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Abstract: We compute the excitation of double giant resonances due to a relativistic coulomb field using a one dimensional quantic oscillator model. We show that the introduction of small anharmonicities in the hamiltonian of the oscillator and small non-linearities in the external exciting field changes the cross section of the two-phonon states up to a factor 2. The results obtained within this very schematic model indicate that small anharmonicities and non-linearities could bring the theoretical predictions of the cross section of double giant resonances close to the experimental value.

1. Introduction

Giant resonances are collective vibrations of the nucleus. Since their discovery in 1947, different vibrating modes of nuclei have been observed and studied. In particular, those of lower angular momentum L are the monopole mode or "breathing mode" ($L = 0$), in which the nucleus undergoes a compression and a dilatation; the dipole mode ($L = 1$), where protons and neutrons oscillate one against the other and the quadrupole mode ($L = 2$), where the nuclear shape changes from oblate to prolate and viceversa (fig.1). A typical evidence of the excitation of a giant dipole resonance is given by the photoabsorption cross section in a nucleus (fig.2)[1].

If we suppose that the potential of the nucleus is harmonic in the general deformation parameters, then we can think of a giant resonance as the first excited state (or 1-phonon state) of an oscillator and a double giant resonance as the second excited state (or 2-phonon state). In 1977, an indication of the existence of excited states where a giant resonance is built on top of another one was reported for the first time [2]. Since then the existence of two phonon states has been clearly established. These states can be excited either through double charge exchange reactions or through heavy ion collisions at intermediate and relativistic energies (fig.3). The latter in particular are appropriate to strongly excite two dipole giant resonances.

In the last years , a quite huge systematics has been collected on the properties of 2-phonon states, that is their energy, their width and their cross section. From this

Fig. 1: Three different modes of vibration of the nucleus which correspond to the Giant Monopole Resonance (GMR), the Giant Dipole Resonance (GDR) and the Giant Quadrupole Resonance (GQR).

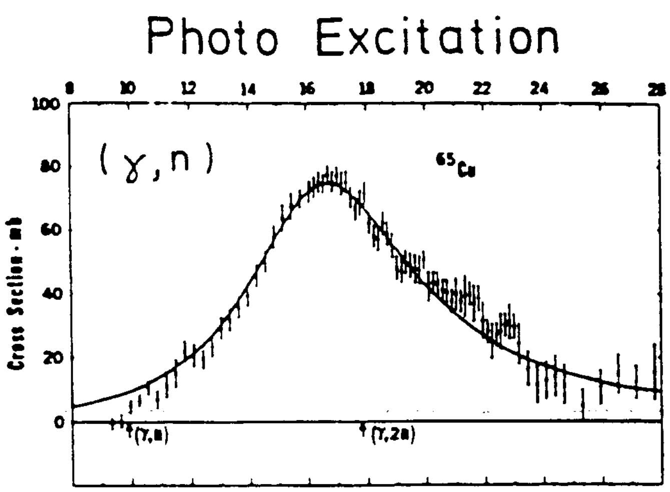

Fig. 2: Total photo neutron cross section for the ^{65}Cu showing the strong resonance associated with a GDR [1].

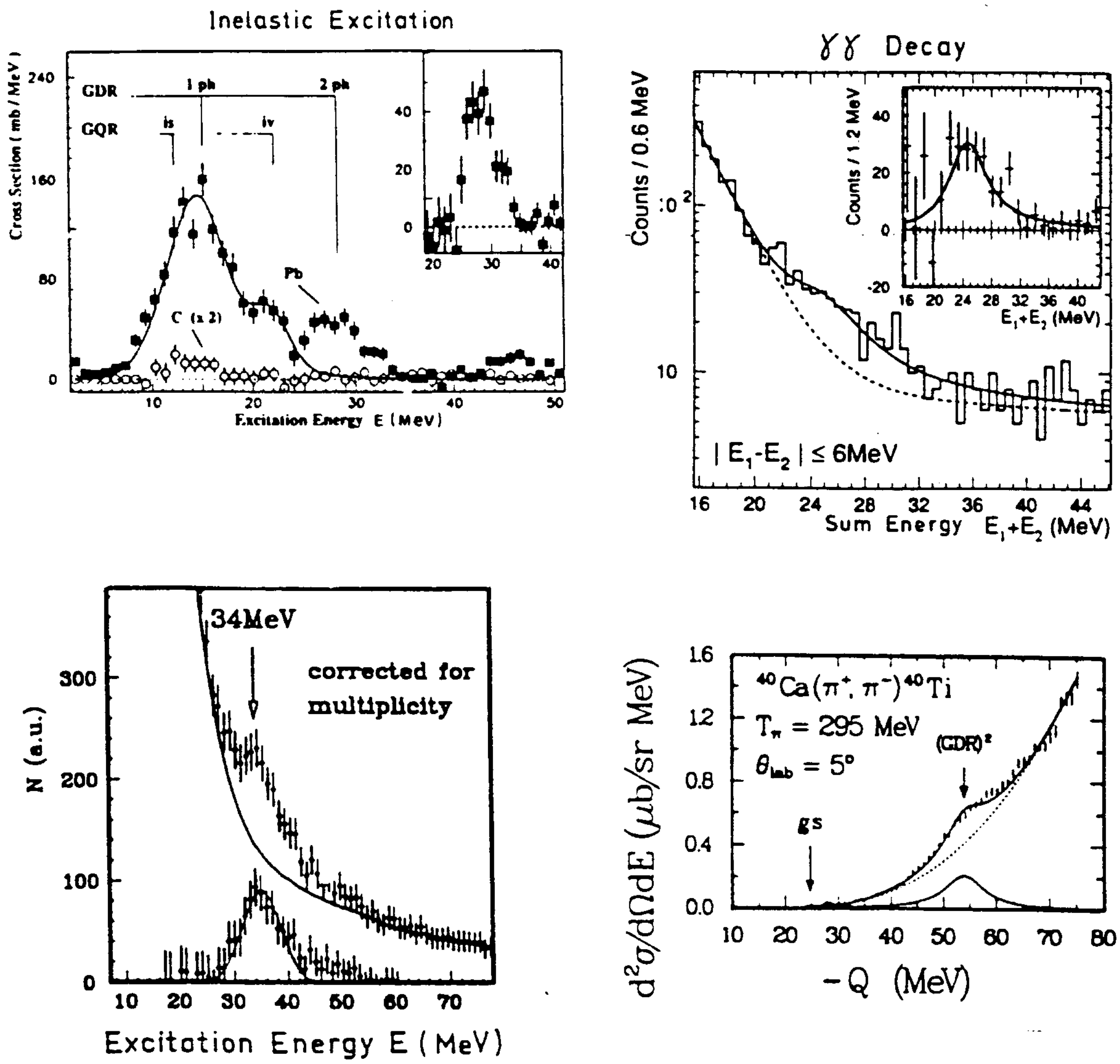

Fig. 3: *upper left:* Excitation energy spectrum obtained for ^{136}Xe projectile interacting with a ^{208}Pb target and a ^{12}C target (circles) at 700 MeV/n (squares). The second bump (2-ph) is associated to the excitation of a two phonon state [4]. *upper right:* Sum energy of coincident photon pairs with an energy difference less than 6 MeV for peripheral events from ^{209}Bi target at 1 GeV/n on ^{208}Pb. The shoulder is assigned to a two phonon state [5]. *lower left:* Inelastic spectrum corrected for proton multiplicity from $^{40}Ca + ^{40}Ca$ at 44 MeV/n. The peak has been shown to be the excitation of a $GQR \otimes GQR$ [6]. *lower right:* Double-differential cross section for the $^{40}Ca(\pi^+,\pi^-)^{40}Ti$ at $T_\pi = 295 MeV$. The peak corresponds to a $GDR \otimes GDR$ [7].

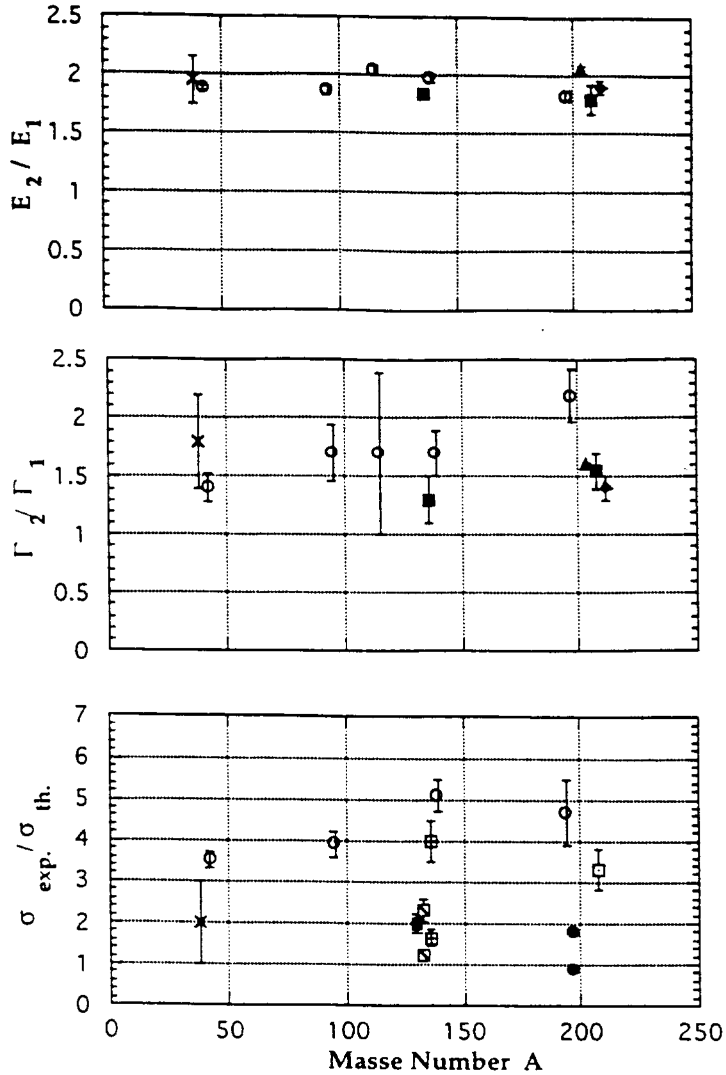

Fig. 4: Systematics on the properties of two phonon states. The first two figures on the top show as a function of the mass number A, the ratio of the energy of the second phonon to the energy of the first phonon and the ratio of the width of the second phonon to the width of the first phonon. These two ratios are consistent with a harmonic picture of the nucleus. The last figure on the bottom shows the ratio of the experimental to the theoretical cross section of the second phonon calculated with this harmonic approximation. As we can see, the experimental value of this cross section is two to four time larger than the theoretical one [3].

systematics, we can see that the ratio of the energy of the two-phonon state to the energy of the one-phonon state is almost two (anharmonicities are at maximum of the order of 10%) and that the width of the two-phonon state is about the width of the one-phonon state times the square root of two (fig.4). These results are consistent with the hypothesis that the nucleus behaves as a harmonic oscillator. Now, the cross section of the two-phonon state, calculated with different approaches which use this harmonic picture happens to be two to four times smaller than the experimental value (fig.4, [3]). In the present contribution, we will show with a very simple model that the introduction of small anharmonicities on the energies of a quantic oscillator and small non-linearities in the exciting coulomb field can change the cross section of the 2-phonon states up to a factor 2.

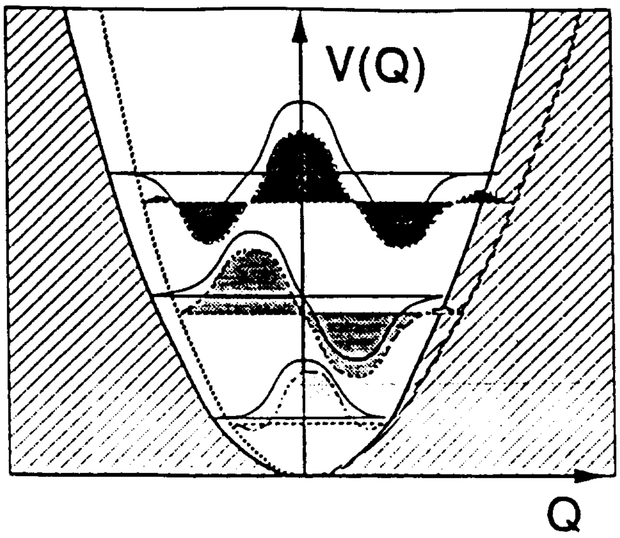

Fig. 5: Schematic representation of a harmonic oscillator potential (solid line) and of an anharmonic one (dashed line). The introduction of anharmonicities changes the energy levels and the corresponding wave functions.

2. A simple model

In our schematic model, we consider a quantic oscillator whose hamiltonian is

$$\hat{H}_{AN} = \frac{\hat{P}^2}{2m} + \frac{1}{2}k\hat{X}^2 + \frac{\alpha}{3}\hat{X}^3 + \frac{\beta}{4}\hat{X}^4 \tag{1}$$

and where the operator $\hat{X}$ measures the deformation. For example, in the case of a GDR, $\hat{X}$ is the distance between the centre of charge of protons and neutrons. The higher order terms $\frac{\alpha}{3}\hat{X}^3$ and $\frac{\beta}{4}\hat{X}^4$ in the harmonic oscillator potential have two effects. On one hand they change the wavefunctions because they introduce a mixing between eigenstates with different number of phonons (fig.5) and on the other hand they change the eigenvalues of the oscillator. The breaking of the harmonicity of the energy spectrum depends on the sign and the values of the coefficients α and β. We have evaluated these coefficients by imposing the matrix elements $< 2|\frac{\alpha}{3}\hat{X}^3|1 >$ and $< 2| : \frac{\beta}{4}\hat{X}^4 : |2 >$ equal to 1MeV, as expected from microscopic calculations [8]. As far as the eigenenergies are concerned, the sign of α is irrelevant whereas a negative or a positive sign of β makes the harmonic potential stiffer or softer, increasing or decreasing the eigenenergies. We have chosen a negative β because, in the case of two-GDR phonon states, their energy should be lower than twice the energy of a GDR because the anharmonicities are expected to produce a reduction of the collectivity of the RPA states [8]. Now, when we consider a heavy-ion collision at relativistic energies, the strongest component of the electric field is the transverse one. If we assume that the projectile of charge Z_P is travelling on a straight line trajectory defined by an impact parameter b and a constant velocity v associated with the Lorentz contraction factor γ [9], the excitation of the transverse GDR degrees of freedom in a nucleus of mass A_T, charge Z_T and neutron number N_T, in the linear response approximation, can be simulated in our one dimensional oscillator model by

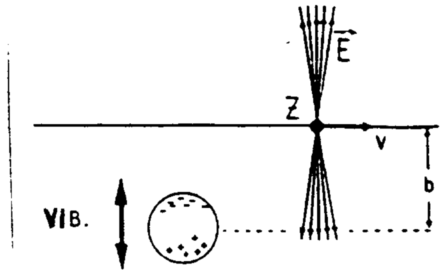

Fig. 6: Schematic representation of a heavy ion collision.

the linear external field

$$\hat{W}(t) = -\frac{Z_T N_T e}{A_T} E_{\perp}(t)\hat{X} = -\frac{Z_T N_T}{A_T} \frac{Z_P e^2 \gamma b}{(b^2 + (\gamma v t)^2)^{\frac{3}{2}}} \hat{X} \tag{2}$$

which corresponds to the excitation of a GDR with 100 % of the energy weighted sum rule (fig.6). The hamiltonian of our oscillator then becomes

$$\hat{H}(t) = \hat{H}_{AN} + \hat{W}(t). \tag{3}$$

In order to find the evolution of our anharmonic oscillator, we can write its wavefunction $|\Psi(t)>$ as

$$|\Psi(t)> = \sum_{\alpha} A_{\alpha}(t) e^{-iE_{\alpha}t} |\phi_{\alpha}> \tag{4}$$

where the basis $|\phi_\alpha>$ and the corresponding eigenergies E_α can be obtained by solving the stationary Schroedinger equation for the hamiltonian (1). The amplitudes $A_\alpha(t)$ which are the projections of the wave function $|\psi(t)>$ on the anharmonic basis $|\phi_\alpha>$ are obtained by solving the time-dependent Schroedinger equation associated with $\hat{H}(t)$ and the boundary condition $|\psi(-\infty)> = |0>$. Then, the probability to excite at a given impact parameter b the first, P_{1b}, or the second P_{2b}, excited state of the anharmonic oscillator will be given by

$$P_{1b} = |A_{1b}(t = +\infty)|^2 \quad \text{and} \quad P_{2b} = |A_{2b}(t = +\infty)|^2 \tag{5}$$

and the integrated cross section σ_α, $\alpha = 1, 2$, can be calculated as

$$\sigma_\alpha = 2\pi \int_{b_0}^{\infty} P_\alpha(b) b db \tag{6}$$

where b_0 has to be taken such that contributions from the nuclear part of the external field can be neglected. We have chosen $b_0 = r_0(A_P^{1/3} + A_T^{1/3})$ with $r_0 = 1.5 fm$.

3. Results

In order to see the influence of anharmonicities on the cross section of the two-phonon states, we have considered the reaction $^{136}Xe(0.7AGeV)$ on ^{208}Pb. The measured energies of the one- and the two-phonon states in ^{136}Xe are $E_{GDR} = 15.2$MeV and $E_{GDR\otimes GDR} = 28.3 \pm 0.7$MeV and the cross sections $\sigma_1 = 1485 \pm 100$mb and $\sigma_2 = 215 \pm 50$mb, respectively [4]. In the reference calculation of a harmonic oscillator ($\alpha = \beta = 0.$) linearly excited, the cross sections result $\sigma_{1ref} = 1153$mb and $\sigma_{2ref} = 33$mb. When we introduce a 10% of anharmonicity in the hamiltonian of the oscillator, requiring $< 2|\frac{\alpha}{3}\hat{X}^3|1 >\approx 1$MeV and $< 2| : \frac{\beta}{4}\hat{X}^4 : |2 >\approx 1$MeV (that means $\alpha = -38.6$MeV/fm^3, $\beta = -188.18$MeV/fm^4), we get, for the ^{136}Xe, $E_{GDR} = 15.2$MeV and $E_{GDR\otimes GDR} = 28.3$MeV, in agreement with the experimental data. The first value has been kept constant by renormalizing the hamiltonian (1). The cross section of the two-phonon state, σ_2, is then a factor 1.67 greater than the value obtained in the reference calculation, whereas the cross section of the one-phonon, σ_1, keeps almost constant (first line of table 1). In order to understand the origin of the large increase of σ_2, we have calculated P_1 and P_2 within perturbation theory. In first order perturbation theory, P_1 is equal to

$$P_1 \simeq |A_1|^2 = |\int_{-\infty}^{+\infty} < \phi_1|\hat{W}(t)|\phi_0 > e^{iE_1t}dt|^2 \tag{7}$$

and can be factorised as

$$P_1 = | < \phi_1|\hat{X}|\phi_0 > |^2|\tilde{F}_1(E_1)|^2 \tag{8}$$

where the first term is a transition matrix element which depends only on the wave functions and the second term, $\tilde{F}_1(E_1)$, is the Fourier transform of the external field evaluated at the energy of the one-phonon E_1. In a similar way, we can write P_2, in second order perturbation theory, as

$$P_2 \simeq |A_2|^2 = \frac{1}{4}| < \phi_2|\hat{X}|\phi_1 >< \phi_1|\hat{X}|\phi_0 > |^2|\tilde{F}_2(E_2, E_1)|^2 \tag{9}$$

where $\tilde{F}_2(E_2, E_1)$ is a double Fourier transform of the external field evaluated at the energies E_1 and E_2 of the one-phonon and two-phonon state respectively. The first order contribution of P_2 which is relative to direct transitions from the ground state to the two-phonon state, has been evaluated and is negligeable. If we consider now the ratio P_2/P_1^2, we have

$$\frac{P_2}{P_1^2} \simeq \frac{1}{4}\frac{| < \phi_2|\hat{X}|\phi_1 > |^2}{| < \phi_1|\hat{X}|\phi_0 > |^2}\frac{|\tilde{F}_2(E_2, E_1)|^2}{|\tilde{F}_1(E_1)|^4} \tag{10}$$

In the harmonic case ($E_2 = 2E_1$), $< \phi_2|\hat{X}|\phi_1 >= \sqrt{2} < \phi_1|\hat{X}|\phi_0 >$ and $\tilde{F}_2(E_2, E_1) = |\tilde{F}_1(E_1)|^2$. Therefore, the above ratio is equal to 1/2, while when anharmonicities are taken into account, it departs from this value. The first factor contains the effects of the anharmonicities present in the wavefunctions, while the second factor contains the effects of the anharmonicities of the energy spectrum. It turns out that both effects are quite important. To show this, in fig. 1, we report the ratio P_2/P_1^2, as a function of the impact parameter b, as obtained from four different calculations. The first one, corresponding to the thick full line, is the complete calculation. The second one, dashed line, is a calculation in which the wave functions are those of the harmonic

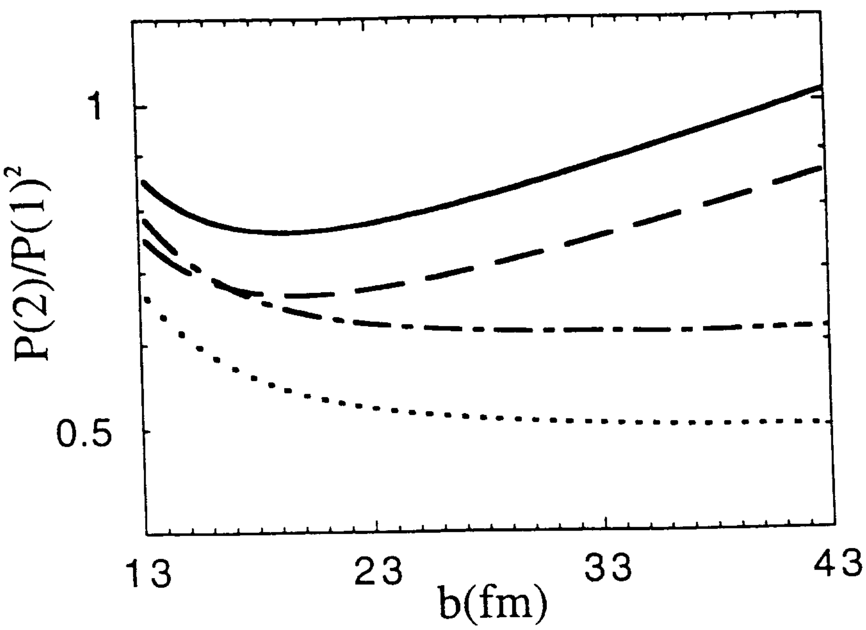

Fig. 7: Ratio of the excitation probability of the two- to the one-phonon states, P_2/P_1^2, as a function of the impact parameter b. The thin solid line shows the results of the reference calculation (harmonic and linear), the thick solid line those of the complete calculation. The dashed line represents the results of the calculation in which only the anharmonicities in the energy spectrum are included. Finally, the dot-dashed line refers to the case where only the mixing of the wavefunctions is included.

hamiltonian while the energies are the anharmonic ones. The third one, dash-dotted line, was obtained by considering harmonic energies while the wavefunctions are those of the anharmonic hamiltonian. For comparison, the results concerning the linear excitation of the harmonic oscillator are also plotted. Looking at the figure, we see that the anharmonicities in the wave functions give rise to a sizeable enhancement of the ratio with respect to the harmonic case, whereas a very large effect comes from the anharmonicities in the energy spectrum, especially for large impact parameters. Now, if we turn to the cross section, we observe that the change of the matrix elements due to the anharmonicities leads to $\sigma_2/\sigma_{2ref} = 1.44$, while the modification of the energy spectrum alone gives $\sigma_2/\sigma_{2ref} = 1.22$. Therefore both effects equally contribute to the factor 1.67 of increase of the two-phonon cross section.

As a final step, we can introduce in the external exciting field non-linearities of the type $\delta\hat{W}(t)\hat{X}$, where the coefficient δ is equal to 4.04fm^{-2} as expected from microscopic calculations [10]. [1] In this complete calculation, a non-linear excitation of an anharmonic oscillator, σ_2 increases by a factor 2 (table 1).

In conclusion, we have seen that the introduction of small anharmonicities in the hamiltonian of an oscillator and non linearities in the relativistic coulomb field change significantly the cross section of the 2-phonon state. In particular, a 10% of anharmonicity and a 17% of non-linearity can increase σ_2 by a factor 2. This important modification of the calculated σ_2 obtained with our simple model indicate that, in a realistic calculation, this could be a way to reduce the discrepancy between

[1] For a microscopic justification of the anharmonic and non-linear terms, see ref.[11].

	σ_1/σ_1^{ref}	σ_2/σ_2^{ref}
lin. and anhar.	1.1	1.67
non-lin. and anhar.	1.17	2.04

Table 1: Ratios of the cross sections of the first and second phonon, σ_1 and σ_2, calculated in the case of a linear excitation of an anharmonic oscillator (first row) and of a non-linear excitation of an anharmonic oscillator (second row) to the reference values, σ_{1ref} and σ_{2ref}, obtained in the case of a linear excitation of a harmonic oscillator.

the theoretical and the experimental cross section of double giant resonances.

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