A natural model of spontaneous CP violation

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ABSTRACT: We examine the possibility of building a natural non-supersymmetric model of spontaneous CP violation equipped with the Nelson-Barr (NB) mechanism to address the strong CP problem. Our approach is to utilize a doubly composite dynamics where the first confinement of the CFT occurs at the scale of spontaneous CP violation (SCPV) and the second confinement at the TeV scale. A holographic dual description of this 4D set-up via a warped extra dimension with three 3-branes provides an explicit realization of this idea. In this model, radiative corrections to the strong CP phase are well under control, and the coincidence of mass scales, which we generally encounter in NB models, is addressed. Our model also provides an explanation to the quark Yukawa hierarchies, and a solution to the gauge hierarchy problem just as in the usual Randall-Sundrum model with the Higgs being localized on the TeV brane.

Contents

1   Introduction

The strong CP problem, namely why CP violation in the strong interaction is so small, i.e. $\bar{\theta} \lesssim 10^{-10}$, is one of the outstanding puzzles in particle physics. There is no symmetry protecting $\bar{\theta}$ to be small within the Standard Model (SM), similar to the case of the Higgs mass. But, even worse than the case of the Higgs mass, there is no known direct anthropic solution to it. The most well known solution is the Peccei-Quinn solution with the QCD axion [1, 2]. As an alternative, spontaneous violation of the CP symmetry [3–13] is a promising paradigm of physics beyond the SM, which also realizes the Cabibbo-Kobayashi-Maskawa (CKM) phase of the quark-mixing matrix.[1] The most popular realization of the idea is known as the Nelson-Barr (NB) mechanism [8–10] where the QCD vacuum angle $\bar{\theta}$ is protected under the spontaneous CP violation at the classical level by the structure of an extended quark mass matrix. A minimal model was proposed by Bento, Branco and Parada (BBP) [17], which introduces a vector-like pair of $SU(2)_L$ singlet quarks and SM singlet scalar fields breaking the CP symmetry spontaneously.

However, the NB models encounter several theoretical challenges (see, for example, a well documented summary in [18]). First, to suppress higher-dimensional operators destroying the NB mechanism, the scale of spontaneous CP violation must be hierarchically smaller than a cutoff scale of the theory. In non-supersymmetric models, fine-tuning caused by quadratically divergent radiative corrections to the masses of the CP-violating scalar fields is even worse than that of the original strong CP problem. Second, although the NB mechanism works at the classical level, an unacceptably large value of $\bar{\theta}$ is generated radiatively in general. These issues can be addressed when we consider supersymmetric NB models where supersymmetry breaking is provided by gauge mediation. Loop effects on $\bar{\theta}$

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[1]Spontaneous CP violation also plays a role in suppressing contributions to electric dipole moments in supersymmetry [14–16].

in this case have been estimated in ref. [19] (see also refs. [18, 20–23]). Another approach to protect a hierarchically small scale of spontaneous CP violation is to utilize a strong dynamics which makes the CP-violating scalar fields composite, while additional gauge symmetries may be introduced to suppress the contribution to $\bar{\theta}$ (see e.g. [24, 25] for recent works).

In this paper, we explore a non-supersymmetric NB model based on the BBP model [17] to realize a hierarchically small scale of spontaneous CP violation and to sufficiently suppress radiative corrections to $\bar{\theta}$. We introduce a warped extra dimension with three 3-branes whose typical energy scales are given by the Planck scale, the scale of spontaneous CP violation and the TeV scale, respectively. Warped extra dimension models with multiple branes have been explored in various studies [26–49] and the radion stabilization for multiple branes has been established [50]. According to the AdS/CFT correspondence [51–53], a 5D three-brane model is dual to a nearly-conformal strongly-coupled 4D field theory where spontaneous breaking of the conformal symmetry takes place via confinement twice [42]. Three branes are essential to forbid dangerous operators leading to a large $\bar{\theta}$. We find that the model can solve the strong CP problem without fine-tuning. Furthermore, the model accommodates the mechanism to explain fermion mass hierarchies through localized profiles of bulk SM fermions [54, 55]. This feature does not exist in supersymmetric models and delivers a unique advantage to our model.

The rest of the paper is organized as follows. In section 2, we review the NB mechanism and the BBP model. Section 3 presents our warped extra dimension model with three 3-branes to solve the strong CP problem without fine-tuning. In section 4, we discuss corrections to $\bar{\theta}$ in our model. Section 5 is devoted to conclusions. Details about a 5D fermion in the three 3-brane setup are summarized in appendix A.

2 Nelson-Barr mechanism

Let us start with a review of the NB mechanism [8–10] and the BBP model [17] as its simplest realization. We assume that our Lagrangian is invariant under the CP transformation. The CP symmetry is only spontaneously broken by vacuum expectation values (VEVs) with nonzero phases of SM singlet complex scalar fields η_a ($a \geq 2$). We also introduce a vector-like pair of quarks $q_d, \bar{q}_d$ where $\bar{q}_d$ is in the same representation as the right-handed down-type quarks $\bar{d}$ under the SM gauge group. To suppress unwanted terms, a $\mathbb{Z}_N$ ($N \geq 3$) symmetry is imposed. The newly introduced fields $q_d, \bar{q}_d, \eta$ have charges $+1, -1, -1$, respectively, i.e. they transform by $q \to e^{2\pi i/N} q$, $\bar{q} \to e^{-2\pi i/N} \bar{q}$, $\eta_a \to e^{-2\pi i/N} \eta_a$, while the SM fields are neutral.[2] The relevant part of the Lagrangian is then given by

$$\mathcal{L}_{\mathrm{NB}} = \mu_d q_d \bar{q}_d + \sum_{a,f} a^d_{af} \eta_a q_d \bar{d}_f + \sum_{f,f'} Y^d_{ff'} H Q_f \bar{d}_{f'}\,, \tag{2.1}$$

where μ_d is a real parameter with a mass dimension one, $f = 1, 2, 3$ denotes the flavor index and a^d_{af}, $Y^d_{ff'}$ are real dimensionless couplings. The third term is the ordinary Yukawa

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[2]We may consider a $\mathbb{Z}_2$ symmetry instead of the $\mathbb{Z}_N$ ($N \geq 3$). In that case, a CP phase can be provided into the SM sector with only one η_a ($a = 1$) by $\mathcal{L} \sim \sum_f (a_f \eta_1 + a'_f \eta_1^*) q_d \bar{d}_f$. In addition, e.g. the first term in eq. (2.4) is not dangerous anymore with only one η_a, but now we have another dangerous operator like $\mathcal{L} \supset \frac{\eta_1}{\Lambda} \eta_1 \bar{q}_d q_d$.

interaction with the left-handed quark Q and the Higgs field H. We focus on the parameter space with $\mu_d, a^d_{af}\langle\eta_a\rangle > 1\,\text{TeV}$. Then, the first and second terms in eq. (2.1) make one linear combination of $\bar{q}$ and $\bar{d}$ massive, while the other combinations give the SM down-type quarks that obtain nonzero masses from the third Yukawa term.

The observed CKM phase is provided when μ is comparable to $|a^d_{af}\eta_a|$ [24, 56]. On the other hand, the QCD vacuum angle,

$$\bar{\theta} = \theta - \arg(\det\hat{M}_u \cdot \det\hat{M}_d)\,, \tag{2.2}$$

is not generated at the tree level. Here, $\hat{M}_u$ is the 3×3 up-type quark mass matrix and $\hat{M}_d$ denotes the 4×4 down-type quark mass matrix including the vector-like quark components,

$$\hat{M}_d = \begin{pmatrix} \mu_d & \sum_a a^d_{af}\langle\eta_a\rangle \\ 0 & Y^d_{ff'}\langle H\rangle \end{pmatrix}\,. \tag{2.3}$$

Note that only one off-diagonal block includes complex phases due to $\langle\eta_a\rangle$ while the other off-diagonal block is zero, leading to a real $\det\hat{M}_d$.

The BBP model, however, suffers from corrections to the down-type quark mass $\hat{M}_d$ which generally induce a non-zero $\bar{\theta}$. The discrete $\mathbb{Z}_N$ symmetry cannot forbid higher dimensional operators such as

$$\mathcal{L} \supset \frac{\eta^*_b}{\Lambda}\eta_a\bar{q}_d q_d + \frac{\eta^*_a}{\Lambda}HQ\bar{q}_d\,, \tag{2.4}$$

where Λ denotes a UV cutoff scale and coefficients of the terms are omitted. If Λ is given by the Planck scale M_{Pl}, the absolute values of $\langle\eta_a\rangle$ must be much smaller than the Planck scale, $|\langle\eta_a\rangle|/M_{\text{Pl}} < 10^{-10}$, to satisfy $\bar{\theta} < 10^{-10}$. Since the theory is non-supersymmetric, this leads to another naturalness problem which is more serious than the strong CP problem itself. Furthermore, a large $\bar{\theta}$ can be generated by radiative corrections. We can write down

$$\mathcal{L} \supset \gamma_{ab}\eta^*_a\eta_b H^\dagger H + \gamma_{abcd}\eta_a\eta_b\eta^*_c\eta^*_d + \text{h.c.}\,, \tag{2.5}$$

where γ_{ab} and γ_{abcd} are dimensionless coupling constants. The first and second terms in eq. (2.5), respectively, lead to corrections to $\bar{\theta}$ at one and two loops with the interactions in eq. (2.1), which may be estimated as [18]

$$\delta\bar{\theta} \sim \left|\frac{\gamma_{bc}a^d_{af}a^d_{bf}\eta_a\eta^*_c}{16\pi^2 M^2_{\text{CP}}}\right|\,, \tag{2.6}$$

$$\delta\bar{\theta} \sim \left|\frac{g^2\gamma_{abcd}a^d_{af}a^d_{cf}\eta^*_b\eta_d}{(16\pi^2)^2 M^2_{\text{CP}}}\right|\,, \tag{2.7}$$

where M_{CP} represents the scale of spontaneous CP violation, g denotes a SM gauge coupling and the complex phases of η_a are assumed to be $\mathcal{O}(1)$. In the next section, we will present a simple solution to suppress these dangerous contributions to $\bar{\theta}$.
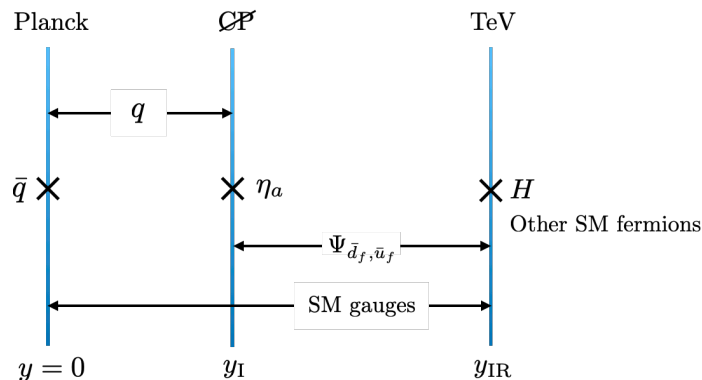
Figure 1. A schematic picture of the setup. Here, q denote q_u and q_d while $\bar{q}$ denote $\bar{q}_u$ and $\bar{q}_d$.

3 The model

We consider a 5D model whose extra dimension is compactified on the S^1/Z_2 orbifold. The spacetime geometry is described by the metric,

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\mu - dy^2. \tag{3.1}$$

Here, $y \in [0, y_{\rm IR}]$ denotes the fifth coordinate and $A(y)$ is a function of y. Three 3-branes are introduced: two of them are located at the orbifold fixed points $y = 0, y_{\rm IR}$ which we call the UV and IR branes, respectively. The other brane is at $y = y_{\rm I}$ $(0 < y_{\rm I} < y_{\rm IR})$, which we call the intermediate brane. The subregions $1, 2$ are defined as the bulk region of $0 < y < y_{\rm I}$ and the region of $y_{\rm I} < y < y_{\rm IR}$, respectively. The warp factor $A(y)$ is different for each subregion,

$$A(y) = \begin{cases} k_1 y & \text{(subregion 1)} \\ k_2(y - y_{\rm I}) + k_1 y_{\rm I} & \text{(subregion 2)} \end{cases}, \tag{3.2}$$

where $k_{1,2}$ are constant with mass dimension 1 and we assume $k_2 > k_1$ to avoid a tachyonic radion corresponding to the intermediate brane tension. See ref. [50] for a more detail discussion about the three 3-brane setup and the radion stabilization.

To implement the NB mechanism, we assume that all the complex scalar fields η_a are localized at the intermediate brane. These fields obtain CP-violating VEVs around the mass scale $\sim k_1 e^{-k_1 y_{\rm I}}$ owing to the warped geometry.

As for the vector-like quarks, we introduce q_d as a chiral zero mode from a 5D fermion Ψ_{q_d} living in the subregion 1 and $\bar{q}_d$ as a 4D fermion localized at the UV brane. The 5D down-type quarks $\Psi_{\bar{d}_f}$ including $\bar{d}_f$ as zero modes live in the subregion 2. In addition to those quarks, we introduce another vector-like pair of quarks by putting a 5D fermion Ψ_{q_u} in the subregion 1 and a 4D fermion $\bar{q}_u$, which has the same charge as the right-handed up-type quarks, at the UV brane. The 5D up-type quarks $\Psi_{\bar{u}_f}$ including $\bar{u}_f$ as zero modes live in the subregion 2. The SM gauge fields live in the whole bulk region. The other SM fermions, and the SM Higgs field live on the IR brane. A schematic picture of our model is shown in figure 1. The reason that $SU(2)_L$ doublet quarks Q live only on the IR brane is

to avoid potentially large radiative corrections to $\bar{\theta}$. For the same reason, we assume that $\Psi_{\bar{d}_f}$ and $\Psi_{\bar{u}_f}$ live only in the subregion 2 instead of the whole bulk region. It is also noted that a naive extension of the BBP model only requires $\Psi_{\bar{d}_f}$ live in the subregion 1 and the right-handed up-type quarks $\bar{u}$ live at the IR brane. However, again, to suppress corrections to $\bar{\theta}$, we do not take this option. We discuss in more detail about the motivation to adopt our setup in the next section.

The first term in eq. (2.1) is introduced by a UV brane-localized coupling,

$$
\begin{aligned}
S &\supset \int d^4x \sqrt{|g_{\text{in}}|}\, \hat{\mu}_d \psi_{q_d} \bar{q}_d + \text{h.c.}\Big|_{y=0} \\
&\supset \int d^4x\, \mu_d q_d \bar{q}_d + \text{h.c.}
\end{aligned}
\tag{3.3}
$$

Here, g_{in} denotes the determinant of the induced metric on the UV brane, i.e. $|g_{\text{in}}| = 1$, and ψ_{q_d} is the left-handed component of Ψ_{q_d}, which contains $\psi_{q_d}(x,y) \ni f_{q_d}(y)q_d(x)$ with the bulk profile of the zero mode $f_{q_d}(y)$ whose normalization is determined so that q_d is canonically normalized. The parameter $\hat{\mu}_d$ is naturally given by $\hat{\mu}_d \sim k_1^{1/2}$, while the parameter μ_d can be hierarchically different from k_1 depending on the bulk profile of q_d,

$$
\mu_d = \hat{\mu}_d f_{q_d}(0) = \hat{\mu}_d \sqrt{\frac{(1-2c_{q_d})k_1}{e^{(1-2c_{q_d})k_1 y_{\text{I}}}-1}}\,,
\tag{3.4}
$$

with a dimensionless bulk mass parameter c_{q_d} when the bulk mass of ψ_{q_d} is parametrized in units of k_1. See appendix A for more details about a 5D fermion in the three 3-brane setup. The second term of eq. (2.1) is obtained on the intermediate brane,

$$
\begin{aligned}
S &\supset \int d^4x \sqrt{|g_{\text{in}}|} \sum_{a,f} \hat{a}_{af}^d \eta_a \psi_{q_d} \psi_{\bar{d}_f} + \text{h.c.}\Big|_{y=y_{\text{I}}} \\
&\supset \sum_{a,f} \int d^4x\, a_{af}^d \eta_a q_d \bar{d}_f + \text{h.c.}\,,
\end{aligned}
\tag{3.5}
$$

where $\hat{a}_{af}^d$ is a constant with mass dimension -1 and $\psi_{\bar{d}_f}$ denotes the right-handed component of the 5D fermion $\Psi_{\bar{d}_f}$. The order of magnitude of $|a_{af}^d \eta_a|$ may be suppressed compared to $|\eta_a| \approx k_1 e^{-k_1 y_{\text{I}}}$ depending on the bulk profiles of q_d and $\bar{d}_f$. As discussed in the previous section, the observed CKM phase can be provided for $\mu_d \approx |a_{af}^d \eta_a|$. In fact, in our setup, the mass parameter μ_d can be much smaller than the Planck scale by assuming that q_d is localized toward the intermediate brane, so that we can realize $|a_{af}^d \eta_a| \approx \mu_d$. The Yukawa interaction terms in eq. (2.1) are obtained on the IR brane where the Higgs is localized.[3]

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[3]Note that there does not exist terms involving only the down-type quarks and η_a,

$$
S \sim \int d^5x \sqrt{|g_{\text{in}}|}(\eta_a \bar{\Psi}_{\bar{d}_f} \Psi_{\bar{d}_f} \delta(y-y_{\text{I}}) + \text{h.c.})\,,
$$

because of the Dirichlet boundary condition for $\Psi_{\bar{d}_f}$ on the intermediate brane.

We also have the similar couplings for $\Psi_{\bar{u}_f}$, Ψ_{q_u} and $\bar{q}_u$, i.e.

$$S \supset \int d^4 x \sqrt{|g_{\text{in}}|}\, \hat{\mu}_u \psi_{q_u} \bar{q}_u + \text{h.c.}\Big|_{y=0}$$

$$\supset \int d^4 x\, \mu_u q_u \bar{q}_u + \text{h.c.}\,, \tag{3.6}$$

$$S \supset \int d^4 x \sqrt{|g_{\text{in}}|} \sum_{a,f} \hat{a}^u_{af} \eta_a \psi_{q_u} \psi_{\bar{u}_f} + \text{h.c.}\Big|_{y=y_{\text{I}}}$$

$$\supset \sum_{a,f} \int d^4 x\, a^u_{af} \eta_a q_u \bar{u}_f + \text{h.c.} \tag{3.7}$$

Here, $\hat{\mu}_u \sim k^{1/2}$, μ_u is a mass parameter determined via $\hat{\mu}_u$, and a^u_{af} are dimensionless couplings.

We encounter the coincidence of two apparently uncorrelated mass scales, namely μ and $a_{af}\langle\eta\rangle$, in a generic NB model.[4] However, in our framework, the existence of the intermediate brane together with the localization of the q-field towards that brane provides a prescription to the coincidence problem. The intermediate brane naturally provides the mass scale of $\langle\eta\rangle$. Meanwhile, the overlap of the UV brane-localized $\bar{q}$ and (quasi-)intermediate brane-localized q determines μ, which is controlled by the intermediate brane mass scale as seen in eq. (3.4) for $c_q = -\mathcal{O}(1)$. Furthermore, in terms of the dual 4D description, $\bar{q}$ is elementary whereas q is (mostly) composite. Hence, the prescription to the coincidence problem is similar in nature to the solution for the hierarchy problem in warped extra dimension models.

Compared to the original BBP model, our model contains KK excitation modes corresponding to $q_{u,d}$, $\bar{d}$, $\bar{u}$, and the SM gauge bosons. These KK excitation modes lead to new diagrams contributing to $\bar{\theta}$ together with flavor violation effects as we will discuss in detail in the next section. To avoid potentially large corrections to $\bar{\theta}$ and make our analysis simple, we introduce a flavor protection mechanism in the basis shown in eqs. (3.3), (3.5).[5] We assume $\text{U}(1)^3 = \text{U}(1)_1 \times \text{U}(1)_2 \times \text{U}(1)_3$ gauge symmetries in the whole bulk region where each U(1) rotates quarks with the same flavor index $f (= 1, 2, 3)$. Under the $\text{U}(1)_f$, $\bar{d}_f$ has charges $(n\delta_{f1}, n\delta_{f2}, n\delta_{f3})$ with a positive integer n, Q_f has charges $(-n\delta_{f1}, -n\delta_{f2}, -n\delta_{f3})$, and $\bar{u}_f$ has charges $(n\delta_{f1}, n\delta_{f2}, n\delta_{f3})$. The $\text{U}(1)^3$ symmetries are spontaneously broken in the subregion 1 by VEVs of three flavons ϕ_f living only in the subregion 1. Here, ϕ_f has charges $(\delta_{1f}, \delta_{2f}, \delta_{3f})$ under the $\text{U}(1)_f$. The $\text{U}(1)^3$ symmetries are also explicitly broken on the UV brane.[6] Since the $\text{U}(1)^3$ symmetries are unbroken in the subregion 2, the quark Yukawa couplings are given by diagonal matrices.[7] The intermediate brane-localized terms in eqs. (3.5), (3.7) are suppressed by the flavon VEVs. By taking e.g. $n = 5$ and $\langle\phi_f\rangle/M_5|_{y=y_{\text{I}}} \approx 10^{-1}$, we obtain $a^{u,d}_{af} \approx 10^{-5}$ which is enough to suppress corrections to $\bar{\theta}$

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[4]See [25] for a recent attempt to address the coincidence problem.

[5]The kinetic terms are canonically normalized. We call this basis as the gauge basis.

[6]If the explicit breaking terms are suppressed, we have axions [57, 58].

[7]Note that the Yukawa couplings after integrating out the heavy vector-like quarks are not diagonal, and the CKM structure is obtained from the diaognal Yukawa terms and the $q - d$ ($q - u$) mixing terms on the intermediate brane.
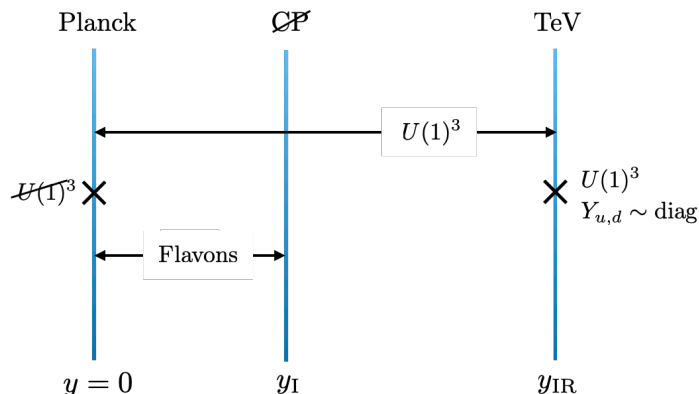
Figure 2. A schematic picture of our setup for the $U(1)^3$ flavor symmetries. The $U(1)^3$ are spontaneously broken by VEVs of flavons in the subregion 1. The quark Yukawa couplings on the IR (TeV) brane are given by diagonal matrices due to the unbroken $U(1)^3$ symmetries. At the intermediate brane, the flavons couple to $\bar{u}$ and $\bar{d}$, which lead to the terms in eqs. (3.5), (3.7). The $U(1)^3$ symmetries are explicitly broken on the UV brane.

as we will discuss in the next section. Figure 2 shows a schematic picture of our setup for the flavor symmetries. At the intermediate brane, we can write down flavor off-diagonal kinetic terms of the $SU(2)_L$ singlet quarks with some powers of the flavon VEVs. However, these terms are suppressed well for e.g. $n = 5$ and $\langle\phi_f\rangle/M_5|_{y=y_I} \approx 10^{-1}$, i.e. the ratio of the diagonal and off-diagonal kinetic terms is given by $\approx 10^{-10}$.

The quark mass hierarchies and the CKM structure are given from the diagonal Yukawa terms on the IR brane, and the terms on the intermediate brane of eqs. (3.5) and (3.7) in which the flavor symmetries are broken. As a demonstration, we assume the following couplings,

$$
\begin{aligned}
(Y_{11}^d, \ Y_{22}^d, \ Y_{33}^d) &\approx (1.8 \times 10^{-4}, \ 9.4 \times 10^{-4}, \ 4.3 \times 10^{-2}) \,, \\
(Y_{11}^u, \ Y_{22}^u, \ Y_{33}^u) &\approx (2.0 \times 10^{-4}, \ 2.6 \times 10^{-2}, \ 1) \,, \\
(a_{a1}^d\langle\eta_a\rangle, \ a_{a2}^d\langle\eta_a\rangle, \ a_{a3}^d\langle\eta_a\rangle) &\approx (0.19 + i0.092, \ -0.091 + i0.18, \ 0.65 - i0.41)m_{\mathrm{I}} \,, \\
(a_{a1}^u\langle\eta_a\rangle, \ a_{a2}^u\langle\eta_a\rangle, \ a_{a3}^u\langle\eta_a\rangle) &\approx (-0.27 - i0.0084, \ 0.81 + i0.39, \ 0)m_{\mathrm{I}} \,, \\
\mu_d &\approx 0.034 m_{\mathrm{I}} \,, \ \mu_u \approx 0.0093 m_{\mathrm{I}} \,,
\end{aligned}
\tag{3.8}
$$

where $Y_{ff'}^u$ denote the up-type quark Yukawa couplings, the off-diagonal elements of $Y^{d,u}$ are zero due to the flavor symmetries, and m_{I} is a mass parameter given by $m_{\mathrm{I}} \approx \langle\eta_a\rangle(\langle\phi_f\rangle/M_5)^n|_{y=y_I}$. These couplings are reasonably reproduced by taking e.g. $k_1 \approx k_2 \approx k$, $ky_{\mathrm{IR}} = 33$, $ky_{\mathrm{I}} = 17$, $n = 5$, $\langle\phi_f\rangle/M_5|_{y=y_I} \approx 10^{-1}$, and the following dimensionless bulk mass parameters of $\psi_{\bar{d}_f}, \psi_{\bar{u}_f}$ and $\psi_{q_{u,d}}$ in units of k,

$$
c_d \approx \{1, 0.93, 0.66\} \,, \quad c_u \approx \{1, 0.7, 0\} \,, \quad c_{q_{u,d}} \approx -1.2 \,.
\tag{3.9}
$$

With the assumption of eq. (3.8), the up and down-type quark masses and the CKM

matrices are, respectively, given as

$$m_u \approx (1\,\text{MeV},\ 1.3\,\text{GeV},\ 173\,\text{GeV})\,,$$

$$m_d \approx (5\,\text{MeV},\ 120\,\text{MeV},\ 2.7\,\text{GeV})\,,$$

$$|V_{\text{CKM}}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \tag{3.10}$$

$$\approx \begin{pmatrix} 0.98 & 0.18 & 0.0074 \\ 0.18 & 0.98 & 0.039 \\ 0.015 & 0.037 & 0.99 \end{pmatrix}\,.$$

We also get the Jarlskog invariant, $J \approx 1.2 \times 10^{-5}$.

4 Correction to $\bar{\theta}$

Let us first discuss how our warped extra dimension model can suppress contributions to $\bar{\theta}$ which are problematic in the non-supersymmetric BBP model. The model forbids the higher dimensional operators presented as in eq. (2.4) at the tree level because $\bar{q}_{u,d}$ and η_a live on different branes. The first term in eq. (2.5) is also forbidden at the tree level by the same reason. The quartic interactions of η_a in eq. (2.5), however, cannot be erased in the similar way because they live on the same brane. The contribution to $\bar{\theta}$ from those interactions was estimated in eq. (2.7), and a simple solution to satisfy the condition $\delta\bar{\theta} \leq 10^{-10}$ is to assume the coefficients $a_{af}^{u,d}$ are much smaller than $\mathcal{O}(1)$ like $a_{af}^{u,d} \lesssim 10^{-3}$. Such small $a_{af}^{u,d}$ are realized by the flavor symmetries as discussed in the previous section.

We now explore radiative corrections to $\bar{\theta}$ that are specific to our 5D multi-brane model. It is important to note that all CP phases given from eq. (3.5) can be moved into the up-quark sector of eq. (3.7) completely (or vise versa) by a phase rotation, $Q_f \rightarrow e^{i\alpha_f} Q_f$, $\bar{d}_f \rightarrow e^{-i\alpha_f} \bar{d}_f$ and $\bar{u}_f \rightarrow e^{-i\alpha_f} \bar{u}_f$ where α_f are determined to erase the phases in $a_{af}^d \langle \eta_a \rangle$. Note that only a pair of vector-like quarks is introduced in each of the down- and up-type sectors, and the parameters $\mu_{u,d}$ are 1×1 matrices, which make it possible to move all the CP phases in the down-sector into the up-type sector. This implies that one needs both up and down-type quarks in a diagram to obtain a non-zero correction to $\bar{\theta}$. We define corrections to the tree-level up and down-type quark mass matrices $M_{u,d}$ as $\delta M_{u,d}$ so that

$$\bar{\theta} = \arg \det(M_u + \delta M_u) \det(M_d + \delta M_d)$$

$$= \text{ImTr}(R_u^\dagger \delta M_u L_u \mu_u^{-1} + R_d^\dagger \delta M_d L_d \mu_d^{-1}) + \mathcal{O}(\delta M_u^2, \delta M_d^2, \delta M_u \delta M_d)\,, \tag{4.1}$$

where $R_{u,d}, L_{u,d}$ denote unitary matrices diagonalizing $M_{u,d}$, i.e. $R_u^\dagger M_u L_u = \mathcal{M}_u$ and $R_d^\dagger M_d L_d = \mathcal{M}_d$ where $\mathcal{M}_u$ and $\mathcal{M}_d$ are real and diagonal matrices. Note that $M_{u,d}$ here are different from $\hat{M}_{u,d}$ introduced around eq. (2.3), i.e. $M_{u,d}$ are the up-type and down-type quark matrices including $q_{u,d}$, $\bar{q}_{u,d}$, the SM up and down-type quarks and their KK modes, respectively. The last line of eq. (4.1) denotes terms with higher orders of $M_{u,d}$. Since we have not found important corrections to $\bar{\theta}$ through these terms, we focus on corrections through the leading order terms in the following discussion.
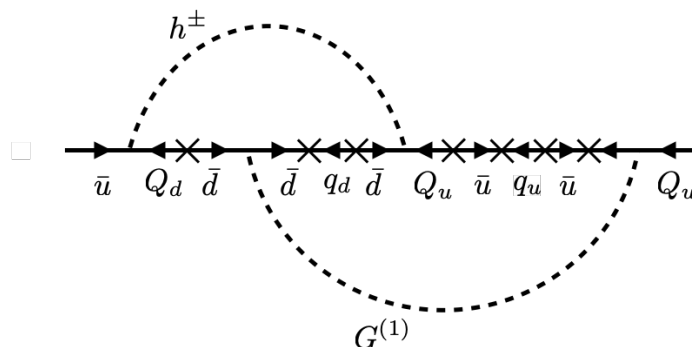
Figure 3. A two-loop diagram involving $h^{\pm}$ and the first KK excitation of the gluon $G^{(1)}$. Here, $\bar{d}$, $\bar{u}$, q_d and q_u include KK excitation modes as well as zero modes. $Q_{u,d}$, respectively, denote the up and down-type quark components of Q. We have omitted the flavor indices for notational simplicity. The solid lines represent fermion lines, and the dashed lines denote boson lines including gauge bosons and Higgs.

At the one-loop level, there is no diagram containing both up and down-type quarks together with SM gauge bosons or their KK excitation modes. A correction to δM_d may come from diagrams involving $h^{\pm}$ (NG bosons), which are however leading to [59]

$$
\begin{aligned}
\bar{\theta} &\propto \sum_{n=0}^{\infty} \mathrm{Im}\mathrm{Tr}\left(R_d^{\dagger} Y_d M_u^{\dagger}(M_u M_u^{\dagger})^n Y_u L_d \mathcal{M}_d^{-1}\right) \\
&\propto \sum_{n=0}^{\infty} \mathrm{Im} \sum_i (\mathcal{M}_d L_d^{\dagger} P_0 L_d \mathcal{M}_d^{2n+2} L_d^{\dagger} P_0 L_d)_{ii}/(\mathcal{M}_d)_{ii} \\
&= 0\,.
\end{aligned}
\tag{4.2}
$$

Here, Y_d is the SM down-type Yukawa matrix and P_0 denotes a matrix projecting $\mathcal{M}_d$ on vY_d, i.e. $vY_d \equiv R_d \mathcal{M}_d L_d^{\dagger} P_0$. In the last equality of eq. (4.2), we have used the fact that $V_{ij}V_{ji}^{\dagger}$ is real for a matrix V.[8] A one-loop correction to δM_u involving $h^{\pm}$ also vanishes by the same reason. Therefore, we conclude that there is no sizable correction to $\bar{\theta}$ at the one-loop level.

At the two-loop level, the number of diagrams which may contribute to $\bar{\theta}$ drastically increases. To find relevant diagrams involving both up and down-type quarks, we require at least one $h^{\pm}$ or $W^{\pm}$ in a diagram. In our warped extra dimension model, KK excitation modes of the SM gauge bosons, up and down-type quarks and $q_{u,d}$ can enter into a diagram. Figure 3 shows a diagram which may lead to an important contribution to $\bar{\theta}$. The diagram is given in the gauge basis and contains $h^{\pm}$ and the first KK excitation of the gluon $G^{(1)}$ in the loops. A correction to $\bar{\theta}$ generated by this diagram is however suppressed by two SM Yukawa couplings and three SM quark mass insertions. Moreover, the bulk profiles of the down-type quark zero modes to explain the mass hierarchies give a suppression for their couplings to the KK gluon whose profile is localized toward the IR brane. Therefore, this diagram safely leads to $\bar{\theta} < 10^{-10}$. Figure 4 is a diagram with different topology which contains the first KK excitation mode of the down-type quark (the Dirac partner

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[8]In other words, a one-loop diagram cannot contain both terms of eqs. (3.5), (3.7).
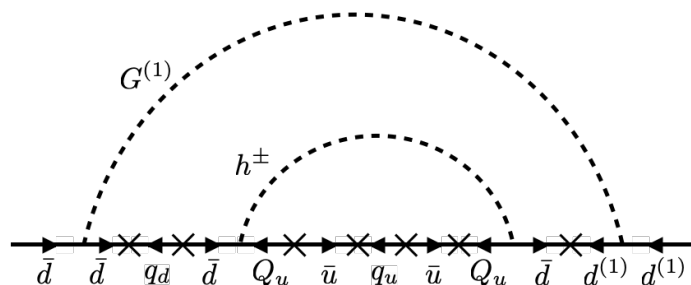
Figure 4. A two-loop diagram which contains the first KK excitation mode of the down-type quark $d^{(1)}$ (the Dirac partner of $\bar{d}^{(1)}$). The flavor indices are also omitted.

of $\bar{d}^{(1)}$). This diagram also includes $\eta \bar{d}_f^{(1)} q_d$, $\eta \bar{u}_3^{(0)} q_u$ and $\eta \bar{u}_f^{(1)} q_u$ couplings suppressed by their profiles. To obtain a sufficient suppression, we require $\exp(-ky_{\mathrm{IR}})/\exp(-ky_{\mathrm{I}}) \ll 10^{-5}$ where we assume $k_1 \approx k_2 \equiv k$. Consequently, we require two conditions to realize $\bar{\theta} < 10^{-10}$ at the two-loop level: the mass hierarchies of SM quarks are explained by their bulk profiles, and a distance between the intermediate and IR branes is large enough, $e^{k(y_{\mathrm{I}}-y_{\mathrm{IR}})} \ll 10^{-5}$. Under these conditions, we have also investigated diagrams involving e.g. $h^\pm - h^\pm$, $h^\pm - h^0$, $h^\pm - W^\pm$, $W^\pm - h^0$ and found no dangerous corrections to $\bar{\theta}$.

The kinetic term of $q_{u,d}$ can be modified by interactions with η_a on the intermediate brane,

$$\mathcal{L} \supset iZ_{u,d}(\eta_a/M_{\mathrm{I}}, \eta_a^*/M_{\mathrm{I}}) \, q_{u,d}^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu q_{u,d} \,, \tag{4.3}$$

where $M_{\mathrm{I}} \approx k_1 e^{-k_1 y_{\mathrm{I}}}$ represents a cutoff scale of the intermediate brane. Since $Z_{u,d}(\langle\eta_a\rangle/M_{\mathrm{I}}, \langle\eta_a^*\rangle/M_{\mathrm{I}})$ is a real number, it does not provide a new correction to $\bar{\theta}$. However, $Z_{u,d}(\eta_a/M_{\mathrm{I}}, \eta_a^*/M_{\mathrm{I}})$ may contribute to $\bar{\theta}$ through diagrams such as the one shown in figure 5. This diagram leads to a correction to the $\mu q_d \bar{q}_d$ term, i.e. a CP phase is provided in the diagonal component of M_d. The correction to $\bar{\theta}$ is estimated as

$$|\delta\bar{\theta}| \sim \left| \sum_{a,b,c,f} \left(\frac{1}{16\pi^2}\right)^3 \frac{\langle\eta_a\rangle\langle\eta_b\rangle a_{af}^d a_{cf}^d}{\Lambda_{\mathrm{I}}^2} \right| \,, \tag{4.4}$$

where we have assumed coefficients in $Z_d(\eta_a/M_{\mathrm{I}}, \eta_a^*/M_{\mathrm{I}})$ are $\mathcal{O}(1)$. This correction can be sufficiently suppressed by $a_{af}^d \lesssim 10^{-3}$ that is required to suppress the contribution from eq. (2.7). We note that modified kinetic terms of $\bar{u}$ and $\bar{d}$ do not give relevant corrections because their off-diagonal components are suppressed by the flavor symmetries introduced in section 3.

Let us comment on corrections from KK modes of the U(1)^3 flavor gauge fields and flavons. The former corrections are given by replacing the KK gluon in figure 3 with a KK mode of the U(1)^3 gauge field. The corrections depend on the U(1)^3 gauge couplings, and if the couplings are tiny, they are suppressed. The flavons ϕ couple to $\bar{d}$, $\bar{u}$ in eqs. (3.5), (3.7) like $\mathcal{L} \sim \phi^{*5} \eta \psi_{\bar{d}} \psi_{q_d} + \phi^{*5} \eta \psi_{\bar{u}} \psi_{q_u} + \mathrm{h.c.}$, as well as the $\bar{d}$, $\bar{u}$ kinetic terms. The corrections are suppressed for small flavon VEVs, $\langle\phi\rangle/M_5|_{y=y_{\mathrm{I}}} \ll 1$, and large flavor charges of quarks, $n \gg 1$. Therefore, we conclude that the corrections from KK modes of the flavor gauge bosons and flavons can be sufficiently small and hence do not pose a problem.
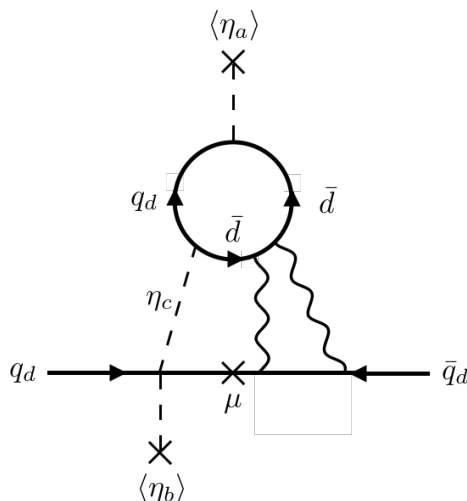
Figure 5. An example diagram contributing to $\bar\theta$. The wavy lines denote the SM gauge bosons.

5 Conclusions

We have explored a doubly composite dynamics via a holographic warped extra dimension model with three 3-branes that realizes the NB mechanism to address the strong CP problem. The scale of spontaneous CP violation was introduced as the scale of the intermediate brane. We have shown that three branes are essential to forbid dangerous operators leading to a large $\bar\theta$. With the help of flavor symmetries for quarks, the model can solve the strong CP problem without fine-tuning. The SM quark masses and mixings are naturally explained by the bulk profiles of the quarks.

Acknowledgments

A 5D fermion in three 3-brane models

We consider a Dirac fermion Ψ living in the whole 5D spacetime whose metric is given in eq. (3.1) in the main text. The action is

$$
\begin{aligned}
S = &\int_0^{y_{\mathrm{I}}} dy \int d^4x \sqrt{g} \left[\frac{i}{2} \left(\bar\Psi \Gamma^M \partial_M \Psi - \partial_M \bar\Psi \Gamma^M \Psi \right) - m_{\Psi,1} \bar\Psi \Psi \right] \\
&+ \int_{y_{\mathrm{I}}}^{y_{\mathrm{IR}}} dy \int d^4x \sqrt{g} \left[\frac{i}{2} \left(\bar\Psi \Gamma^M \partial_M \Psi - \partial_M \bar\Psi \Gamma^M \Psi \right) - m_{\Psi,2} \bar\Psi \Psi \right],
\end{aligned}
\tag{A.1}
$$

where $M = (\mu, y)$ and we have defined

$$\Gamma^M \equiv e^M_A \gamma^A, \qquad g^{MN} \equiv e^M_A e^N_B \eta^{AB}, \tag{A.2}$$

with the vielbein e^M_A which is non-zero for diagonal components, i.e. $e^\mu_\alpha = e^{A(y)} \delta^\mu_\alpha$, $e^y_5 = 1$. The gamma matrices are

$$\gamma^A \equiv (\gamma^\mu, \ -i\gamma_5), \quad \gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \quad \gamma_5 \equiv \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}, \tag{A.3}$$

$$\sigma^\mu \equiv (\mathbf{1}, \ \boldsymbol{\sigma}), \qquad \bar\sigma^\mu \equiv (\mathbf{1}, -\boldsymbol{\sigma}). \tag{A.4}$$

Here, $\boldsymbol{\sigma}$ denote the Pauli matrices. Note that the dependence on the spin connection is canceled in the equation of motion as in the case of the two 3-brane setup.

The bulk equation of motion is obtained from the variation of the action (A.1),

$$e^{k_{1,2}y} i\gamma^\mu \partial_\mu \Psi_- - \partial_5 \Psi_+ + k_{1,2}(-c_{1,2} + 2)\Psi_+ = 0, \tag{A.5}$$

$$e^{k_{1,2}y} i\gamma^\mu \partial_\mu \Psi_+ + \partial_5 \Psi_- - k_{1,2}(c_{1,2} + 2)\Psi_- = 0, \tag{A.6}$$

where $m_{\Psi_{1,2}} \equiv c_{1,2} k_{1,2}$ and $\Psi_\pm$ denote the left and right-handed spinors defined as

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \ \Psi_+ = \begin{pmatrix} \psi_+ \\ 0 \end{pmatrix}, \ \Psi_- = \begin{pmatrix} 0 \\ \psi_- \end{pmatrix}, \tag{A.7}$$

$$\Psi_\pm = \mp\gamma_5 \Psi_\pm. \tag{A.8}$$

By using the Dirac equation $i\gamma^\mu \partial_\mu \Psi^{(n)}_\pm = m_n \Psi^{(n)}_\pm$, the above equations are further reduced to

$$\partial_5 \psi^{(n)}_+ + k_{1,2}(c_{1,2} - 2)\psi^{(n)}_+ = e^{k_{1,2}y} m_n \psi^{(n)}_-, \tag{A.9}$$

$$-\partial_5 \psi^{(n)}_- + k_{1,2}(c_{1,2} + 2)\psi^{(n)}_- = e^{k_{1,2}y} m_n \psi^{(n)}_+. \tag{A.10}$$

In the variation of the action, the boundary terms on the UV and IR branes vanish when

$$\delta\bar\Psi_+ \Psi_- = 0 \quad \text{and} \quad \delta\bar\Psi_- \Psi_+ = 0 \tag{A.11}$$

are satisfied while the boundary term on the intermediate brane vanishes for

$$\delta\bar\Psi_+ [\Psi_-]|_{y=y_\mathrm{I}} = 0 \quad \text{and} \quad \delta\bar\Psi_- [\Psi_+]|_{y=y_\mathrm{I}} = 0, \tag{A.12}$$

where $[X]|_{y=y_\mathrm{I}} \equiv \lim_{\epsilon \to 0} X(y_\mathrm{I} + \epsilon) - X(y_\mathrm{I} - \epsilon)$. Thus, we require

$$\psi_+|_{y=y_\mathrm{UV,IR}} = 0 \quad \text{or} \quad \psi_-|_{y=y_\mathrm{UV,IR}} = 0, \tag{A.13}$$

$$[\psi_+]|_{y=y_\mathrm{I}} = 0 \quad \text{and} \quad [\psi_-]|_{y=y_\mathrm{I}} = 0, \tag{A.14}$$

as boundary conditions at the three branes. By taking the Dirichlet condition on the UV and IR branes for Ψ_+ (Ψ_-), the zero mode solution for Ψ_+ (Ψ_-) becomes trivial, i.e. $\Psi_+ = 0$ ($\Psi_- = 0$) for the whole space. Then, only the right-handed (left-handed) spinor appears as a massless mode as given in the two 3-brane case.

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