned an initial linac followed by an RLA that could provide 5 GeV muons for neutrino factory applications [23]. A second RLA would then take the beams to 63 GeV to enable an s-channel Higgs Factory option. A series of Rapid Cycling Synchrotrons (RCS) would be used to reach to reach TeV-scale energies.

Collider designs were developed for an s-channel Higgs factory, as well as 1.5, 3.0 and 6.0 TeV collision energies [42]. There are several notable features associated with the design of a muon collider ring. First, the luminosity performance of a muon collider is proportional to the dipole field that is used in the ring. Next, muon decays within the collider ring require large aperture superconducting magnets with shielding around the beam-pipe to prevent excessive radiation load on the magnets themselves. Finally, the use of straight sections in the ring must be minimised to prevent tightly focused beams of neutrinos from creating radiation issues.

In the collider and accelerator rings of the high energy complex both muon beams will pass through the same magnet apertures moving in opposite directions; single aperture magnets are sufficient.

Longitudinal beam dynamics is the key to high luminosity. Each muon beam consists of one high-charge bunch and the accelerating cavities must be designed to have an acceptable single-bunch beam loading. This is more demanding at high energies where shorter bunches are required to boost the luminosity. A global lattice design for the high energy complex should be developed, including start-to-end simulations of key systems, taking into account the need to move the magnets in order to mitigate neutrino radiation. Particular attention should be paid to longitudinal collective effects such as beam loading. Consideration should be made of RF cavity design and effective beam-loading compensation schemes.

In the baseline scheme, acceleration to 10 TeV centre-of-mass energies requires ∼30 km of 2 T fast-ramping normal-conducting magnets, which are interleaved with fixed-field superconducting magnets. The magnets for acceleration to high energies are a large-scale system that can have significant impact on the cost and power consumption of the facility. Design and prototyping should be performed for these magnets. Alternative options based on high-temperature superconductor (HTS) should be explored.

The collider ring arc magnets have to combine high dipole field, to maximise the collision rate, and large aperture, to allow shielding in the magnet bore to protect the cold mass from the 500 W/m of high energy electrons and positrons produced by muon beam decay. Combined function magnets are essential to minimise the neutrino flux and the field-free gap between magnets must be minimised for the same reason. Shielding of the collider ring magnets from muon decay products drives the aperture and consequently the maximum field that can be achieved. Particular attention needs to be given to optimisation of the aperture in order to yield the best performance.

The quadrupoles of the 3 TeV final focus pose similar challenges to the ones of HL-LHC or FCC-hh. At 10 TeV larger aperture and higher magnetic field in the aperture are required and call for HTS. The design of the correction system to achieve the required bandwidth for the final focus system is a key challenge to ensure that the luminosity per beam power can increase with energy. The final focus magnets should be developed, paying attention to the needs of the detector and any beam-induced background.

5.7 Delivery plan

The muon collider is expected to provide a sustainable long-term path toward high-energy, high-luminosity lepton collisions. The goal of the study is to assess and develop the concept to a level that allows the next ESPPU to make fully informed decisions about the role of the muon collider in the future of particle physics. In particular, based on the study outcome and strategic decisions, a conceptual design and demonstration programme could then be launched.

Two energy scales are currently considered: 10 TeV and 3 TeV. This should allow a better understanding of the trade-off of risk and cost compared to performance. Also, the 3 TeV option could be the first step toward the implementation of a 10 TeV machine. The latter would require an additional accelerator ring and a new collider ring. All of the 3 TeV option could be reused with the exception of its 4.5 km-long collider ring. The cost of the 3 TeV stage might be roughly half the cost of the full machine. The 3 TeV stage could be implemented faster, since it is more compact and is currently assumed to use magnets in the collider ring with fields similar to those that are developed for the HL-LHC, but with larger aperture for the dipoles. The R&D programme will focus on the 10 TeV collider but naturally encompass all challenges of the 3 TeV stage. Dedicated studies of the 3 TeV option will only be made where this is required in view of the more aggressive timeline.

It is expected that with this strategy a 3 TeV stage could be realised as the next European high-energy collider project in case that a Higgs factory is realised by other means. At this moment, no insurmountable obstacle has been identified that would prevent realising the technically limited timeline shown in Fig. 5.3, with a start of commissioning before 2045. This is an ambitious scenario that requires early investment, in particular into the muon-cooling technology, including the solenoids and RF, and into the fast-ramping magnet technology. A significant further ramp-up of effort is required for the full range of technologies after the next ESPPU.

In the following, two R&D scenarios are described: The aspirational programme, which allows the collaboration to reach its ambition by the next ESPPU; and the minimal programme, which contains a sub-set of the R&D activities. Both programmes require more resources than are currently committed.

5.7.1 Main deliverables

Three main deliverables are foreseen:

- a project evaluation report that assesses the muon collider potential as input to the next ESPPU;
- an R&D plan that describes a path towards the collider;
- an interim report by the end of 2023 that documents progress and allows the wider community to update their view of the concept and to give feedback to the collaboration.

The associated timeline is shown in Figs. 5.4, 5.5, and 5.6. The availability of the Interim Report will coincide with the expected time when the strategy process in the US will arrive at its conclusion.

The timeline is made under the assumption that the decision making bodies want to maintain the momentum and the option of a fast muon collider implementation by supporting efforts during the

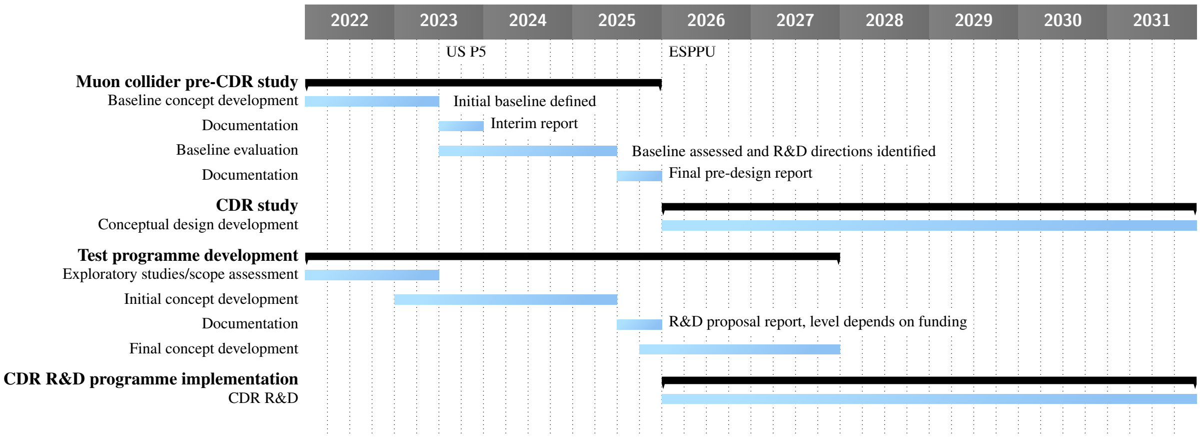

Fig. 5.4: Overall timeline for the R&D programme.

strategy and decision making process in 2026 and 2027, beyond the resource-loaded schedule presented below. This includes limited cost efforts such as the construction of a prototype cooling cell module and the legal procedures to prepare the demonstrator construction. An important ramp-up of resources could occur in 2028 following the strategy process and the subsequent decisions and would include the construction of the demonstrator.

5.7.1.1 Project evaluation report

The project evaluation report will contain an assessment of whether the 10 TeV muon collider is a promising option and identify the required compromises to realise a 3 TeV option by 2045. In particular the questions below would be addressed.

- What is a realistic luminosity target?
- What are the background conditions in the detector?
- Can one consider implementing such a collider at CERN or other sites, and can it have one or two detectors?
- What are the key performance specifications of the components and what is the maturity of the technologies?
- What are the cost drivers and what is the cost scale of such a collider?
- What are the power drivers and what is the power consumption scale of the collider?
- What are the key risks of the project?

5.7.1.2 R&D plan

The R&D plan will describe the R&D path toward the collider, in particular during the CDR phase, and will comprise the elements below.

- An integrated concept of a muon cooling cell that will allow construction and testing of this key novel component.
- A concept of the facility to provide the muon beam to test the cells.
- An evaluation of whether this facility can be installed at CERN or another site.
- A description of other R&D efforts required during the CDR phase including other demonstrators.

This R&D plan will allow the community to understand the technically limited timeline for the muon collider development after the next ESPPU.

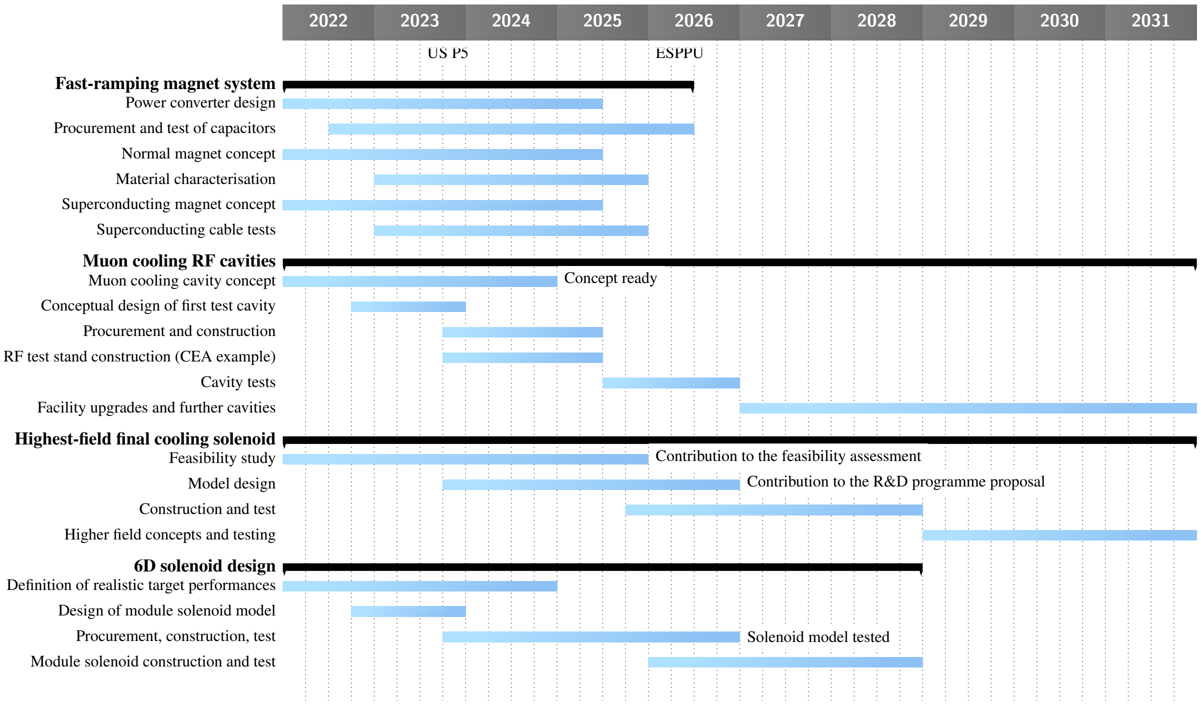

Fig. 5.5: Timeline for the technology R&D part of the programme. The solenoid model testing aims to develop the technology and will be followed by a programme to develop full performance models. The 6D solenoid models and the RF cavity tests provide input to the design choice for the prototype module.

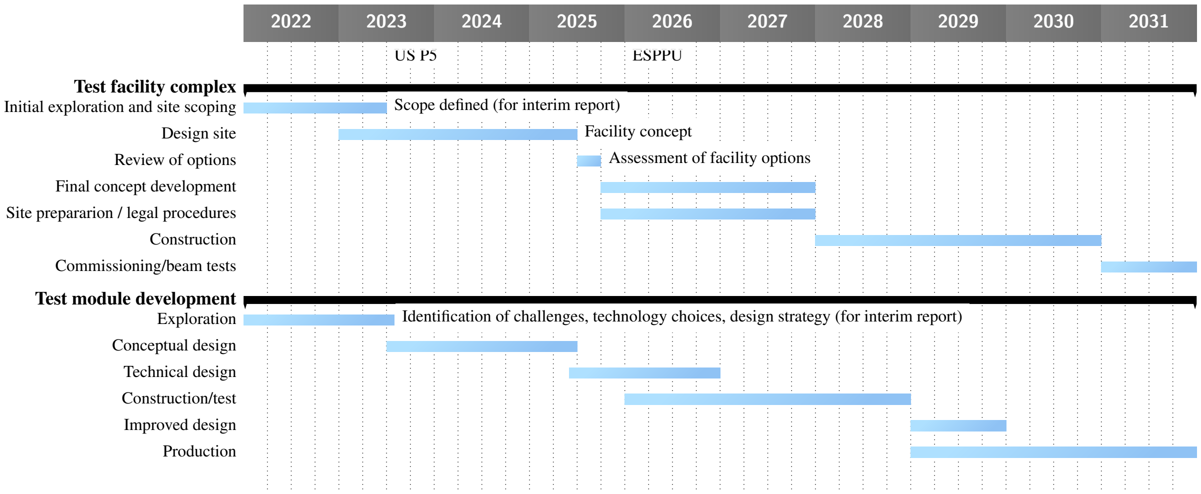

Fig. 5.6: Timeline for the demonstrator R&D. The long-lead-time procurement of prototype module components would start in 2026, while the technical design continues, aiming at a prototype module to be ready by end of 2028. Within the framework of the demonstrator design an infrastructure to test the module with power will be developed.

5.7.1.3 Interim report

The Interim Report at the end of 2023 will allow the community to gauge the progress of the concept well in advance of the next ESPPU. It will also provide an opportunity for additional feedback to the collaboration.

5.7.2 Scope of the aspirational scenario

The aspirational scenario contains theoretical studies of the accelerator design and the technologies in order to define key functional specifications of the collider complex and components that allow achievement of the performance goals and that are realistic targets for the technology developments. This effort will be supported by a limited experimental programme to improve the reliability of the estimates:

- component tests for the unique fast-ramping magnet system and its powering, to demonstrate sufficient muon energy reach with appropriate cost and power efficiency;
- construction of models for the superconducting solenoids of the muon cooling complex;
- construction of a test stand to measure the performance of the normal conducting muon cooling cavities in high field;
- test of components for the mechanical neutrino flux mitigation system and its alignment;
- tests of materials for the target of the muon production complex.

In this scenario a further R&D programme will be prepared, which will cover the development of individual components but also integrated demonstrations. In particular, the following would be included in the R&D plan:

- a conceptual design of one muon cooling cell module;
- a conceptual design of a demonstrator facility that allows testing of the muon-cooling technology with beam;
- a concept to demonstrate the target of the muon production complex.

The list of work packages of the aspirational scenario is presented below. Labels in brackets indicate for each activity if it is only in the aspirational (*ASP*) or also in the minimal programme (*MIN*).

MC.SITE Site considerations and layout

The goal is to assess whether one can consider implementing a muon collider at CERN or another site. A key consideration is the decay of the muons in the collider ring that produces a dense flux of neutrinos, which might limit the choice of site. The goal is to develop a mitigation method that reduces the impact of the neutrino flux to the public, if possible to the same level as the LHC.

- Verifying requirements and models of the impact of the neutrino flux. (*MIN*)
- Assessing whether the mechanical system to mitigate the neutrino flux from the arcs can fulfil legal requirements and the above goal. (*MIN*)
- Verifying that the system would not compromise the beam operation. (*MIN*)
- Defining the strategy to mitigate the neutrino flux from the experimental insertions. (*MIN*)
- Developing a tool to identify the surface areas that would show neutrino radiation based on the lattice design. (*MIN*)
- Identifying a potential orientation of the collider ring considering neutrino flux and geology. Estimating the civil engineering cost scale. (*MIN*)

MC.NF Neutrino flux mitigation system

The design of the proposed mechanical neutrino flux mitigation system will be assessed to ensure its performance. In particular:

- Developing a concept of the mechanical flux mitigation system and of the alignment system required to control it. This includes high-accuracy, large stroke movers, alignment of the tunnel reference system to the surface and mechanical deformations and misalignments of the beam line components due to the movers. (*ASP*)

As part of the programme development, appropriate discussion is necessary with ring designers and experts on the technology systems to understand requirements and tolerances of the system.

MC.MDI Machine–detector interface

Muons decaying close to the detector and beam-beam effects can create background in the detector. This will be addressed by:

- Further developing the simulation tools to predict the background in the detector. (*MIN*)
- Further developing the masking system to mitigate the background in the detector. (*MIN*)
- Developing a tool to study the beam-beam background. (*MIN*)
- Developing the interaction region lattice considering the impact on background. (*MIN*)

This effort relies on a strong support from the physics and detector community, in addition to close collaboration with collider ring designers.

MC.ACC Accelerator design

The goal is to develop concepts of the accelerator systems of the complex and to provide key functional component specifications and beam studies supporting realistic performance targets. Key expected results are:

- A lattice for the experimental insertion and arcs of the collider ring addressing the key high-energy challenges (*MIN*):
    - Maintaining the very short bunch length, which decreases with energy.
    - Achieving the very small beta-function, which decreases with energy.
    - Mitigating the beam loss in the magnets due to muon decay.
- A lattice for the arcs of the pulsed synchrotrons that accelerate the muon beam to full energy. (*MIN*)
- An improved concept for the final muon cooling system, which failed to achieve the emittance target by a factor two in the MAP study. (*MIN*)
- An improved and chained concept for the cooling systems before the final cooling, which achieve the largest emittance reduction factor. (*MIN*)
- Exploration of alternatives for the final muon cooling. (*ASP*)
- Consideration of the engineering aspects of the muon cooling module design and its impact on beam dynamics. (*ASP*)
- Assessment of the limitations arising from collective effects along the whole complex. (*MIN*)
- A concept for the system of linacs that provide the initial acceleration after the muon cooling. (*ASP*)

- A concept for the key systems of the proton complex, and in particular the systems that combine the bunches from the proton beam pulses into single, high-charge bunches. (*ASP*)
- Exploration of alternative concepts for muon and proton acceleration and the collider ring, in particular using FFAs. (*ASP*)

The accelerator design will require communication with most of the other areas of the facility, to ensure realistic hardware parameters and proper interfaces with the components of the muon source.

MC.HFM High-field magnet technologies

The goal is to develop realistic targets for the high-field magnet specifications and to develop an R&D programme to demonstrate them, where they are beyond the state of the art. The emphasis is on high-field solenoids in the muon production and cooling complex since they are unique for colliders. In particular:

- Assessment of realistic target parameters for the superconducting collider ring magnets. This contains theoretical studies that translate the progress of the High-field Magnet programme into the specific case of the muon collider.
- Assessment of realistic target parameters for the superconducting final muon cooling solenoids, aiming well beyond 30 T and ideally for 50 T. The solenoids have small apertures and the luminosity will be roughly proportional to their field. This includes theoretical studies using input from the High-Field Magnet programme and other developments.
- Assessment of realistic target parameters for the 6D muon cooling solenoids, which form the main part of the system. The goal is to use HTS solenoids instead of Nb_3Sn technology for field strength of 20 to 25 T, well above the level in the MAP study. This may allow a shorter system and improve both the muon survival rate and the emittance. (*MIN*)
- Assessment of realistic target parameters for the solenoid system around the target in order to understand the strong constraints arising from the large aperture and the high-radiation environment. Higher field corresponds to a higher capture rate of muons. (*MIN*)
- Testing and characterisation of cables and potentially the design and construction of models for the target solenoid at lower fields (around 30 T) to improve the understanding of the technology and to prepare the development of prototypes. (*MIN*)
- Testing and characterisation of cables and potentially the construction of models for the 6D solenoid. The closer packing, larger aperture but lower field places different demands on the technology than for the final solenoids. (*ASP*)
- Design of the solenoid for the test module in **MC.MOD**. This might use less ambitious specifications and technologies than the 6D cooling solenoid models. (*ASP*)
- Conceptual design of the target solenoid. (*ASP*)

MC.FR Fast-ramping magnet technologies

The goal is to develop realistic targets for the functional specifications of the fast-ramping magnet systems including their powering. These systems form the longest technical system of the collider and are critical for the cost and power consumption. The large stored energy in the magnets and the large power flow during the ramp requires the development of efficient and cost-effective solutions. Particular efforts are required in the following areas.

- A concept for the power converters and the power distribution system focusing on cost and power recovery efficiency. (*MIN*)
- A concept for a normal-conducting fast-ramping magnet. (*MIN*)

- Characterisation of the magnet material to understand the linearity of the magnetic field during the ramp and the maximum practical field. (*MIN*)
- A concept for an alternative fast-ramping magnet using superconducting cables. This can be superferric or with air coils to reach higher magnetic fields and shorten the length of the system but demanding larger stored energy and power flow. (*ASP*)
- Testing of superconducting cables to assess if the required high ramp speeds can be obtained. (*ASP*)

These efforts have to be tightly integrated with the development of the RF systems for the high-energy acceleration, as both need to be synchronised, and with the beam studies of the accelerator ring.

MC.RF Radio frequency technologies

The goal is to develop realistic targets for the functional specifications of the normal-conducting RF system in the muon cooling complex and the superconducting RF system in the high-energy complex. The muon cooling RF is unique as it has to operate in a very high magnetic field. The high-energy RF has to address exceptionally high transient beam loading.

- A concept for the normal-conducting accelerating cavities of the muon cooling complex, in particular choices for the frequencies and shapes along the cooling chain. These have to balance beam loading effects and RF power requirements. Initially, they would be based on the two cavities that have been tested in the past. (*MIN*)
- A concept for the longitudinal beam dynamics and the RF systems in the high-energy muon beam acceleration complex, which uses superconducting cavities. The very high bunch charge and short bunch length require mitigation of single bunch beamloading effects. The RF has to be synchronised with the fast-ramping magnet system with due consideration of the lattice limitations. The study will link to measurements of the achievable gradients in superconducting cavities within the RF R&D Programme and world-wide. (*MIN*)
- Design and construction of a test stand that allows measurement of the gradient and breakdown rate of the muon cooling cavities in a high magnetic field. This test stand is instrumental to make technology choices and to develop the cavity design. The cost and specification of the test stand depends depends on the availability of existing equipment. Two different examples have been assessed during the roadmap process, with 3 T and 7 T field strength respectively. Only the much cheaper 3 T option is included in the resource estimate. Currently two fundamentally different cavity technologies exist, one filled with high-pressure hydrogen the other using beryllium. Copper structures at 50 to 70 K could also be considered. However, it is currently not possible to predict the performance of different technologies theoretically and the need to operate them in high magnetic field adds to the uncertainty. Measurements are therefore mandatory. (*ASP*)
- The cavity design for the test module in **MC.MOD**. (*ASP*)
- A powering system concept for the muon cooling and acceleration system. In particular, the muon cooling requires short, high-peak-power pulses, similar to the CLIC drive beam. The high-efficiency klystron development at CERN will be important. A high-power klystron will have to be developed for an upgrade of the RF test stand and the module tests.(*ASP*)

For the studied examples, the construction of the test stand could start early in 2024, when the required klystrons become available, for operations from mid-2025 and first test results shortly before the ESPPU.

MC.TAR Target facility and technologies

Significant proton beam power is required in the target of the muon production complex. The current estimate is 2 MW, but the specification may change once the muon survival rate can be estimated more precisely based on the accelerator chain design. A liquid mercury target has been demonstrated in MERIT. For safety reasons a solid graphite target would be preferred, which appears possible at 2 MW. Targets using liquid metal other than mercury, or fluidised powder, can also be considered and would provide some margin in muon production.

- Assessment of feasibility of the target, specifically (*MIN*):
    - estimation of heat load and radiation in magnets and design of shielding;
    - a preliminary study of the target area design;
    - estimation of the shock wave and pion yield.
- Development of a target concept (*ASP*):
    - optimisation of a graphite target for yield;
    - consideration of non-solid targets such as power jet or liquid metal;
    - conceptual design of the critical target cooling system.
- Design of the target including (*ASP*):
    - essential engineering aspects of the target including remote handling;
    - a concept for demonstration of target power capability;
    - an engineering design of target.
- The experimental programme (*ASP*):
    - verification of the impact of radiation (building upon the HFM programme);
    - measurements of the impact of shocks on the material in HiRadMat and similar facilities. (*ASP*)
- The development of a programme to demonstrate target performance in the CDR phase and beyond. This could use infrastructures at CERN or ESS. (*ASP*)

The available proton beam will impact the target system design, while the field profile directly influences the eventual pion and muon beam distributions and the longitudinal capture system.

MC.MOD Muon cooling cell module technology design

The muon cooling technology is unique and requires very tight integration of high-field solenoids and their cooling system with the RF cavities and their powering. Compactness is instrumental in achieving necessary emittance and high muon survival rates. The cooling cell will thus be the heart of the demonstration programme. A conceptual design of the cell will allow identification of challenges resulting from integration of subcomponents and is instrumental to prepare a timely start of the demonstration programme after the next ESPPU.

- Assessment of technological challenges of 6D cooling cell. (*MIN*)
- Conceptual design of technical systems for 6D cooling cell (*ASP*):
    - mechanical engineering;
    - adaptation of RF design;
    - adaptation of magnet design;
    - cryogenics design;
    - vacuum design;
    - beam instrumentation.
- Integrated conceptual design of the 6D cooling cell. (*ASP*)

This package is intimately linked to the HFM and RF R&D programmes, which provide the conceptual design of the key components and to **MC.ACC**, which provides the accelerator physics design of the cell.

MC.DEM Muon cooling demonstrator

The muon cooling technology will need to be tested with all systems powered and ultimately with beam. This requires a facility that can produce a muon beam and measure its properties before and after the cooling cells. This facility will be the core of the demonstration programme during the CDR phase. A conceptual design will enable accurate estimation of the cost and complexity of this demonstrator facility and allow its timely implementation.

- Definition of the scope of the cooling demonstrator facility. The goal is to demonstrate significant 6D cooling of the muon beam and to show the ability to reliably predict the equilibrium emittance. (*MIN*)
- Identification of at least one potential suitable site. (*MIN*)
- A conceptual design for the demonstrator facility (*ASP*), including
    - transfer of the proton beam from the existing complex;
    - the pion-production target;
    - the capture and transport system;
    - the beam preparation system;
    - the upstream beam diagnostics system;
    - the cooling system;
    - the final beam diagnostics system.
- A concept for a facility to test single modules with proton beam will be developed and sites explored. This could be either integrated with the demonstrator facility or be independent. (*ASP*)

Currently rough dimensions of the facility have been identified and two sites at CERN are being explored that can use proton beam from the proton synchrotron (PS).

MC.INT Integration

The integration package coordinates the different efforts and defines the collider baseline and the alternatives that will be maintained.

- The fundamental parameters of the concept. (*MIN*)
- The layout and site considerations in collaboration with the work package **MC.SITE**. (*MIN*)
- The optimisation of the concept. (*MIN*)
- The cost scale of the key components and the civil engineering. (*MIN*)
- Alternative approaches to the muon collider will also be considered. In particular the LEMMA scheme could use much smaller beam currents than the proton-driven baseline. However, solutions to some fundamental limitations would need to be developed. (*MIN*)

5.7.3 Scope of the minimal scenario

The minimal scenario addresses selected key challenges and design drivers of the muon collider. It comprises a subset of the aspirational scenario that it is important to address at the earliest stage, and which will allow an efficient ramp-up of the subsequent effort. The definition of the minimal scenario has been made, considering the following factors for each R&D item.

- What is the risk of the challenge and the level of resources required to address it? For example the neutrino flux and the machine detector interface can fundamentally limit the energy and physics reach.
- Is the R&D required early to provide specifications for other parts of the collider? For example the accelerator chain from the muon production to the collision point defines the number of muons that have to be produced and hence the required proton beam.
- Is R&D performed outside of the collaboration that will advance the maturity of the technology and inform the community of likely performance? This is for example the case for high-field dipoles, which are developed in the magnet programme.
- Based on existing expertise, can one hope to address the uncovered challenges rapidly if demanded by the European Strategy and if resources become available later? For example one can expect to be able to design the proton complex more rapidly than the muon cooling complex.

In particular the following R&D is not covered by the minimal scenario.

- No conceptual design will be developed for the system to move the beam line in order to mitigate the neutrino flux and of the associated alignment system.
- The alternative design of the fast-ramping magnet system that uses superconducting cables would not be studied.
- The conceptual design of several collider systems would not be covered, in particular
  - the linac system that accelerates the muon beam after the muon cooling system into the accelerating rings;
  - the target complex;
  - the proton complex;
  - alternative designs for the final cooling system;
  - the high-energy FFA as an alternative to the pulsed synchrotrons;
  - alternatives to the collider ring design.
- No studies would be carried out to consider the engineering of the muon cooling cells of the collider.
- No test stand would be constructed to develop the muon cooling accelerating cavities.
- No conceptual design of a muon cooling cell for the test programme would be developed.
- No conceptual design of a muon cooling demonstrator facility would be developed.
- No concept of the power sources for the muon cooling and high-energy acceleration would be developed.
- No design and construction of models to foster the muon cooling solenoid technology would be performed. Only a very limited theoretical effort would be maintained to explore realistic performance specifications.

The minimal scenario will make key design decisions possible but important technology choices will remain. For example, the choice of RF technology for the muon cooling complex requires experimental input that could not be obtained. The programme can provide realistic targets for key component performance specifications but will rely almost completely on experimental programmes outside of the study that have a different focus. An important example is the solenoid development, which can profit from the HFM R&D programme, but where the latter is focused on dipoles that have somewhat different requirements than solenoids. The minimal scenario will provide beam studies that provide evidence for the performance of key parts of the collider system, but with no start-to-end study. The cost scale will remain approximate.

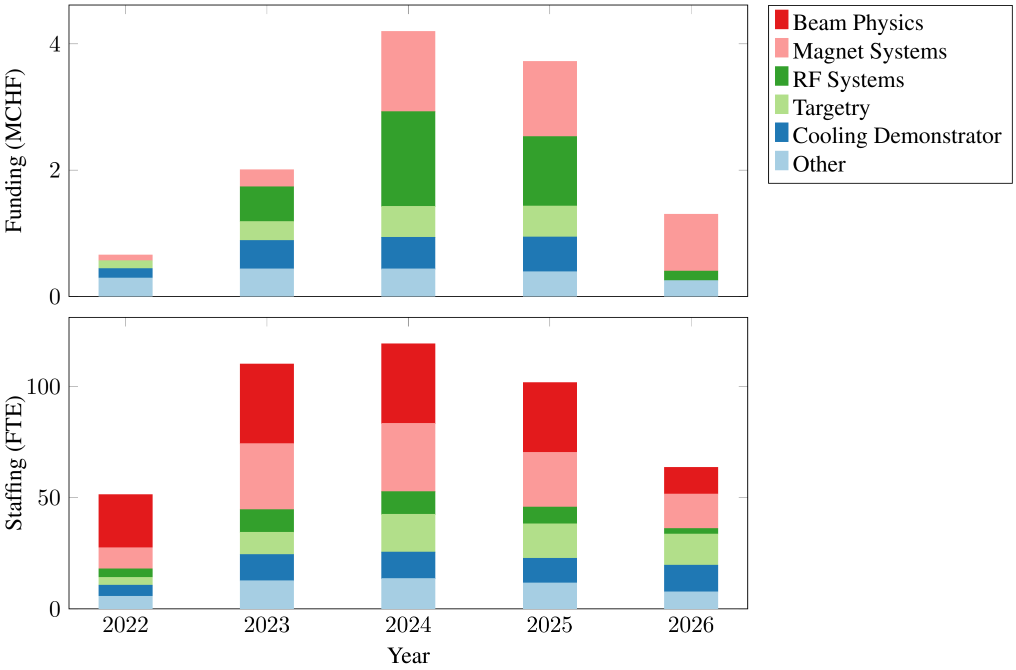

Fig. 5.7: Muon resource requirement profile in the aspirational programme.

5.7.4 *Intermediate scenario*

Between the aspirational and minimal scenario, an intermediate scenario would include with highest priority the design of the cooling module and the experimental programme. In particular the RF test stand and cavity development and the solenoid development would minimise delays to the R&D programme after the next ESPPU. This work would also support the choice of technologies and the assessment of realistic goals for the solenoids and the cavities.

5.7.5 *Resource estimate*

The estimated resources for the two example scenarios are given in Table 5.5 and the resource time profile is shown in Fig. 5.7.

The minimal programme would extend over the years 2022–2025, since the theoretical study should be documented in the reports. The intermediate and aspirational scenarios would extend into 2026, since updates of experimental results and designs could still be considered during the strategy process. Only activities towards the three stated main deliverables are accounted for. As a consequence, the resources are reduced toward the end of the period. In practice, it will be necessary to preserve the effort at a constant or increasing level to maintain the momentum into the CDR phase.

5.8 Facilities, demonstrators and infrastructure

5.8.1 *Demonstrator requirements*

Demonstrations are required both for the muon source and the high energy complex. The compact nature of the muon cooling system, high gradients and relatively high-field solenoids present some unique challenges that require demonstration. The high-power target also has a number of challenges that should be evaluated using irradiation facilities or single impact beam tests. The issues in the high energy complex arise from the muon lifetime. Fast acceleration systems and appropriate handling of decay products result in unique challenges for the equipment.

The following new facilities are required and will be developed or constructed as part of the pro-

Table 5.5: The resource requirements for the two scenarios. The personnel estimate is given in full-time equivalent years and the material in kCHF. It should be noted that the personnel contains a significant number of PhD students. Material budgets do not include budget for travel, personal IT equipment and similar costs. Colours are included for comparison with the resource profile shown in Fig. 5.7.

Label	Begin	End	Description	Aspirational		Minimal	
				[FTEy]	[kCHF]	[FTEy]	[kCHF]
MC.SITE	2021	2025	Site and layout	15.5	300	13.5	300
MC.NF	2022	2026	Neutrino flux mitigation system	22.5	250	0	0
MC.MDI	2021	2025	Machine-detector interface	15	0	15	0
MC.ACC.CR	2022	2025	Collider ring	10	0	10	0
MC.ACC.HE	2022	2025	High-energy complex	11	0	7.5	0
MC.ACC.MC	2021	2025	Muon cooling systems	47	0	22	0
MC.ACC.P	2022	2026	Proton complex	26	0	3.5	0
MC.ACC.COLL	2022	2025	Collective effects across complex	18.2	0	18.2	0
MC.ACC.ALT	2022	2025	High-energy alternatives	11.7	0	0	0
MC.HFM.HE	2022	2025	High-field magnets	6.5	0	6.5	0
MC.HFM.SOL	2022	2026	High-field solenoids	76	2700	29	0
MC.FR	2021	2026	Fast-ramping magnet system	27.5	1020	22.5	520
MC.RF.HE	2021	2026	High energy complex RF	10.6	0	7.6	0
MC.RF.MC	2022	2026	Muon cooling RF	13.6	0	7	0
MC.RF.TS	2024	2026	RF test stand + test cavities	10	3300	0	0
MC.MOD	2022	2026	Muon cooling test module	17.7	400	4.9	100
MC.DEM	2022	2026	Cooling demonstrator design	34.1	1250	3.8	250
MC.TAR	2022	2026	Target system	60	1405	9	25
MC.INT	2022	2026	Coordination and integration	13	1250	13	1250
			Sum	445.9	11875	193	2445

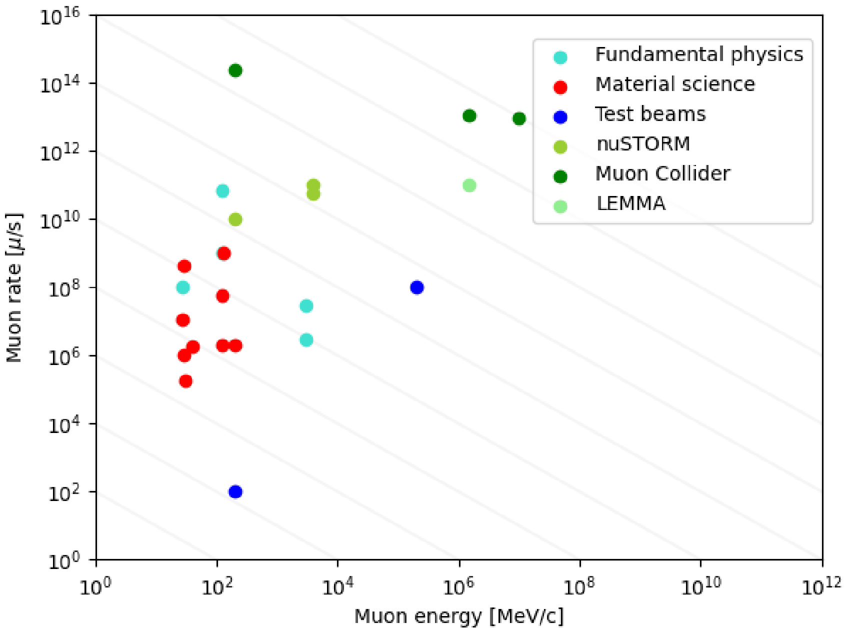

Fig. 5.8: Existing and proposed muon sources as a function of muon rate and muon energy. Diagonal lines show contours of equal beam power. Where available, muon rate data is taken near to the target. For muon collider and nuSTORM, multiple values are shown corresponding to different design options and regions of the facility.

gramme:

- a demonstration of fast-ramping magnet and power converter systems;
- a demonstration of muon cooling module solenoids;
- a demonstration of high-gradient normal-conducting muon cooling cavities operating in a high magnetic field; and
- an integrated demonstration of the muon cooling module as an engineering prototype, as an intensity demonstrator with protons, and as a cooling demonstrator with muons.

The following existing facilities are essential for the successful execution of the programme:

- facilities to demonstrate radiation and shock resistance of materials such as targets and superconducting cables;
- facilities to demonstrate high gradient superconducting RF cavities;
- facilities to demonstrate high-field dipoles.

Further details are given below.

5.8.2 Ionisation cooling demonstrator and related facilities

Ionisation cooling is a novel technology and there are a number of tests which are required before the scheme discussed in Section 5.5 can be realised. In particular, MICE only demonstrated transverse cooling without re-acceleration and operated at relatively high emittance. Further tests must be performed to demonstrate the 6D cooling principle at low emittance and including re-acceleration through several cooling cells.

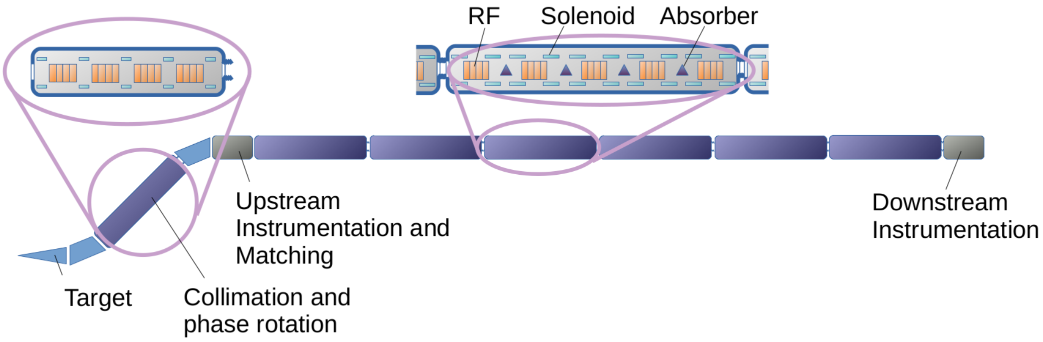

Fig. 5.9: Schematic diagram of a possible implementation of the muon cooling demonstrator. A pion-production target is followed by a collimation and phase rotation section where a low emittance muon beam is created. Instrumentation upstream and downstream of the cooling region is used to determine muon beam properties before and after a number of ionisation cooling cryostats, each containing a series of solenoids and RF cavities.

1. An RF test stand is required to test normal conducting RF cavity operation in the strong magnetic fields required by the cooling lattice.
2. Superconducting magnet fabrication and testing facilities are required to develop and test superconducting cables and solenoids operating at the highest fields and in challenging configurations.
3. A cooling cell prototype is required to test integration of the individual components.
4. Beam tests at low intensity using muons are required to test the beam physics of muons passing through several cooling cells.
5. Beam tests at high intensity using protons may be required to study potential intensity effects.

While the construction of an ionisation cooling demonstrator is not foreseen in the next five years, design of such a facility and necessary preparatory activities will need to begin so that the eventual muon collider can be delivered by 2045.

5.8.2.1 Ionisation cooling demonstrator

A test facility with beam is required to demonstrate the ability of the muon collider to deliver the requisite luminosity. Achieving high luminosity rests on the solution of two critical issues; the ability to create a high-flux muon beam from pions created at the target, and the ability to efficiently cool the beam in the six phase-space dimensions. This technology represents the single most novel system of the muon collider and requires unique customisation of key accelerator technologies. A cooling demonstrator may be able to contribute to a cutting-edge physics programme and this possibility should be exploited [43].

The construction and operation of the cooling demonstrator that can explore the full potential of relevant accelerator technologies will be required. Initial explorations are ongoing at CERN to identify a site the demonstrator could be situated at any laboratory where access to a suitable proton source with high instantaneous beam power could be provided. At CERN, preliminary studies indicate that construction of a junction cavern to the existing proton complex may be required in the next long shutdown in order to meet the timeline of the muon collider.

In addition to site studies, early design studies have been made for the demonstrator. The rectilinear 6D cooling lattices developed as part of the MAP studies have been identified as a good candidate for cooling experiments [28]. These lattices will enable demonstration of cooling with the low β^* required to get good equilibrium emittance. The rectilinear B5 and B8 lattices have received particular attention. Both lattices yield an excellent cooling performance; the B8 lattice will deliver cooling at the lowest longitudinal emittance, but the challenges in the magnet system may make this lattice more appropriate for offline prototyping with beam tests possible at a later stage in the programme. The B5 lattice would cool to slightly higher emittances, but would still enable a full programme of study of beam physics issues including performance as well as addressing practical issues such as the commissioning of such a novel system.

The RF systems for the demonstrator are particularly challenging. The B5 lattice was designed with RF cavities operating at 650 MHz. However no suitable klystron exists at such a frequency. Klystrons operating at 704 MHz are available, used for example by ESS, but the peak power available is only 2 MW. The most suitable existing klystron would be at 1 GHz, with peak power output of 20 MW. Effort is required to understand whether such a frequency would be suitable for the cooling system; the bunch length would need to be very short, with impact on RF bucket size and longitudinal emittance. The transverse aperture of such RF cavities will be relatively small, and this may impinge on the physical acceptance of the lattice.

In order to realise the cooling demonstrator, an appropriate pion and muon source must be identified. Most sources have relatively long pulses, whereas the cooling demonstrator requires extremely short pulses so that the number of muons in each RF bucket is sufficient to yield an appropriate signal for the beam instrumentation. Even so, in order to meet the initial emittance requirements of the muon cooling system very low emittance muon beams are required, which can only be delivered using a collimation system to yield the appropriate transverse emittance and a phase rotation system to yield the appropriate longitudinal emittance. Event rates between 10^5 and 10^7 muons per pulse have been estimated for a source based on the CERN PS. The actual event rate depends on the configuration of the target and collimation system. Such a low event rate may be challenging for conventional beam instrumentation, and a dedicated study is required to understand potential solutions.

The possibility to share a pion source with another high energy physics facility has been explored. Particular interest has been expressed by the community surrounding the proposed nuSTORM facility. nuSTORM requires a high momentum pion beam, with energy in the range 1 to 6 GeV. Studies to investigate whether a beam could be shared with nuSTORM are inconclusive. During nuSTORM operations, the target horn system would be tuned for high energy pions and the rate at low energy would be compromised. The possibility to develop a momentum selection chicane that could capture both high and low momentum pions simultaneously has been investigated. Dedicated study would be required to understand the feasibility of combined operations. Even if this were not possible, appropriate sharing of beam time would enable the two facilities to operate using the same target.

The benefit of sharing a facility is significant. Successful operation of nuSTORM would demonstrate the highest power muon beam ever produced, albeit two orders of magnitude below the muon power in the front end of a muon collider. nuSTORM itself would yield a high impact physics programme comprising cross scattering measurements enabling full realisation of the capabilities of the international neutrino oscillation programme and beyond-the-standard-model physics searches.

5.8.2.2 Prototype cooling system

Many of the challenges are associated with integration issues of the magnets, absorbers and RF cavities. For example, operation of normal conducting cavities near to superconducting magnets may compromise the cryogenic performance of the magnet. As discussed elsewhere, operation of RF cavities in strong magnetic fields may lead to a lower breakdown threshold for the RF field gradient. Installation of absorbers, particularly using liquid hydrogen, may be challenging in such compact assemblies. In order to understand and mitigate the associated risks, an offline prototype cooling system will be required. Such a system will require an assembly and testing area, with access to RF power and support services. This could be integrated with the demonstrator facility, as it will need an area for staging and offline testing of equipment prior to installation on the beamline.

5.8.2.3 Intensity studies of ionisation cooling

The possibility to perform intensity studies with a muon beam are limited owing to the challenges with collecting a high-brightness muon beam in the absence of the full muon collider capture system. However, there are a number of technical issues that may arise in the presence of high beam currents, for example heating of absorbers, beam loading of RF cavities and space-charge effects in the vicinity of beam intersecting devices. In the first instance such effects should be studied using simulation tools. If such studies reveal potential technical issues, beam studies in the presence of a high intensity source will be necessary, for example using protons. In order to achieve this, a suitable beam will be required having an appropriate momentum. Protons lose more energy when passing through material than muons having the same momentum, so appropriate scaling will be required for proton momentum or absorber thickness.

5.8.3 Ionisation cooling RF development

In addition to the cooling demonstrator, a dedicated programme of component development will be required. This programme will run in parallel to, and inform development of, the demonstrator facility itself. The cooling systems require normal-conducting RF cavities that can operate with high gradient in strong magnetic fields without breakdown. The likelihood of RF breakdown can only be estimated from empirical observations relating the frequency and field gradient and further informed by cavity materials, surface preparation and environmental factors such as external magnetic fields. No satisfactory theory exists to predict the phenomenon.

Considerable effort was made by MAP to develop high-gradient RF cavities. Two test cavities have been developed that can exceed the required performance. The first cavity was filled with gas at very high pressure [26]. Electrons originating from RF breakdown lose significant energy in the gas, so that it acts as an insulator. Muons, on the other hand, lose relatively little energy in the gas compared to the absorbers already present in the cooling channel. By using a gas comprising low atomic number material, such as hydrogen, the gas can in some circumstances even contribute to the ionisation cooling. The second cavity used beryllium walls [27]. Beryllium is both hard and low-density, so that it absorbs relatively little energy from electron beamlets that develop during breakdown and the damage is relatively weaker.

Alternative concepts may yield even higher gradients. Operation of normal-conducting RF cavities at liquid nitrogen temperature has been demonstrated to reduce multipacting. Additional benefits may include reduced power requirements and reduced cooling requirements for the superconducting magnets, which are situated close to the RF cavities. Use of a shorter RF pulse may enable beam acceleration before the breakdown can fully develop. The muon pulse is less than 100 ns long, which is short compared to the RF pulses used during previous cavity tests. Operation of copper cavities at low temperatures has also been shown to enable increased field gradient.

In order to test these concepts and others, a dedicated test facility is required. An RF source having high peak power at the appropriate frequency and a large aperture solenoid that can house the RF cavity will be needed. No such facility exists at present.

5.8.4 Cooling magnet tests

Development of a more effective 6D cooling system may yield improved performance. The longitudinal and transverse emittance delivered is limited by the available magnets. An improved cooling system would yield lower longitudinal and transverse emittances, resulting in a shorter final cooling system and potentially less longitudinal emittance growth. Overall the system performance and luminosity would improve.

In order to improve performance high field magnets are required with opposing-polarity coils very close together. The possibility to implement high-field magnets (including those based on HTS) will be investigated, with appropriate design studies leading to the construction of high-field solenoid magnets having fields in the range 20 to 25 T. Techniques for integration with RF cavities will be studied and tests of operation in the presence of RF cavities will be performed.

Very high field magnets are required for the final cooling system. In this system, the ultimate transverse emittance is reached using focusing in the highest-field magnets As a first step, a 30 T magnet, corresponding to the MAP baseline, would be designed and constructed. Feasibility studies towards a 50 T magnet would also be desirable, which may include testing of cables in high field magnets. These very demanding magnets are envisaged to be developed separately to the cooling demonstrator. Eventually they could be tested in beam if it was felt to be a valuable addition to the programme.

In order to support this magnet R&D, appropriate facilities will be required. Testing of cables requires a suitable test area having access to services such as cryogenics and power supplies along with access to high field magnets. Magnet fabrication will also require these facilities in addition to access to appropriate winding capabilities.

5.8.5 Acceleration RCS magnets

Acceleration within the muon lifetime is rather demanding. The baseline calls for magnets that can cycle through several T on a time scale of a few ms. The exact specification will be defined during the design work, but it is clear that a resonant circuit will be required to power the magnets and work on a prototype is anticipated [42, 44]. Studies will be made to examine the available capacitors and performance under various loads. The cost and sustainability of RCS magnets is a concern, due to resistive power loss in the conductor and magnetisation loss in the magnet cores. In order to study the effect of eddy currents in the magnets, prototyping of novel very thinly laminated cores will be performed.

Superconducting RCS magnets are challenging to realise owing to heating arising from energy dissipation in the conductor during cycling [45]. This heating can lead to demands on the cryogenic systems that outweigh the benefits over normal-conducting magnets. Recent prototypes have been developed using HTS that can operate at higher temperatures, and in configurations leading to lower AC losses, yielding improved performance. This is a promising research direction that will be developed as part of the study. In order to continue this research, magnet tests with rapid pulsed power supplies and cryogenic infrastructure will be required.

5.8.6 Effects of radiation on material

The high beam power incident on the target and its surroundings is very demanding. Practical experience from existing facilities coupled with numerical studies indicate that there will be challenges in terms of target temperature and lifetime. Instantaneous shock load on the target will also be challenging. Tests are foreseen to study behaviour of target material under beam in this instance. Tests are desirable both for instantaneous shock load and target lifetime studies.

Additionally, the effect of radiation on superconducting wire is an important parameter in the target region. Studies have been performed as part of the HL-LHC work. As the target solenoid design matures, additional studies may be required taking into account the magnet arrangement, conductor design and estimates of radiation levels.

In order to realise such tests, facilities having both instantaneous power and integrated protons on target equivalent to the proton beam parameters assumed for this study are desirable. Preliminary studies indicate that existing facilities such as HiRadMat at CERN can yield sufficient instantaneous power.

5.8.7 Superconducting RF

Development of efficient superconducting RF with large accelerating gradient is essential for the high energy complex. Initially work will focus on cavity design; however eventually a high gradient prototype at 300–400 MHz frequency will be required. In order to realise such a device, appropriate superconducting cavity production and test facilities will be required including surface preparation techniques and a capability for high power tests.

5.8.8 FFA magnets

Instead of ramping the synchrotron magnets, the use of FFA-style magnets has been considered. In FFAs the orbit moves to regions of higher field as the energy increases, but the magnets themselves are fixed. Vertical orbit excursion FFAs have been considered, which have a path length that does not vary with energy. In the ultra-relativistic regime this would yield an isochronous beam. VFFAs are novel, but are under consideration for the next generation of neutron spallation sources. Initially, scalings will be made from magnets designed as part of the associated R&D activity [46]. If FFAs seem promising for the muon collider, dedicated magnet fabrication and testing will eventually be required. Owing to the complicated nature of the field, such fabrication requires challenging magnet windings which may require novel winding facilities and dedicated tests for the specific parameters chosen for the muon collider.

5.9 Collaboration and organisation

5.9.1 The international Muon Collider Collaboration

Following the 2020 ESPPU, the international Muon Collider Collaboration (MCC) was established by CERN. Its goal is to establish whether the investment into a full CDR and demonstrator for a muon collider is scientifically justified. The MC Study will provide a baseline concept for a muon collider, well-supported performance expectations and assess the associated key risks as well as the cost and electricity consumption drivers. It will also identify an R&D path to demonstrate the feasibility of the collider and support its performance claims. The focus of study will be a collider at 3 TeV and a collider at $\geq$10 TeV.

An International Collaboration Board (ICB) oversees the MC study and channels contributions from the participants. CERN is the initial host organisation for the MC Study. An International Advisory Committee will be established whose mandate is to review the scientific and technical progress of the Study typically on an annual basis and to submit recommendations to the ICB. The ICB will appoint a MC study leader who organises and guides the study, establishes collaborations, ensures coherent communications, coordinates the resources and organises workshops, conferences and meetings where relevant. He or she will be appointed by the ICB and guided by its decisions, and will act under the authority of the head of the host organisation. The term of office of the Study Leader will be three years, renewable.

Studies on the detector and physics reach of the collider are an essential part of the study; however they are not within the scope of the Accelerator R&D Roadmap presented here. The MCC is coordinating and integrating these efforts.

The international MCC has representation from regions outside Europe. In particular, the collaboration is supporting closely the Snowmass process in the US.

5.9.2 Relationship to other fields

The ambitious programme of R&D necessary to deliver the muon collider has the potential to enhance the science that can be done at other muon-beam facilities.

nuSTORM and ENUBET have the potential to offer offer world-leading precision in the measurement of neutrino cross-sections and exquisite sensitivity to sterile neutrinos and physics beyond the standard model. nuSTORM in particular will require capture and storage of a high-power pion and muon beam, and management of the resultant radiation near to superconducting magnets. The target and capture system for nuSTORM and ENUBET may also provide a testing ground for the technologies required at the muon collider and as a possible source of beams for the essential 6D cooling-demonstration experiment.

Technologies required to deliver the muon collider are important in a number of fields.

- A multi-MW proton source is at the heart of neutron spallation facilities. Long-pulse facilities such as ESS use linacs while short pulse machines such as SNS, JPARC and ISIS accumulate protons either before or after acceleration. The protons are delivered to a target when neutrons are used for material studies. In Europe ESS and ISIS are both studying options for upgrades to MW-class short-pulse proton production.
- High power targets are of interest in a number of fields, for example neutrino physics and neutron physics. The solenoid focusing that is the baseline for the muon collider will also be employed by the next generation of charged lepton flavour violation experiments.
- High field solenoids required for the muon cooling systems have application in a broad range of sciences. In particular, the high field solenoids envisaged for final stage muon cooling are of great interest in applications such as MRI.
- Rapid-cycling synchrotrons are of interest for high-power proton users such as neutrino and neutron users. Novel fast-ramping synchrotrons can enable higher repetition rates and hence higher beam powers.
- FFAs have been proposed as a route to high proton beam power for secondary particle sources such as neutron spallation sources, owing to the potential for high repetition rate and lower wall plug power compared to other facilities. An FFA is under study as a possible means to upgrade the ISIS neutron and muon source.
- The potential to deliver high quality muon beams could enhance the capabilities of muon sources such as those at PSI and ISIS. The use of frictional cooling to deliver ultra-cold positive and negative muon beams is under study at PSI and may be applicable to the muon collider.
- High gradient RF is of interest to the linear collider community. Linear colliders are limited by the achievable real-estate gradient and development here could improve performance. There is considerable potential for collaboration with industry in the development of novel RF power supplies.
- High-gradient normal-conducting RF cavities are used by electron sources, often near to high field solenoids.

5.9.3 Training and human resources

Training is an essential part of the muon accelerator programme. The neutrino factory conference series supports a regular school in essentials of accelerator and neutrino physics, with a significant component dedicated to muon accelerators. The collaboration will continue to support similar endeavours, as well as direct training through PhDs, internships and university-based training.

Communication and outreach is a core part of our effort, both in peer reviewed journals, conferences, workshops and the broader media in collaboration with the appropriate groups in collaboration institutes.

The muon collider collaboration is a global one, and it is important to the project success to include collaboration members from a wide range of backgrounds. The collaboration will continue to support this effort.

5.10 Conclusion

The muon collider presents enormous potential for fundamental physics research at the energy frontier. Previous studies, in particular the MAP study, have demonstrated feasibility of many critical components of the facility. A number of proof-of-principle experiments and component tests, such as MICE, EMMA and the MuCool RF programme, have been carried out to practically demonstrate the underlying technologies.

The muon collider is based on novel concepts and is not as mature as the other high-energy lepton collider options, in particular also compared to the highest energy option CLIC. However, it promises a unique opportunity to deliver physics reach at the energy frontier on a cost, power consumption and time scale that might improve significantly on other energy-frontier colliders. At this stage the panel, building on significant prior work, has not identified any showstopper in the concept.

The panel has identified a development path that can address the major challenges and deliver a 3 TeV muon collider by 2045. The panel has identified the R&D effort that it considers essential to address these challenges during the next five years to a level that allows estimation of the performance, cost and power consumption with adequate certainty. Execution of this R&D is required in order to maintain the timescale described in this document. Ongoing developments in underlying technologies will be exploited as they arise in order to ensure the best possible performance. This R&D effort will allow the next ESPPU to make fully informed recommendations, and will similarly benefit equivalent strategy processes in other regions. Based on these decisions, a significant ramp-up of resources could be made to accomplish construction by 2045 and exploit the enormous potential of the muon collider.

Bright muon beams are also the basis of neutrino physics facilities such as NuSTORM and ENUBET. These could potentially share an important part of the complex with a muon cooling demonstrator.

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