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Deep inelastic scattering off quark-gluon plasma and its photon emissivity

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The photon emissivity of quark-gluon plasma probes the interactions in the medium and differs qualitatively between a weakly coupled and a strongly coupled plasma in the soft-photon regime. The photon emissivity is given by the product of kinematic factors and a spectral function associated with the two-point correlator of the electromagnetic current at lightlike kinematics. A certain Euclidean correlator at imaginary spatial momentum can be calculated in lattice QCD and is given by an integral over the relevant spectral function at lightlike kinematics. I present a first exploratory lattice calculation of this correlator. Secondly, I show how Euclidean correlators at imaginary spatial momenta can also be used to probe the regime of deep inelastic scattering off quark-gluon plasma, which reveals its parton distribution function.

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1. Introduction

In this presentation I will be concerned with the vector spectral functions of strong-interaction matter at non-vanishing temperature, which can be defined as (in diag$(+,-,-,-)$ metric)

$$\rho^{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x}\, \frac{1}{Z}\sum_n e^{-E_n/T}\,\langle n|[j^\mu(x), j^\nu(0)]|n\rangle, \qquad Z = \sum_n e^{-E_n/T}. \tag{1}$$

Here Z is the canonical partition function, and j^μ is a conserved vector current, the case of the electromagnetic current $j^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s + \dots$ being particularly important for the phenomenology of heavy-ion collisions [1, 2] and the physics of the early universe (see e.g. [3]), since it is via this current that the medium interacts with photons. Therefore we will have this case in mind in the following.

The components of $\rho^{\mu\nu}(q)$ can be parametrized by two independent kinematic variables, which can be chosen to be the virtuality q^2 and the photon energy q^0 in the rest frame of the thermal medium. At vanishing virtuality $q^2 = 0$, the spectral functions describe the rate at which the medium emits photons (see Eq. (10) below), and for positive virtuality, they describe the production rate of lepton pairs via a timelike photon [4]. At spacelike virtualities, the vector spectral functions measure the ability of the medium to convert the energy stored in external electromagnetic fields into heat [5]. A conceivable way to create electromagnetic fields with frequencies on the order of a GeV is to scatter a lepton of energy E on the medium. The process is illustrated in Fig. 1. In the one-photon exchange approximation, the cross-section for scattering off the medium of volume L^3 reads

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4\,L^3}{8\pi^2 Q^4}\left(\frac{E'}{E}\right)\ell_{\mu\nu}W_>^{\mu\nu}(u,q), \tag{2}$$

in the rest frame of the medium, with E' the final-state energy of the lepton and Ω the solid angle of its outgoing momentum relative to the incident momentum [6]. Here $\ell_{\mu\nu}$ is the (exactly known) leptonic tensor and $W_>^{\mu\nu}$ the 'hadronic' tensor, names that we borrow from the literature on deep-inelastic scattering on the nucleon. The hadronic tensor is defined as

$$W_>^{\mu\nu}(u,q) = \frac{1}{4\pi\, Z}\sum_n e^{-\beta E_n}\int d^4x\, e^{iq\cdot x}\,\langle n|j^\mu(x)\, j^\nu(0)|n\rangle, \tag{3}$$

with u the four-velocity of the medium, and is related by the Kubo–Martin-Schwinger relation to the spectral functions in the rest frame of the medium [7],

$$W_>^{\mu\nu}(u,q) = \frac{1}{4\pi(1-e^{-\beta q^0})}\,\rho^{\mu\nu}(q^0,\vec{q}), \qquad u = (1,\vec{0}). \tag{4}$$

Particularly for studying the deep-inelastic scattering (DIS) limit, a suitable tensor decomposition is

$$\begin{aligned} W_>^{\mu\nu}(u,q) \;\; = \;\; & F_1(u\cdot q, Q^2)\Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big) \\ & + \frac{T}{u\cdot q}F_2(u\cdot q, Q^2)\Big(u^\mu - (u\cdot q)\frac{q^\mu}{q^2}\Big)\Big(u^\nu - (u\cdot q)\frac{q^\nu}{q^2}\Big), \end{aligned} \tag{5}$$

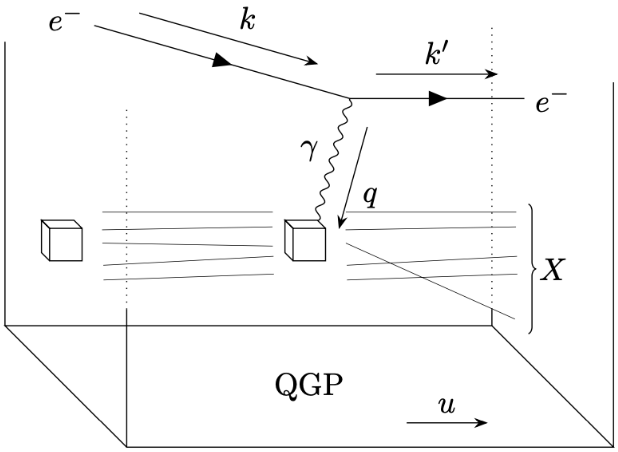

Figure 1: Scattering of a lepton on quark-gluon plasma at global thermal equilibrium moving with a four-velocity u in the one-photon exchange approximation. The picture shows the interaction occurring with a cubic fluid cell, producing an unobserved QCD final-state X.

with F_1, F_2 being the structure functions of the medium. We introduce the Bjorken variable

$$x = \frac{Q^2}{2Tq^0}, \tag{6}$$

in which the temperature T represents the energy scale of the medium used here to make x dimensionless and $Q^2 = -q^2$. Note that x is not bounded from above by unity in the present context.

2. Probing the structure functions with Euclidean correlation functions

We focus here on the spatially transverse channel represented by the structure function F_1 and define the corresponding spectral function

$$\sigma^T(q^0, Q^2) = 8\pi(1 - e^{-\beta q^0})F_1(q^0, Q^2), \tag{7}$$

viewed as a function of the photon energy and virtuality. Choose now a Matsubara frequency $\omega_n = 2\pi nT$ and a particular value of $Q^2 \leq \omega_n^2$. In [6], it is shown that the q^0-dependence of this spectral function can be probed via the Euclidean correlator

$$H_E^T(\omega_n; Q^2) \;\; = \;\; -2\int_0^\beta dx_0 \int d^3x \; e^{i\omega_n x_0 + x_3\sqrt{\omega_n^2 - Q^2}} \; \big\langle j_1(x)\, j_1(0)\big\rangle. \tag{8}$$

This correlator contains a logarithmic ultraviolet divergence[1], which cancels in the difference on the left-hand side of Eq. (9). Indeed this difference would vanish in the vacuum, since in the latter state H_E^T only depends on Q^2. In order to reach these conclusions, it is sufficient to Taylor-expand around $x = 0$ the weight function with which the position-space vector correlator is multiplied, taking into account that this correlator is even under the coordinate transformation $x_0 \to -x_0$ as well as

[1]An exception to this statement is the case of $Q^2 = 0$, when the theory is regularized in a Lorentz-invariant way.

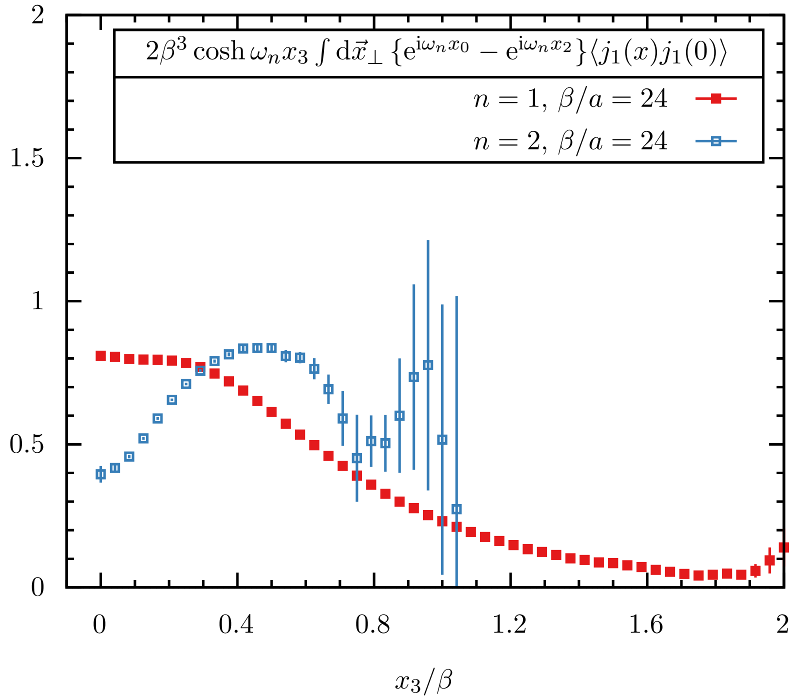

Figure 2: The x_3-integrand to obtain $H_E^T(\omega_n, 0)$ based on Eq. (11) for the isovector current with quark charges $Q_u = -Q_d = 1/\sqrt{2}$, after integrating over $d\vec{x}_\perp \equiv dx_0 dx_1 dx_2$. The area under both displayed correlators would be $T^2/2$ for free massless quarks [9]. In general, the non-negativity of σ^T implies that the area can be no smaller for the $n = 2$ than for the $n = 1$ correlator. The calculation is performed on a 24×96^3 lattice QCD ensemble with two dynamical flavours of light quarks and $T \simeq 254\,\text{MeV}$, the X7 ensemble described in [5]. The displayed data is based on 1331 configurations, with 64 point sources used per configuration. At the largest x_3, the effect of the periodicity of this variable can be seen in the $n = 1$ correlator.

under $x_3 \to -x_3$. For $\omega_n^2 \geq Q^2$, note the 'imaginary spatial momentum' involved in computing the correlator H_E^T. This correlator admits the fixed-virtuality dispersive representation

$$H_E^T(\omega_n; Q^2) - H_E^T(\omega_r; Q^2) = \int_0^\infty \frac{dq^0}{\pi}\, q^0\, \sigma^T(q^0, Q^2) \left[\frac{1}{(q^0)^2 + \omega_n^2} - \frac{1}{(q^0)^2 + \omega_r^2}\right]. \quad (9)$$

Since $\sigma^T(q^0, Q^2 = 0) \sim (q^0)^{1/2}$ at weak coupling [8], one subtraction is expected to be sufficient to guarantee convergence of the dispersion relation. In practice, one has the option to compute the left-hand side of Eq. (9) using at long distances $e^{x_3\sqrt{\hat\omega_n^2 - Q^2}}$ with $\hat\omega_n^2 = \omega_n^2 - \mathrm{O}(a^2)$ slightly reduced in magnitude, in order to avoid convergence issues in the infrared at non-zero lattice spacing.

Of special interest is the case of zero virtuality, since the differential photon emissivity of the medium is given by

$$\frac{d\Gamma_\gamma}{dq^0} = \frac{\alpha}{\pi}\, \frac{q^0}{e^{\beta q^0} - 1}\, \sigma^T(q^0, Q^2 = 0). \quad (10)$$

At $Q^2 = 0$, H_E^T is ultraviolet-finite in a regularisation respecting Lorentz symmetry. On the lattice however, it is necessary to perform a subtraction in order to obtain the correct continuum limit for

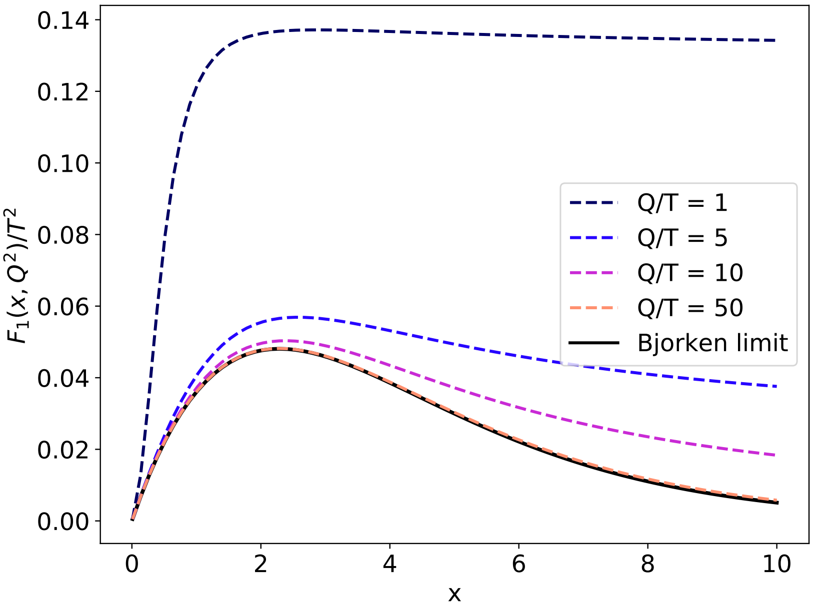

Figure 3: The structure function $F_1(x, Q^2)$ of the free plasma for different values of Q^2, together with the Bjorken limit Eq. (14). Here we have set $N_c = 3$ and $\sum_f Q_f^2 = 1$.

the correlator $H_E^T(\omega_n)$ [9]. For that purpose, we introduce here the estimator

$$H_E^T(\omega_n, 0) = -2 \int_0^\beta dx_0 \int d^3x\, (e^{i\omega_n x_0} - e^{i\omega_n x_2})\, e^{\omega_n x_3} \left\langle j_1(x) j_1(0) \right\rangle. \tag{11}$$

Note that the subtraction term $(e^{i\omega_n x_2})$ vanishes in the continuum thermal theory, but on the lattice removes an ultraviolet divergence associated with the lack of Lorentz symmetry at finite lattice spacing. This is easiest to see from the fact that estimator (11) of $H_E^T(\omega_n)$ automatically vanishes on the ($T = 0$) infinite lattice. With estimator (11), no cumbersome subtraction of vacuum correlators is needed, which would require costly simulations. The integrand of Eq. (11) in a lattice QCD calculation at $T \approx 250\,\mathrm{MeV}$ is illustrated in Fig. 2. Similar to the comment below Eq. (9), the exponential weight factor $e^{\omega_n x_3}$ can be replaced by a factor with a slightly ($\mathrm{O}(a^2)$) reduced exponent, for instance by substituting $\omega_n \longrightarrow \frac{2}{a} \sin \frac{a\omega_n}{2}$. Given that $H_E^T(0, 0) = 0$, we can write a simplied dispersion relation for the light-like case,

$$H_E^T(\omega_n, 0) = -\frac{\omega_n^2}{\pi} \int_0^\infty \frac{dq^0}{q^0} \frac{\sigma^T(q^0)}{(q^0)^2 + \omega_n^2}\,. \tag{12}$$

3. Thermal medium structure functions in the DIS regime

As in the case of the nucleon, one can show that the structure functions admit a partonic interpretation in the DIS limit [6]. With u being the four-velocity of the fluid, let $f_f(\xi)\, d\xi$ represent the number of partons of type f in the fluid cell that carry momentum $\xi\, T\, u$. This number is of order the cell volume, L^3. In the DIS limit, the structure function reads

$$F_1(u\cdot q, Q^2) \overset{Q^2\to\infty}{\longrightarrow} \frac{1}{4L^3T}\sum_f Q_f^2\; f_f(x), \tag{13}$$

with $x = Q^2/(2Tu\cdot q)$ kept fixed. In words, $4\,F_1\cdot dx$ is the square-electric-charge weighted number of partons carrying a momentum xT times the fluid four-velocity u^μ per unit transverse area in a slab of fluid which in its rest frame has thickness $1/T$ in the longitudinal direction.

In the theory of free quarks, in which the spectral functions can be computed analytically [10], one finds in the DIS limit

$$\lim_{Q^2\to\infty} F_1(x, Q^2) = \frac{(\sum_f Q_f^2)N_cT^2}{4\pi^2} x\log(1+e^{-x/2}), \tag{14}$$

thus showing that the parton distribution is proportional to $x\log(1+e^{-x/2})$, and normalized such that the partons altogether carry the entire momentum of the aforementioned fluid cell as given by its rest-frame enthalpy, consistently with the expectation from ideal hydrodynamics [6]. The approach to the DIS limit as a function of x is illustrated in the free-quark theory in figure 3. One can see that it is only in the limit $Q^2 \to \infty$ that the structure function acquires the normalizable form of a probability distribution in the variable x. This represents a qualitative difference with respect to ordinary DIS on the nucleon.

How can the DIS regime of the structure functions be addressed using lattice QCD? Using the twist expansion of the product of two vector currents, one finds the moment sum rules obeyed by the structure functions [6],

$$\int_0^\infty dx\, x^{n-1}\, [F_1(x, Q^2)]_{\text{leading-twist}} = \frac{1}{2}\sum_{f,j} Q_f^2 M_{fj}(Q,\tilde\mu)\langle O_{nj}\rangle\,, \quad n = 2, 4, \ldots\,, \tag{15}$$

where the M_{fj} are the coefficients parametrizing the operator mixing and the $O_{nj}^{\mu_1\ldots\mu_n}$ are the usual twist-two operators of mass dimension $(n+2)$, whose thermal expectation values are parametrized as follows,

$$\langle O_{nj}^{\mu_1\ldots\mu_n}\rangle = T^n\,[u^{\mu_1}\ldots u^{\mu_n} - \text{traces}]\langle O_{nj}\rangle\,. \tag{16}$$

The index j runs over the quark flavours and the gluon operator. The right-hand side of the moment sum rule is computable on the lattice for the lowest few values of n. A difference with ordinary DIS is however that the leading-twist component of F_1 must be taken before calculating the x^{n-1} moment in Eq. (15), as the integral otherwise does not converge, as can be seen from Fig. 3. Alternatively to computing the lowest x-moments of the structure functions, the approach to the DIS limit can be studied by computing the correlators $H_E^T(\omega_n; Q^2)$ for a sequence of Matsubara frequencies ω_n. Performing the change of variables $x = Q^2/(2Tq^0)$ in Eq. (9), one obtains

$$H_E^T(\omega_n; Q^2) - H_E^T(\omega_r; Q^2) = \int_0^\infty \frac{dx}{\pi}\, x\,\hat\sigma^T(x, Q^2)\frac{a_r^2 - a_n^2}{(1+a_n^2x^2)(1+a_r^2x^2)}\,, \tag{17}$$

with $a_n = 2T\omega_n/Q^2$. Thus if a_n and a_r are kept fixed as Q^2 is varied, the only Q^2 dependence on the left-hand side comes from the spectral function $\hat\sigma^T(x, Q^2) \equiv \sigma^T(q^0, Q^2)$. The extent to which it becomes independent of Q^2 in the Bjorken limit can thus be probed in this way.

4. Conclusion

Dispersion relations at fixed spacelike virtuality, rather than at fixed spatial momentum, open up new perspectives on the thermal spectral functions. Here we have shown that certain moments of the spectrum of emitted photons can be computed in lattice QCD without solving an inverse problem. The general lesson is that when an analytic continuation in one variable of a Euclidean correlation function depending on several kinematic variables is performed, which numerically amounts to solving an inverse problem, it plays an important role which of these kinematic variables are kept fixed.

We have also seen that moments of the in-medium structure functions can be computed in lattice QCD at fixed spacelike virtuality. In this kinematic regime, the structure functions describe the scattering of a lepton on the medium, and, at sufficiently high virtuality in the Bjorken regime, should reveal its quark-gluon constituents. Such a regime is however difficult to reach with the current simulation techniques, due to the real exponential $e^{x_3\sqrt{\omega_n^2-Q^2}}$ emphasizing the large distances in the spatial directions. One must also remember that even in the theory of free quarks, the Bjorken scaling is only reached after 'higher-twist' ($\mathrm{O}(1/Q^2)$) contributions are sufficiently suppressed.

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References

[1] P. Braun-Munzinger, V. Koch, T. Schaefer, and J. Stachel, *Properties of hot and dense matter from relativistic heavy ion collisions*, *Phys. Rept.* **621** (2016) 76–126, [arXiv:1510.0044].

[2] G. David, *Direct real photons in relativistic heavy ion collisions*, *Rept. Prog. Phys.* **83** (2020), no. 4 046301, [arXiv:1907.0889].

[3] T. Asaka, M. Laine, and M. Shaposhnikov, *On the hadronic contribution to sterile neutrino production*, *JHEP* **06** (2006) 053, [hep-ph/0605209].

[4] L. D. McLerran and T. Toimela, *Photon and Dilepton Emission from the Quark - Gluon Plasma: Some General Considerations*, *Phys.Rev.* **D31** (1985) 545.

[5] M. Cè, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato, *The rate of photon production in the quark-gluon plasma from lattice QCD*, *Phys. Rev. D* **102** (2020), no. 9 091501, [arXiv:2001.0336].

[6] M. Cè, T. Harris, H. B. Meyer, and A. Toniato, *Deep inelastic scattering on the quark-gluon plasma*, *JHEP* **03** (2021) 035, [arXiv:2012.0752].

[7] H. B. Meyer, *Transport Properties of the Quark-Gluon Plasma: A Lattice QCD Perspective*, *Eur.Phys.J.* **A47** (2011) 86, [arXiv:1104.3708].

[8] S. Caron-Huot, P. Kovtun, G. D. Moore, A. Starinets, and L. G. Yaffe, *Photon and dilepton production in supersymmetric Yang-Mills plasma*, *JHEP* **0612** (2006) 015, [hep-th/0607237].

[9] H. B. Meyer, *Euclidean correlators at imaginary spatial momentum and their relation to the thermal photon emission rate*, *Eur. Phys. J.* **A54** (2018), no. 11 192, [arXiv:1807.0078].

[10] M. Laine, *NLO thermal dilepton rate at non-zero momentum*, *JHEP* **1311** (2013) 120, [arXiv:1310.0164].

[11] Jülich Supercomputing Centre, *JUWELS: Modular Tier-0/1 Supercomputer at the Jülich Supercomputing Centre*, *Journal of large-scale research facilities* **5** (2019), no. A135.