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Form factors for semileptonic $B \to \pi$, $B_s \to K$ and $B_s \to D_s$ decays

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We report on our determinations of $B \to \pi \ell \nu$, $B_s \to K \ell \nu$ and $B_s \to D_s \ell \nu$ semileptonic form factors. In addition we discuss the determination of R-ratios testing lepton-flavor universality and suggest an improved ratio. Our calculations are based on the set of 2+1 flavor domain-wall Iwasaki gauge field configurations generated by the RBC/UKQCD collaboration with three lattice spacings of $1/a = 1.78$, 2.38, and 2.79 GeV. We use the relativistic heavy quark action for b quarks and charm quarks are simulated with the Möbius domain-wall fermion action.

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1. Introduction

Semileptonic decays of $B_{(s)}$ mesons play an important role in testing and constraining the Standard Model (SM) of elementary particle physics. Focusing on exclusive semileptonic decays, we report on our work for $B \to \pi\ell\nu$, $B_s \to D_s\ell\nu$ and $B_s \to K\ell\nu$ decays. Each of these processes can be described by two form factors, f_+ and f_0, which parametrize the semileptonic decay rate. For the semileptonic decay of pseudoscalar meson $B_{(s)}$ of mass M and momentum p to pseudoscalar meson P of mass m and momentum k, with $q = p - k$,

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \eta \frac{G_F^2|V_{xb}|^2}{24\pi^3} \frac{(q^2-m_\ell^2)^2|\vec{k}|}{(q^2)^2} \left[\left(1+\frac{m_\ell^2}{2q^2}\right)\vec{k}^2\,|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}\frac{(M^2-m)^2}{M^2}\,|f_0(q^2)|^2\right], \quad (1)$$

where m_ℓ is the mass of the outgoing charged lepton ℓ and η is an isospin factor. The form factors f_+ and f_0 appear in the decomposition

$$\langle P(k)|\mathcal{V}^\mu(0)|B_{(s)}(p)\rangle = 2\, f_+(q^2)\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right) + f_0(q^2)\frac{M^2-m^2}{q^2}q^\mu, \quad (2)$$

where $\mathcal{V}^\mu = \bar{x}\gamma^\mu b$, with $x = u$ or c.

Compared to our earlier results for $B \to \pi$ and $B_s \to K$ decays [1], we have added calculations of $B_s \to D_s$ form factors and have an additional, third, lattice spacing. With results for $B_s \to K$ and $B_s \to D_s$ from the same ensembles, we will be able to compute the ratio of partially integrated decay rates (minus CKM factors) in the region $q^2 \geq 7\,\mathrm{GeV}^2$ for the two decays and combine with recent LHCb results [2] to determine $|V_{ub}/V_{cb}|$. We also consider R ratios of branching fractions with τ leptons in the final state to those with light final-state leptons, sensitive to violations of lepton-flavor universality (for $B \to D^{(*)}\ell\nu$ there is tension between Standard Model predictions and experimental results for $R(D^{(*)})$ [3–12]). We propose a modified ratio with smaller uncertainty when evaluated using lattice-determined form factors.

2. Lattice calculation

We use a subset of six RBC/UKQCD 2+1-flavor domain-wall fermion (DWF) and Iwasaki gauge field ensembles with three lattice spacings $a \sim 0.11, 0.08, 0.07$ fm, and pion masses spanning $267\,\mathrm{MeV} < M_\pi < 433\,\mathrm{MeV}$. The ensembles are listed in Table 1. Light and strange quarks are simulated with the Shamir DWF action with $M_5 = 1.8$. Lattice spacings are determined from combined RBC/UKQCD analyses [13–15]. Our calculations are described briefly below; for more details see [1].

Bottom quarks are simulated with the relativistic heavy quark (RHQ), action, which is the Columbia variant [18, 19] of the Fermilab heavy-quark action [20], with three nonperturbatively-tuned parameters (m_0a, c_P, ζ) [21]. A new tuning was performed for this analysis. Charm quarks are simulated with the Möbius DWF action with $M_5 = 1.6$ [14, 15, 22, 23]. We use three masses below m_c^{phys} on the C ensembles and two masses which bracket m_c^{phys} on M and F. Light and

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	L	T	L_s	a^{-1}/GeV	$am_l^{\rm sea}$	$am_s^{\rm sea}$	M_π/MeV	# cfgs	# sources
C1	24	64	16	1.785	0.005	0.040	340	1636	1
C2	24	64	16	1.785	0.010	0.040	433	1419	1
M1	32	64	16	2.383	0.004	0.030	302	628	2
M2	32	64	16	2.383	0.006	0.030	362	889	2
M3	32	64	16	2.383	0.008	0.030	411	544	2
F1S	48	96	12	2.785	0.002144	0.02144	267	98	24

Table 1: Ensembles used for the simulations reported here [13, 14, 16, 17]. $am_l^{\rm sea}$ and $am_s^{\rm sea}$ are the sea light and strange quark masses and M_π is the unitary pion mass. Valence strange quarks are near their physical mass, with the mistuning accounted for in our systematic errors.

strange quarks have point sources; while the b and c quarks use Gauss-smeared sources and point or smeared sinks.

Renormalized, $\mathcal{V}_\mu$, and lattice, V_μ, currents are related by the 'partially nonperturbative' procedure [24, 25], using

$$\langle P|\mathcal{V}_\mu|B_s\rangle = Z_{V_\mu}^{bx}\langle P|V_\mu|B_s\rangle, \tag{3}$$

with $Z_{V_\mu}^{bx} = \rho_{V_\mu}^{bx}\sqrt{Z_V^{xx}Z_V^{bb}}$ and

$$V_0 = V_0^0 + c_t^3 V_0^3 + c_t^4 V_0^4, \qquad V_i = V_i^0 + c_s^1 V_i^1 + c_s^2 V_i^2 + c_s^3 V_i^3 + c_s^4 V_i^4. \tag{4}$$

Here, $\rho_{V_\mu}^{bx}$ and the coefficients $c_{t,s}^n$ of the $O(a)$ current-improvement operators are computed perturbatively at one-loop [26], while Z_V^{bb} is computed nonperturbatively from the forward matrix element

$$Z_V^{bb}\langle B_{(s)}|V_0(0)|B_{(s)}\rangle = 2M \tag{5}$$

and Z_V^{xx} is computed nonperturbatively using the relation $Z_V^{xx} = Z_A^{xx} + O(am_{\rm res})$ for DWF fermions [17] (for our systematic error analysis for $B_s \to D_s$ decays, we also compare to using $Z_V^{cc} = Z_A^{ll}$ [14]).

To extract the form factors we first calculate the matrix elements

$$f_\parallel(E) = \frac{\langle P|\mathcal{V}^0(0)|B_{(s)}\rangle}{\sqrt{2M}}, \qquad f_\perp(E) = \frac{\langle P|\mathcal{V}^i(0)|B_{(s)}\rangle}{k^i\sqrt{2M}}, \tag{6}$$

with a $B_{(s)}$ meson at rest, where E is the energy of the outgoing pseudoscalar meson, from which we determine

$$f_0(q^2) = \frac{\sqrt{2M}}{M^2 - m^2}\left[(M-E)f_\parallel(E) + (E^2 - m^2)f_\perp(E)\right], \tag{7}$$

$$f_+(q^2) = \frac{1}{\sqrt{2M}}\left[f_\parallel(E) + (M-E)f_\perp(E)\right]. \tag{8}$$

To find $f_\parallel$ and $f_\perp$, we evaluate a correlator ratio

$$R_{3,\mu}(t, t_{\rm snk}, \vec{k}) = \frac{C_{3,\mu}(t, t_{\rm snk}, \vec{k})}{\sqrt{C_2^P(t,\vec{k})C_2^{B_{(s)}}(t_{\rm snk}-t,\vec{0})}}\sqrt{\frac{2E}{e^{-Et-M(t_{\rm snk}-t)}}}, \tag{9}$$

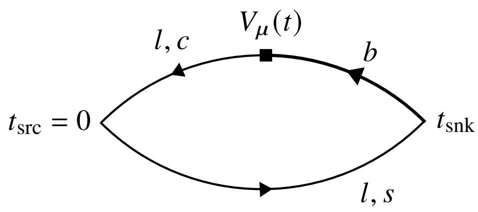

Figure 1: Three-point correlator used in form-factor determinations.

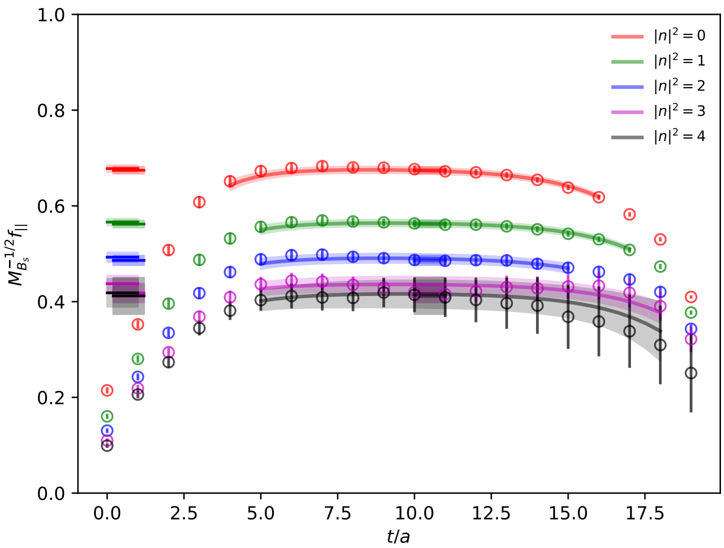

Figure 2: Extraction of $f_\parallel$ for $B_s \to K$ on the coarse ensemble C1 from the ratio $R_{3,0}$. The different colors denote different three-momenta $2\pi\vec{n}/L$ injected at the current, labelled by n^2. The plot shows a ground-state-only fit together with a fit over an extended range of times for each momentum once excited state terms are included for the current matrix elements in the numerator of $R_{3,0}$. The horizontal bars near the left axis show the values for $f_\parallel$ from the ground-only and from the excited-state fits.

where $C_2^{P,B_{(s)}}$ are two-point correlators and $C_{3,\mu}$ is the three-point correlator shown schematically in figure 1. For large time separations between source, sink and current insertion, we obtain

$$f_\parallel^{\rm bare}(\vec{k}) = \lim_{0 \ll t \ll t_{\rm snk}} R_{3,0}(t, t_{\rm snk}, \vec{k}), \qquad f_\perp^{\rm bare}(\vec{k}) = \lim_{0 \ll t \ll t_{\rm snk}} \frac{1}{p_P^i} R_{3,i}(t, t_{\rm snk}, \vec{k}). \tag{10}$$

Figure 2 illustrates the determination of $f_\parallel$ for $B_s \to K$ on the coarse, C1, ensemble.

For $B_s \to K$ and $B \to \pi$ we extrapolate the renormalized lattice form factors to vanishing lattice spacing and to the physical light-quark mass, and interpolate in the kaon(pion) energy, using next-to-leading order SU(2) heavy-meson chiral perturbation theory (HMχPT) in the "hard-kaon(pion)"

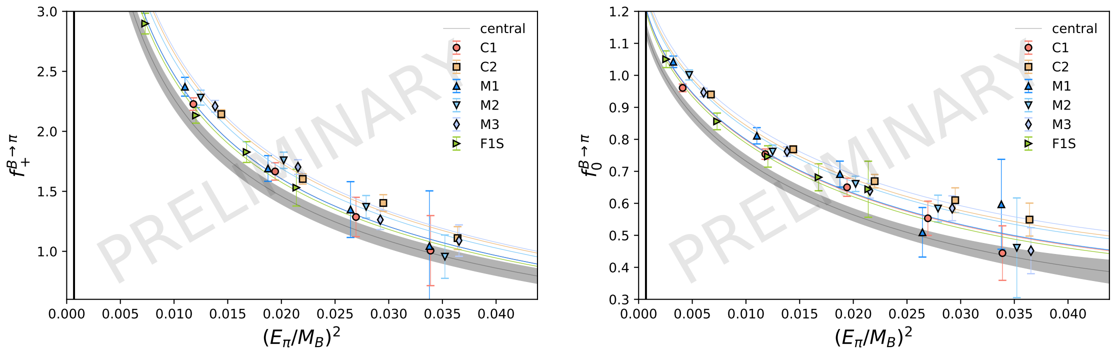

Figure 3: Chiral-continuum extrapolation for the $B \to \pi$ form factors f_+ (left) and f_0 (right). The colored data points show the underlying data. The colored lines show the result of the fit evaluated at the parameters of the respective ensembles. The grey bands display the form factors in the chiral-continuum limit and the associated statistical uncertainty.

limit [27–29]. The function we use, with P denoting kaon or pion, is

$$f^{B_{(s)}\to P}(M_\pi, E_P, a^2) = \frac{\Lambda}{E_P + \Delta}\left[c_0\left(1 + \frac{\delta f(M_\pi^{\text{sea}}) - \delta f(M_\pi^{\text{phys}})}{(4\pi f_\pi)^2}\right) + c_1\frac{\Delta M_\pi^2}{\Lambda^2} + c_2\frac{E_P}{\Lambda} + c_3\frac{E_P^2}{\Lambda^2} + c_4(a\Lambda)^2\right], \quad (11)$$

where M_π^{sea} is the simulated pion mass on a given ensemble, M_π^{phys} is the physical pion mass, $\Delta M_\pi^2 = (M_\pi^{\text{sea}})^2 - (M_\pi^{\text{phys}})^2$ and $\Lambda = 1\,\text{GeV}$ is the renormalization scale appearing in the one-loop chiral logarithm in δf, and is also used as a dimensionful scale to render the fit coefficients dimensionless. $\Delta = M_{B^*} - M_{B_{(s)}}$ and the B^* is a $\bar{b}u$ flavor state with $J^P = 1^-$ for f_+, or $J^P = 0^+$ for f_0. For f_+ this is the vector meson B^* with mass $M_{B^*} = 5.32470(22)\,\text{GeV}$ [30], while for f_0 there is a theoretical estimate for the 0^+ state, $M_{B^*(0^+)} = 5.63\,\text{GeV}$ [31]. The term δf also contains an estimate for finite volume effects. Figure 3 shows the fit for $B \to \pi$ and Figure 4 shows the fit for $B_s \to K$.

For $B_s \to D_s$ form factors, we combine a chiral-continuum fit with an extra-/inter-polation in the charm mass with a fit form

$$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + \sum_{j=1}^{n_{D_s}} c_{1j}\, h\left(\frac{M_{D_s}}{\Lambda}\right)^j + c_2(a\Lambda)^2\right] P_{a,b}(q^2/M_{B_s}^2), \quad (12)$$

where

$$h\left(\frac{M_{D_s}}{\Lambda}\right) = \frac{M_{D_s}}{\Lambda} - \frac{M_{D_s}^{\text{phys}}}{\Lambda} \quad \text{and} \quad P_{a,b}(x) = \frac{1 + \sum_{i=1}^{a} a_i x^i}{1 + \sum_{i=1}^{b} b_i x^i}. \quad (13)$$

Figure 5 shows the fit. We use only one of the charm masses for F1S in this fit because of the strong correlations between results for the two charm masses on that ensemble.

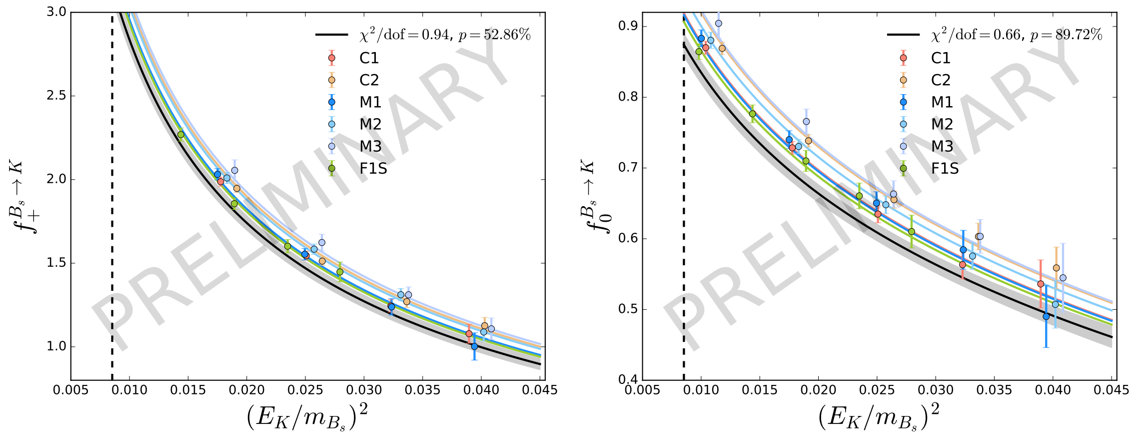

Figure 4: Chiral-continuum extrapolation for the $B_s \to K$ form factors f_+ (left) and f_0 (right). The colored data points show the underlying data. The colored lines show the result of the fit evaluated at the parameters of the respective ensembles. The grey bands display the form factors in the chiral-continuum limit and the associated statistical uncertainty.

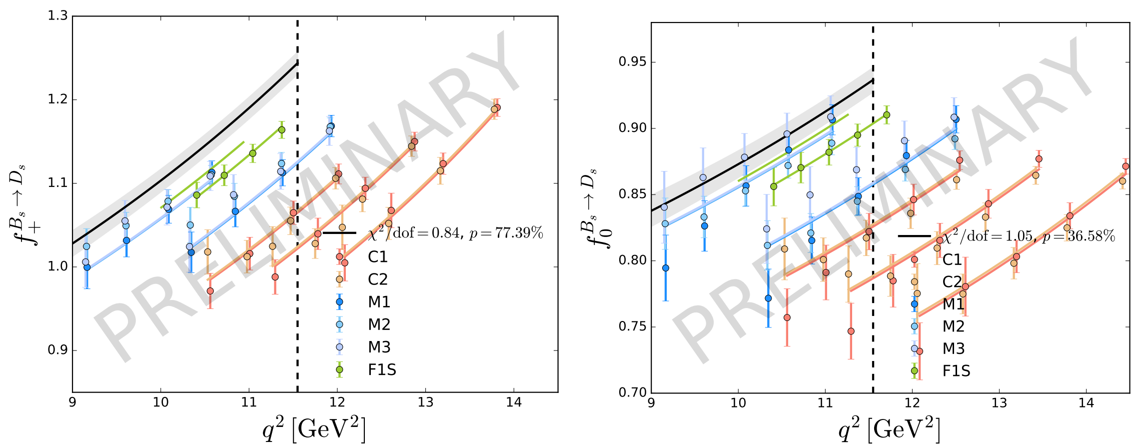

Figure 5: Chiral-continuum extrapolation for the $B_s \to D_s$ form factors.

3. z-fits

After extrapolating our results to the continuum and physical masses, our strategy is to generate synthetic data points for the form factors, with all errors included, which can then be used in standard z-fits to extrapolate over the full q^2 range for the physical form factors. In figure 6, we illustrate the cumulative statistical plus systematic error budgets for the f_+ form factor for $B_s \to K$ and $B_s \to D_s$ decays. Figure 7 shows results for z-fits for $B \to \pi$ and $B_s \to K$ form factors. These are Bourrely-Caprini-Lellouch (BCL) fits [32], where we have included the 1^- B^* vector meson pole for fitting f_+ and no pole for f_0 in both cases.

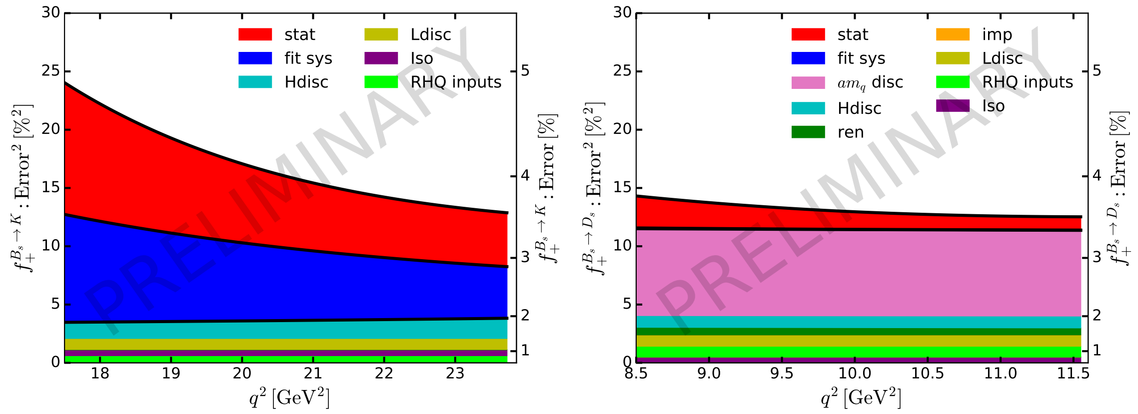

Figure 6: Cumulative error budgets for statistical and systematic errors for f_+ for $B_s \to K$ and $B_s \to D_s$. The q^2 ranges of the plots correspond to the ranges in which we generate synthetic data points for subsequent z-fits. The plots are for squared percentage errors, but an additional scale on the right shows the corresponding percentage error. The legends label those errors visible on the plots, but other error sources with sub-percent effects were considered. For $B_s \to K$, statistical errors (red) and systematic errors from the chiral-continuum extrapolation (blue) dominate. For $B_s \to D_s$, the dominant error (pink) is from discretization for the charm quark.

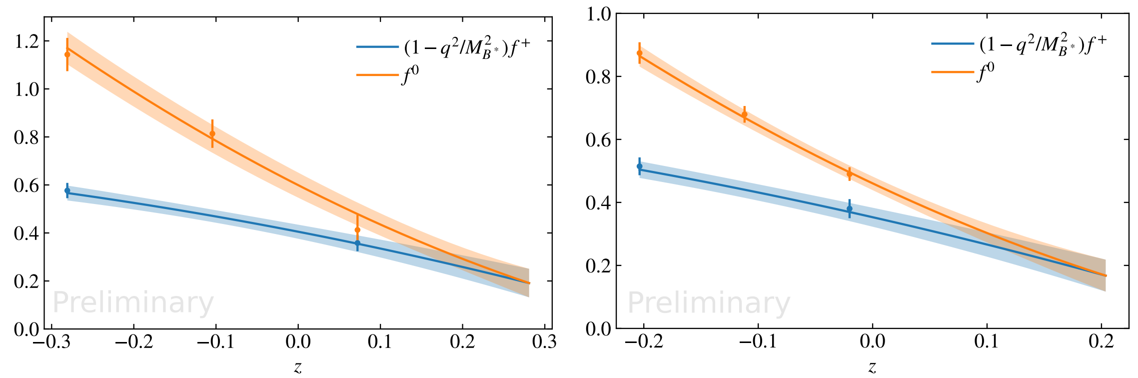

Figure 7: BCL z-fits for $B \to \pi$ (left) and $B_s \to K$ (right) form factors.

4. Ratios for testing lepton flavor universality

Ratios of decay rates or partially integrated decay rates with tau leptons in the final state to those with light leptons in the final state are of great interest in looking for violations of the universality of lepton couplings in the Standard Model. For the semileptonic decays considered here, such a ratio is $R(P)$ given by

$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}}, \tag{14}$$

where ℓ in the denominator can be μ or e. We have considered modifying this ratio to look for a sharper test of lepton flavor universality (see discussion in [33] on optimising observables). In particular, we try to reduce the uncertainty in the ratio coming from uncertainties in the form factors taken from our lattice simulations. To this end, we consider the modified ratio

$$R^{\mathrm{new}}(P) = \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2 \frac{d\Gamma(B_{(s)}\to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2 \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)}\to P\ell\bar{\nu}_\ell)}{dq^2}}, \tag{15}$$

following the recipe already applied to $B_{(s)} \to V$ decays (with a vector meson instead of a pseudoscalar meson in the final state) by Isidori and Sumensari [34]. The ingredients are

- Use a common integration range for numerator and denominator, with $q^2_{\mathrm{min}} \geq m_\tau^2$ [35–37].
- Reweight the integrand in the denominator to make the contributions from the vector form factor the same in numerator and denominator.

Write the differential decay rate from equation (1) in the form

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \Phi(q^2)\omega_\ell(q^2)\left[F_V^2 + (F_S^\ell)^2\right], \tag{16}$$

where now ℓ can be any lepton flavor, with

$$\Phi(q^2) = \eta \frac{G_F^2 |V_{xb}|^2}{24\pi^3} |\vec{k}|, \tag{17}$$

$$\omega_\ell(q^2) = \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left(1 + \frac{m_\ell^2}{2q^2}\right), \tag{18}$$

$$F_V^2 = \vec{k}^2 |f_+(q^2)|^2, \tag{19}$$

$$(F_S^\ell)^2 = \frac{3}{4} \frac{m_\ell^2}{m_\ell^2 + 2q^2} \frac{(M^2 - m^2)^2}{M^2} |f_0(q^2)|^2. \tag{20}$$

If we drop the scalar contribution, $(F_S^\ell)^2$, in the denominator, with $\ell = \mu$ or e again, then relying on $m_\ell^2/2q^2 \leq m_\mu^2/2m_\tau^2 = 0.002$ for the light leptons, we expect in the Standard Model,

$$R^{\mathrm{new,SM}}(P) = 1 + \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2\, \Phi(q^2)\omega_\tau(q^2)(F_S^\tau)^2}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2\, \Phi(q^2)\omega_\tau(q^2)F_V^2}. \tag{21}$$

Evaluating the new ratio from equation (15) using the z-fit for our lattice form factors, we find a reduced uncertainty compared to evaluating the original ratio in equation (14). It would be interesting to see how the ratios compare if evaluated using experimental differential decay rates.

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