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The semileptonic B_s and Λ_b widths

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We present estimates of the semileptonic widths of the B_s and Λ_b hadrons. We employ the latest fit to the B inclusive semileptonic data, the heavy quark expansion and lattice QCD results. Our results suffer from large uncertainties that could be reduced with dedicated measurements and new lattice QCD calculations.

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The determination of the Cabibbo-Kobayashi-Maskawa element V_{cb} from inclusive semileptonic B decays has been recently updated [1] with the inclusion of new $O(\alpha_s^3)$ contributions to the width and to the relation between the b quark mass in the $\overline{\text{MS}}$ and in the kinetic scheme [2], leading to

$$|V_{cb}| = 42.16(51) \times 10^{-3},$$

with a 30% reduction in the total uncertainty. Indeed, the three-loop calculation brings the perturbative effects under better control. The new value for $|V_{cb}|$ implies a $\sim 4\sigma$ tension with the determination based on $B \to D^*\ell\nu$ data and lattice QCD [3]. The detailed results of the fit to inclusive semileptonic data can be found in Table I of Ref. [1]. In these proceedings we employ them to provide updated estimates of the total semileptonic widths of the B_s and Λ_b, which can be useful in semileptonic studies and in the determination of f_s/f_d at LHCb.

1. The semileptonic width

As is well-known, the semileptonic width can be written as a double expansion in α_s and Λ_{QCD}/m_b,

$$\begin{aligned}\Gamma_{\text{sl}}(B_q) = \Gamma_0 f(\rho)\Bigg[1 + a_1 a_s + a_2 a_s^2 + a_3 a_s^3 - \left(\frac{1}{2} - p_1 a_s\right)\frac{\mu_\pi^2(B_q)}{m_b^2} + (g_0 + g_1 a_s)\frac{\mu_G^2(B_q)}{m_b^2} \\ + (d_0 + d_1 a_s)\frac{\rho_D^3(B_q)}{m_b^3} - g_0\frac{\rho_{LS}^3(B_q)}{m_b^3} + \dots\Bigg],\end{aligned} \tag{1}$$

where $a_s = \alpha_s(m_b)/\pi$ and the subscript q accounts for the down, up or strange spectator quark in the meson. The ellipses stand for higher order contributions not included in the reference fit of [1]. We adopt the kinetic scheme for the b quark mass and for the expectation values μ_π^2, μ_G^2, ρ_D^3 and ρ_{LS}^3, with cutoff $\mu = 1\,\text{GeV}$. Unlike the perturbative corrections, the latter depend on the spectator quark and differ between $B \equiv B_{d,u}$ and B_s. The charm quark mass is evaluated in the $\overline{\text{MS}}$ scheme at 2 GeV. The current experimental data on the moments of the kinematic distributions in semileptonic B decays allow us to extract the non-perturbative parameters for a light spectator, see Table I of Ref. [1]. As similar experimental data are not yet available for B_s decays, to estimate $\Gamma_{\text{sl}}(B_s)$ one has to resort to the Heavy Quark Expansion (HQE) of meson masses and to the heavy quark sum rules [4]. In the following, we update the analysis of Ref. [4] using the results of Ref. [1], the latest PDG values for the meson masses [5], and recent lattice QCD data on meson masses obtained with both d and s spectators [6, 7].

Before diving into the discussion, we report the contribution of the various non-perturbative parameters to the B semileptonic width based on [1], relative to the lowest order B width:

$$\begin{aligned}\frac{\delta_{\mu_\pi^2}\Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} &= -0.9(1)\%\,, \qquad \frac{\delta_{\mu_G^2}\Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} = -3.2(5)\%\,, \\ \frac{\delta_{\rho_D^3}\Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} &= -3.2(5)\%\,, \qquad \frac{\delta_{\rho_{LS}^3}\Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} = -0.3(2)\%\,.\end{aligned} \tag{2}$$

Notice that the expectation values in (1) refer to physical B mesons and not to their infinite mass limit.

2. SU(3) breaking effects in the HQE parameters

We now follow closely the arguments of Ref. [4]. For later convenience, we recall here the HQE for the heavy hadron masses

$$M_H = m_b + \bar{\Lambda}(H) + \frac{\tilde{\mu}_\pi^2(H) - c_G\,\tilde{\mu}_G^2(H)}{2m_b} + O\Big(\frac{1}{m_b^2}\Big)\,, \tag{3}$$

where $\tilde{\mu}^2_{\pi,G}(H)$ are the expectation values in the infinite mass limit for the heavy quark. For $H = B$ they are related to the quantities entering Eq. (1) by $\mu_\pi^2(B) = \tilde{\mu}_\pi^2(B) - (\rho^3_{\pi\pi} + 1/2\rho^3_{\pi G})/m_b$ and $\mu_G^2(B) = \tilde{\mu}_G^2(B) + (\rho_s^3 + \rho_A^3 + 1/2\rho^3_{\pi G})/m_b$, where the power suppressed contributions are given by expectation values of non-local operators that also appear in the $O(1/m_b^2)$ terms of Eq. (3).

The expectation value of $\tilde{\mu}_G^2$ can be extracted from the hyperfine splitting,

$$\frac{\tilde{\mu}_G^2(B_s)}{\tilde{\mu}_G^2(B)} \simeq \frac{m_{B_s^*} - m_{B_s}}{m_{B^*} - m_B} = 1.08 \pm 0.04\,, \tag{4}$$

up to power corrections. On the other hand, the HQE fits to meson masses computed on the lattice for different heavy quark masses and $q = d, s$ [6, 7] give

$$\frac{\tilde{\mu}_G^2(B_s)}{\tilde{\mu}_G^2(B)} \simeq 1.20 \pm 0.10\,. \tag{5}$$

The fits of [6, 7] also find large power corrections to (4), subject to substantial $SU(3)$ breaking. The numerical value given in (4) should therefore be considered with care. Moreover, what enters Eq. (1) are the expectation values in the physical mesons, while those appearing in the HQE of the meson masses refer to the infinite mass limit. We therefore average the values in Eq. (4) and Eq. (6) and assign a large uncertainty that covers both values:

$$\frac{\mu_G^2(B_s)}{\mu_G^2(B)} \simeq 1.14 \pm 0.10\,. \tag{6}$$

Recalling Eq. (2), we then expect

$$\delta_{\mu_G^2}\frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B)} = (-0.4 \pm 0.3)\%\,. \tag{7}$$

Let us now consider the spin-averaged masses, defined by

$$\overline{m}_B = \frac{3m_{B^*} + m_B}{4}\,, \tag{8}$$

and similarly for different spectators, in terms of which we get

$$\Delta m_B = \overline{m}_{B_s} - \overline{m}_B = (89.9 \pm 1.2)\,\text{MeV}\,, \quad \Delta m_D = \overline{m}_{D_s} - \overline{m}_D = (101.1 \pm 0.3)\,\text{MeV}\,. \tag{9}$$

The spin-averaged masses admit the expansions

$$\overline{m}_B = m_b + \bar{\Lambda} + \frac{\tilde{\mu}_\pi^2(B)}{2m_b} + O(1/m_b^2)\,, \quad \overline{m}_{B_s} \simeq m_b + \bar{\Lambda}_s + \frac{\tilde{\mu}_\pi^2(B_s)}{2m_b} + O(1/m_b^2)\,, \tag{10}$$

and similarly for the D system, with the replacement $m_b \to m_c$. We can now use Eq. (9) and Eq. (10) to extract the values for $\bar{\Lambda}_s - \bar{\Lambda}$ and $\tilde{\mu}^2_\pi(B_s) - \tilde{\mu}^2_\pi(B)$,

$$\bar{\Lambda}_s - \bar{\Lambda} = \frac{m_b \Delta m_B - m_c \Delta m_D}{m_b - m_c}\,, \qquad \tilde{\mu}^2_\pi(B_s) - \tilde{\mu}^2_\pi(B) = \frac{2 m_b m_c}{m_b - m_c}(\Delta m_B - \Delta m_D)\,. \tag{11}$$

Using the values of $m_{b,c}$ extracted in [1], we find

$$\bar{\Lambda}_s - \bar{\Lambda} \simeq 86\,\mathrm{MeV}\,, \qquad \tilde{\mu}^2_\pi(B_s) - \tilde{\mu}^2_\pi(B) \simeq 0.032\,\mathrm{GeV}^2\,. \tag{12}$$

This should be compared with the results reported in Ref. [7]:

$$\bar{\Lambda}_s - \bar{\Lambda} \simeq (84 \pm 20)\,\mathrm{MeV}\,, \qquad \tilde{\mu}^2_\pi(B_s) - \tilde{\mu}^2_\pi(B) \simeq 0.11 \pm 0.03\,\mathrm{GeV}^2\,. \tag{13}$$

The discrepancy in $\tilde{\mu}^2_\pi(B_s) - \tilde{\mu}^2_\pi(B)$ might again be explained by higher power corrections, which are neglected in the derivation of Eq. (12). We take this and the different definition of the HQE parameters in (1, 10) into account and use

$$\mu^2_\pi(B_s) - \mu^2_\pi(B) = (0.1 \pm 0.1)\,\mathrm{GeV}^2\,. \tag{14}$$

We can then estimate

$$\delta_{\mu^2_\pi} \frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B)} = (-0.2 \pm 0.1)\%\,. \tag{15}$$

Let us now consider the contributions proportional to ρ^3_D. Using the QCD equations of motion and the vacuum saturation approximation one expects

$$\frac{\rho^3_D(B_s)}{\rho^3_D(B)} \simeq \frac{f^2_{B_s} m_{B_s}}{f^2_B m_B} = 1.48 \pm 0.06\,, \tag{16}$$

where the value $f_{B_s}/f_B = 1.206 \pm 0.023$ has been used [10]. We can also use the heavy quark sum rules to estimate the same ratio

$$\frac{\rho^3_D(B_s)}{\rho^3_D(B)} \simeq \left(\frac{\mu^2_\pi(B_s)}{\mu^2_\pi(B)}\right)^2 \frac{\bar{\Lambda}}{\bar{\Lambda}_s} = 1.30 \pm 0.10\,, \tag{17}$$

where we employed (14) and the results of [1]. Averaging the values in Eq. (16) and Eq. (17) we arrive at

$$\frac{\rho^3_D(B_s)}{\rho^3_D(B)} \simeq 1.39 \pm 0.15\,. \tag{18}$$

The fits of Refs. [6, 7] do not extract ρ^3_D, but a linear combination of different terms:

$$\frac{\rho^3_D(B_s) + \rho^3_{\pi\pi}(B_s) - \rho^3_S(B_s)}{\rho^3_D(B) + \rho^3_{\pi\pi}(B) - \rho^3_S(B)} = 1.33 \pm 0.25\,, \tag{19}$$

which confirms a sizeable increase of the $1/m_b^3$ terms. In summary, the impact of Eq. (18) on the semileptonic width is quantifiable as

$$\delta_{\rho^3_D} \frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B)} = (-1.2 \pm 0.6)\%\,. \tag{20}$$

Since the impact of ρ_{LS}^3 on the rate is rather small, we will not discuss its $SU(3)$ breaking. Concerning higher power corrections, we have indications that $O(1/m_Q^4)$ and $O(1/m_Q^5)$ are suppressed with respect to lower power corrections [11], but they might be more sensitive to $SU(3)$ breaking. We therefore assign an additional 0.5% uncertainty to the ratio of the semileptonic widths, which together with Eqs. (7,14,20) leads to our final estimate

$$\frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B_d)} - 1 = -(1.8 \pm 0.8)\% \,. \tag{21}$$

3. The Λ_b case

The Λ_b case can be dealt with in a similar fashion, keeping in mind that in the Λ_b baryon μ_G^2 and ρ_{LS}^3 vanish at the leading order in $1/m_b$. Subtracting these terms will give the largest difference in the semileptonic width. From the fit in Ref. [1] we get

$$\frac{\delta_{\mu_G^2}\Gamma_{\rm sl}(B) + \delta_{\rho_{LS}^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -(3.5 \pm 0.6)\% \,. \tag{22}$$

For the other contributions we basically confirm the estimates of Ref. [4] (see also [14]) and find

$$\delta_{\mu_\pi^2}\frac{\Gamma_{\rm sl}(\Lambda_b)}{\Gamma_{\rm sl}(B)} = (-0.2 \pm 0.2)\% \,, \qquad \delta_{\rho_D^3}\frac{\Gamma_{\rm sl}(\Lambda_b)}{\Gamma_{\rm sl}(B_d)} = (0.8 \pm 1.3)\% \,. \tag{23}$$

Putting together the above results and including an additional 0.7% uncertainty due to subleading effects of the chromomagnetic operator and to higher power corrections, we find

$$\frac{\Gamma_{\rm sl}(\Lambda_b)}{\Gamma_{\rm sl}(B_d)} - 1 = (4.1 \pm 1.6)\% \,, \tag{24}$$

where the large uncertainty is dominated by the value of $\rho_D^3(\Lambda_b)$.

4. Summary

We have estimated the B_s and Λ_b semileptonic widths, updating Ref.[4] in various ways. These semileptonic widths are an important input in a few LHCb analyses. The leading uncertainty in our estimates is related to the Darwin term ρ_D^3 and only a better knowledge of this parameter in the B_s and Λ_b can improve them. In particular, a measurement of the moments of kinematic distributions such as the hadronic mass distribution in B_s semileptonic decays at LHCb could improve the current theoretical prediction of $\Gamma_{\rm sl}(B_s)$.

The inclusive semileptonic width can now also be computed on the lattice [12, 13], but we do not have yet results for physical values of m_b and a complete assessment of the lattice systematic uncertainties is still missing. In the future, however, the ratio $\Gamma_{\rm sl}(B_s)/\Gamma_{\rm sl}(B)$ might be determined with an accuracy similar or better than in (21) as many systematic uncertainties will cancel out between numerator and denominator. For what concerns the Λ_b semileptonic width, an improvement could come from a study similar to that of Ref. [6] for heavy baryons.

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References

[1] M. Bordone, B. Capdevila and P. Gambino, Phys. Lett. B **822** (2021), 136679 doi:10.1016/j.physletb.2021.136679 [arXiv:2107.00604 [hep-ph]].

[2] M. Fael, K. Schönwald and M. Steinhauser, Phys. Rev. D **104** (2021) no.1, 016003 doi:10.1103/PhysRevD.104.016003 [arXiv:2011.13654 [hep-ph]]; M. Fael, K. Schönwald and M. Steinhauser, Phys. Rev. Lett. **125** (2020) no.5, 052003 doi:10.1103/PhysRevLett.125.052003 [arXiv:2005.06487 [hep-ph]].

[3] A. Bazavov *et al.* [Fermilab Lattice and MILC], [arXiv:2105.14019 [hep-lat]].

[4] I. I. Bigi, T. Mannel and N. Uraltsev, JHEP **09** (2011), 012 doi:10.1007/JHEP09(2011)012 [arXiv:1105.4574 [hep-ph]].

[5] P. A. Zyla *et al.* [Particle Data Group], PTEP **2020** (2020) no.8, 083C01 doi:10.1093/ptep/ptaa104

[6] P. Gambino, A. Melis and S. Simula, Phys. Rev. D **96** (2017) no.1, 014511 doi:10.1103/PhysRevD.96.014511 [arXiv:1704.06105 [hep-lat]].

[7] P. Gambino, V. Lubicz, A. Melis and S. Simula, J. Phys. Conf. Ser. **1137** (2019) no.1, 012005 doi:10.1088/1742-6596/1137/1/012005

[8] P. Gambino, T. Mannel and N. Uraltsev, Phys. Rev. D **81** (2010), 113002 doi:10.1103/PhysRevD.81.113002 [arXiv:1004.2859 [hep-ph]].

[9] D. Pirjol and N. Uraltsev, Phys. Rev. D **59** (1999), 034012 doi:10.1103/PhysRevD.59.034012 [arXiv:hep-ph/9805488 [hep-ph]].

[10] S. Aoki *et al.* [Flavour Lattice Averaging Group], Eur. Phys. J. C **80** (2020) no.2, 113 doi:10.1140/epjc/s10052-019-7354-7 [arXiv:1902.08191 [hep-lat]].

[11] P. Gambino, K. J. Healey and S. Turczyk, Phys. Lett. B **763** (2016), 60-65 doi:10.1016/j.physletb.2016.10.023 [arXiv:1606.06174 [hep-ph]].

[12] P. Gambino and S. Hashimoto, Phys. Rev. Lett. **125** (2020) no.3, 032001 [arXiv:2005.13730 [hep-lat]].

[13] P. Gambino, S. Hashimoto, S. Mächler, M. Panero, F. Sanfilippo, S. Simula, A. Smecca and N. Tantalo, [arXiv:2203.11762 [hep-lat]].

[14] P. Colangelo, F. De Fazio and F. Loparco, JHEP **11** (2020), 032 doi:10.1007/JHEP11(2020)032 [arXiv:2006.13759 [hep-ph]].