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THE PRINCIPLE OF GLOBAL RELATIVITY[*] [**]

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In memoriam Prof. M.J.G. Veltman
for Tini's 90th birthday [m1][1]

We describe the non-minimal Standard Model, consisting of minimalistic extensions of the Standard Model, which for all we know is the theory of the universe, able to describe all of the universe from the beginning of time. Extensions discussed are an extra neutrino and a new Higgs model. We introduce the principle of global relativity and discuss how the theory can be largely derived from this principle. One is led to the unification of forces into SU(5) and a form of dark matter. We discuss the limitations of the theory, showing that it is not the theory of everything. However, we argue that it is the only part that is within conceivable reach of physical experiment or astronomical observation. It is argued that at the Planck scale the universe is effectively three-dimensional.

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1. Introduction — theory of the universe

Underlying the endeavour of fundamental physics is the conviction that nature can be described by unbreakable fundamental laws that take the form of mathematical equations. To find out what these laws are, one uses the so-called scientific method. This method consists of performing experiments, guided by theoretical ideas and inventing theoretical ideas, guided by experiment. This all follows common sense. This must be so. After all, it is easy to do senseless experiments or to invent crazy theories. In certain popular presentations and books on the "philosophy" of science this is not always clear. The scientific method has recently been enormously successful

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[1] Citations [m1], [m2], *etc.* refer to the paragraphs in the last, memoir section.

in the discovery of the Higgs boson and of gravitational waves. The first follows from a combination of the special theory of relativity and quantum mechanics, the second from the combination of special relativity and gravity. Thus, one gains confidence that one may have found at least a large part of the laws of nature. In this paper, I will argue that we know enough by now to say that we know the laws of nature sufficiently to explain all of the universe that we can see. We know, so to say, the theory of the universe. Assuming this to be true, we can try to move up a level and ask whether we can find a reason why precisely these laws are the right ones. This has been tried rather unsuccessfully in the past, where people attempted to find a theory of everything. Here, we will make a new attempt and argue that we may have come halfway.

A theory of the universe should be able to describe, at least in principle, how the universe developed from the beginning of time to its present form. Since the time I started studying physics in Utrecht in 1974, we have come a long way in this direction. There are actually four questions that should be answered:

(1) Why are the experiments so consistent with the Standard Model?
(2) How did the baryon number of the universe arise?
(3) How did the universe come into existence from inflation?
(4) What is the dark matter?

There are many proposals in the literature that attempt to explain these four points. These can take various forms of complexity, involving sophisticated concepts, like string theory or supersymmetry that will predict many more things than what we see in nature. These are maximalistic models that tend to predict phenomena that simply are not there, in conflict with point (1). The point of view we take here is that already the simplest possible extensions, minimalistic extensions, of the Standard Model are sufficient. By minimalistic extensions I mean extensions that do not change the fundamental chiral gauge structure of the Standard Model. Examples are non-chiral fermions, inert multiplets, Stückelberg-like vector bosons and singlets in general. I call this class of models the Non-Minimal Standard Model (NMSM). In discussions, one often hears about the Standard Model (SM), the minimal Supersymmetric Standard Model (MSSM) and the non-minimal Supersymmetric Standard Model (NMSSM). As the SM is not complete and the MSSM and the NMSSM have severe phenomenological difficulties, the NMSM may be the truth [m2]. The favorite explanation for the baryon number of the universe nowadays is leptogenesis, whereby heavy neutrinos generate a lepton number in the early universe that subsequently gets transformed into baryons through sphaleron processes [1–3]. In Section 2, we will study precision experiments to see if there is evidence for the existence of extra neutrinos. One of the most popular theories of inflation is Higgs-inflation, where the Higgs boson acts as the inflaton [4]. In Section 3, we will extensively study Higgs physics and construct minimalistic extensions that can work for inflation as well. In Section 5, we introduce a new principle, with which we address the question why the Standard Model should be the low-energy theory. We are led to a model with a unification into SU(5) that implies a fairly unique answer to question (4). Therefore, we arrive altogether at a consistent view of the world, whereby at low energies indeed a form of the NMSM describes nature in a way sufficient to describe the development of the universe as far as we can study it. Beyond that, we found a reason, why this should be the case. In the final section, we discuss the limitations of the theory and make some speculations about a theory of everything.

2. Precision predictions

The core of Tini's work is the calculation of radiative corrections due to the weak interactions. Without the possibility of performing such calculations, one cannot speak of a theory. This was the situation before Tini entered the field. The situation now is different. Calculations due to weakly interacting particles have in the meantime been done even to the three-loop level in certain cases. This is of course only possible in a renormalizable theory. Such calculations would not have been possible without the development of computer algebra programs, of which Tini's SCHOONSCHIP was the first program that could handle the large expressions appearing in quantum field theory calculations. Comparing precise calculations with precise measurements can under circumstances even give an indication of the existence of new particles. I will give two examples of this.

2.1. ρ parameter

One of the quantities that is sensitive to effects of heavy particles is the so-called ρ parameter [5]. That is the ratio between the neutral and the charged Fermi constant $\rho = G_{\rm F}^0/G_{\rm F}^+$. A peculiar feature of the Standard Model in comparison with other gauge models is that at the tree level ρ is equal to one. This is due to an accidental O(4) symmetry of the Higgs sector that is larger than the symmetry SU(2) × U(1) of the Standard Model as a whole. It receives radiative corrections from mass splittings within fermion multiplets and through hypercharge couplings. In particular, the effects of a heavy top quark can become quite large. This is due to the fact that the mass of the top quark is proportional to its Yukawa coupling, which becomes large, so that we have strong interactions present. As a consequence, even at the one-loop level, effects growing like m_t^2 are present [6]. Two-loop effects grow like m_t^4 [7] and three-loop effects like m_t^6 [8]. The results can be summarized in an effective Lagrangian [9]. However, just like in the Fermi theory, this Lagrangian cannot be used for loop calculations, since the loop effects can only be determined in the underlying renormalizable theory. This is a rather common problem for effective Lagrangians. Similar large top mass effects appear in couplings of the Z-boson to bottom quarks. In principle, having precise enough data at low energy, one could have predicted the top mass. This is not quite how things happened historically. There were some indications of a heavy top quark in the bottom data, but not as large as 170 GeV where it was found at Fermilab. The LEP data only agreed with the Fermilab result after a reanalysis of the data.

2.2. Lepton non-universality

With the mass of the top quark known, one can try to predict the mass of the Higgs boson from the precision measurements. For this the so-called blue-band plot was invented, which listed the $(\Delta\chi)^2$ from the best fit for the Higgs mass. However, this was always controversial, since the overall χ^2 was large and so the fit was not good. The problem lies with the bottom quark asymmetry in the decay of the Z-boson. Leaving out this measurement from the data, arguments for the existence of New Physics were made [10, 11]. However, because the Higgs mass was unknown, these attempts were inconclusive. After the discovery of the Higgs boson, things changed.

The importance of the results at the LHC is that the Higgs boson mass has been determined and that no new particles carrying weak charges appear to exist. This implies that we can compare theory predictions with data to a much higher level of precision than before. Precise predictions in the theory are sensitive to radiative corrections, dependent on the Higgs boson mass. Before the discovery of the Higgs boson, the data were used to constrain the range of the mass for the Higgs boson. Now that all parameters of the model are known the theory predictions are essentially exact, so one can look for much smaller deviations than before. To look for possible deviations in the precision data, we consider a model with n neutral sterile fermions that only mix with the neutrinos of the Standard Model. Such particles can play a role in cosmology, *i.e.* in leptogenesis or as dark matter candidates. The consequence is that the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix is part of a more general mixing matrix.

Taking into account the Standard Model neutrinos and the extra neutrinos, we find that the mass eigenstates $(\nu_1 \cdots \nu_{3+n})$ and flavour basis $(\alpha = e, \mu, \tau)$: $\{\nu_i = \nu_{L_\alpha},\, N_n\}$ are connected by a unitary $(3+n) \times (3+n)$ matrix

$$\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_{3+n} \end{pmatrix} = \begin{pmatrix} \mathrm{PMNS} & \mathcal{W} \\ \mathcal{W}^\dagger & \mathcal{V} \end{pmatrix} \begin{pmatrix} \nu_{L_e} \\ \vdots \\ N_n \end{pmatrix} .$$

As a consequence, the PMNS matrix, being a submatrix, is not necessarily unitary. We describe the deficit from unitarity by the ϵ parameters

$$\epsilon_\alpha = \sum_{i>3} |\mathcal{U}_{\alpha i}|^2 = 1 - \sum_{\beta=e,\mu,\tau} |\mathcal{U}_{\alpha\beta}|^2 \,.$$

As a consequence, low-energy parameters are affected by the ϵ parameters. For example, the Fermi constant in muon-decay is modified by the following relation:

$$G_\mu^2 = G_\mathrm{F}^2(1-\epsilon_e)(1-\epsilon_\mu)\,,$$

with G_μ the Fermi constant measured in muon decay, and G_F the Fermi parameter, derived from the Standard Model theory without ϵ parameters.

Other corrections appear in meson-decays and in precision measurements at LEP; we are, therefore, in the lucky position that we can combine low-energy and high-energy (LEP) measurements. When we do this, we find that the most precise data cannot be well fitted to the model [12, 13], even allowing for the presence of the ϵ parameters. The origin of the problem was tracked to a single measurement, namely the forward–backward asymmetry of bottom quarks at LEP. This measurement would lead to a large and unphysical negative value for ϵ_e. The other measurements are in good agreement with each other, leading to a value of $\epsilon_e \approx 2.10^{-3}$, excluding $\epsilon_e = 0$ at the 2–3 σ level. A number of experiments that can test this result are underway, new measurements of $\sin^2_\mathrm{eff}(\theta_W)$, an improvement on M_W, meson decays in b and τ factories, the ratio $W \to e/W \to \mu$, and a precise lattice evaluation of f_π. In combination, these could lead to a 5σ discovery.

3. Higgs physics

3.1. Heavy Higgs

With the construction of the Standard Model, it became clear that in some way one would have to look for the Higgs boson itself in colliders [m3]. The problem was that there was no way to tell what its mass was. Therefore, it was somewhat unclear, what sort of collider one needed. An upper mass for the Higgs boson of about 1 TeV was found in [14], based on tree-level unitarity violation, indicating the start of possible strong interaction phenomena like multiple vector boson production. Actually, the more relevant scale is probably $4\pi v \approx 3$ TeV, where interaction effects become large. Many people at the time thought the Higgs boson would not really be fundamental, but more of an effective description coming from a strongly interacting underlying theory, somewhat analogous to QCD. Of course, a simple scalar was also possible, but in general, people were somewhat suspicious of the mechanism [m4]. As a conclusion, it was decided that one would have to build a large energy hadron collider, SSC or LHC, in order to cover the full range of possibilities; in the end, only the LHC was built. The situation was summarized in the so-called no-lose scenario: building such a machine would either lead to the discovery of the Higgs boson or to the finding of new strong interactions [15]. Of course, the most important point of such scenarios is how to avoid them, which was indeed possible but not so easy. With strong enough mixing, the HEIDI-models to be described below, would have made the Higgs boson undetectable at the LHC. It is somewhat debatable, whether one should call finding the Higgs boson a discovery. After all, barring miraculous fine-tunings in other theories, the Standard Model was the only quantum mechanically consistent theory of the weak interaction, so one is actually only confirming quantum mechanics. But no one seriously doubts the validity of quantum mechanics. Not finding the Higgs boson, however, would have been a real discovery.

Complementary to large effects at high energy, there are large effects at low energies through radiative corrections. Thus, a large Higgs mass that leads to large cross sections at high energy, also will give rise to effects that rise with the Higgs mass in loop effects. This was the subject of the first discussion I had with Tini in 1979. He asked me how one could see a strongly interacting Higgs. This was in the first instance meant in radiative corrections [16]. Within the Standard Model that would mean a heavy Higgs. Though in the end the Higgs boson is light, the question has a number of interesting aspects, for instance related to the Landau pole and to the decoupling theorem. From the technical point of view, having a heavy Higgs, makes higher-loop graphs simpler. Starting with an arbitrary Higgs mass, would have been prohibitively difficult given the technical possibilities of the time. Even now, two-loop graphs with arbitrary masses are still at the edge of technique; examples are [17, 18]. For massless fields, one can go to much higher order [19]. This is only possible due to the method of dimensional regularization [20].

The radiative corrections dependent on the Higgs mass form a special case, different from the effects of other heavy particles. Normally, particles that are heavy decouple from the theory, since their effects are suppressed by the square of the mass. Typical is, for instance, the muon that does not affect atomic physics. The other extreme is the top quark, where the mass is proportional to a coupling, and so one gets strong effects for a heavy mass. The Higgs case is in between. The theory without a Higgs boson is close to renormalizable and at the one-loop level, one only gets logarithmically divergent effects. The theory without a Higgs particle can be seen as the limit of infinite Higgs mass, keeping the vacuum expectation value of the Higgs field constant. This means taking the self-coupling to infinity. This can be done at the tree level without problem; at the one-loop level, one gets logarithmically divergent corrections. However, due to the strong coupling, this is not sufficient for an ultimate conclusion and one needs a more detailed analysis. As a starting point, one can calculate two-loop effects for the ρ parameter [21] [m5,m6]. One finds effects growing like m_H^2. Also other quantities such as mass-shifts of the vector bosons or anomalous self-couplings behave in the same way; this is also true for the subsequent calculations. A first bound on the Higgs self-coupling was found in [22]. A new aspect arises in the four-vector boson couplings. Here, also Higgs-reducible diagrams contribute [23]. In these calculations, special numbers like $Cl(\pi/3)$ appear that play a role in number theory, having connections with elliptic functions [24]. Altogether, the effects were rather small. Thus, the next step was to look at the Higgs propagator itself at the two-loop level [25]. Here larger effects appeared. A logical next step is then to take a non-perturbative approach by making a $1/N$ expansion of the theory, whereby $N = 4$, given the fact that the Higgs sector is an O(4) model. The lowest order is trivial to calculate, but not very accurate. The next order was calculated in [26]. This was highly non-trivial, introducing new features like tachyon subtraction. The approach also goes by the name of renormalon physics or causal perturbation theory, and is more familiar in QCD. The two-loop and the second-order $1/N$ result agreed rather well, giving the following picture. The Higgs becomes very wide with a mass peak at about 1 TeV [27]. However, it is the width that is the measure of the coupling strength. The one-loop corrections to the Higgs propagator are small, however. That also explains why the two-loop corrections to the low-energy parameters are small. The qualitative reason is that the s-dependent width of the Higgs first appears at the two-loop order in the propagator. Thus, it became necessary to calculate the three-loop correction to the ρ parameter [28]. This was indeed larger than the two-loop one. Therefore, one finally has a consistent picture of a heavy Higgs. The Higgs gets wide, which means that the Källén–Lehmann spectral density of the Higgs field has a high mass component. In the radiative corrections, one then practically has a one-loop correction with a Higgs mass replaced by the spectral density [29]. This shows that a strongly interacting Higgs will lead to large effects in the radiative corrections. The strong interactions enhance the one-loop effects and cannot cancel them. One loophole is still left. Maybe cross exchange between the Higgs and the vector boson in the loop could become large, leading to bound states between vector bosons and Higgs particles. This was studied in [30]. Qualitatively, the situation is clear. In order to compensate for the one-loop $\log(m_H)$ effects, one would need low-lying bound states mixing with the vector bosons, which one should have seen. The conclusion is therefore clear. If precision measurements based on a one-loop analysis indicate that the Higgs is light, it must be light. Strong interactions cannot compensate for the one-loop effects.

3.2. Effective field theories

With the discovery of the Higgs particle, the impression exists that high-energy physics is finished and that there is no real reason to build a new accelerator. However, it is argued that the measurement of the Higgs self-coupling is necessary to further establish the Standard Model. This is one of the main arguments for extending the LHC to a higher luminosity, the HL-LHC. However, the argument is somewhat weak because the precision with which one can measure the three-Higgs self-coupling is rather small, the four-Higgs self-coupling is completely out of reach. So this is a weak test, in particular since it is difficult to make models that would generate a large Higgs self-coupling. Anomalous Higgs self-couplings can be generated by loop effects, for instance through the exchange of singlet scalars. However, these are small and effects would have shown up in other precision experiments before. Therefore, changes in the Higgs sector tend to be described by so-called effective field theories (EFT's) [31]. What one does here is to add higher-dimensional point-like interactions that can parametrize deviations from the Standard Model couplings. The term is a bit of a misnomer. Traditionally, one describes couplings by writing general Lorentz-invariant amplitudes, also allowing for form factors. In an EFT, form factors are made point-like and one compares them with the data at the tree level. This is not a very consistent procedure, since it is not possible to do loop corrections in a way that can be compared with electroweak precision measurements. In order to really have a quantum mechanical theory, one needs to impose the Schwinger–Dyson equations that have to be made finite first. In order to do this, one has to change the propagators too, generating a cut-off to the theory. In general, things become quite complicated and one loses effectively any predictivity. An example of such an analysis for anomalous vector boson couplings is given in [32]. Another example that we discussed before, is the Standard Model without a top quark.

Moreover, the energy range of the LHC is very large and one sees no deviations from the Standard Model. Therefore, the EFT's effects should be too small to be measured. The approach actually violates Veltman's theorem, which says that a low-energy theory must be either renormalizable or strongly interacting. This means one should work with a renormalizable theory from the start, which is the reason Veltman and 't Hooft received a Nobel prize in the first place. Nonetheless, not all hope is lost. At least for a limited class of EFT's, it is possible to show that they are the limit of renormalizable models, where some couplings become infinite. At the tree level, this can be done, but at the quantum level, one must take the original model. The situation is very similar to taking the infinite Higgs-mass limit within the Standard Model. EFT's where such a construction is not possible most likely cannot correspond to real physics; these are only a parametrization.

3.3. The Hill model

In order to test how good a theory like the Standard Model is, it is good practice to compare with a model that is different in the most minimal way. The simplest possible renormalizable extension of the Standard Model is actually the Hill model [34], having only two extra parameters [m7]. The Hill model is described by the following Lagrangian:

$$\begin{aligned}\mathcal{L} = \quad & -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) - \frac{1}{2}\left(\partial_\mu H\right)^2 \\ & -\frac{\lambda_0}{8}\left(\Phi^\dagger\Phi - f_0^2\right)^2 - \frac{\lambda_1}{8}\left(2f_1H - \Phi^\dagger\Phi\right)^2 .\end{aligned}$$

Working in the unitary gauge, one writes $\Phi^\dagger = (\sigma, 0)$, where the σ-field is the physical Standard Model Higgs field. Both the SM Higgs field σ and the Hill field H receive vacuum expectation values and one ends up with a two-by-two mass matrix to diagonalize, thereby ending with two masses m_- and m_+ and a mixing angle α. There are two equivalent ways to describe this situation. One is to say that one has two Higgs fields with reduced couplings g to Standard Model particles

$$g_- = g_{\mathrm{SM}}\cos(\alpha)\,, \qquad g_+ = g_{\mathrm{SM}}\sin(\alpha)\,.$$

The Standard Model would correspond to $\alpha = 0$, with the light Higgs being the Standard Model Higgs. The other way, which has some practical advantages, is not to diagonalize the propagator, but simply keep the σ–σ propagator explicitly. One simply replaces the Standard Model Higgs propagator, in all calculations of experimental cross section, by

$$D_{\sigma\sigma}\left(k^2\right) = \frac{\cos^2(\alpha)}{\left(k^2 + m_-^2\right)} + \frac{\sin^2(\alpha)}{\left(k^2 + m_+^2\right)}\,.$$

The generalization to an arbitrary set of fields H_k is straightforward, one simply replaces the singlet–doublet interaction term by

$$L_{H\Phi} = -\sum\frac{\lambda_k}{8}\left(2f_kH_k - \Phi^\dagger\Phi\right)^2 .$$

For a finite number of fields H_k, no essentially new aspects appear, however dividing the Higgs signal over even a small number of peaks, can make the study of the Higgs field at the LHC somewhat challenging. Having an infinite number of Higgs fields, one can also make a continuum [35, 36]. A mini-review of this type of models is given in [37].

3.4. A new Higgs model

Here, we will derive the Hill model backwards, starting from an effective field theory and show some extra possibilities. We start with the Standard Model and add the following effective terms to the Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2M^2}\partial_\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) - \frac{\lambda_3}{6M^2}(\Phi^\dagger\Phi)^3 - \frac{\lambda_4}{24M^4}(\Phi^\dagger\Phi)^4 \,.$$

Here, Φ is the ordinary Higgs field. Now, we introduce a new composite field H through the formula

$$MH = \Phi^\dagger\Phi \,.$$

In its form, the Lagrangian now is a simple singlet–doublet model, however with the constraint above. In the Lagrangian, that would correspond to a delta function for the fields. In quantum field theory, this is represented by a steep potential, so we get an extra term in the Lagrangian

$$\delta(MH - \Phi^\dagger\Phi) \to \lim_{\lambda_\delta\to\infty} \lambda_\delta\,(MH - \Phi^\dagger\Phi)^2 \,.$$

If we now take λ_δ finite, we have an ordinary singlet–doublet model, so we see that the EFT is a singular limit of a renormalizable theory, in which radiative corrections can be calculated. For the analysis of the data, one should of course use the renormalizable theory. In the case of $\lambda_3 = \lambda_4 = 0$, one gets back the Hill model.

An interesting Källén–Lehmann spectral density can be generated by assuming that the Hill field H moves in more than four dimensions [35, 36], which can be taken to be infinite and flat. We call such models HEIDI models because of the German pronunciation of high-D(imensional). In this case one is led to the following propagator:

$$D_{\sigma\sigma}\left(q^2\right) = \left(q^2 + M^2 - \frac{\mu^{8-d}}{(q^2+m^2)^{\frac{6-d}{2}} \pm \nu^{6-d}}\right)^{-1} .$$

In this expression, d is the number of dimensions which should satisfy $d \leq 6$, in order to ensure renormalizability. Actually, d does not necessarily have to be an integer to have a proper propagator. The parameter μ describes the mixing of the higher-dimensional Hill field with the Standard Model Higgs; indeed, putting $\mu = 0$, one gets the ordinary Higgs propagator with Higgs boson mass M. The parameter m is a higher-dimensional mass term and the parameter ν describes the mixing between higher-dimensional modes. Depending on the parameters, this propagator describes zero, one or two peaks plus a continuum. The continuum would correspond to a part of the

Higgs field moving away in the extra dimensions; experimentally, this would be interpreted as an invisible decay. The HEIDI models can also stabilize the Higgs potential. For example, one could have a 90% Standard Model Higgs at 125 GeV, a 5% Standard Model-like Higgs at 142 GeV, and a 5% invisible continuum with an average Higgs mass-squared around (180 GeV)^2. This would lead to a flat Higgs potential at the Planck mass. Therefore, it is important to measure the properties of the 125 GeV Higgs boson as precisely as possible, in particular, the overall cross section normalized to the Standard Model is of interest. One should also look for further Standard Model-like Higgs bosons. In these models, the branching ratios to Standard Model particles are the same as in the Standard Model for a Higgs boson with the same mass. The invisible continuum might be hard to see.

In this analysis, it was assumed that $\lambda_3 = 0$. For renormalizability, this is not necessary. Up to six dimensions, an H^3 self-interaction of the Hill field stays renormalizable, but the tree-level potential becomes unbounded from below. This may actually not be a problem in perturbation theory. After all, also the quantum potential of the Higgs field is possibly unstable. The presence of the H^3 self-interactions will after diagonalization affect the Higgs triple-coupling as well. This is, therefore, a new and renormalizable model that in principle allows for large deviations in the self-coupling of the Higgs. Maybe these are large enough to be measured at the HL-LHC. A further study is needed.

4. Colliders

Having constructed the HEIDI models, it is now clear that one cannot claim to have established the Standard Model in the Higgs sector, without having measured the complete Källén–Lehmann spectral density of the Higgs propagator. So far, the limits on this model are quite weak. Even if one measures the ratio of the Standard Model cross section and branching ratios within 10%, still 10% of the spectral density could be hidden elsewhere. For the coupling constants, one would have to take the square root, so one gets about a 0.3 limit at the coupling constant level, which is no precision at all. The term precision Higgs physics as usually used is, therefore, a misnomer. CERN has basically decided to build the HL-LHC, which will not improve the situation much at all. Therefore, one will need a lepton collider in order to get much better information on the Higgs sector. There are actually three reasons to build a lepton collider [m8]. First, there is the need to redo the precision measurements at the Z-pole in order to clarify the discrepancies in the LEP,data that we described in the section on neutrinos. Secondly, there is a 2.3σ 10% Higgs-like signal at 98 GeV that one should check. A hadron machine cannot do this, since the branching ratio into photons is too small. Thirdly, one should scan the Higgs spectrum, looking among others for the non-decaying "invisible" part of the propagator and in particular measuring the Higgs line shape. Strictly speaking, one has only fully established the Standard Model, when one has measured the line shape of the Higgs boson. Since the Standard Model width is 4 MeV, this is highly challenging.

There are basically two approaches to this problem. One is to build a muon collider. It appears possible to build a muon collider that would be able to measure the width of the Standard Model Higgs with moderate precision. However, this would leave out the challenge of looking for an invisible, maybe 10% partial Higgs boson at another mass. In order to look for such a signal, one needs to radiate off an extra photon, which reduces the cross section by a factor of the fine-structure constant. Thus, one would need a muon collider with a luminosity that is about a thousand times larger than usually considered. This does not look feasible.

The other option is to build a very large electron–positron collider, where one can measure the Higgs spectrum from the recoil spectrum in the process $e^+e^- \to ZH$, looking only at the outgoing Z-boson [m9]. This makes extreme demands on the machine. One would have to know the collision energy within 1 MeV. Also the detectors should be able to measure the outgoing muons with a precision of $\Delta p/p \approx 10^{-5}$. A very naive rescaling from LEP gives a size of 230 km for the required ring. Such a ring might (barely) be built, for instance in Fermilab, if the whole world would work on this in a united way. It is clearly too large for a single region. For organizational purposes, one would need a structure like CERN, but not with countries, but with regions as units. One could divide the world into 9 regions that could contribute according to their level of development: North America, South America, Western Europe, Eastern Europe, China, India, East-Asia-Pacific, Middle-East, Subsaharan Africa. Given the present state of the planet and humanity, it is unlikely that humanity will ever build such a machine; so we will never know for sure whether the simple Standard Model describes the Higgs sector correctly. The present plans, like FCC and CEPC, are actually not sufficient to settle the question. Moreover, we are interested in having a machine now, not after the LHC. So at present, the easiest way forward would be to go for the ILC in Japan that is ready to be built. This is clearly not the ultimate machine, but can settle a number of questions nonetheless. However, it should be designed to go to 300 GeV, so one can study the Higgs propagator up to the $2Z$-threshold, below which hadron colliders are not very good. Except for reasons of national pride, there is no particular reason why other countries could not combine to pay for at least half the cost of the ILC.

5. The principle of global relativity

After this discussion, we are tempted to conclude that the NMSM is indeed the theory of the universe . So we give in to some hubris and assume that we know the laws of nature well enough. Therefore, we will make an attempt to get to the next level: is there a way to understand why this would be the case? Are the laws of nature unique? To put it differently: Are there a small number of principles that would be sufficient to determine the form of the fundamental laws of nature? Einstein paraphrased the question in the following way: Did God have a choice when He created the world?

We will not try to answer everything at the same time, but will consider the theory without the Higgs boson. Then one is interested in the following only: Is there a reason for the choice of gauge group and representations? That is not an unreasonable limitation as a start. After all, before worrying about the breaking of a symmetry, it may be necessary to understand the symmetry itself first. It is actually an old question, going back to the discovery of the muon, summarized by the famous question of I. Rabi: who ordered that?

What of a handle exists in quantum field theory for such a question? The only one we know of are anomalies. Since pure gauge theories with fermions can exist as fundamental theories, when they are asymptotically free, anomalies in the gauge sector cannot be enough. So we are naturally led towards the consideration of anomalies involving gravity. The question of gravitational anomalies has been studied in great detail in [46, 47]. The main focus of these papers is on anomalies in higher dimensions, with applications towards Kaluza–Klein theories. Here, I will take the view that one should maybe go to lower, actually three, dimensions.

In three dimensions it is possible to have, besides the ordinary Yang–Mills or Einstein–Hilbert action, a parity-violating Chern–Simons term in the action [50–54]. This gives rise to a parity-violating mass term that is actually quantized for topological reasons. Radiative corrections give contributions to the Chern–Simons mass and, as a consequence, there are restrictions on the number of fermion fields. Alternatively, in the case of massless fermions, there can be a non-perturbative parity anomaly. Most of the discussion in the literature was focused on the Yang–Mills case. Here also, the gauge bosons themselves can renormalize the Chern–Simons term [55]. There was some discussion on the three-dimensional parity anomaly in [46], however, this part was limited to gauge anomalies. Regarding gravity it was said: We do not know under what conditions such phenomena occur in general relativity.

Actually, in three dimensions there is not only a parity anomaly, also known as the induced Chern–Simons term, from the fermions, there is also one coming from the gauge bosons themselves. Subsequently, a number of calculations addressed this question [56–60], where only [56] addressed both the fermion and the vector boson contribution to the induced gravitational Chern–Simons term [m10]. The opposite calculation gives that there is no contribution to the gauge Chern–Simons term from graviton exchange.

Just looking at three-dimensional anomalies will of course not do, since we are living in four dimensions. However, cosmologically speaking this is not so clear. In the simplest scenario, the universe basically starts as a point. Going back in time, everything shrinks in the same way, but cosmology gives more possibilities.

Relevant here is [47], which ends with the following sentence: The choice of S4 corresponds to treating four-dimensional space-time as Minkowski space. In the long run, a more delicate choice will be necessary to accommodate cosmological considerations. It may be that eventually, global anomalies will have cosmological applications, restricting the large-scale topology of space-time.

This is one way to look at the problem. If one knows what particle types exist in the universe, certain conditions on the topology of space-time can surely be derived. However, one can also turn the statement around. We could assume that “all” topologies are allowed, also in a cosmological context. Hereby “all” is to be defined in detail, as we would want to allow for the existence of fermions for example. Allowing all topologies, but having the same matter content always present in some form, we can reformulate. The laws of nature, *i.e.* the matter content of gauge fields and fermions, should be the same in all topologies allowed by the Einstein equations. Formulated this way, the term “principle of global relativity” appears unavoidable.

If this principle is valid, it could imply that the possible matter content is constrained, in the best case being unique. In order to see what is possible, I will describe a possible cosmological scenario that might correspond to the actual universe. For the principle to apply, it is not strictly necessary that the universe follows the model; it is sufficient that the model is potentially possible.

Observationally, there are limits on the topology of the universe [48]. Within the Λ cold dark matter model of the universe, the “slab” universe is the least constrained. However, real solid conclusions on the topology of the universe are difficult to obtain, because of inhomogeneities that grow into galaxy distributions at recent epochs [49].

The idea is that one starts with a “slab” universe, where the compact dimension shrinks faster, going backwards in time, than the other two. At a certain point in time, it reaches the Planck scale and disappears. Then one is in three dimensions and can use the results from the calculations in [61, 62]. One finds that the dimension of the gauge group must be a multiple of eight; SU(5) has three times eight generators. The fermions must come in multiplets of sixteen; a generation has sixteen fermions. To cancel fermions against bosons, one needs precisely three generations. At first sight, this looks a bit like numerology. However, it is a rather unique possibility within group theory and, as far as I know, the only argument where something close to nature comes out. Also the possible symmetry breaking pattern points towards a breaking pattern leading to the Standard Model. The physics I used is fairly well-established mathematical physics that has applications in solid state physics as well. So I did not need to introduce a large superstructure of New Physics, like superstrings in order to derive the results; it is a conservative approach.

At first sight, it looks strange that one assumes that the universe starts from a lower-dimensional space, but is it really stranger than starting the universe from a point, like in normal cosmology? Or is it stranger than assuming that the present universe started from a previous one, as is described by Prof. Penrose in his Nobel prize talk? In the end, it is hard to prove things one way or the other, as with such transitions one is in the realm, where quantum gravity/geometry is fundamentally important [m11,m12].

At least the factor sixteen for the fermions has been found directly in a four-dimensional context [63]. If in a quantum-gravitational context the transition from three to four dimensions could be understood, one would have a formal derivation that the Standard Model with three generations is the only possible low-energy theory. This is, of course, only valid for the chiral sector, which is why the allowed class of models is the NMSM, with minimalistic extensions of the Standard Model.

Actually, there is work on quantum gravity [65–67], following a more or less canonical approach that indicates that at large energies far above the Planck scale, space-time effectively becomes two dimensional [m13]. Thus, at low energies, space-time is four dimensional, as we know; at scales far above the Planck scale it is two dimensional. Logically speaking, it is natural to assume that at the Planck scale space-time is effectively three dimensional. Altogether, one finds this way a rather consistent picture of the universe including matter fields. In the geometrical approaches, one is mostly focused on gravity and less on the matter fields. How to truly unify these may involve new ideas; the factors eight and sixteen are intriguing. A question is whether we can find some experimental verification of the ideas. There is some hint that there is indeed a preferred direction in the universe that might be related to a compactified dimension, but there is little possibility for a definite conclusion, since one is always affected by the question of cosmic variance. More promising is, therefore, to look at what the prediction of unification into SU(5) tells us.

6. SU(5) and dark matter

We have now shown, with all caveats that the gauge group of nature must be SU(5). This immediately raises a problem. It is well known that the Standard Model particle content cannot lead to a unification of forces when one lets the coupling constants run as a function of the scale. It is known that supersymmetry can change this, but supersymmetry is not seen and is not in the class of minimalistic extensions. Thus, one has to start with different additions to the model. The easiest way is to introduce a **24** of fermions of SU(5). Introducing one Majorana **24** helps with unification, but is not really successful. This also does not satisfy the condition that the number of fermions must be a multiple of 16. The easiest way out is to start with a Dirac **24** [68]. Then unification is easy, because different patterns of mass splitting are possible after symmetry breaking. While a Dirac fermion **24** appears the most natural, any combination of an even number of real **24** representations would satisfy the 16-fold condition. With these extra fermions, the scale of unification can be varied over a wide range. The scale can be such that proton decay is out of range or just around the corner for proton decay experiments.

Another feature is that after symmetry breaking, a **24** leads to triplet fermions. These are candidates for the dark matter of the universe [69]. Actually, at the tree level, the charged and the neutral fermions have the same mass, but through radiative corrections, the charged one is 165 MeV heavier. It decays into the neutral one with emission of a charged pion. This makes this type of dark matter particularly hard, probably impossible to see at the LHC. The signal would be a soft pion with missing energy. WIMP triplet fermions are in a sense preferred, because they do not couple directly to ordinary matter via Z or Higgs exchange, but only through loops, so their cross section is much smaller. WIMP dark matter with direct exchange is strongly constrained by experiment [70]. The triplets are more constrained through indirect limits, meaning dark matter annihilation leading to γ rays. Having a single Dirac fermion appears to be ruled out by the HESS experiment, the reason being a large Sommerfeld enhancement in the annihilation cross section, precisely in the predicted mass range, needed to explain dark matter. This can be cured by having two Dirac triplets, or more general, an even number of real triplets [71]. Also, the role of the singlet in the **24** is not quite clear. The new information that apparently a large part of dark matter consists of black holes, may allow for more possibilities. The Čerenkov telescope array (CTA) should be able to clarify the situation.

7. Not the theory of everything

The idea of a theory of everything is, of course, not to describe everything that happens, for instance all craziness in human affairs, in detail. It is known, that even for simple systems this is not possible, because of the phenomenon of chaotic behaviour. The idea is to derive the fundamental rules from simple principles and, in particular, to calculate the constants of nature from the theory. Even though, as we have seen above, we have some idea of the general structure of the world, all attempts to calculate coupling constants and masses in the literature have been a miserable failure. Also, all attempts to quantize gravity have been less than successful. Both these questions are not addressed in the theory above. Both these questions have also been discussed in thousands of papers. I will argue that these questions lie beyond the range of physics, understood as an experimental science.

The theory described above is a good theory to describe the universe as we know it, but cannot explain the coupling constants and masses. Actually, in a way the theory appears to be too good. Calculations show that, for instance, the Higgs potential becomes essentially flat precisely near the Planck scale, which surely means something. This feature would be spoiled if there were a mechanism, whereby the masses of the particles are determined by new dynamics below the Planck scale. Of course, we do not know of such a mechanism anyway, but the argument shows that high-energy colliders cannot be expected to see anything interesting in flavour dynamics. Therefore, any dynamics that determines the coupling constants and masses must come from physics beyond the Planck scale. Is there any way we could conceivably probe this region of nature? The answer appears to be a resounding NO. In inflationary theories, even when one enhances quantum gravitational effects with non-minimal couplings, these quantum gravitational effects have far too small an impact to be seen in observable quantities like density fluctuations [72].

So what can we do? Astronomy and physics can test the precise form the theory of the universe will take. As mentioned above, a number of experiments and observations are underway or could in principle be performed, though they would be quite expensive. Regarding the theory of everything, one can only hope to find a deeper mathematical principle that is sufficiently strong to determine the dynamics in a unique way. However, history tells us that, without experimental guidance, this could well be a fool's errand [m14].

This work would not have been possible without Tini, who was like a father to me. Secondly, I am thankful to my gymnasium teacher Jan van de Putte, who introduced relativity to me and to my drs.-thesis advisor Gerard 't Hooft, for teaching me about topology in field theory. Further thanks are due to my 54 collaborators on scientific papers and the roughly 600–800 people with whom I worked on reports or had fruitful scientific discussions.

Memories, history and remarks

[m1] On Tini's 80th birthday I gave a talk about Higgs physics [38]. Tini told me he had rather have heard something about SU(5). So I prepared this talk for his 90th birthday. I have discussed the subjects in this paper with him. Normally, he would find faults in my ideas, but this time he only found it a pity that these theories could not be measured. This is not really true, but it will be quite expensive. The terminology "principle of global relativity" was first used in a discussion during my talk at the AEI in Potsdam on January 26, 2018. In writing, it first appeared at the 15th Patras meeting in Freiburg on June 3, 2019.

[m2] Actually MNSM, which was originally a typing error, might be a more interesting name for the theory. It would stand for Minimalistic Non-Standard Model.

[m3] Tini himself was not directly involved in calculations for the production of particles at the LHC. But SCHOONSCHIP was used in the first calculation of boosted Higgs production [39]. Moreover, from 1987–1989, there were two groups systematically calculating the production of weakly interacting particle pairs from gluon fusion. One was in CERN, consisting of E.W.N. Glover and myself. We used SCHOONSCHIP on an Atari computer, to express the amplitudes in irreducible scalar integrals. The others were D.A. Dicus and Chung-Kao, who used Tini's FORMF program that contained numerical routines also for tensor integrals. An example is Higgs pair production [40, 41].

[m4] The idea that there would be strong interactions or that there would be more than just the simple Standard Model was rather wide-spread at the time. For instance, I had an early discussion with G. 't Hooft:
G.: *How can the Higgs be so light? There is no symmetry to protect its mass!*
J.: *Scale invariance?*
G.: *Yes, but that is not a quantum symmetry!*
J.: *OK, how about supersymmetry?*
G.: *Yes, but that does not exist!*
J.: *So what could it be?*
G.: *It has to be like with the strong interactions, but a bit different.*
This might be true. The Higgs field in the HEIDI models is much like the σ-field in QCD, but more weakly interacting. Also scale invariance may still play a role somewhere and supersymmetry appears not to exist.

[m5] Traditionally, doctoral theses in the Netherlands are printed as a small book. As my thesis defence was in Utrecht, but my work had been done in Ann Arbor and there were some timing problems, my booklet was printed in Ann Arbor. For the local printers, this was an interesting

experience and I got a tour of the factory. Anyway, because of this, the thesis was imported into the Netherlands and I had to pay import duty on it. Tini was quite indignant about this: *Jochum that is all wrong, they should have called me and I would have told them that it is not worth anything.*

[m6] Working together with Tini could be an interesting experience. As the two-loop calculation we were doing was quite difficult for that time, things were a bit tense sometimes. He once threw me out of his office, which with other students and supervisors could be problematic. I just found it curious. We had gotten side-tracked in a discussion on the meaning of the renormalization of the Faddeev–Popov ghost fields. I interpreted this as a redefinition of the basis in the Lie algebra of the gauge-symmetry. As the ghost fields do not contribute to the order we calculated, this may have sounded smart, but was beside the point. Near the end of the calculation, things did not fit. Tini said that he was at the end of his Latin. As I have a classical gymnasium education, I answered: *No problem, we continue in Greek.* The bug was quickly found, we had to expand some integrals a bit further.

[m7] Alfred Hill was a German physicist who studied in Groningen and did his doctorate with Tini in Michigan. Besides the Hill model, which was his thesis, he wrote an unpublished paper on higher-dimensional anomalies. His Dutch was so perfect that from this you could tell he was not really Dutch. He died in Lockerbie.

[m8] Tini and I often discussed the question what accelerators should be built. He has often told me that he had considered the energy of LEP to be too small. He was right there; a bit more would have allowed LEP to discover the Higgs boson and study it in precise detail. But, of course, there were engineering limitations and no one knew where the Higgs mass would be. Also, there was for a long time a fairly general expectation that the Higgs would be heavy. For a very heavy Higgs boson, only indirect effects would be visible, for instance in 4W couplings. Tini and I were together in DESY in 1990. We there often discussed the possibility of building a linear collider. For an energy, we came up with 500 GeV. The precise reason for this energy is a bit unclear, but at this scale, 4W couplings become measurably different from the Standard Model due to radiative corrections from a heavy Higgs boson. Also one comes in the region where one can study triple vector boson production. Anyway, 500 GeV became the standard, with an option to go to 800 GeV. For these energies, circular colliders are no option.

[m9] I first considered the option of a large circular e^+e^- collider as a Higgs factory in 2009. This was as a consequence of my studies on the possibility of not seeing the Higgs boson at the LHC, which I presented in Moriond 2007 and 2008, and Blois 2009. I then mentioned the possibility at the LC forum in DESY Hamburg, June 14, 2010 and München, Juli 14, 2011. However, here the focus was more on a 300 GeV linear collider. At the time, there was no design for a 300 GeV collider; all was concentrated on 500 GeV. I had some correspondence with Prof. Brian Foster on the possibility of lowering the energy. I also mentioned the possibility at the Veltman 80th birthday meeting in NIKHEF on June 24, 2011. Things became more official after the Moriond meeting March 3–10, 2012, where there were also contributions by experimentalists from CERN. The considerations became serious after the European Strategy meeting in Kraków, September 2012, where Nigel Glover and I also sent in a contribution.

[m10] Sumathi Rao was my officemate in Fermilab. After she and Rob Pisarski had finished their Chern–Simons Yang–Mills calculation [55], we got into a discussion about the gravitational case. Rob thought this would be too difficult. However, I had experience with SCHOONSCHIP, Tini's algebraic manipulation program, so I knew it would not be too bad. I also used Tini's lectures on gravity from the 1975 Les Houches School for the vertices.

[m11] Of course, Tini was a pioneer in quantum gravity calculations [42, 43]. While gravity involves the manipulation of many indices, Tini never left the underlying physics out of sight. Typical is the discussion he had with M. Gell-Mann. They were discussing quarks and Gell-Mann said: *Tini, these are just indices*, upon which Tini started to jump up and down: *This is just indices?*

[m12] When I started as a doctoral student with Tini, I was of course full of optimism, convinced that we would soon solve quantum gravity and everything else. I even wrote a small paper [44]. Tini, who had been around a bit longer, told me: *Jochum, quantum gravity is a black hole; when you jump in, you will never get out!* For me at the time that was wise advice and I did not jump in. This in contrast to my former student Eugen Radu [45].

[m13] Tini used poles in $d = 2$ as a gauge-invariant way to define quadratic divergences [64]. I try here a somewhat more dynamical approach to dimensional reduction. Some modern approaches towards quantum gravity point in this direction as well [65–67].

[m14] The last subjects I discussed with Tini, not long before he died, were some ideas along these lines. And of course, he told me they were crazy. He was probably right, but maybe not; we will see or maybe not.

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