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Higgs and super-Higgs effects with naturally vanishing vacuum energy [1]

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Abstract

We construct $N = 1$ supergravity models where the gauge symmetry and supersymmetry are both spontaneously broken, with naturally vanishing classical vacuum energy and unsuppressed Goldstino components along gauge non-singlet directions. We discuss some physically interesting situations where such a mechanism could play a role, and identify the breaking of a grand-unified gauge group as the most likely possibility. We show that, even when the gravitino mass is much smaller than the scale m_X of gauge symmetry breaking, important features can be missed if we first naively integrate out the degrees of freedom of mass $\mathcal{O}(m_X)$, in the limit of unbroken supersymmetry, and then describe the super-Higgs effect in the resulting effective theory. We also comment on possible connections with extended supergravities and realistic four-dimensional string constructions.

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1. If space-time supersymmetry plays a role in the unification of all fundamental interactions (for a review and references, see e.g. [1]), the major obstacle to the construction of a predictive theory beyond the Standard Model is the problem of supersymmetry breaking. Whilst useful theoretical tools can be developed by studying models with global supersymmetry, the only realistic framework for the discussion of such a problem is $N = 1$ supergravity, regarded as the low-energy limit of a consistent quantum theory including gravity.

In supergravity, gravitational interactions are *always* relevant in the discussion of the super-Higgs phenomenon, and we must face the highly non-trivial requirement of a sufficiently small cosmological constant. In this respect, promising starting points are the $N = 1$ supergravity models characterized by a positive–semi-definite classical potential, with all minima corresponding to broken supersymmetry and vanishing vacuum energy, and the gravitino mass sliding along some flat direction [2,3].

Along this line of thought, attention has mainly concentrated on the case in which both the Goldstino field and the flat directions are singlets under the full gauge group. Only recently, the possibility was considered of breaking supersymmetry and $SU(2) \times U(1)$ at once, with naturally vanishing vacuum energy [4]: an explicit model of this kind was produced, but the gauge and Yukawa interactions of the Goldstino were suppressed down to gravitational strength, $\mathcal{O}(m_{3/2}/M_{\rm P})$, by mixing effects involving some singlet moduli fields.

In this paper, we examine the possibility of breaking the gauge symmetry together with supersymmetry, with a naturally vanishing classical vacuum energy and unsuppressed Goldstino components along gauge non-singlet directions. In section 2, we present a toy model that provides an existence proof for this possibility and allows a number of issues of general relevance to be discussed in a simplified setting. In section 3, we discuss how our results could be extended to more realistic situations: the breaking of the electroweak symmetry, of a grand-unified symmetry, or of a gauge symmetry of a strongly interacting hidden sector. In section 4, we conclude with some comments on the possible connections with extended supergravities and four-dimensional string models. To improve the readability of the text, we have collected some useful formulae in an Appendix.

2. Consider an $N = 1$ supergravity model containing three chiral superfields, whose complex spin 0 components parametrize the Kähler manifold:

$$\left[\frac{SU(1,1)}{U(1)}\right]_S \times \left[\frac{SO(2,2)}{SO(2)\times SO(2)}\right]_{T,U} \simeq \left[\frac{SU(1,1)}{U(1)}\right]^3_{S,T,U} . \qquad (1)$$

The Kähler potential can be conveniently written as[1]

$$K = -\log Y , \qquad (2)$$

[1]Unless otherwise stated, we use the standard supergravity conventions where $M_{\rm P} \equiv 1/\sqrt{8\pi G_N} = 1$.

where, using one of the parametrizations of $SO(2,2)/[SO(2)\times SO(2)] \simeq [SU(1,1)/U(1)]^2$ discussed in the Appendix,

$$Y = \left(S+\overline{S}\right)\left(T+\overline{T}\right)\left(U+\overline{U}\right) . \tag{3}$$

This parametrization has the advantage that a constant superpotential, $w = k \neq 0$ (where it is not restrictive to choose k real and positive), gives an identically vanishing classical potential with a non-vanishing gravitino mass

$$e^{\mathcal{G}} = \frac{k^2}{\left(S+\overline{S}\right)\left(T+\overline{T}\right)\left(U+\overline{U}\right)} . \tag{4}$$

On the other hand, if one sticks to this parametrization one cannot introduce any gauge symmetry acting linearly but non-trivially on the fields.

As discussed in the Appendix, one can move to an alternative parametrization,

$$Y = \left(S+\overline{S}\right)\left(1-|H_1|^2\right)\left(1-|H_2|^2\right) . \tag{5}$$

The constant superpotential $w = k$ would now become

$$w = \frac{k}{2}(1+H_1)(1+H_2) . \tag{6}$$

The Kähler potential corresponding to eq. (5) is invariant under two continuous $U(1)$ groups, whose generators will be denoted by X_1 and X_2, acting linearly but non-trivially on the fields H_1 and H_2. One could think of gauging some non-anomalous combination of them, but such an attempt must face the fact that the superpotential of eq. (6) would explicitly break gauge-invariance.

A possible way out is to replace the superpotential of eq. (6) by

$$w = k\left(1+\sqrt{H_1 H_2}\right)^2 . \tag{7}$$

The ambiguity in the relative phase between the two terms within brackets can be removed by a phase redefinition of the $H_{1,2}$ fields. The model defined by eqs. (5) and (7) admits the gauge group $G_0 = U(1)_X$ if one assigns to (S, H_1, H_2) the (arbitrarily normalized) charges, corresponding to the combination $X \equiv X_1 - X_2$:

$$X(S) = 0 , \qquad X(H_1) = -1/2 , \qquad X(H_2) = +1/2 . \tag{8}$$

We fix the arbitrariness in the choice of the gauge kinetic function by taking, for the time being, $f = S$ (alternative choices will be discussed at the end of this section). Then a well-behaved gauge coupling and Kähler metric require $s \equiv S+\overline{S} > 0$ and either $|H_1|, |H_2| < 1$ or $|H_1|, |H_2| > 1$. Moreover, analyticity of the superpotential excludes from the acceptable field configurations the lines $H_1 = 0$ and $H_2 = 0$. The continuous $[SU(1,1)]^3$ symmetry of the Kähler manifold is explicitly broken by the superpotential w and by the gauge kinetic function f, with the exception of the $U(1)_X$ gauge symmetry. It is also interesting to notice that the discrete transformations $(H_1 \to 1/H_1, H_2 \to 1/H_2)$ and $(H_1 \to H_2, H_2 \to H_1)$, which are not contained in $[SU(1,1)]^3$, are also symmetries of the model.

The full classical potential for our model reads $V_0 = V_F + V_D$, where

$$V_F = \frac{k^2}{s} \frac{(|H_1| - |H_2|)^2 (1 + |H_1||H_2|)|1 + \sqrt{H_1 H_2}|^2}{|H_1||H_2|(1 - |H_1|^2)(1 - |H_2|^2)} \, , \tag{9}$$

$$V_D = \frac{1}{4s} \frac{(|H_1|^2 - |H_2|^2)^2}{(1 - |H_1|^2)^2 (1 - |H_2|^2)^2} \, . \tag{10}$$

It is easy to see that V_0 is positive semi-definite, and admits a continuum of degenerate minima with broken gauge symmetry, broken supersymmetry and vanishing vacuum energy, corresponding to arbitrary $|H_1| = |H_2|$ and S. It may be useful to reinterpret these flat directions in terms of continuous symmetries of the classical potential and of its minimization conditions. The only continuous symmetry of V_0, besides the gauged $U(1)_X$, is the non-compact $U(1)$ corresponding to imaginary translations of the S field [the $U(1)_{\hat{X}}$ associated with $\hat{X} \equiv X_1 + X_2$ is an invariance of V_D but not of V_F]. However, the minimization conditions $V_F = V_D = 0$ defining the classical vacua inherit as symmetries the full $[SU(1,1)]_S$, and common phase rotations and rescalings of H_1 and H_2, corresponding to the complexification of $U(1)_{\hat{X}}$: we then expect four massless real spin-0 degrees of freedom, besides the would-be Goldstone boson of the broken $U(1)_X$.

In order to examine the classical moduli space of our theory, we recall that there are in principle three independent gauge-invariant VEVs, $|H_1|$, $|H_2|$ and $\theta \equiv \arg(H_1 H_2)$. The minimization condition $V_0 = 0$ requires $|H_1| = |H_2| \equiv h$, so we need h and θ to label the physically inequivalent vacua (apart from the residual redundancy due to the unbroken discrete symmetries).

The order parameters for gauge and supersymmetry breaking are the vector boson mass

$$m_X^2 = \frac{2h^2}{s(1 - h^2)^2} \, , \tag{11}$$

and the gravitino mass

$$m_{3/2}^2 = \frac{k^2 |1 + e^{i\theta/2} h|^4}{s(1 - h^2)^2} \, , \tag{12}$$

respectively. In the spin-0 sector, the only massive state corresponds to $\mathrm{Re}\,[(H_1 - H_2) e^{-i\theta/2}]$, with mass

$$m_0^2 = m_X^2 + m_{3/2}^2 \frac{2(1 + h^2)(1 - h^2)^2}{h^2 |1 + h e^{i\theta/2}|^2} \, . \tag{13}$$

In the spin-1/2 sector, a 4×4 mass matrix describes the mixing of the fields $\tilde{S}$, $\tilde{H}_1$, $\tilde{H}_2$ and $\tilde{X}$ (gaugino), which here are understood to be already canonically normalized. It is particularly convenient to introduce the symmetric and antisymmetric higgsino combinations, $\tilde{H}_S \equiv (\tilde{H}_1 + \tilde{H}_2)/\sqrt{2}$ and $\tilde{H}_A \equiv (\tilde{H}_1 - \tilde{H}_2)/\sqrt{2}$, because the 4×4 fermionic mass matrix can then be decomposed into two 2×2 blocks, one for $(\tilde{S}, \tilde{H}_S)$ and the other for

$(\tilde{H}_A, \tilde{X})$. The first block has a vanishing eigenvalue corresponding to the Goldstino and another eigenvalue $m_1^2 = m_{3/2}^2$. The (canonically normalized) Goldstino can be written as $\tilde{\eta} = (\tilde{S} + \sqrt{2} e^{i\phi} \tilde{H}_S)/\sqrt{3}$, where $e^{i\phi} = (1 + e^{i\theta/2} h)/(1 + e^{-i\theta/2} h)$. Notice that, since the Goldstino component along $\tilde{H}_S$ is unsuppressed, we obtain a gravitino with interactions of gauge strength via its $\pm 1/2$ helicity components [5]. The second block has eigenvalues

$$m_{2,3}^2 = m_X^2 + m_{3/2}^2 \left[1 + \frac{(1+h^2)(1-h^2)^2}{2h^2 |1 + h e^{i\theta/2}|^2} \right] \pm \frac{1+h^2}{h} m_{3/2} \sqrt{m_{3/2}^2 \frac{(1-h^2)^4}{4h^2 |1 + h e^{i\theta/2}|^4} + m_X^2} \, . \tag{14}$$

Observe that, in the model under consideration,

$$\mathrm{Str}\, \mathcal{M}^2(h, \theta, S) \equiv \sum_i (-1)^{2J_i} (2J_i + 1) m_i^2(h, \theta, S) = -10 m_{3/2}^2 \, , \tag{15}$$

where the only dependence on the variables h, θ and S is the implicit one through the gravitino mass. Such a property is phenomenologically welcome, since it may allow for a natural cancellation of the quadratically divergent quantum corrections to the vacuum energy from other sectors of the theory [3]. For example, we could add n chiral superfields z, with canonical kinetic terms and superpotential at least quadratic in z: in this case we would obtain, around the minima with $z = 0$, an additional contribution $\Delta\, \mathrm{Str}\, \mathcal{M}^2 = 2\, n\, m_{3/2}^2$.

We would like to stress that, as expected, the mass spectrum is invariant under the discrete transformation $h \to 1/h, \theta \to -\theta$, so it will not be restrictive to study it for $0 < h < 1$.

Some interesting limits of our model are $h \to 0$ (equivalent to $h \to \infty$) and $h \to 1$ (with θ, k and s fixed).

For $h \to 0$ we obtain $m_X^2 \to 0$, $m_{3/2}^2 \to k^2/s$, i.e. unbroken gauge symmetry with broken supersymmetry[2]. Observe that, in this limit, $m_0^2 \to 2 m_{3/2}^2/h^2 + \ldots$, $m_2^2 \to m_{3/2}^2/h^2 + \ldots$, where the dots stand for terms that do not diverge in the limit. Reintroducing explicitly the Planck mass for clarity, for $h \ll \sqrt{m_{3/2} M_{\mathrm{P}}}$ there are supersymmetry-breaking mass splittings Δm^2 much larger than $m_{3/2}^2$, with couplings of order $\Delta m^2/(m_{3/2} M_P) \sim m_{3/2} M_P / h^2 \gg 1$, and we end up with a strongly interacting Goldstino.

For any fixed $\theta \neq 2\pi$, the limit $h \to 1$ corresponds (formally) to maximally broken gauge symmetry and supersymmetry, i.e. $m_X^2, m_{3/2}^2 \to \infty$. This is not the case for the special value $\theta = 2\pi$, for which $h \to 1$ corresponds to $m_X^2 \to \infty$ but $m_{3/2}^2 \to 0$.

It may be useful to rephrase the previous results in the alternative (T, U) parametrization: the classical vacua correspond to $T = U$, and the singular points to $T = U = 1$ and $T + \overline{T} = U + \overline{U} = 0, \infty$.

Another physically interesting limit is the case in which $m_{3/2} \ll m_X$ (which is realized, for example, for $k \ll 1$ and h generic). In this case one can write down a low-energy

[2] Notice that the superpotential w, restricted to the classical moduli space, is singular at $h = 0$, with monodromy $(H_1 \to -H_1, H_2 \to -H_2)$, corresponding to $(T \to 1/T, U \to 1/U)$ in the alternative parametrization.

effective field theory for the light modes. Such a theory has no residual gauge symmetry, and its chiral supermultiplet content consists only of S and $H_S \equiv (H_1 + H_2)/\sqrt{2}$. After an innocuous rescaling $H_S \to \sqrt{2} H_S$, its Kähler potential and superpotential are given by

$$Y = (S + \overline{S})(1 - |H_S|^2)^2 \, , \qquad w = k(1 + H_S)^2 \, , \tag{16}$$

and give an identically vanishing classical potential. Notice that this effective theory cannot correctly reproduce the singular behaviour of the full theory for $h \to 0$: as trivial as it sounds, this may be interpreted as a warning for the discussion of modular covariant superpotentials in superstrings effective supergravities. Notice also that, when $m_{3/2} \ll m_X$, in the effective theory below the scale m_X we would find Str $\mathcal{M}^2 = -6 m_{3/2}^2$, which differs from eq. (15): this is just reminding us that Str $\mathcal{M}^2$ is a physically meaningful object, in relation with the stability of the flat background and of possible gauge hierarchies, only when computed over all states of the fundamental theory that get supersymmetry-breaking mass splittings.

One could also consider more complicated limits involving combinations of k, s, h and θ, but we shall not pursue this type of considerations further.

Before leaving our toy model for the discussion of more realistic situations, we would like to comment on some possible variants. One may ask if there are forms of the gauge kinetic function f, more general than $f = S$, that respect gauge invariance and allow for Str $\mathcal{M}^2 = (\text{constant}) m_{3/2}^2$. On the vacua with $\theta = 0$ and S real, a class of functions satisfying this requirement is

$$f = \left(S \frac{1 - \sqrt{H_1 H_2}}{1 + \sqrt{H_1 H_2}} \right)^{-c/2} \cdot \varphi \left(S \frac{1 + \sqrt{H_1 H_2}}{1 - \sqrt{H_1 H_2}} \right) \, , \tag{17}$$

where c is an arbitrary real constant and $\varphi(z)$ is an arbitrary holomorphic function. The original choice $f = S$ is recovered for $c = -1$ and $\varphi(z) = \sqrt{z}$. For the general gauge kinetic function of eq. (17), the supertrace formula of eq. (15) becomes

$$\text{Str} \, \mathcal{M}^2(h, \theta = 0, S) = -2(4 + c^2) m_{3/2}^2 \, . \tag{18}$$

As a curiosity, observe that, choosing $\varphi(z) = z^{c/2}$, we get $f = [(1 + \sqrt{H_1 H_2})/(1 - \sqrt{H_1 H_2})]^c$. The transformation $(H_1 \to -H_1, H_2 \to -H_2)$, associated with the monodromy of w around $h = 0$, would correspond in this case to a weak/strong coupling duality $f \to 1/f$.

Another possibility is to look for different gaugings of the sigma model under consideration. For example, one could make the additional field redefinition $S = (1 - z)/(1 + z)$, and introduce the superpotential $w = k[1 + (z H_1 H_2)^{1/3}]^3$. This would allow two independent $U(1)$ factors to be gauged, producing a positive–semi-definite potential, broken supersymmetry at all classical vacua, and less flat directions than in the model defined by eqs. (5) and (7). As a candidate form for the gauge kinetic function f_{ab} $(a, b = 1, 2)$, it is interesting to consider in this case $f_{ab} = k_a \delta_{ab} \{[1 + (z H_1 H_2)^{1/3}]/[1 - (z H_1 H_2)^{1/3}]\}^r$, which gives, on the vacua with $z = H_1 = H_2 \in \mathbf{R}^+$, a gaugino mass $m_{1/2} = r \, m_{3/2}$, and has also interesting properties with respect to weak/strong coupling duality.

Yet another variant would consist in removing the S field (either explicitly or by introducing a superpotential that gives a VEV to its scalar component without giving a VEV to its auxiliary component), and in assigning to the fields (H_1, H_2) the Kähler potential $K = -(3/2)\log[(1-|H_1|^2)(1-|H_2|^2)]$ and the superpotential $w = k(1+\sqrt{H_1 H_2})^3$. Choosing $f = L[(1+\sqrt{H_1 H_2})/(1-\sqrt{H_1 H_2})]^c$, with L arbitrary constant and $c \in \mathbf{R}$, would give a gaugino mass $m_{1/2} = c\, m_{3/2}$ at all minima with $H_1 = H_2 \in \mathbf{R}$; the choice $c = \pm 1$ and $L \in \mathbf{R}$ would guarantee $m_{1/2}^2 = m_{3/2}^2$ at all minima, corresponding to $|H_1| = |H_2|$, but would break the discrete invariance under $(H_1 \to 1/H_1, H_2 \to 1/H_2)$.

3. Supergravity models of the type considered in the previous section, with gauge symmetry and $N = 1$ supersymmetry both spontaneously broken, and naturally vanishing classical vacuum energy, can be obtained by the following procedure. First, one selects a Kähler manifold for the symmetry-breaking sector. For Kähler manifolds of the type G/H, where H is the maximal compact subgroup of G, one chooses the gauge group G_0 as a subgroup of H (this can be obviously generalized to a factorized manifold of the type $G/H \times M$, where M is a sub-manifold parametrized by some gauge-singlet fields). To ensure manifest gauge-invariance, it is convenient to work in a parametrization of G/H where H is linearly realized. For example, in the case where $G = SU(m,n)$ and $H = SU(m) \times SU(n) \times U(1)$, the scalar fields can be described by an $m \times n$ complex matrix Z, with the Kähler potential for G/H given by [6]

$$K = -\log \det \left(1 - Z Z^\dagger\right) , \tag{19}$$

where 1 denotes the unit $m \times m$ matrix; K is manifestly invariant under the transformations

$$Z' = e^{i\alpha} U Z V^\dagger , \tag{20}$$

where α is a real parameter, and U and V are $SU(m)$ and $SU(n)$ matrices, respectively. Another important example, which appears in the effective supergravity theories of many four-dimensional string constructions, corresponds to $G = SO(2,n)$ and $H = SO(2) \times SO(n)$. The scalar fields are described by the n-dimensional complex vector y. The Kähler potential reads [8]

$$K = -\log \det \left(1 - 2 y^\dagger y + |y^T y|^2\right) , \tag{21}$$

and is manifestly invariant under the transformations

$$y' = e^{i\alpha} O y , \tag{22}$$

where α is a real parameter and O is an $SO(n)$ matrix. In the parametrizations specified by eqs. (19) and (21), the full H subgroup of G is linearly realized and the Kähler potential is strictly gauge-invariant. One then looks for a gauge-invariant superpotential w that breaks simultaneously supersymmetry and the gauge symmetry with naturally vanishing vacuum energy. Needless to say, additional physical criteria can be used to constrain the possible forms of the superpotential: we have in mind, for example, generalized duality symmetries and singularity structure of strings effective supergravities (for a review and references see e.g. [7]). One can then couple additional sectors of the theory, which do not take part in the symmetry breaking mechanism, by specifying their contributions to the Kähler potential and to the superpotential.

We now discuss some physically relevant situations where the general mechanism discussed above may be at work.

The first possibility that comes to mind is to associate the breaking of supersymmetry with the breaking of the electroweak gauge symmetry, $SU(2)_L \times U(1)_Y$, in the Minimal Supersymmetric extension of the Standard Model (MSSM) coupled to $N=1$ supergravity. Since the MSSM Higgs sector contains the two doublets $H_1 \equiv (H_1^0, H_1^-)$ and $H_2 \equiv (H_2^+, H_2^0)$, a natural choice is to consider the Kähler manifold

$$\frac{SO(2,4)}{SO(2)\times SO(4)} \simeq \frac{SU(2,2)}{SU(2)\times SU(2)\times U(1)}\,, \tag{23}$$

parametrized by the 2×2 complex matrix[3]

$$Z \equiv \left(\begin{array}{cc} H_1^0 & H_2^+ \\ H_1^- & H_2^0 \end{array} \right) \,, \tag{24}$$

with the Kähler potential of eq. (19). To recover the usual $SU(2)_L \times U(1)_Y$ gauge transformations of the doublets H_1 and H_2, with parameters α_A ($A=1,2,3$) and α_Y, one must consider eq. (20) with $\alpha=0$, $U=\exp(i\alpha_A\tau^A/2)$, and $V=\exp(i\alpha_Y\tau^3/2)$. Inspired by the structure of string effective supergravities and by the analogy with our toy model, we also introduce a singlet field S, parametrizing a factorized $SU(1,1)/U(1)$ manifold; we assume a gauge-invariant superpotential of the form

$$w_0 = k\left(1+\sqrt{\det Z}\right)^2 \,, \tag{25}$$

which represents the obvious generalization of the one of eq. (7). This leads to a positive–semi-definite tree-level potential, identically vanishing for arbitrary s and

$$Z = h e^{i\theta/2} A\,. \tag{26}$$

In eq. (26), h and θ are arbitrary real numbers, and A is an arbitrary $SU(2)$ matrix, which can be reabsorbed by an $SU(2)_L \times U(1)_Y$ gauge transformation. Thus the classical moduli space in the Z sector, describing the broken phase in which only $U(1)_{em}$ survives, can be parametrized in terms of h and θ. With the choice $f_{ab}=\delta_{ab}S$ ($g^2=g'^2=2/s$), the spectrum is an obvious generalization of the toy-model one. Notice that, for $\mathcal{O}(1)$ gauge couplings, to obtain $m_{W,Z}/M_{\rm P} \sim 10^{-16}$ one must choose $h/M_{\rm P} \sim 10^{-16}$,

[3]An analogous description, in the absence of gravity, can be found in [9].

leading to $m_{3/2}/M_{\rm P} \sim k/\sqrt{s}$. The tree-level supersymmetry-breaking mass splittings in the gauge-Higgs sector are either vanishing or of order $\Delta m^2 \sim m_{3/2}^2 M_{\rm P}^2/h^2$. Thus for $m_{3/2} \sim h^2/M_{\rm P} \sim 10^{-4}$ eV the non-vanishing splittings are of order h and the Goldstino couplings are of order unity, whereas for $m_{3/2} \sim h$ one gets non-vanishing splittings of order $M_{\rm P}$ and Goldstino couplings outside the perturbative regime.

To complete the model, one should also specify the Kähler potential and the superpotential involving the quark and lepton superfields z. If $m_{3/2} \sim h^2/M_{\rm P}$, one has to face the same problem as in the models with spontaneously broken global supersymmetry: one typically obtains at least one squark of charge 1/3 lighter than the corresponding quark [10], which is excluded by the present experimental bounds. If $m_{3/2} \sim h$, one can obtain an acceptable spectrum of squarks and sleptons, for example choosing canonical kinetic terms and a superpotential

$$w = w_0 \left(1 + \frac{h^U Q U^c H_2 + h^D Q D^c H_1 + h^E L E^c H_1}{\sqrt{\det Z}} \right) , \tag{27}$$

where w_0 is the superpotential of eq. (25). However, the presence of huge mass splittings of order $M_{\rm P}$ in the gauge-Higgs sector, associated with non-perturbative Goldstino couplings, does not allow us to control the quantum corrections. If we naively compute the one-loop corrections to the effective potential, imagining $m_{3/2}$ fixed and considering only the leading h-dependence, we find that the classical degeneracy is removed to give $h \sim m_{3/2}$ at the one-loop minimum, but we cannot trust this result in the absence of tools to control higher order and non-perturbative effects.

In summary, the structure discussed for the toy model does not seem suitable for a direct application to $SU(2)_L \times U(1)_Y$ breaking. For a more satisfactory description of the latter, one may be forced to introduce some extra G_{SM}-singlets as in ref. [4].

A second, more intriguing possibility is to associate the breaking of supersymmetry with the breaking of a grand-unified gauge group[4] G_U down to the MSSM gauge group $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. Various realizations could be possible, depending on the choice of G_U and of the Kähler manifold for the Higgs sector. We do not commit ourselves here to any specific example, but we just use the toy model as a guideline for a qualitative discussion. Given the approximate phenomenological relation $M_U \sim g_U M_{\rm P}$, also suggested by four-dimensional string models, we need h to be of order $M_{\rm P}$. Assuming as before $f = S$, supersymmetry-breaking mass splittings will then be of order $m_{3/2}$, signalling a Goldstino with interactions of gravitational strength if we take $m_{3/2}$ at the electroweak scale as usual. In this case a perturbative study of the dynamical determination of M_U and $m_{3/2}$ could be possible, and one may also find applications to the doublet-triplet splitting problem.

The previous list does not exhaust the physically interesting possibilities. For example, one may imagine a strongly interacting hidden sector where non-perturbative phenomena

[4]The combined breaking of supersymmetry and of a grand-unified gauge symmetry was previously considered in [11], but the vanishing of the classical vacuum energy was achieved there by fine-tuning some superpotential parameters.

break supersymmetry as well as the gauge symmetry G_{hid} down to a subgroup H_{hid}.

4. The new class of supergravity models discussed in the previous sections has in our opinion rather intriguing properties (including some formal similarities with recent and less recent results results on non-perturbative phenomena in globally supersymmetric theories [12]), but suffers from two main unsatisfactory aspects. The first is connected with the apparent arbitrariness of the construction: at the level of $N = 1$ supergravity, we are practically free to choose the gauge group, the number of chiral superfields, the Kähler manifold, the embedding of the gauge group in the isometry group of the Kähler manifold, and finally the gauge kinetic function and the superpotential that breaks supersymmetry. The second is connected with the fact that, at the level of $N = 1$ supergravity, we are essentially bound to a classical treatment, given the ambiguities of an effective, non-renormalizable theory in the control of quantum corrections, both perturbative and non-perturbative. One may hope to improve in both directions by establishing some connections with extended $N > 1$ supergravity theories and especially with four-dimensional superstring models.

To obtain a realistic $N = 1$ supergravity model, only the candidate quark and lepton superfields need to transform in chiral representations of the gauge group. It is then conceivable that the sector involved in the Higgs and super-Higgs effects can be obtained, by some suitable projection, from the gauge and gravitational sectors of an extended supergravity model. Indeed, spontaneous supersymmetry breaking with vanishing classical vacuum energy can be associated, in $N = 2$ [13], $N = 4$ [14] and $N = 8$ [15] supergravity, with the gauging of a non-compact subgroup of the duality group. The examples we are aware of give gauge-singlet Goldstinos in the resulting $N = 1$ theory, but one could look for models where the projected $N = 1$ Goldstino transforms non-trivially under the $N = 1$ gauge group: such models would satisfy highly non-trivial constraints, due to the underlying extended supersymmetry.

Further constraints could be obtained by deriving models of the type discussed in this paper as low-energy effective theories of four-dimensional string models with spontaneously broken $N = 1$ supersymmetry. This looks like a natural possibility: we know many examples of singlet moduli appearing in the effective string supergravities that are indeed flat directions breaking an underlying gauge group, restored only at points of extended symmetry. Unfortunately, the only existing examples [16] are those in which supersymmetry is broken at the string tree-level, via coordinate-dependent orbifold compactifications, and correspond to cases where the Goldstino direction is a gauge singlet. It should be possible to extend these constructions to models where the gauge symmetry and supersymmetry are both spontaneously broken. This could lead to some progress in the control of perturbative quantum corrections, since, working at the string level and not in the effective field theory, one can compute the full spectrum of states that contribute to the one-loop partition function.

However, the previous approach looks hopeless as far as the dynamical determination of the dilaton VEV is concerned, since the latter must involve some non-perturbative mechanism. Still, one could use the knowledge of some effective string supergravities in the limit of unbroken supersymmetry, even in the version including infinitely many lattice states [17], and parametrize possible non-perturbative effects with suitable modifications of the superpotential and of the gauge kinetic function, respecting the quantum symmetries of the underlying string theory.

One could take, as a modest but concrete example, one of the $N = 1$ four-dimensional fermionic string constructions [18] that give gauge groups such as $SO(10) \times \ldots$ or flipped $SU(5) \times U(1) \times \ldots$ [19], where the dots stand for some hidden-sector gauge group. In the limit of unbroken $N = 1$ supersymmetry, their classical effective theories are known [20]. One could then look for gauge-invariant superpotential modifications that break the gauge symmetry down to G_{SM} and supersymmetry at the same time, with naturally vanishing vacuum energy.

We hope to return to these problems in a future publication.

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Appendix

We collect here some useful formulae for the Kähler manifold

$$\frac{SO(2,2)}{SO(2)\times SO(2)} \simeq \left[\frac{SU(1,1)}{U(1)}\right]^2 . \tag{28}$$

Its Kähler potential can be written in general as [20]

$$K = -\log Y \,, \qquad Y = 2\left(|a|^2 + |b|^2 - |\varphi_1|^2 - |\varphi_2|^2\right) \,, \tag{29}$$

where $(a, b, \varphi_1, \varphi_2)$ are analytic functions of two unconstrained complex fields, satisfying the requirement

$$a^2 + b^2 - \varphi_1^2 - \varphi_2^2 = 0 \,. \tag{30}$$

One useful parametrization is

$$Y^{(i)} = \left(T + \overline{T}\right)\left(U + \overline{U}\right) \,, \tag{31}$$

corresponding to $a = (1 + UT)/2$, $b = i(U + T)/2$, $\varphi_1 = (1 - UT)/2$, $\varphi_2 = i(U - T)/2$. In the parametrization of eq. (31), each of the two $SU(1,1)$ factors acts as follows:

$$T \longrightarrow \frac{aT - ib}{icT + d} \qquad (ad - bc = 1) \,, \tag{32}$$

and modifies the Kähler potential by a Kähler transformation

$$T + \overline{T} \longrightarrow \frac{T + \overline{T}}{|icT + d|^2} \,. \tag{33}$$

In particular, $Y^{(i)}$ is strictly invariant under the continuous $U(1)$ associated with imaginary translations, $T \to T - ib$. Similar relations can be obtained for the U field. Another continuous invariance of the Kähler potential corresponds to the rescalings

$$T \to \lambda T \,, \qquad U \to \frac{1}{\lambda} U \,. \tag{34}$$

Finally, $Y^{(i)}$ is strictly invariant under some additional discrete transformations that do not belong to $[SU(1,1)]^2$:

$$T \to -T \,, \;\; U \to -U \,, \tag{35}$$

$$T \to U \,, \;\; U \to T \,. \tag{36}$$

In the parametrization of eq. (31), the Kähler potential is well defined in the two domains

$$(T + \overline{T}), (U + \overline{U}) > 0 \,, \qquad (T + \overline{T}), (U + \overline{U}) < 0 \,. \tag{37}$$

A second useful parametrization is

$$Y^{(ii)} = \left(1 - |H_1|^2\right)\left(1 - |H_2|^2\right) \,, \tag{38}$$

corresponding to $a = (1 + H_1H_2)/2$, $b = i(1 - H_1H_2)/2$, $\varphi_1 = -(H_1 + H_2)/2$, $\varphi_2 = i(H_1 - H_2)/2$. Equations (31) and (38) are connected by the field redefinitions $T = (1 - H_1)/(1 + H_1)$ $[H_1 = (1 - T)/(1 + T)]$, and similarly for U and H_2. Notice that the two Kähler potentials are equivalent only up to a Kähler transformation, corresponding to a multiplicative superpotential modification:

$$T + \overline{T} = \frac{1 - |H_1|^2}{\left|\frac{1+H_1}{\sqrt{2}}\right|^2}, \qquad 1 - |H_1|^2 = \frac{T + \overline{T}}{\left|\frac{1+T}{\sqrt{2}}\right|^2}, \tag{39}$$

and similarly for U and H_2. This is consistent with the fact that $(a, b, \varphi_1, \varphi_2)$ are defined only up to a universal multiplicative function of the unconstrained fields. In the parametrization of eq. (38), each of the $SU(1,1)$ factors acts as follows

$$H_1 \longrightarrow \frac{\xi H_1 + \eta}{\overline{\eta} H_1 + \overline{\xi}}, \qquad (|\xi|^2 - |\eta|^2 = 1), \tag{40}$$

$$\xi = \frac{(d + a) + i(b - c)}{2}, \qquad \eta = \frac{(d - a) + i(b + c)}{2}, \tag{41}$$

and modifies the Kähler potential by a Kähler transformation

$$1 - |H_1|^2 \longrightarrow \frac{1 - |H_1|^2}{|\overline{\eta} H_1 + \overline{\xi}|^2}. \tag{42}$$

In particular, the Kähler potential is strictly invariant under the continuous $U(1)$ associated with phase rotations, $H_1 \to e^{i\theta} H_1$. Similar relations can be obtained for the H_2 field. The continuous invariance of eq. (34) is not realized in a simple form. On the other hand, the discrete invariances of eqs. (35) and (36) take the suggestive forms

$$H_1 \to \frac{1}{H_1}, \quad H_2 \to \frac{1}{H_2}, \tag{43}$$

$$H_1 \to H_2, \quad H_2 \to H_1. \tag{44}$$

The two domains in which the Kähler potential is well defined become

$$|H_1|, |H_2| < 1, \qquad |H_1|, |H_2| > 1. \tag{45}$$

The parametrizations of eqs. (31) and (38) make explicit the factorization property of the manifold: to make connection with the general parametrization of $SO(2, n)/[SO(2) \times SO(n)]$ manifolds, we can make the field redefinitions $(H_1 = y_1 + iy_2, H_2 = y_1 - iy_2)$ and $(T = \rho + \sigma, U = \rho - \sigma)$, which give

$$Y^{(i)} = 1 - 2\left(|y_1|^2 + |y_2|^2\right) + |y_1^2 + y_2^2|^2 \tag{46}$$

and

$$Y^{(ii)} = (\rho + \overline{\rho})^2 - (\sigma + \overline{\sigma})^2. \tag{47}$$

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