Subnuclear Physics: Past, Present and Future
Pontifical Academy of Sciences, Scripta Varia 119, Vatican City 2014
www.pas.va/content/dam/accademia/pdf/sv119/sv119-altarelli.pdf

The Mystery of Neutrino Mixing

Guido Altarelli

Dipartimento di Fisica, Universita' di Roma Tre
Rome, Italy
and
CERN, Department of Physics, Theory Division
CH-1211 Genève 23, Switzerland
E-mail: guido.altarelli@cern.ch

ABSTRACT

In the last years we have learnt a lot about neutrino physics. A long list of models have been formulated to understand neutrino masses and mixings. Along the way, with the continuous improvement of the data, most of the models have been discarded by experiment. At present, the surviving models still span a wide range of possibilities, going from a maximum of symmetry, described by discrete non-abelian flavour groups, to the opposite extreme of anarchy. In particular, discrete flavour groups have been studied in connection with special patterns of neutrino mixing suggested by the data, like Tri-Bimaximal mixing (groups A_4, S_4...) or Bi-Maximal mixing (group S_4....) etc. We briefly summarize a number of models based on various patterns and symmetries and compare them with the experimental data.

1. Introduction

So far the main theoretical lessons from ν mass and mixing[1),2)] are that ν's are not all massless but their masses are very small; probably their masses are small because ν's are Majorana fermions with masses inversely proportional to the large scale M of interactions that violate lepton number (L) conservation. From the see-saw formula [3)], the observed atmospheric oscillation frequency and a Dirac mass m_D of the order of the Higgs VEV, it follows that the Majorana mass scale $M \sim m_{\nu R}$ is empirically close to $10^{14} - 10^{15}$ GeV $\sim M_{GUT}$, so that ν masses fit well in the SUSY GUT picture. Decays of ν_R with CP and L violation can produce a sizable B-L asymmetry that survives instanton effects at the electroweak scale thus explaining baryogenesis as arising from leptogenesis. There is still no direct proof that neutrinos are Majorana fermions: detecting neutrino-less double beta decay ($0\nu\beta\beta$) would prove that ν's are Majorana particles and that L is violated. It also appears that the active ν's are not a significant component of dark matter in the Universe.

2. Experimental Highlights

On the experimental side the two main recent developments were, first, that substantial evidence for a non vanishing value of the smallest mixing angle θ_{13} is building up and, second, the coming back of some hints of sterile neutrinos. As well known, the T2K run was suddenly interrupted by the devastating earthquake that hit Japan on March 11, 2011 just minutes away from the scheduled presentation of the first T2K data. Later T2K released the first publication on their data[4)], reporting a 2.5σ signal for $\sin^2 2\theta_{13}$. The T2K result[4)], based on the observation of 6 electron events when 1.5 ± 0.3 are expected for $\theta_{13} = 0$, is converted into a confidence interval $0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34)$ at 90% c.l. for $\sin^2 2\theta_{23} = 1$, $|\Delta m^2_{23}| = 2.4\ 10^{-3} eV^2$, $\delta_{CP} = 0$ and for normal (inverted) neutrino mass hierarchy. Also the MINOS Collaboration released[5)] their corresponding 90% c.l. range as $0(0) \leq \sin^2 2\theta_{13} \leq 0.12(0.19)$, which is displaced towards smaller values with respect to that of T2K. Finally DOUBLE CHOOZ[6)] finds (with only the far detector in operation): $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ at 1σ. Additional input on $\sin^2 2\theta_{13}$ is derived from comparing the 3-neutrino fits with the separate 2-neutrino fits for solar and atmospheric oscillations. These results on $\sin\theta_{13}$ have very important implications on neutrino oscillation physics. First, it is very good news for the possibility of detecting CP violation in neutrino oscillations. Second, the relatively large central values for $\sin^2\theta_{13}$ in the fits of Table 1 have a strong impact in discriminating models of neutrino mixing. In fact, these values correspond to $\sin\theta_{13} \sim 0.158$ or 0.114, which is comparable to $\lambda_C = \sin\theta_C \sim 0.226$ or perhaps to $\lambda_C^2 \sim 0.051$.

On the evidence for sterile neutrinos a number of hints have been reported recently. They do not make yet a clear evidence but certainly pose an experimental problem that needs clarification. First, there is the MiniBooNE experiment[7)] that in the antineutrino channel reports an excess of events supporting the LSND oscillation signal (originally observed with antineutrinos). The MiniBooNE best fit point falls in an excluded area but there is an overlap with the LSND signal in an allowed region. In the neutrino channel MiniBooNE did not observe a signal in the LSND domain. However, in these data there is a unexplained excess at low energy over the (reliably?) estimated background. Consequently, in the neutrino data sample, for the search of a LSND-like signal, only the events with neutrino energy above a threshold value E_{th} were used, leaving the issue of an explanation of the low energy excess unanswered. In the antineutrino channel most of the support to the LSND signal appears to arise from an excess above E_{th} but quite close to it, so that there is, in my opinion, some room for perplexity. More recently an update of the MiniBooNE data in the antineutrino channel shows less supporting evidence[8)]. Then there is the reactor anomaly: a reevaluation of the reactor flux[9)] produced an apparent gap between the theoretical expectations and the data taken at small distances from the reactor ($\lesssim 100$ m). The discrepancy is of the same order of the quoted systematic error whose estimate, detailed in the paper, should perhaps be reconsidered. Similarly the Gallium anomaly[10] depends on the assumed cross-section which could be questioned. The reactor anomaly and the Gallium anomaly do not really agree on the oscillation parameters that they point to: the Δm^2 values are compatible but the central values of $\sin^2 2\theta$ differ by about an order of magnitude, with Gallium favouring the larger angle. Cosmological data allow the existence of one sterile neutrino, while the most stringent bounds arising form nucleosynthesis disfavour two or more sterile neutrinos[11]. Over all, only a small leakage from active to sterile neutrinos is allowed by present neutrino oscillation data[12]. If all the indications listed above were confirmed (it looks unlikely) then 1 sterile neutrino would not be enough and at least 2 would be needed with sub-eV masses. Establishing the existence of sterile neutrinos would be a great discovery. In fact a sterile neutrino is an exotic particle not predicted by the most popular models of new physics. A sterile neutrino is not a 4th generation neutrino: the latter is coupled to the weak interactions (it is active) and heavier than half the Z mass. A sterile neutrino would probably be a remnant of some hidden sector. The issue is very important so that new and better experimental data are badly needed.

In neutrino oscillations the leakage from the three active species towards the sterile neutrinos is any case small and, in fact, the bulk of oscillation phenomena is well described in terms of 3-neutrino models. In the following we will neglect this possible small leakage to sterile neutrinos and concentrate on 3-neutrino models. The results of two fits of all the present data are summarised in Table(1)[13],[14].

Quantity	Fogli et al[13]	Schwetz et al[14]
Δm^2_{sun} (10^{-5} eV^2)	$7.58^{+0.22}_{-0.26}$	$7.59^{+0.20}_{-0.18}$
Δm^2_{atm} (10^{-3} eV^2)	$2.35^{+0.12}_{-0.09}$	$2.50^{+0.09}_{-0.16}$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.016}$	$0.312^{+0.017}_{-0.015}$
$\sin^2\theta_{23}$	$0.42^{+0.08}_{-0.03}$	$0.52^{+0.06}_{-0.07}$
$\sin^2\theta_{13}$	0.025 ± 0.007	$0.013^{+0.007}_{-0.005}$

Table 1:
Fits to neutrino oscillation data. The results correspond to the new reactor fluxes. The fit of Schwetz et al[14] refers to the normal hierarchy case (in the inverse hierarchy case the main difference is that $\sin^2\theta_{13} = 0.016 + 0.008 - 0.006$)

For the near future the most important experimental challenges on neutrino oscillation experiments are more precise measurements of the absolute scale of neutrino mass (KATRIN, MARE), the search for neutrinoless double beta decay ($0\nu\beta\beta$ (CUORE, GERDA, ...), the accurate determination of θ_{13} (from MINOS, T2K and the reactor experiments DOUBLE CHOOZ, Daya Bay and RENO) and of the shift

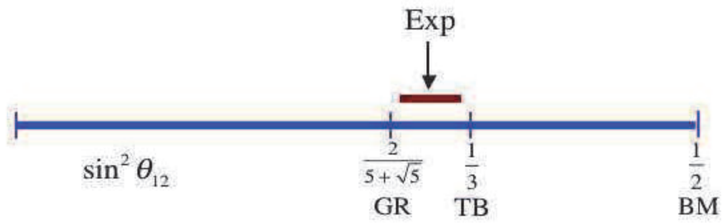

Figure 1: The values of $\sin^2\theta_{12}$ for TB or GR or BM mixing are compared with the data

from maximal of θ_{23}, the fixing of the sign of Δm^2_{23} (normal or inverse hierarchy) (e.g. NOνA) and the detection of CP violation in ν oscillations. Related to neutrino physics is the issue of the non conservation of the separate e, μ and τ lepton numbers. The recent new limit $\mathrm{Br}(\mu \to e\gamma) \lesssim 2.4 \cdot 10^{-12}$ obtained by the MEG experiment[15] is largely satisfied in the SM but it imposes a strong constraint on SUSY-GUT models.

3. Models of Neutrino Mixing

To illustrate the impact of the new results on θ_{13} on models of neutrino mixing, we consider the case of models based on discrete flavour groups that have received a lot of attention in recent years[16]. There are a number of special mixing patterns that have been studied in this context. These mixing matrices all have $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 0$ and differ by the value of $\sin^2\theta_{12}$ (see Fig. 1). The corresponding mass matrices are 2-3 symmetric , i.e. $\mu - \tau$ symmetric (see, as examples, the early work in ref.[17] and the recent paper ref.[18]). The observed value of $\sin^2\theta_{12}$[13],[14] the best measured mixing angle, is very close, from below, to the so called Tri-Bimaximal (TB) value[19] which is $\sin^2\theta_{12} = 1/3$. Alternatively it is also very close, from above, to the Golden Ratio (GR) value[20],[21],[22] which is $\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5+\sqrt{5}} \sim 0.276$, where $\phi = (1+\sqrt{5})/2$ is the GR (for a different connection to the GR in this context, see[23],[24]). Thus, a possibility is that one or the other of these coincidences is taken seriously and this leads to models where either TB or GR mixing is naturally predicted as a good first approximation. On a different perspective, one has considered models with Bi-Maximal (BM) mixing, with $\sin^2\theta_{12} = 1/2$, i.e. also maximal, as the value before diagonalization of charged leptons. This is in line with the well known empirical observation that $\theta_{12}+\theta_C \sim \pi/4$, a relation known as quark-lepton complementarity[25]. Probably the exact complementarity relation becomes more plausible if replaced with $\theta_{12} + \mathcal{O}(\theta_C) \sim \pi/4$ (which we could call "weak" complementarity). One can think of models where, because of a suitable symmetry, BM mixing holds in the neutrino sector at leading order and the necessary, rather large, corrective terms for θ_{12} arise from the diagonalization of charged lepton masses[25].

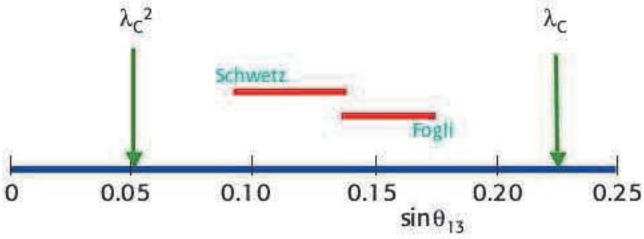

Figure 2: The experimental values of $\sin\theta_{13}$ (world averages derived from Table(1)[13),14)]) are numerically intermediate between $\mathcal{O}(\lambda_C^2)$ and $\mathcal{O}(\lambda_C)$

Thus, a possibility is that one of these coincidences is taken seriously and this leads to models where TB or GR or BM mixing is naturally predicted as a good first approximation. In the following we will mainly refer to TB or BM mixing which are the most studied first approximations to the data. The simplest symmetry that, in leading order (LO), leads to TB is A_4 while BM can be obtained from S_4. In the literature A_4 models have been widely studied (for a review and a list of references, see[16)]). At LO the typical A_4 model leads to exact TB mixing. The LO approximation is then corrected by non leading effects. Given the set of flavour symmetries and having specified the field content, the non leading corrections to TB mixing, arising from higher dimensional effective operators, can be evaluated in a well defined expansion. In the absence of specific dynamical tricks, in a generic model, all three mixing angles receive corrections of the same order of magnitude. Since the experimentally allowed departures of θ_{12} from the TB value, $\sin^2\theta_{12} = 1/3$, are small, numerically not larger than $\mathcal{O}(\lambda_C^2)$, it follows that both θ_{13} and the deviation of θ_{23} from the maximal value are also expected to be typically of the same general size. The same qualitative conclusion also applies to A_5 models for GR mixing. This generic prediction of θ_{13} small, numerically of $\mathcal{O}(\lambda_C^2)$ can now be confronted with the most recent data. The central values $\sin\theta_{13} \sim 0.16$ or 0.11 that can be derived from the experimental results in the two columns of Table(1), respectively, are in between $\mathcal{O}(\lambda_C^2) \sim \mathcal{O}(0.05)$ and $\mathcal{O}(\lambda_C) \sim \mathcal{O}(0.23)$. Although models based on TB (or GR) mixing tend to lead to a rather small value of θ_{13} one can argue that they are still viable with preference for the lower side of the experimental range (see Fig. 2).

It is to be stressed in this context that, of course, one can introduce some additional theoretical input to enhance the value of θ_{13}. In the case of A_4, one particularly interesting example is provided by the Lin version of the A_4 model[26)], formulated before the T2K, MINOS and DOUBLE CHOOZ results were known. In the Lin model the A_4 symmetry breaking is arranged, by suitable additional Z_n parities, in such a way that, not only at LO but also at next-to-the-leading (NLO), the corrections to the charged lepton and the neutrino sectors are kept separate. Then the contributions to neutrino mixing from the diagonalization of the charged leptons can be of $\mathcal{O}(\lambda_C^2)$ while those in the neutrino sector can be of $\mathcal{O}(\lambda_C)$. In addition, in the Lin model these large corrections do not affect θ_{12} and satisfy the relation $\sin^2\theta_{23} = 1/2 + 1/\sqrt{2}\cos\delta\sin\theta_{13}$, with δ being an unspecified phase. Thus in the Lin model the NLO corrections to the solar angle θ_{12} and to the reactor angle θ_{13} can naturally be of different orders.

Alternatively one can think of models where, because of a suitable symmetry, BM mixing holds in the neutrino sector at LO and the corrective terms for θ_{12}, which in this case are necessarily rather large, arise from the diagonalization of charged lepton masses[25)]. These terms numerically of order $\mathcal{O}(\lambda_C)$ from the charged lepton sector would then generically also affect θ_{13} and the resulting value could well be compatible with the present experimental values of θ_{13}. An explicit model of this type based on the group S_4 has been developed in ref.[27)] . An important feature of this model is that only θ_{12} and θ_{13} are corrected by terms of $\mathcal{O}(\lambda_C)$ while θ_{23} is unchanged at this order. This model is compatible with present data and clearly prefers the upper range of the present experimental interval for θ_{13}. Recently the model was extended to include quarks in a $SU(5)$ Grand Unified version[28)].

It is important to keep in mind that the implications of lepton flavour violating processes for the three classes of possibilities, e.g. TB mixing in a typical A_4 model, the Lin version of A_4 and BM in S_4, are quite different and the present bounds pose severe constraints on the respective models. In particular we refer to the recent improved MEG result[15)] on the $\mu \to e\gamma$ branching ratio and to other similar processes like $\tau \to (e \text{ or } \mu)\gamma$. It appears[16)] that the safest class of models is one where no large corrective terms of order $\mathcal{O}(\lambda_C)$ are present in either the charged or the neutral lepton sectors. The most dangerous case is that of the models where large terms directly appear in the off diagonal terms of the charged lepton mass matrix.

We now briefly turn to models that do not take seriously any of the coincidences described above (the proximity of the data to the TB or GR patterns or the quark-lepton complementarity: these indications cannot all be relevant and it is possible that none of them is so) and are therefore based on a less restrictive flavour symmetry. There are many possible models that fit the data on mixing angles well and yet have no TB or GR or BM built in in their structure (for a largely incomplete list of examples see[29)]). It is clear that the T2K hint that θ_{13} may be large is great news for the most extreme position of this type, which is "anarchy"[30)]: no symmetry at all in the lepton sector, only chance. This view predicts generic neutrino mixing angles, so the largest θ_{23} should be different than maximal and the smallest θ_{13} should be as large as possible within the experimental bounds. Anarchy can be formulated in a $SU(5) \otimes U(1)$ context by taking different Froggatt-Nielsen[31)] charges only for the $SU(5)$ tenplets (for example 10: (3,2,0), where 3 is the charge of the first generation, 2 of the second, zero of the third) while no charge differences appear in the $\bar{5}$: $\bar{5}$: (0,0,0). This assignment is in agreement with the empirical fact that the mass hierarchies are more pronounced for up quarks in comparison with down quarks and charged leptons. In a non see-saw model, with neutrino masses dominated by the contribution of the dimension-5 Weinberg operator [32)], the $\bar{5}$ vanishing charges directly lead to random neutrino mass and mixing matrices. In anarchical see-saw models also the charges of the SU(5) singlet right-handed neutrinos must be undifferentiated. Anarchy can be mitigated by assuming that it only holds in the 2-3 sector: e.g $\bar{5}$: (2,0,0) with the advantage that the first generation masses and the angle θ_{13} are naturally small (see also the recent revisiting in ref.[33)]). In models with see-saw one can alternatively play with the charges for the right-handed SU(5) singlet neutrinos. If, for example, we take 1: (1, -1, 0), together with $\bar{5}$: (2,0,0), it is possible to get a normal hierarchy model with θ_{13} small and also with $r = \Delta m^2_{solar}/\Delta m^2_{atm}$ naturally small (see, for example, ref.[34)]). In summary anarchy and its variants, all based on chance, offer a rather economical class of models that are among those encouraged by the new θ_{13} result.

4. Conclusion

In the last decade we have learnt a lot about neutrino masses and mixings. A list of important conclusions have been reached. Neutrinos are not all massless but their masses are very small. Probably masses are small because neutrinos are Majorana particles with masses inversely proportional to the large scale M of lepton number violation. It is quite remarkable that M is empirically not far from M_{GUT}, so that neutrino masses fit well in the SUSY GUT picture. Also out of equilibrium decays with CP and L violation of heavy RH neutrinos can produce a B-L asymmetry, then converted near the weak scale by instantons into an amount of B asymmetry compatible with observations (baryogenesis via leptogenesis)[35)]. It has been established that most probably active neutrinos are not a significant component of dark matter in the Universe. We have also understood there there is no contradiction between large neutrino mixings and small quark mixings, even in the context of GUTs.

This is a very impressive list of achievements. Coming to a detailed analysis of neutrino masses and mixings a long collection of models have been formulated over the years. With continuous improvements of the data and more precise values of the mixing angles most of the models have been discarded by experiment. Still the surviving models span a wide range going from a maximum of symmetry, with discrete non-abelian flavour groups, to the opposite extreme of anarchy. By now, besides the detailed knowledge of the entries of the V_{CKM} matrix, we also have a reasonable determination of the neutrino mixing matrix U_{PMNS}. The data appear to suggest some special patterns (recall Fig. 1) like TB or GR or BM mixing to be valid in some leading approximation, corrected by small non leading terms. If one takes these "coincidences" seriously, then non-abelian discrete flavour groups emerge as the main road to an understanding of this mixing pattern. Indeed the entries of e.g. TB mixing matrix are clearly suggestive of "rotations" by simple, very specific angles. It is remarkable that neutrino and quark mixings have such a different qualitative pattern. An obvious question is whether some additional indication for discrete flavour groups can be obtained by considering the extension of the models to the quark sector, perhaps in a Grand Unified context. The answer appears to be that, while the quark masses and mixings can indeed be reproduced in models where TB or BM mixing is realized in the leptonic sector through the action of discrete groups, there are no specific additional hints in favour of discrete groups that come from the quark sector[16)]. Further important input could come from $\mu \to e\gamma$ and in general from lepton flavour violating processes, from $b \to s\gamma$ and from LHC physics. In fact, new physics at the weak scale could have important feedback on the physics of neutrino masses and mixing.

It is expected that in the near future, we will know the value of θ_{13} with a good accuracy, from the continuation of T2K and from the reactor experiments DOUBLE CHOOZ, Daya Bay and RENO. Many existing models will be eliminated and the surviving ones will be updated to become more quantitative in order to cope with a precisely known mixing matrix. A sizable θ_{13} will encourage the planning of long baseline experiments for the detection of CP violation in neutrino oscillations. Along the way the important issue of the existence of sterile neutrinos must be clarified. The on going or in preparation experiments on the absolute value of neutrino masses, on $0\nu\beta\beta$, on $\mu \to e\gamma$, on the search for dark matter etc can also lead to extremely important developments in the near future. So this field is very promising and there all reasons to expect an exciting time ahead of us.

Finally, one could have imagined that neutrinos would bring a decisive boost towards the formulation of a comprehensive understanding of fermion masses and mixings. In reality it is frustrating that no real illumination was sparked on the problem of flavour. We can reproduce in many different ways the observations, in a wide range that goes from anarchy to discrete flavour symmetries) but we have not yet been able to single out a unique and convincing baseline for the understanding of fermion masses and mixings. In spite of many interesting ideas and the formulation of many elegant models the mysteries of the flavour structure of the three generations of fermions have not been much unveiled.

5. Acknowledgments

It is a very pleasant duty for me to most warmly thank Professors Werner Arber and Antonino Zichichi, Monsignor Marcelo Snchez Sorondo and the whole Organising Committee for their kind invitation and for the great hospitality offered to all of us in the Vatican. I am also glad to acknowledge interesting discussions on this subject

with Ferruccio Feruglio, Davide Meloni and Luca Merlo.

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