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Global Analysis of Leptophilic Z' Bosons

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Abstract: New neutral heavy gauge bosons (Z') are predicted within many extensions of the Standard Model. While in case they couple to quarks the LHC bounds are very stringent, leptophilic Z' bosons (even with sizable couplings) can be much lighter and therefore lead to interesting quantum effects in precision observables (like $(g-2)_\mu$) and generate flavour violating decays of charged leptons. In particular, $\ell \to \ell' \nu\bar{\nu}$ decays, anomalous magnetic moments of charged leptons, $\ell \to \ell'\gamma$ and $\ell \to 3\ell'$ decays place stringent limits on leptophilic Z' bosons. Furthermore, in case of mixing Z' with the SM Z, Z pole observables are affected. In light of these many observables we perform a global fit to leptophilic Z' models with the main goal of finding the bounds for the Z' couplings to leptons. To this end we consider a number of scenarios for these couplings. While in generic scenarios correlations are weak, this changes once additional constraints on the couplings are imposed. In particular, if one considers an $L_\mu - L_\tau$ symmetry broken only by left-handed rotations, or considers the case of $\tau-\mu$ couplings only. In the latter setup, on can explain the $(g-2)_\mu$ anomaly and the hint for lepton flavour universality violation in $\tau \to \mu\nu\bar{\nu}/\tau \to e\nu\bar{\nu}$ without violating bounds from electroweak precision observables.

Contents

1 Introduction

In 2012, the LHC confirmed the predictions of the Standard Model (SM) of particle physics by discovering the (Brout-Englert) Higgs boson [1, 2]. However, so far no particles beyond the ones of the SM have been observed in high energy searches. In particular, the bounds from di-jet [3, 4] and di-lepton [5–7] searches on particles that can be produced resonantly in the s-channel are very stringent. This also puts tight bounds on heavy neutral gauge bosons (Z's), which are predicted by many new physics models (see [8–14]), in case they have sizable couplings to quarks.

However, in case the resonances are neutral and couple (to a good approximation) only to leptons, mostly LEP searches apply and the bounds are much weaker [15], i.e. significantly below the TeV scale. Therefore, such leptophilic Z' bosons can have sizable couplings while at the same time being quite light. They can thus lead to relevant quantum corrections to leptonic precision observables and generate lepton flavour violating decays of leptons that are extremely suppressed in the SM since they vanish in the limit of massless neutrinos. The $(g-2)_\mu$ discrepancy [16, 17], recently reinforced by the $g-2$ experiment at Fermilab [18–21], with a tension of $4.2\,\sigma$ compared to the SM prediction [22], can be explained with a Z' boson heavier than the electroweak (EW) scale if it couples flavour violatingly to the second and third lepton generation [23–41].

At first sight, new neutral gauge bosons coupling only to leptons and not to quarks might appear artificial. But as already in the SM gluons couple only to quarks and not to leptons, it is actually an interesting possibility that for a Z' boson the situation could be reversed. For example, gauged abelian flavour symmetries in the lepton sector, such as $L_\mu - L_\tau$ [42–44], can naturally generate the observed pattern of the PMNS matrix [45–47] and lead by definition to leptophilic Z' bosons, which, after the breaking of the symmetry, can also induce charged LFV processes such as $\tau \to 3\mu$ [29, 48, 49] and $h \to \mu\tau$ [49–51].

Such scenarios are particularly interesting as within recent years several hints for the violation of lepton flavour universality (LFU) have been acquired. These include $\tau \to \mu\nu\bar{\nu}/\tau \to e\nu\bar{\nu}$, $\tau \to \mu\nu\bar{\nu}/\mu \to e\nu\bar{\nu}$ [52, 53] and the Cabibbo angle anomaly [54–57], which can also be interpreted as a sign of LFUV [58–69]. Furthermore, even though in this case also (small) couplings to bottom and strange quarks are necessary, Z' bosons are among the prime candidates for explaining the discrepancies between the SM predictions and data in $b \to s\ell^+\ell^-$ transitions [12, 33, 49, 51, 65, 70–104]. Here, LHCb measurements [105, 106] indicate a deficit in muons with respect to electrons, i.e. LFUV with a combined significance of $\approx 4\sigma$ [107–117]. This is consistent with many other measurements involving the same current, in particular with angular observables [118, 119], where data also shows a deficit in muonic channels [120, 121] such that the most up-to-date global analysis finds several NP scenarios to be preferred over the SM at the $5-6\sigma$ level [114–116, 122, 123]. In order to respect LHC bounds, it is again advantageous if the couplings to quarks are small, i.e. if the Z' is to a good approximation leptophilic, which can e.g. be achieved by generating the quark couplings effectively via heavy vector-like quarks [33, 49, 124].

Therefore, it is very interesting to explore the phenomenology of leptophilic Z' bosons. Even in the absence of quark couplings, such a Z' boson affects many observables with the most interesting being

- $\ell \to \ell'\gamma$ decays,
- Anomalous magnetic moments (AMMs) and electric dipole moments (EDMs) of charged leptons,
- $\ell \to 3\ell'$ decays (7 in total),
- $Z \to \ell\ell^{(\prime)}$ decays,
- LFU violation in $\ell \to \ell'\nu\bar{\nu}$
- Neutrino trident production
- LEP searches for contact interactions

Therefore, in order to fully explore the allowed/preferred parameter space of such a new physics scenario, the aim of this article is to perform a global fit to all available data in a number of scenarios for Z' couplings to leptons. For this we will use the publicly available HEPfit code [125] which also allows us to perform a global fit taking into account many degrees of freedom at the same time.

The article is structured as follows: In the next section we will define our setup before we consider the relevant observables, and calculate the relevant Z' contributions to them in Sec. 3. We then perform our phenomenological analysis in Sec. 4 before concluding in Sec. 5. In Appendix A we give some details on the LEP-II bounds and in Appendix B we list the contributions of QED penguins to LFV decays like $\ell \to 3\ell'$ and to $\mu \to e$ conversion in nuclei. In Appendix C we present additional scenarios for LFV couplings beyond those presented in the main text.

2 Setup

We extend the SM by adding a heavy neutral gauge boson Z'_0 (i.e. with a mass above the electroweak symmetry breaking scale). Following Ref. [126, 127] we supplement the SM Lagrangian by a part containing the kinetic terms and the mass terms of the Z'_0-field,

$$\mathcal{L}_{Z'_0} = -\frac{1}{4} Z'_{0,\mu\nu} Z_0^{\prime\mu\nu} + \frac{\mu_Z^{\prime 2}}{2} Z'_{0\mu} Z_0^{\prime\mu} \,, \tag{2.1}$$

where $Z'_{0,\mu\nu} \equiv \partial_\mu Z'_{0\nu} - \partial_\nu Z'_{0\mu}$ is the field strength tensor associated to the Z'_0-field, and a part describing the interactions of the Z'_0-field with the SM fields,

$$\begin{aligned}\mathcal{L}^{int}_{Z'_0} &= g_{Z'} Z'_{0\mu} Z'^{\mu}_0 \phi^\dagger \phi - i g^\phi_{Z'} Z'^{\mu}_0 \phi^\dagger \overleftrightarrow{D}_\mu \phi \\ &+ \bar{\ell}_i \left(g^L_{ij} \gamma_\mu P_L + g^R_{ij} \gamma_\mu P_R \right) \ell_j Z'^{\mu}_0 + \bar{\nu}_i g^L_{ij} \gamma_\mu P_L \nu_j Z'^{\mu}_0 \,,\end{aligned} \tag{2.2}$$

where $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - (\overleftarrow{D}_\mu)^\dagger$ and $g^\phi_{Z'}$ is real. Note that here the subscript 0 refers to the fact that these are not mass, but rather interaction eigenstates. $g^{L/R}_{ij}$ are hermitian and due to $\mathrm{SU}(2)_L$ invariance the coupling to neutrinos is the same as to left-handed charged leptons[1]. ϕ is the SM Higgs $SU(2)_L$ doublet and we use

$$D_\mu = \partial_\mu + i g_2 W^a_\mu T^a + i g_1 Y B_\mu \,, \tag{2.3}$$

as the definition of the covariant derivative.

The coupling $g^\phi_{Z'}$ leads to mixing of the Z'_0-boson with the SM Z. The corresponding mass matrix in the interaction eigenbasis $(Z_0,\, Z'_0)$ is then given by

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{Z_0} & -\frac{y}{c_W} \\ -\frac{y}{c_W} & M^2_{Z'_0} \end{pmatrix} \,, \qquad y \equiv \frac{v^2}{2}\, g_2\, g^\phi_{Z'} \tag{2.4}$$

with $m^2_{Z_0} = \frac{v^2}{4}\left(g_1^2 + g_2^2\right)$ and $\frac{v}{\sqrt{2}} \approx 174\,\mathrm{TeV}$. To order $\frac{v^2}{m^2_{Z'_0}}$ the eigenvalues are

$$m^2_Z \simeq m^2_{Z_0} - \frac{y^2}{c^2_W M^2_{Z'_0}} \equiv m^2_{Z_0}\left(1 + \delta m^2_Z\right) \,, \tag{2.5}$$

$$M^2_{Z'} \simeq M^2_{Z'_0} + \frac{y^2}{c^2_W M^2_{Z'_0}} \,. \tag{2.6}$$

Hence the corrections to the mass of the SM Z_0 can only be destructive. The mass eigenstates $Z^{(\prime)}$ can then be expressed as

$$\begin{pmatrix} Z' \\ Z \end{pmatrix} = \begin{pmatrix} Z'_0 \cos\xi \; - Z_0 \sin\xi \\ Z'_0 \sin\xi + Z_0 \cos\xi \end{pmatrix} \tag{2.7}$$

where

$$\sin\xi \simeq \frac{y}{c_W M^2_{Z'_0}} \tag{2.8}$$

describes the $Z_0 - Z'_0$ mixing. Note that only the relative phase between $\sin\xi$ and $g^{L,R}_{ij}$ is physical. Therefore, one can assume one of the diagonal couplings $g^{L,R}_{ii}$ to

[1]Here we neglected small active neutrino masses and therefore set the PMNS matrix to the unit matrix.

be positive without loss of generality. We write the interactions of the SM Z with fermions as

$$\begin{aligned}\mathcal{L}_{Zff} = &\bar{\ell}_i\gamma_\mu\left(\Delta^{\ell L}_{ij}P_L + \Delta^{\ell R}_{ij}P_R\right)\ell_j Z^\mu + \bar{\nu}_i\gamma_\mu\Delta^{\nu L}_{ij}P_L\nu_j Z^\mu \\ &+ \bar{u}_k\,\gamma_\mu\left(g^{uL}_{\rm SM}P_L + g^{uR}_{\rm SM}P_R\right)u_k\,Z^\mu + \bar{d}_l\,\gamma_\mu\left(g^{dL}_{\rm SM}P_L + g^{dR}_{\rm SM}P_R\right)d_l\,Z^\mu\,,\end{aligned} \tag{2.9}$$

with $i,j = e,\mu,\tau$, $k = u,c,t$, $l = d,s,b$ and

$$\Delta^{\ell L,R}_{ij} \simeq \sin\xi\, g^{L,R}_{ij} + g^{\ell L,R}_{\rm SM}\delta_{ij}\,, \qquad \Delta^{\nu L}_{ij} \simeq \sin\xi\, g^{L}_{ij} + g^{\nu L}_{\rm SM}\delta_{ij}\,, \tag{2.10}$$

where $g^{\ell,\nu\,L,R}_{\rm SM}$ are the SM $Z\bar{\ell}\ell$ and $Z\bar{\nu}\nu$ couplings given by

$$\begin{aligned}
g^{\nu L}_{\rm SM} &= -\frac{e}{2s_W c_W}\,, \\
g^{\ell L}_{\rm SM} &= -\frac{e}{2s_W c_W}\left(-1 + 2s_W^2\right)\,, \qquad & g^{\ell R}_{\rm SM} &= -\frac{e\,s_W}{c_W}\,, \\
g^{uL}_{\rm SM} &= -\frac{e}{s_W c_W}\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\,, \qquad & g^{uR}_{\rm SM} &= \frac{2}{3}\frac{e\,s_W}{c_W}\,, \\
g^{dL}_{\rm SM} &= -\frac{e}{s_W c_W}\left(-\frac{1}{2} + \frac{1}{3}s_W^2\right)\,, \qquad & g^{dR}_{\rm SM} &= -\frac{1}{3}\frac{e\,s_W}{c_W}\,,
\end{aligned} \tag{2.11}$$

with $e = g_1 g_2/\sqrt{g_1^2 + g_2^2} = g_1 c_W = g_2 s_W$ being the electric charge.

Z' scenarios are in general subject to gauge anomalies, which are often assumed to be canceled by additional heavy fields at a higher scale [9, 128, 129]. This is what we will do in the present paper. Gauge anomaly cancellation in Z' models was discussed recently in Refs. [100, 102, 109, 130].

3 Basic Formulae for Observables

3.1 Lepton Flavor Universality

With this setup we are in a position to calculate the effects in the relevant observables. Flavour off-diagonal couplings already generate at tree-level $\ell \to \ell'\nu\bar{\nu}$ amplitudes which interfere with the SM one, while flavour diagonal couplings can only do this via box diagrams (see Fig. 1). Neglecting non-interfering contributions we thus have

$$R\left[\tau \to \mu\right] = \frac{\mathcal{A}(\tau \to \mu\,\nu_\tau\bar{\nu}_\mu)}{\mathcal{A}(\tau \to \mu\,\nu_\tau\bar{\nu}_\mu)_{SM}} = 1 + 2\frac{|g^L_{\mu\tau}|^2}{g_2^2}\frac{m_W^2}{M_{Z'}^2} - \frac{3}{8\pi^2}\,g^L_{\mu\mu}\,g^L_{\tau\tau}\frac{\ln\left(\frac{m_W^2}{M_{Z'}^2}\right)}{1 - \frac{M_{Z'}^2}{m_W^2}}\,, \tag{3.1}$$

where $\mathcal{A}$ denotes the amplitude. Analogous expressions for $\tau \to e\,\nu_\tau\bar{\nu}_e$ and $\mu \to e\,\nu_\mu\bar{\nu}_u$ follow by a straightforward exchange of indices. This has to be compared to

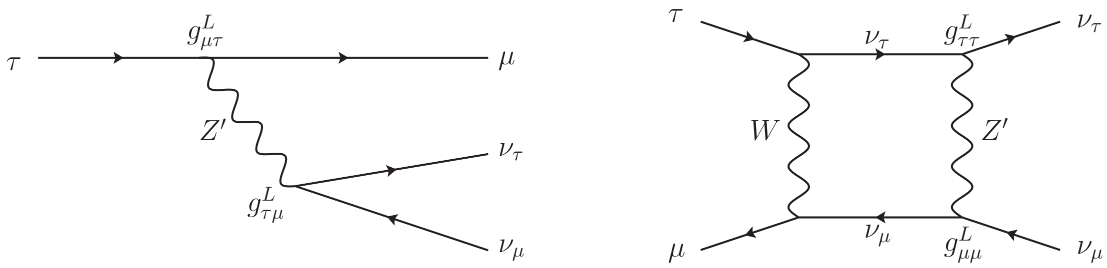


Figure 1. Feynman diagrams illustrating the leading Z'-contributions to the process $\tau \to \mu\nu_\tau\bar{\nu}_\mu$. In the presence of flavour off-diagonal Z'-couplings, the process arises already at tree-level (see left diagram), whereas in the case of flavour-diagonal couplings, corrections to $\tau \to \mu\nu_\tau\bar{\nu}_\mu$ are induced by four boxes, one of which is shown on the right.

the experimental results [53]

$$\begin{aligned} \left.\frac{\mathcal{A}\,[\tau \to \mu\nu\bar{\nu}]}{\mathcal{A}\,[\mu \to e\nu\bar{\nu}]}\right|_{\text{EXP}} &= 1.0029 \pm 0.0014\,, \\ \left.\frac{\mathcal{A}\,[\tau \to \mu\nu\bar{\nu}]}{\mathcal{A}\,[\tau \to e\nu\bar{\nu}]}\right|_{\text{EXP}} &= 1.0018 \pm 0.0014\,, \\ \left.\frac{\mathcal{A}\,[\tau \to e\nu\bar{\nu}]}{\mathcal{A}\,[\mu \to e\nu\bar{\nu}]}\right|_{\text{EXP}} &= 1.0010 \pm 0.0014\,, \end{aligned} \tag{3.2}$$

with the correlation matrix [53]

$$\begin{pmatrix} 1.00 & 0.49 & 0.51 \\ 0.49 & 1.00 & -0.49^{2} \\ 0.51 & -0.49 & 1.00 \end{pmatrix} . \tag{3.3}$$

Furthermore, the effect in $\mu \to e\,\nu_\mu\overline{\nu}_e$ is related to a modification of the Fermi constant which enters not only electroweak precision observables (to be discussed later) but also in the determination of V_{ud} from beta decays, in particular super-allowed beta decays, which allow for the most precise determination of V_{ud}. Here a tension with kaon, tau and D decays has been observed, whose significance depends strongly on the radiative corrections applied to β decays [57, 131–137], but also on the treatment of tensions between $K_{\ell 2}$ and $K_{\ell 3}$ decays [138] and on the bounds from τ decays [53], see Ref. [59] for more details. In the end, quoting a significance of 3σ should provide a realistic representation of the current situation, and for definiteness

[2] The HFLAV collaboration reports -0.50, however for the practical usage we choose -0.49 to have a positive semi-definite correlation matrix.

we will thus use the estimate of the first-row CKM unitarity violation from Ref. [139]

$$\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 = 0.9985(5). \tag{3.4}$$

In addition, note that there is also a deficit in the first-column CKM unitarity relation [139]

$$\left|V_{ud}\right|^2 + \left|V_{cd}\right|^2 + \left|V_{td}\right|^2 = 0.9970(18), \tag{3.5}$$

less significant than Eq. (3.4), but suggesting that if the deficits were due to NP, they would likely be related to β decays. This unitarity deficit constitutes the so-called Cabibbo Angle Anomaly (CAA) and could be alleviated by our NP effect given by

$$R\left(\mu \to e\right) = 0.00075 \pm 0.00025, \tag{3.6}$$

with $R(\mu \to e)$ defined in Eq. (3.1) with properly changed flavour indices. Note that in these tests of LFU we can neglect the modifications of the $W\ell\nu$ vertices since these would be loop-induced and thus suppressed by a factor $m_\ell^2/M_Z'^2$.

3.2 Lepton Flavor Violation in $\ell_j \to \ell_i\gamma$

Defining the effective Hamiltonian by

$$\mathcal{H}_{NP} = c_L^{ij}\, \bar{\ell}_i\, \sigma_{\mu\nu} P_L\, \ell_j\, F^{\mu\nu} + \text{h.c.}\,, \tag{3.7}$$

we find

$$c_L^{ij} = \frac{e}{48\pi^2 M_{Z'}^2} \sum_k \left(m_j\, g_{ik}^R\, g_{kj}^R - 3\, m_k\, g_{ik}^R\, g_{kj}^L + m_i\, g_{ik}^L\, g_{kj}^L \right) \tag{3.8}$$

and c_R^{ij}, which can be obtained from c_L^{ij} by interchanging L and R. We find then the branching ratio[3]

$$\text{Br}\left[\ell_j \to \ell_i\gamma\right] = \frac{m_j^3}{4\pi\Gamma_j}\left(|c_L^{ij}|^2 + |c_R^{ij}|^2\right). \tag{3.9}$$

The current experimental limits for lepton flavour violation processes at 90% C.L. are [143–145]:

$$\begin{aligned} \text{Br}[\mu \to e\gamma] &\leq 4.2 \times 10^{-13}\,, \\ \text{Br}[\tau \to \mu\gamma] &\leq 4.4 \times 10^{-8}\,, \\ \text{Br}[\tau \to e\gamma] &\leq 3.3 \times 10^{-8}\,. \end{aligned} \tag{3.10}$$

[3]The coefficients c_L^{ij} in the case of Z' contribution can be obtained from its contribution through the chromomagnetic penguin to $b \to s\gamma$ decay calculated in Ref. [140]. Using formulae in that paper and adjusting the couplings to the case at hand we obtain (at leading order in $m_\ell/m_{Z'}$) consistent results with the generic formula of Ref. [141]. However, our result for the branching ratio is a factor 1/2 smaller than the result of Ref. [142].

Improvements of approximately one order of magnitude for tau decays can be achieved at BELLE II [146] and MeG II will further increase the sensitivity for $\mu \to e\gamma$ [147].

3.3 Anomalous Magnetic Moments and Electric Dipole Moments

Using the coefficients in Eq. (3.8) for the flavour conserving case, we obtain for the NP contributions to anomalous magnetic moments Δa_i and the electric dipole moments d_i,

$$\begin{aligned} \Delta a_i &= -\frac{4\,m_i}{e}\mathrm{Re}\left[c_R^{ii}\right] \,, \\ d_i &= -\,2\,\mathrm{Im}\left[c_R^{ii}\right] \,. \end{aligned} \tag{3.11}$$

These expressions have to be compared with experimental bounds

$$\begin{aligned} \Delta a_e^{\mathrm{Cs}} &= a_e^{\mathrm{exp}} - a_e^{\mathrm{SM,Cs}} = -0.88(36) \times 10^{-12} \,, \\ \Delta a_e^{\mathrm{Rb}} &= a_e^{\mathrm{exp}} - a_e^{\mathrm{SM,Rb}} = 0.48(30) \times 10^{-12} \,, \\ \Delta a_\mu &= a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 251(59) \times 10^{-11} \,, \\ d_e &< 1.1 \times 10^{-29} \text{ e cm} \,, \\ d_\mu &< -0.1(0.9) \times 10^{-19} \text{ e cm} \,, \\ -0.22 \times 10^{-16} \text{ e cm} &< \mathrm{Re}(d_\tau) < 0.45 \times 10^{-16} \text{ e cm} \,, \\ -0.25 \times 10^{-16} \text{ e cm} &< \mathrm{Im}(d_\tau) < 0.08 \times 10^{-16} \text{ e cm} \,. \end{aligned}$$

Here the value of Δa_e extracted from the corresponding measurement [148] and the SM prediction [149, 150], using the fine structure constant from ^{133}Cs [151], is incompatible with the determination using α from ^{87}Rb [152]. Therefore, we quoted both values and we will also distinguish between these cases in our numerical analysis (where relevant). Δa_μ is extracted from the measurement of Ref. [16, 17] and from the recent results from the Fermilab Muon $g-2$ experiment [18–21]. The theory consensus is taken from Ref. [22][4] while the bound on d_μ originates from Ref. [163] and d_e from Refs. [164, 165]. Note that while currently the bounds from tau leptons [166–168] and the muon EDM are not constraining, the latter could be significantly improved by a dedicated experiment proposed at PSI [169].

3.4 Lepton Flavor Violation in $\ell \to 3\ell'$

Three body decays to charged leptons receive the following contributions:

[4]This theory consensus does not include the determination of hadronic vacuum polarization (HVP) from the Budapest-Marseilles-Wuppertal lattice collaboration [153] which would decrease the tension in Δa_μ but differs from HVP determined via $e^+e^- \to$ hadrons [154, 155, 155–160] and would increase the tensions in the global EW fit [161, 162].

1. Tree-level Z' and Z exchanges (in the presence of $Z-Z'$ mixing). The latter contributions to processes with just one flavour transition, such as $\tau^- \to \mu^- e^+ e^-$ or $\tau \to 3\mu$, are proportional to $\sin\xi$ for the Z-Z' interference and proportional to $\sin^2\xi$ for the Z contributions alone. The processes with two flavour transitions, such as $\tau^- \to \mu^- e^+ \mu^-$, are suppressed by higher powers of $\sin\xi$ and neglected in the following. However, for the Z'-mediated tree-level contributions we include the possibility that both vertices are flavour changing.

2. One-loop effects in dipole operators (as defined in Eq. (3.8)) entering via on-shell photon. There contributions again only affect decays with same flavour $\ell^+\ell^-$ pairs in the final states.

3. One-loop contributions generated through the mixing of tree-level induced 4-lepton operators into the operators

$$\begin{aligned}
\mathcal{O}^{LL}_{e\mu} &= (\bar{e}\gamma_\mu P_L \mu)(\bar{\ell}\gamma^\mu P_L \ell), \quad & \mathcal{O}^{LL}_{e\tau} &= (\bar{e}\gamma_\mu P_L \tau)(\bar{\ell}\gamma^\mu P_L \ell),\\
\mathcal{O}^{LL}_{\mu\tau} &= (\bar{\mu}\gamma_\mu P_L \tau)(\bar{\ell}\gamma^\mu P_L \ell), \quad & \mathcal{O}^{RL}_{e\mu} &= (\bar{e}\gamma_\mu P_R \mu)(\bar{\ell}\gamma^\mu P_L \ell),\\
\mathcal{O}^{RL}_{e\tau} &= (\bar{e}\gamma_\mu P_R \tau)(\bar{\ell}\gamma^\mu P_L \ell), \quad & \mathcal{O}^{RL}_{\mu\tau} &= (\bar{\mu}\gamma_\mu P_R \tau)(\bar{\ell}\gamma^\mu P_L \ell),
\end{aligned} \tag{3.12}$$

   and the corresponding ones with L and R interchanged. The details of this mixing through the QED penguin diagrams are discussed in Appendix B. There we list the results for the Wilson coefficients of the operators in question which, due to the vectorial nature of the photon coupling, satisfy the following relations

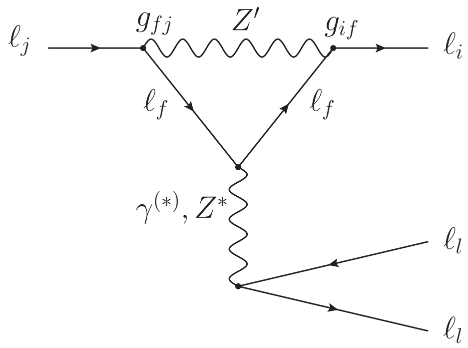
$$C^{LL}_{e\mu}\Big|_{\text{QED}} = C^{LR}_{e\mu}\Big|_{\text{QED}}, \qquad C^{RL}_{e\mu}\Big|_{\text{QED}} = C^{RR}_{e\mu}\Big|_{\text{QED}}, \tag{3.13}$$

   with analogous relations for the remaining coefficients.

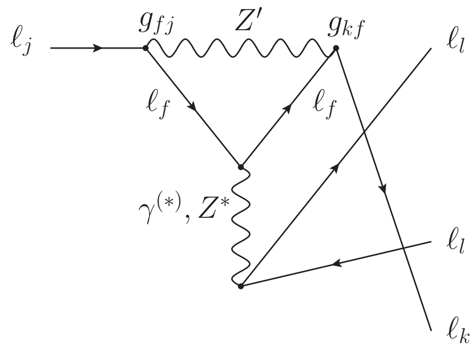
Note that the formulae for the coefficients C^{AB}_{ij} in Appendix B carry no $\ell\ell$ indices (contrary in $C^{AB}_{ij,\ell\ell}$) and consequently apply to all operators. The difference will, however, be present in the parameters $X^{AB}_{ij,kl}$ which are sensitive to the choice of kl. The latter are defined by

$$X^{AB}_{ij,kl} \equiv C^{AB}_{ij}\,\delta_{kl} + \frac{\Delta^A_{ij}\,\Delta^B_{kl}}{m_Z^2}\,\delta_{kl} + \frac{g^A_{ij}\,g^B_{kl}}{M^2_{Z'}}. \tag{3.14}$$

where $A, B = L, R$, which combines the contributions from tree-level Z' and Z exchanges, and from QED penguin contributions. The couplings Δ^A_{ij} are defined in



(a) Penguin



(b) Crossed penguin

Figure 2. Feynman diagrams showing the Z- and the photon-penguin contributions to $\ell_j \to \ell_i\,\bar{\ell}_l\,\ell_k$−processes.

Eq. (2.10). We then find the following branching ratios for flavour-violating τ decays:

$$\begin{aligned}
\mathrm{Br}\left[\tau^\mp \to e^\mp \mu^\pm \mu^\mp\right] = & -\frac{e^2\, m_\tau^3}{48\,\pi^3\,\Gamma_\tau}\left(|c_L^{e\tau}|^2 + |c_R^{e\tau}|^2\right)\left(\ln\left(\frac{m_\mu^2}{m_\tau^2}\right) + 3\right) \qquad (3.15)\\
& + \frac{m_\tau^5}{1536\,\pi^3\,\Gamma_\tau}\left(\left|X^{LL}_{e\tau,\mu\mu} + X^{LL}_{\mu\tau,e\mu}\right|^2 + \left|X^{LR}_{e\tau,\mu\mu}\right|^2 + \left|X^{LR}_{\mu\tau,e\mu}\right|^2 + L \leftrightarrow R\right)\\
& + \frac{e\, {m_\tau}^4}{192\,\pi^3\,\Gamma_\tau}\left(\mathrm{Re}\left[c_R^{e\tau *}\left(X^{LL}_{e\tau,\mu\mu} + X^{LL}_{\mu\tau,e\mu} + X^{LR}_{e\tau,\mu\mu} + X^{LR}_{\mu\tau,e\mu}\right)\right] + L \leftrightarrow R\right)\,,\\
\mathrm{Br}\left[\tau \to 3\mu\right] = & -\frac{e^2\, m_\tau^3}{192\,\pi^3\,\Gamma_\tau}\left(|c_L^{\mu\tau}|^2 + |c_R^{\mu\tau}|^2\right)\left(4\ln\left(\frac{m_\mu^2}{m_\tau^2}\right) + 11\right)\\
& + \frac{m_\tau^5}{1536\,\pi^3\,\Gamma_\tau}\left(2\left|X^{LL}_{\mu\tau,\mu\mu}\right|^2 + \left|X^{LR}_{\mu\tau,\mu\mu}\right|^2 + L \leftrightarrow R\right) \qquad (3.16)\\
& + \frac{e\, m_\tau^4}{192\,\pi^3\,\Gamma_\tau}\left(\mathrm{Re}\left[c_R^{\mu\tau *}\left(2\,X^{LL}_{\mu\tau,\mu\mu} + X^{LR}_{\mu\tau,\mu\mu}\right)\right] + L \leftrightarrow R\right),\\
\mathrm{Br}\left[\tau^\mp \to \mu^\mp e^\pm \mu^\mp\right] = & \frac{m_\tau^5}{1536\,\pi^3\Gamma_\tau}\left(2\,\left|X^{LL}_{\mu\tau,\mu e}\right|^2 + \left|X^{LR}_{\mu\tau,\mu e}\right|^2 + L \leftrightarrow R\right)\,. \qquad (3.17)
\end{aligned}$$

The expression for $\tau \to 3e$ is obtained from $\tau \to 3\mu$ by replacing μ by e, the one for $\mu \to 3e$ from $\tau \to 3\mu$ by replacing index τ by μ and μ by e. Finally the expression for $\tau^\mp \to e^\mp\mu^\pm e^\mp$ is obtained from last formula by interchanging μ and e. The experimental bounds [53, 170–173] at 90% C.L. are:

$$\begin{aligned}
\mathrm{Br}\left[\mu^- \to e^- e^+ e^-\right] &\leq 1.0 \times 10^{-12}\,,\\
\mathrm{Br}\left[\tau^- \to e^- e^+ e^-\right] &\leq 1.4 \times 10^{-8}\,,\\
\mathrm{Br}\left[\tau^- \to e^- \mu^+ \mu\right] &\leq 1.6 \times 10^{-8}\,,\\
\mathrm{Br}\left[\tau^- \to \mu^- e^+ \mu^-\right] &\leq 9.8 \times 10^{-9}\,,\\
\mathrm{Br}\left[\tau^- \to \mu^- e^+ e^-\right] &\leq 1.1 \times 10^{-8}\,,\\
\mathrm{Br}\left[\tau^- \to e^- \mu^+ e^-\right] &\leq 8.4 \times 10^{-9}\,,\\
\mathrm{Br}\left[\tau^- \to \mu^- \mu^+ \mu^-\right] &\leq 1.1 \times 10^{-8}\,.
\end{aligned} \tag{3.18}$$

Here we can expect future improvements in τ decays by BELLE II [146] (and also LHCb [174]) and for $\mu \to 3e$ by Mu3e [175, 176].

3.5 $\mu \to e$ conversion

We define

$$\mathcal{L}_{\text{eff}} = \sum_{q=u,d} \left(C^{LL}_{e\mu,qq}\, O^{LL}_{e\mu,qq} + C^{LR}_{e\mu,qq}\, O^{LR}_{e\mu,qq}\right) + (L \leftrightarrow R) + \text{h.c.}\,, \tag{3.19}$$

with

$$\begin{aligned}
O^{LL}_{e\mu,qq} &= (\bar{e}\gamma^\mu P_L \mu)(\bar{q}\gamma_\mu P_L q)\,, \qquad & O^{RL}_{e\mu,qq} &= (\bar{e}\gamma^\mu P_R \mu)(\bar{q}\gamma_\mu P_L q)\,,\\
O^{LR}_{e\mu,qq} &= (\bar{e}\gamma^\mu P_L \mu)(\bar{q}\gamma_\mu P_R q)\,, \qquad & O^{RR}_{e\mu,qq} &= (\bar{e}\gamma^\mu P_R \mu)(\bar{q}\gamma_\mu P_R q)\,.
\end{aligned} \tag{3.20}$$

In the presence of $Z - Z'$ mixing, the flavour off-diagonal Z'-couplings lead to $\mu \to e$ conversion already at tree-level

$$C^{AB}_{e\mu,qq} = \frac{\sin\xi\; g^A_{e\mu}}{m_Z^2}\; g^{qB}_{\text{SM}}\Big|_{Z-Z'}\,, \tag{3.21}$$

with the $\sin\xi$ given in Eq. (2.7) and $A, B = L, R$. In addition, for small $Z_0 - Z_0'$ mixing, the mixing of four-lepton operators into $O^{V,LL}_{qq}$ and $O^{V,LR}_{qq}$ can be relevant and is obtained analogously to the off-shell photon effects in $\ell \to 3\ell'$ decays (see Appendix B). The standard renormalization group evolution is then performed from scale $M_{Z'}$ down to m_μ, taking into account that the τ-lepton is integrated out at m_τ. Taking into account that for operators with three electrons or three muons two different Wick contractions exist, which leads to a relative factor of 2 w.r.t the τ-leptons case, and only considering the contributions of the hidden operators[5], we

[5]For the contributions of the visible operators we refer to Eq. (B.6) of Appendix B.2 where this naming of operators is explained.

find

$$C^{LL}_{e\mu,qq} = \frac{e^2 Q_q}{16\pi^2} \frac{2}{3M^2_{Z'}} \left(g^L_{e\mu}(2g^L_{\mu\mu} + g^R_{\mu\mu}) \ln\left(\frac{M^2_{Z'}}{m^2_\mu}\right) + \left(g^L_{e\tau} g^L_{\tau\mu} + g^L_{e\mu}(g^L_{\tau\tau} + g^R_{\tau\tau})\right) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right)\right)\Big|_{\text{QED}},$$
$$C^{RL}_{e\mu,qq} = \frac{e^2 Q_q}{16\pi^2} \frac{2}{3M^2_{Z'}} \left(g^R_{e\mu}(2g^R_{\mu\mu} + g^L_{\mu\mu}) \ln\left(\frac{M^2_{Z'}}{m^2_\mu}\right) + \left(g^R_{e\tau} g^R_{\tau\mu} + g^R_{e\mu}(g^L_{\tau\tau} + g^R_{\tau\tau})\right) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right)\right)\Big|_{\text{QED}}, \tag{3.22}$$

and

$$C^{LL}_{e\mu,qq}\Big|_{\text{QED}} = C^{LR}_{e\mu,qq}\Big|_{\text{QED}}, \qquad C^{RL}_{e\mu,qq}\Big|_{\text{QED}} = C^{RR}_{e\mu,qq}\Big|_{\text{QED}}, \tag{3.23}$$

where Q_q is the electric charge of the quarks ($Q_u = +\frac{2}{3}$, $Q_d = -\frac{1}{3}$).

The transition rate $\Gamma^N_{\mu\to e} \equiv \Gamma(\mu N \to eN)$ is given by

$$\Gamma^N_{\mu\to e} = \frac{m^5_\mu}{4} \left| \frac{c^{e\mu}_L}{m_\mu} D_N + 4 \sum_{q=u,d} \left(C^{RL}_{e\mu,qq} + C^{RR}_{e\mu,qq}\right) \left(f^{(q)}_{Vp} V^p_N + f^{(q)}_{Vn} V^n_N\right) \right|^2 + (L \leftrightarrow R)\,. \tag{3.24}$$

The quantities D_N and $V^N_{p/n}$ are related to the overlap integrals between the lepton wave functions and the nucleon densities, and thus depend on the nature of the target N. We use the numerical values [177]

$$D_{\text{Au}} = 0.189, \qquad V^p_{\text{Au}} = 0.0974\,, \qquad V^n_{\text{Au}} = 0.146\,. \tag{3.25}$$

The nucleon vector form factors are the same as the ones measured in elastic electron-hadron scattering, i.e.

$$f^{(u)}_{Vp} = 2, \quad f^{(u)}_{Vn} = 1, \quad f^{(d)}_{Vp} = 1, \quad f^{(d)}_{Vn} = 2\,. \tag{3.26}$$

Finally, the branching ratio of $\mu \to e$ conversion is defined as the transition rate divided by the μ capture rate:

$$\text{Br}\,[\mu \to e] = \frac{\Gamma^{\text{conv}}}{\Gamma^{\text{capt}}}\,, \tag{3.27}$$

and for the latter we use [178]

$$\Gamma^{\text{capt}}_{\text{Au}} = 8.7 \times 10^{-18}\ \text{GeV}\,. \tag{3.28}$$

The experimental limit on $\mu \to e$ conversion from SINDRUM II is [143]

$$\frac{\Gamma^{\text{conv}}_{\text{Au}}}{\Gamma^{\text{capt}}_{\text{Au}}} < 7.0 \times 10^{-13}\,. \tag{3.29}$$

It is expected to be improved by three orders of magnitude by COMET and Mu2e collaborations in the coming years [179].

3.6 Electroweak Precision Observables

The EW sector of the SM has been tested with a very high precision at LEP and Tevatron [15, 180]. Since it can be parametrised by only three Lagrangian parameters, we choose the set with the smallest experimental error consisting of the Fermi constant ($G_F = 1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$ [181]), the mass of the Z boson ($m_Z = 91.1875(21)$ [180]) and the fine structure constant ($\alpha_{em} = 7.2973525664(17) \times 10^{-3}$ [181]).

In our model, the Lagrangian values for G_F and m_Z are shifted with respect to their measurements. In particular, the effect in $\mu \to e\nu\bar{\nu}$ leads to the following relation

$$\frac{G_F}{G_F^{\mathcal{L}}} = 1 + 2\,\frac{|g^L_{e\mu}|^2}{g_2^2}\frac{m_W^2}{M_{Z'}^2} \equiv 1 + \delta G_F\,, \tag{3.30}$$

while the Z boson mass is modified via Eq. (2.5). Moreover, taking into account the tree-level effects in Eq. (2.10) and the loop effects in Ref. [33, 185], we have the following modified Z couplings to leptons

$$\begin{aligned}
\Delta^{\ell L}_{ij}(q^2 = m_Z^2) &= g^{\ell L}_{\mathrm{SM}}\left(\delta_{ij} + \sin\xi\,\frac{g^L_{ij}}{g^{\ell L}_{\mathrm{SM}}} + \sum_k \frac{g^L_{ik}g^L_{kj}}{(4\pi)^2}K_F\left(\frac{m_Z^2}{M_{Z'}^2}\right)\right)\,,\\
\Delta^{\nu L}_{ij}(q^2 = m_Z^2) &= g^{\nu L}_{\mathrm{SM}}\left(\delta_{ij} + \sin\xi\,\frac{g^L_{ij}}{g^{\nu L}_{\mathrm{SM}}} + \sum_k \frac{g^L_{ik}g^L_{kj}}{(4\pi)^2}K_F\left(\frac{m_Z^2}{M_{Z'}^2}\right)\right)\,,\\
\Delta^{\ell R}_{ij}(q^2 = m_Z^2) &= g^{\ell R}_{\mathrm{SM}}\left(\delta_{ij} + \sin\xi\,\frac{g^R_{ij}}{g^{\ell R}_{\mathrm{SM}}} + \sum_k \frac{g^R_{ik}g^R_{kj}}{(4\pi)^2}K_F\left(\frac{m_Z^2}{M_{Z'}^2}\right)\right)\,,
\end{aligned} \tag{3.31}$$

with

$$K_F\,(x) = -\frac{2(x+1)^2(\mathrm{Li}_2(-x) + \ln(x)\ln(x+1))}{x^2} - \frac{7x+4}{2x} + \frac{(3x+2)\ln(x)}{x}. \tag{3.32}$$

For the numerical analysis, we implemented the EW observables shown in Table 1 in HEPfit [125] taking into account the modifications induced by Eq. (2.5), Eq. (3.30) and Eq. (3.31). In addition, the Higgs mass ($M_H = 125.16 \pm 0.13$ GeV [186, 187]), the top mass ($m_t = 172.80 \pm 0.40$ GeV [188–190]), the strong coupling constant ($\alpha_s(M_Z) = 0.1181 \pm 0.0011$ [181]) and the hadronic contribution to the running of α_{em} ($\Delta\alpha_{\mathrm{had}} = 276.1(11) \times 10^{-4}$ [181]) have been used as input parameters, since they enter EW observables indirectly via loop effects.

3.7 $Z \to \ell\ell'$

In the presence of $Z - Z'$ mixing, we obtain a tree-level contribution to $Z \to \ell_i\ell_j$, $i \neq j$, leading to the branching ratios

$$\begin{aligned}
\mathrm{Br}\left[Z \to \ell_i\bar{\ell}_j\right] &= \frac{1}{24\pi}\frac{m_Z}{\Gamma_Z}\left(|\Delta^{\ell L}_{ij}(q^2 = m_Z^2)|^2 + |\Delta^{\ell R}_{ij}(q^2 = m_Z^2)|^2\right)\,,\\
\mathrm{Br}\left[Z \to \nu_i\bar{\nu}_j\right] &= \frac{1}{24\pi}\frac{m_Z}{\Gamma_Z}|\Delta^{\nu L}_{ij}(q^2 = m_Z^2)|^2\,,
\end{aligned} \tag{3.33}$$

Observable	Experimental value
m_W [GeV]	80.379(12)
Γ_W [GeV]	2.085(42)
$\mathrm{BR}(W \to \mathrm{had})$	0.6741(27)
$\mathrm{BR}(W \to \mathrm{lep})$	0.1086(9)
$\sin^2\alpha^{\mathrm{CDF}}_{\mathrm{eff,\,e}}$	0.23248(52)
$\sin^2\alpha^{\mathrm{D0}}_{\mathrm{eff,\,e}}$	0.23146(47)
$\sin^2\alpha^{\mathrm{CDF}}_{\mathrm{eff},\,\mu}$	0.2315(10)
$\sin^2\alpha^{\mathrm{CMS}}_{\mathrm{eff},\,\mu}$	0.2287(32)
$\sin^2\alpha^{\mathrm{LHCb}}_{\mathrm{eff},\,\mu}$	0.2314(11)
P^{pol}_{τ}	0.1465(33)
A_e	0.1516(21)
A_μ	0.142(15)
A_τ	0.136(15)
Γ_Z [GeV]	2.4952(23)
σ^0_h [nb]	41.541(37)
R^0_e	20.804(50)
R^0_μ	20.785(33)
R^0_τ	20.764(45)
$A^{0,e}_{\mathrm{FB}}$	0.0145(25)
$A^{0,\mu}_{\mathrm{FB}}$	0.0169(13)
$A^{0,\tau}_{\mathrm{FB}}$	0.0188(17)
R^0_b	0.21629(66)
R^0_c	0.1721(30)
$A^{0,b}_{\mathrm{FB}}$	0.0992(16)
$A^{0,c}_{\mathrm{FB}}$	0.0707(35)
A_b	0.923(20)
A_c	0.670(27)

Table 1. Electroweak observables [180, 181] used in our fit which are calculated by HEPfit [125] using m_Z^L, α and G_F as input.

with $\Gamma_Z = 2.4952 \pm 0.0023$ GeV [139]. We compare these results to the ATLAS and LEP measurements given in Table 2.

$\mathrm{Br}\left[Z \to e^{\pm}\tau^{\mp}\right]$	$= (-0.1 \pm 3.5(\mathrm{stat}) \pm 2.3(\mathrm{syst})) \times 10^{-6}$	ATLAS: [191]
$\mathrm{Br}\left[Z \to e^{\pm}\tau^{\mp}\right]$	$< 9.8 \times 10^{-6}$	LEP (OPAL): [139, 192]
$\mathrm{Br}\left[Z \to \mu^{\pm}\tau^{\mp}\right]$	$= (4.3 \pm 2.8(\mathrm{stat}) \pm 1.6(\mathrm{syst})) \times 10^{-6}$	ATLAS: [191]
$\mathrm{Br}\left[Z \to \mu^{\pm}\tau^{\mp}\right]$	$< 1.2 \times 10^{-5}$	LEP (DELPHI): [139, 193]
$\mathrm{Br}\left[Z \to e^{\pm}\mu^{\mp}\right]$	$< 7.5 \times 10^{-7}$ (95% CL)	ATLAS: [139, 194]

Table 2. Experimental bounds on $Z \to \ell\ell'$.

3.8 Neutrino Trident Production

Neutrino trident production can be used to constrain couplings of muons to muon neutrinos [33, 35]. Generalizing the formula of Ref. [35] to the case of chiral Z' couplings, we find

$$\frac{\sigma_{SM+NP}}{\sigma_{SM}} = 1 + 8\frac{(1+4s_W^2)\,\frac{g_{22}^L\left(g_{22}^L+g_{22}^R\right)}{g_2^2}\frac{m_W^2}{M_{Z'}^2} - \frac{g_{22}^L\left(g_{22}^R-g_{22}^L\right)^2}{g_2^2}\frac{m_W^2}{M_{Z'}^2}}{\left(1+4s_W^2\right)^2+1}\,. \tag{3.34}$$

This ratio is bounded by the weighted average

$$\sigma_{\mathrm{exp}}/\sigma_{\mathrm{SM}} = 0.83 \pm 0.18\,, \tag{3.35}$$

obtained from averaging the CHARM-II [195], CCFR [196] and NuTeV results [197].

3.9 LEP-II bounds

LEP-II set stringent bounds on 4-lepton operators from $e^+e^- \to \ell^+\ell^-$ (with $\ell = e, \mu, \tau$) [15] for specific chiralities. Some of our more general scenarios cannot be matched to the 4-lepton operators as given in Ref. [15]. For these cases we derive the constraints in Appendix A and provide the formula which we implemented in HEPfit.

4 Phenomenological Analysis

In our phenomenological analysis we perform a global fit taking into account all observables discussed in the last section. This includes EW precision observables,

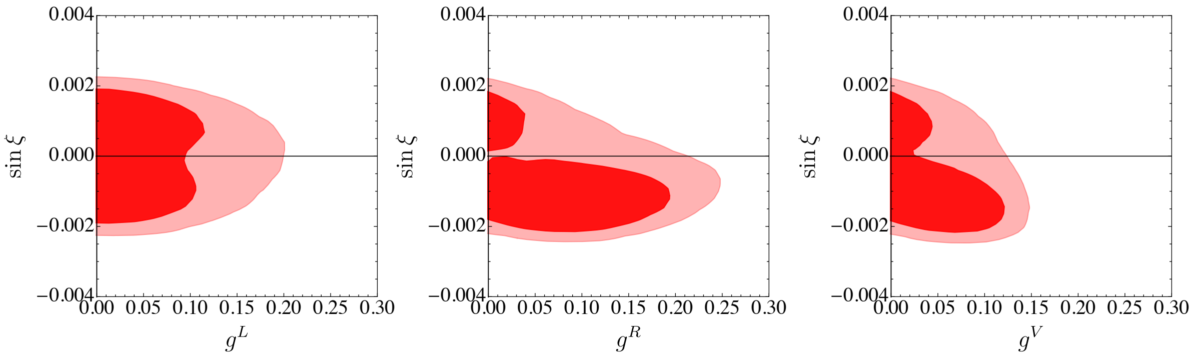


Figure 3. 68% and 95% CL regions for the three LFU cases g^L, g^R and g^V. Note that due to the preference for a slightly lower W mass than predicted in the SM, the origin in the g^R scenario is not within the 68% CL region. The $Z - Z'$ mixing angle is bounded to be $\xi \lessapprox 0.02$ and $g^{L,R} \lessapprox 0.2$, while $g^V \lessapprox 0.1$ for $M_{Z'} = 1\,\text{TeV}$.

as implemented in the HEPfit distribution [125]. To include the other observables discussed previously, we added them to the HEPfit code such that we can perform a Bayesian statistical analysis using the Markov Chain Monte Carlo (MCMC) determination of posteriors of the Bayesian Analysis Toolkit (`BAT`) [198].

With this setup we can now consider several different scenarios. For our numerical analysis we fix $M_{Z'} = 1\,\text{TeV}$ unless stated otherwise and assume real couplings. Note that despite small logarithmic corrections, the results we obtain scale like $g^2/M_{Z'}^2$. However, for the loop-induced modifications of $Z\ell\ell$ couplings in the scenario which aims at an explanation of the data on the anomalous magnetic moment of the muon, these logarithmic corrections to the $g^2/M_{Z'}^2$ scaling can indeed be relevant.

4.1 Lepton Flavour Universality

Here we consider four scenarios which respect lepton flavour universality:

1. Left-handed couplings: $g_{ii}^L = g^L$
2. Right-handed couplings: $g_{ii}^R = g^R$
3. Vectorial couplings: $g_{ii}^L = g_{ii}^R = g^V$
4. Generic chiral couplings: $g_{ii}^L = g^L$, $g_{ii}^R = g^R$

Each of the first three LFU scenarios is two dimensional, with the coupling and the Z-Z' mixing angle ξ being free parameters, and shown in Fig. 3. We can see that for a Z' mass of 1 TeV, the couplings should be smaller than ≈ 0.2 and the $Z - Z'$ mixing is bounded to be less than ≈ 0.002 at 95% CL. In the case of g_L and g_R being

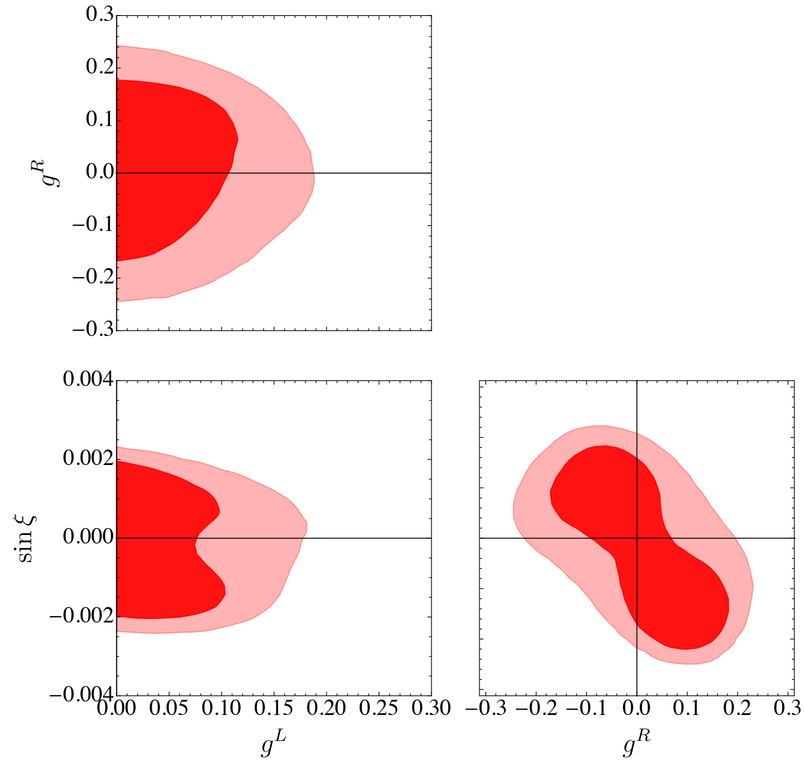


Figure 4. 68% and 95% CL regions for the three-dimensional LFU fit, where g_L, g_R and ξ are free parameters for $M_{Z'} = 1\,\text{TeV}$.

independent of each other, shown in Fig. 4, there is a mild preference for a non-zero mixing angle at the 68% CL, which is due to the slight tension in the W mass prediction within the EW fit. However, neither the tension in $\tau \to \mu\nu\bar{\nu}/\tau \to e\nu\bar{\nu}$ nor in the first row CKM unitarity or $(g-2)_\mu$ can be explained in these LFU setups.

4.2 Lepton Flavour Universality Violation

Here we study the case in which the couplings that are diagonal in flavour space but not proportional to the unit matrix:

1. Vectorial couplings: $g^L_{ii} = g^R_{ii} = g^V_i$
2. Left-handed couplings: $g^L_{ii} = g^L_i$
3. Right-handed couplings: $g^R_{ii} = g^R_i$

These scenarios are shown in Figs. 5-7. The couplings to electrons are very well constrained and can be at most of the order of 0.2 due to the LEP bounds on 4-electron contact interactions. The bounds on muon and tau couplings are less stringent and therefore can be as large as 2 for larger $M_{Z'} = 1\,\text{TeV}$. This is also due to the fact that $\tau \to \mu\nu\bar{\nu}$ prefers larger couplings to muons and taus which enter via

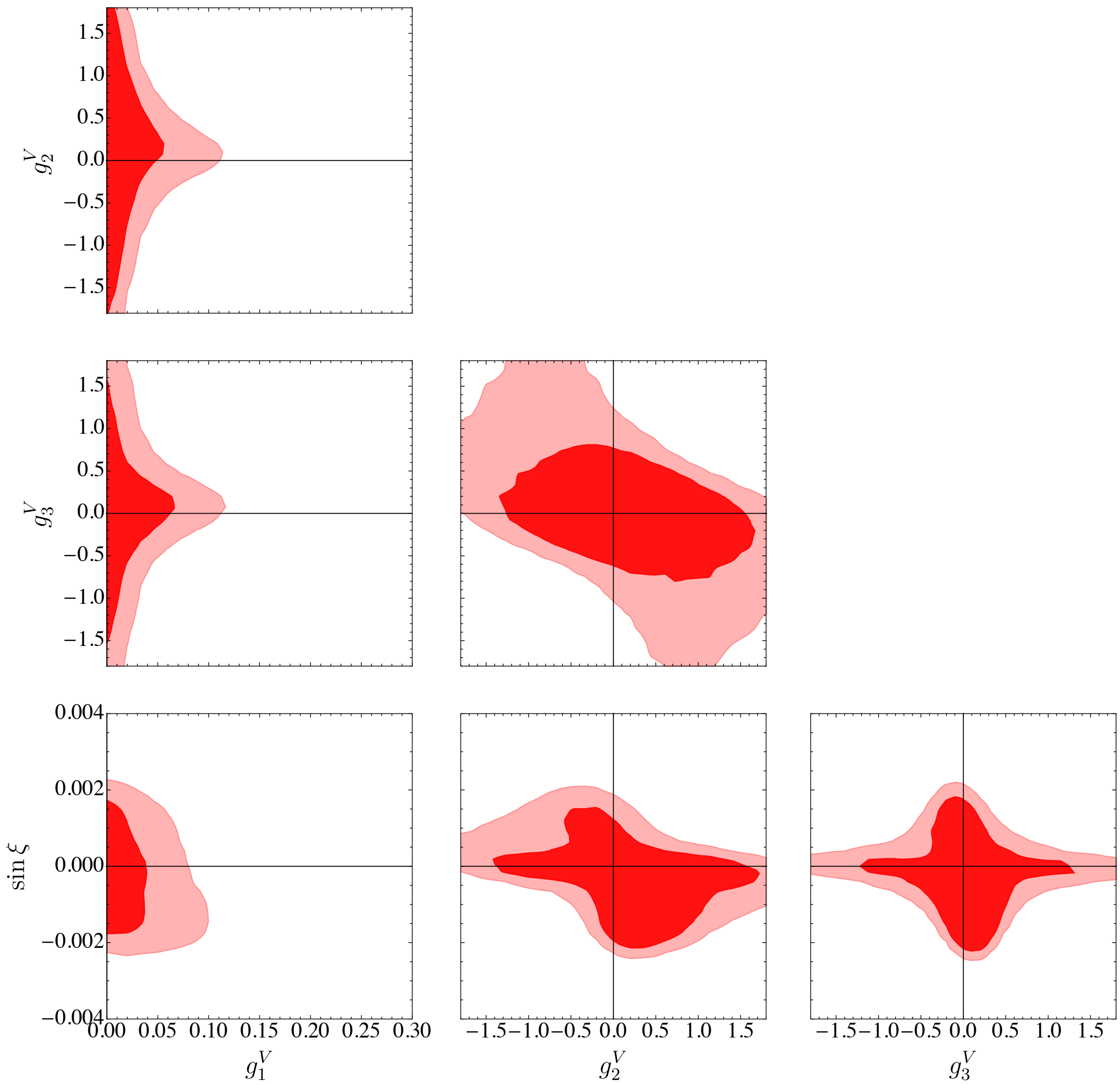


Figure 5. 68% and 95% CL regions for the LFUV case with vectorial couplings $g_{ii}^L = g_{ii}^R = g_i^V$.

the box contributions. However, in this case effects in the EW fit are generated as well, such that no significant preference over the SM fit can be achieved.

4.3 Lepton Flavour Violation

Here we study the following scenarios for the couplings:

1. Flavour violating tau-muon couplings: g_{23}^R and g_{23}^L
2. $L_\mu - L_\tau$ symmetry with left-handed rotations

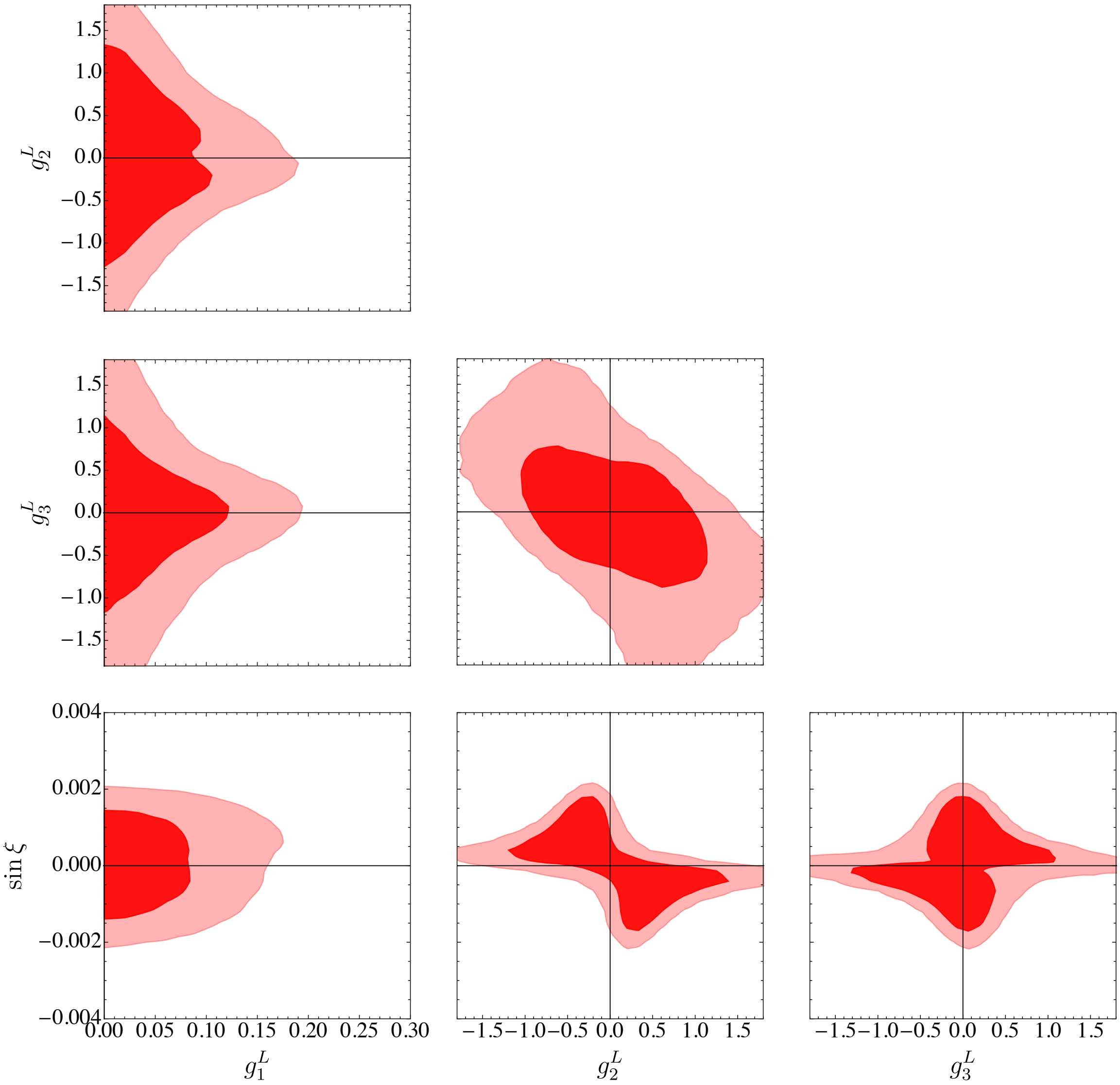


Figure 6. 68% and 95% CL regions for the LFUV case with left-handed couplings $g_i^L = g_{ii}^L$.

3. Vectorial couplings: $g_{ij}^L = g_{ij}^R = g_{ij}^V$
4. Left-handed couplings: g_{ij}^L
5. Right-handed couplings: g_{ij}^R

Scenario 1, which assumes that only g_{23}^L and g_{23}^R are non-zero, allows us to find interesting correlations as shown in Fig. 8. Here one can see that it is possible to explain $\tau \to \mu\nu\bar{\nu}$ and $(g-2)_\mu$ simultaneously, predicting observable effects in $Z \to \mu\bar{\mu}$ and $Z \to \tau\bar{\tau}$ for $M_{Z'} = 1\,\text{TeV}$. However, due to the logarithmic corrections

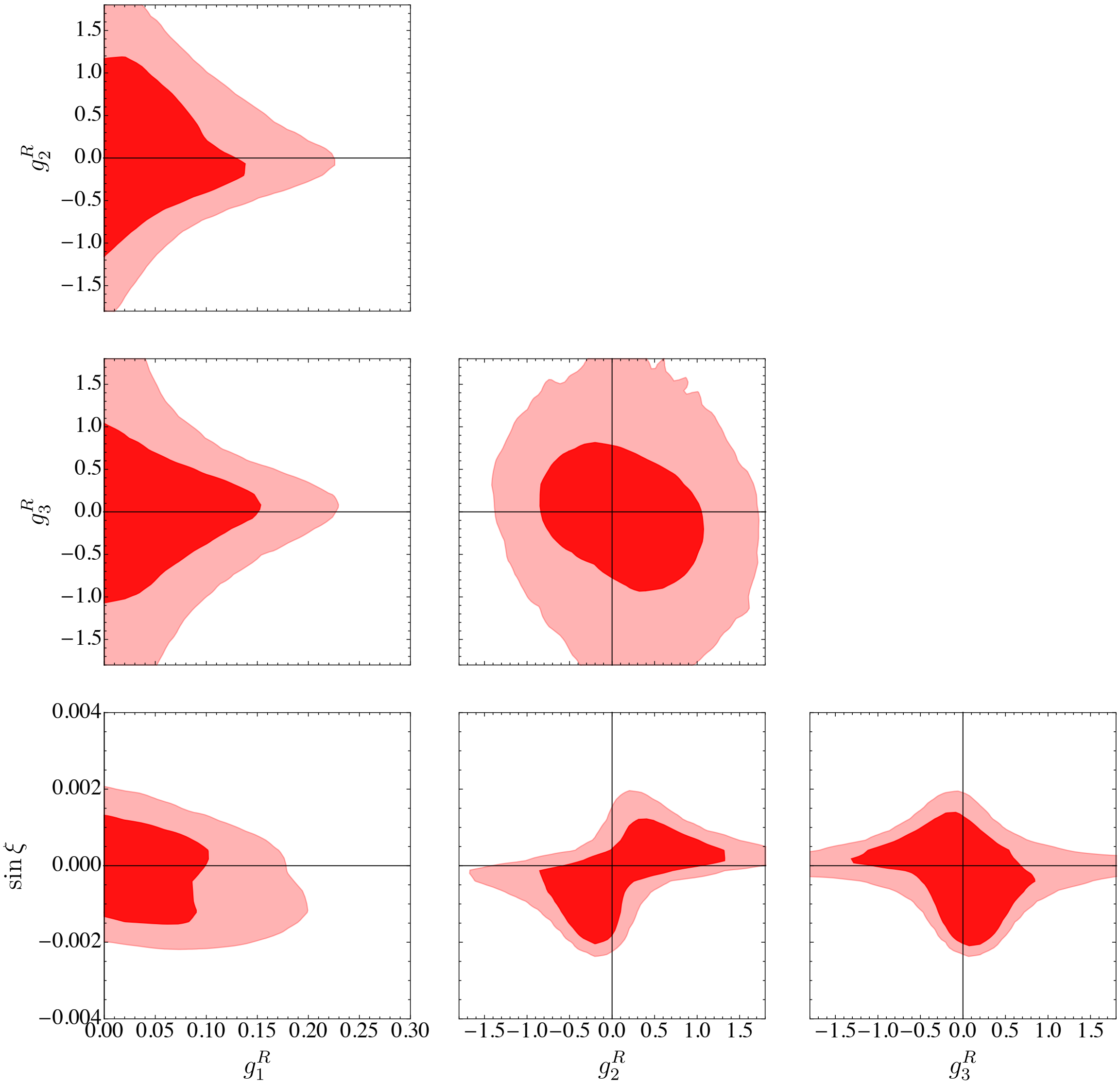


Figure 7. 68% and 95% CL regions for the LFUV case with right-handed couplings $g_i^R = g_{ii}^R$.

involved here, the effects in $Z \to \mu\bar{\mu}$ and $Z \to \tau\bar{\tau}$ become weaker for smaller masses. These effects will allow in the future to conquer the parameter space (assuming an explanation of $(g-2)_\mu$), as direct searches and EW precision constraints test complementary regions in parameter space.

Next, let us consider case 2 with a (broken) $L_\mu - L_\tau$ symmetry. This means that

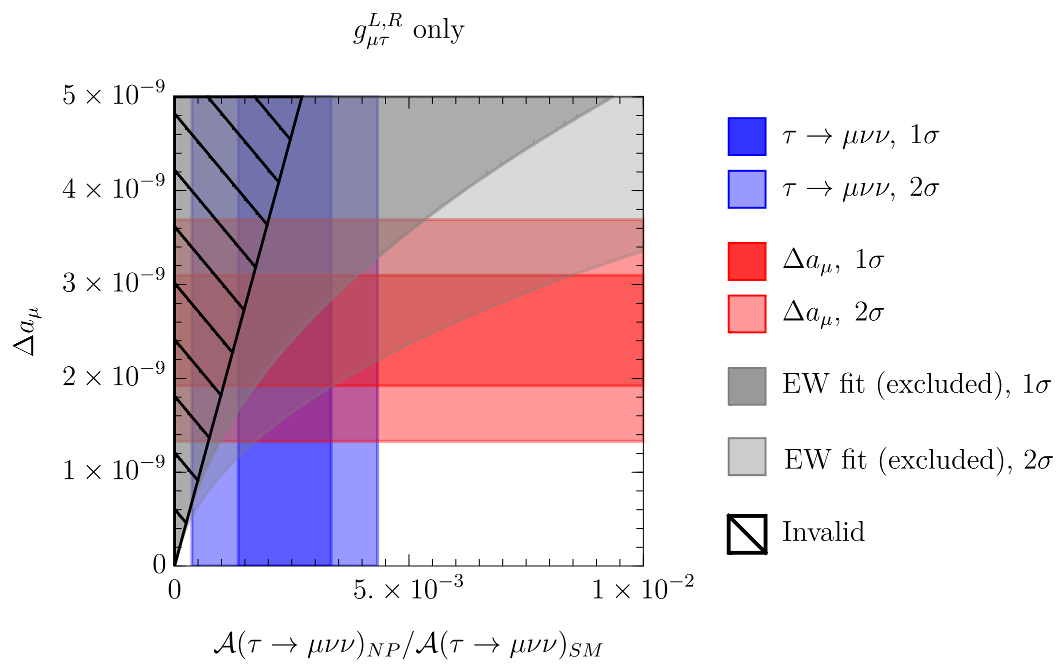


Figure 8. Correlations between $(g-2)_\mu$ and $\mathcal{A}(\tau\to\mu\nu\bar\nu)_{\rm NP}/\mathcal{A}(\tau\to\mu\nu\bar\nu)_{\rm SM}$ in the scenario with zero $Z-Z'$-mixing ($\sin\xi=0$), where all Z' couplings to leptons are set to zero apart from $g_{\mu\tau}^{L,R}\neq 0$. The colored regions are preferred by the anomalous magnetic moment of the muon and by LFUV in tau decays, while the gray region is excluded by electroweak data for $M_{Z'}=1\,$TeV. Note that for lighter Z' masses EW precision observables would be less constraining. The hatched region is excluded in this setup in the sense that points within it cannot be reached in this setup.

the coupling matrix takes the form

$$g_{ij}^V=\begin{pmatrix}0&0&0\\0&g&0\\0&0&-g\end{pmatrix}\tag{4.1}$$

in the interaction basis. Now we assume that $L_\mu-L_\tau$ is broken by the charged lepton Yukawa couplings in the left-handed sector such that, after EW symmetry breaking,

$$g_{ij}^L=\left(\begin{pmatrix}1&0&0\\0&\cos\beta_{23}&\sin\beta_{23}\\0&-\sin\beta_{23}&\cos\beta_{23}\end{pmatrix}\cdot\begin{pmatrix}0&0&0\\0&g&0\\0&0&-g\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&\cos\beta_{23}&-\sin\beta_{23}\\0&\sin\beta_{23}&\cos\beta_{23}\end{pmatrix}\right)_{ij},\tag{4.2}$$

for 2–3 rotations. The analogous formula for 1–2 rotations follows straightforwardly.

For this setup we show in Figs. 9 and 10 the 68% and 95% CL regions for the coupling g and the rotation angle β_{23} and β_{12}, respectively. In Fig. 11 we show the correlations between $\text{Br}[\tau \to \mu\gamma]$ and $\text{Br}[\tau \to \mu\mu\mu]$, which display that in this scenario the present experimental upper bound on $\text{Br}[\tau \to \mu\mu\mu]$ can easily be saturated, while $\text{Br}[\tau \to \mu\gamma]$ is orders of magnitude below the present current bound. Therefore, finding in the coming years $\text{Br}[\tau \to \mu\gamma]$ at the level of 10^{-9} would rule out the $L_\mu - L_\tau$ scenario. Similarly for $\mu \to e$ transitions: Fig. 12 demonstrates the importance of $Z - Z'$ mixing and thereby the role of Z in the enhancement of $\mu \to 3e$ and $\mu \to e$ conversion. However, in this case, in contrast to $\text{Br}[\tau \to \mu\gamma]$, the branching ratio for $\text{Br}[\mu \to e\gamma]$ can easily saturate the present experimental upper bound. MEG, Mu2e, Mu2e and COMET will constrain the allowed space in these plots in the coming years significantly.

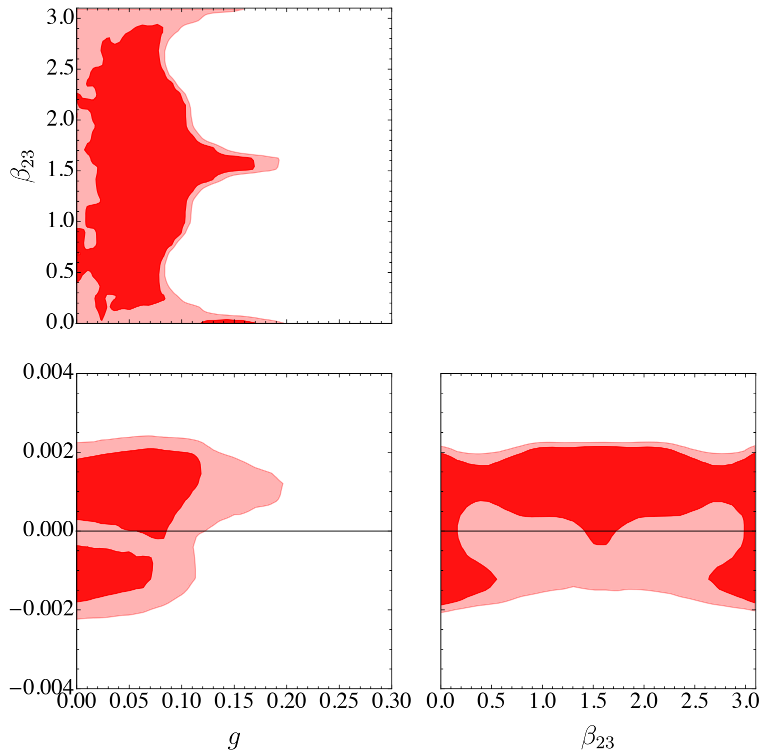


Figure 9. 68% and 95% C.L. regions for the $L_\mu - L_\tau$ coupling g, the rotation angle β_{23} and the mixing angle $\sin\xi$.

We discuss the more general cases 3, 4 and 5, with seven free parameters each, in Appendix C.

5 Conclusions

In this paper we have performed a global fit to leptophilic Z' models with the goal to obtain bounds on the Z' couplings to leptons in multi-dimensional scenarios. In

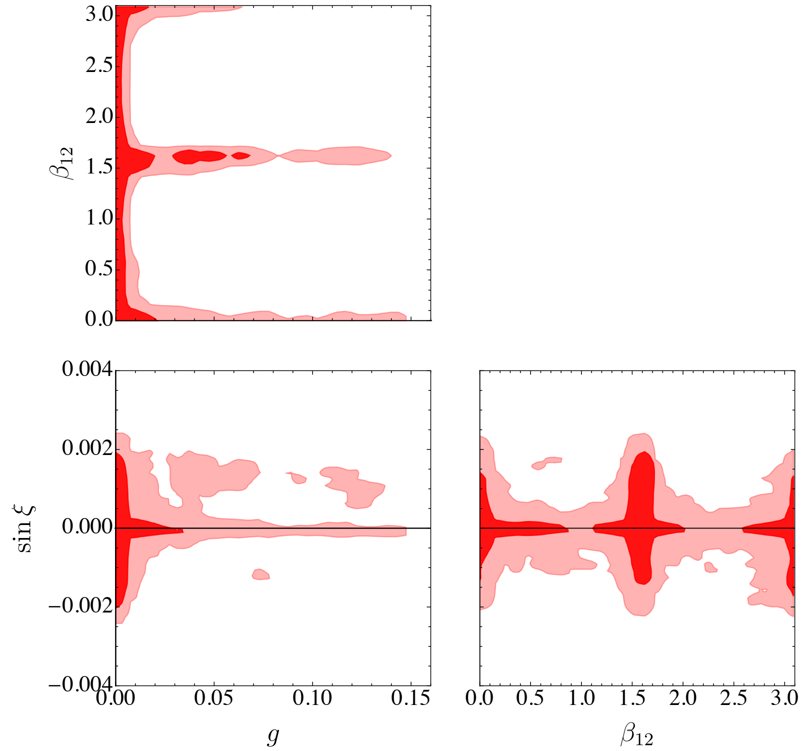


Figure 10. 68% and 95% C.L. regions for the $L_\mu - L_\tau$ coupling g, the rotation angle β_{12} and the mixing angle $\sin\xi$.

our global analysis we took into account a large number of observables, including $\ell \to \ell' \nu \bar{\nu}$ decays, anomalous magnetic moments of charged leptons, $\ell \to \ell' \gamma$, $\ell \to 3\ell'$ decays, $\mu \to e$ conversion, electroweak precision observables, lepton flavour violating Z decays, neutrino trident production and LEP searches for four-lepton contact interactions.

Properly extending the HEPfit code [125] by implementing these observables, and performing a Bayesian statistical analysis, we obtained bounds on the Z' couplings in a number of generic scenarios, as listed in Section 4. The results are presented in Figs. 5-17. The plots in these figures are self-explanatory but the main message is that the couplings involving electrons are much more strongly bounded than those involving muons or tau leptons. These results should turn out to be useful for building models where the patterns for the couplings are governed by flavour symmetries.

In more detail, we find that in the LFU scenario neither the tension in $\tau \to \mu\nu\bar{\nu}/\tau \to e\nu\bar{\nu}$ nor in the first row CKM unitarity or in $(g-2)_\mu$ can be explained. In the LFUV scenario the couplings to electrons are very well constrained and can be at most of the order of 0.2 due to the LEP bounds on 4-electron contact interactions. The bounds on muon and tau couplings are less stringent and therefore can be as

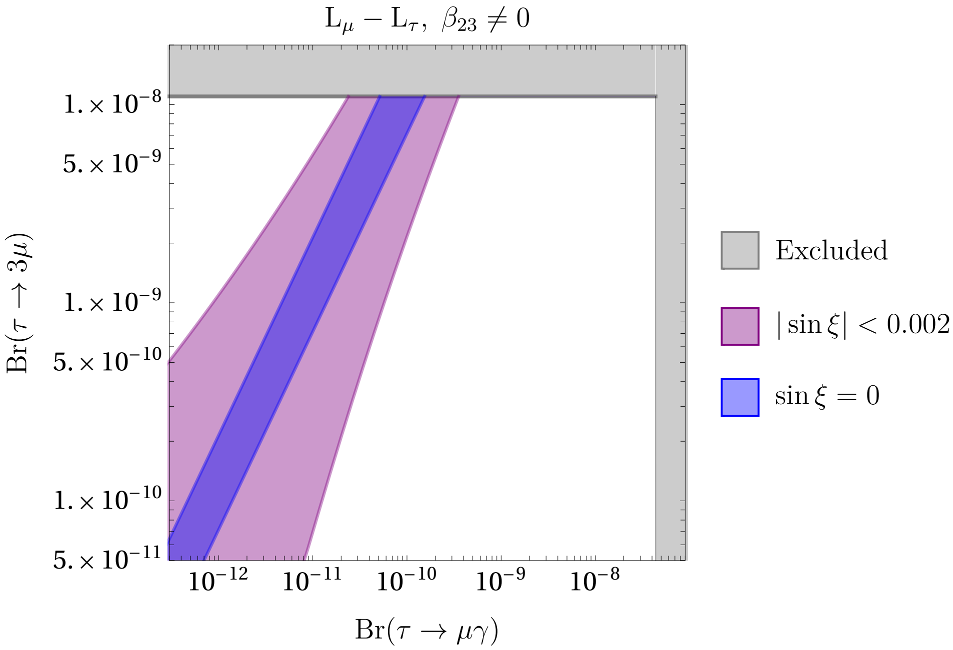


Figure 11. Correlations between $\mathrm{Br}(\tau \to \mu\mu\mu)$ and $\mathrm{Br}(\tau \to \mu\gamma)$ for different values of $\sin\xi$ within the $L_\mu - L_\tau$ scenario with left-handed 2-3 rotations ($\beta_{23} \neq 0$). The colored regions are allowed within this setup.

large as 2 for larger $M_{Z'} = 1\,\mathrm{TeV}$. We then studied more specific scenarios with constrained patterns for the couplings. Here we found that if only g_{23}^L and g_{23}^R are non-zero, the anomaly in the anomalous magnetic moment of the muon and the hint for LFUV in leptonic tau decays can be explained, with interesting predictions for EW precision observables. Furthermore, in scenarios with a $L_\mu - L_\tau$ symmetry we were able to correlate $\tau \to 3\mu$ to $\tau \to \mu\gamma$ (and similarly for $\mu \to e$ transitions). These correlations can be used to test this setup with future experiments.

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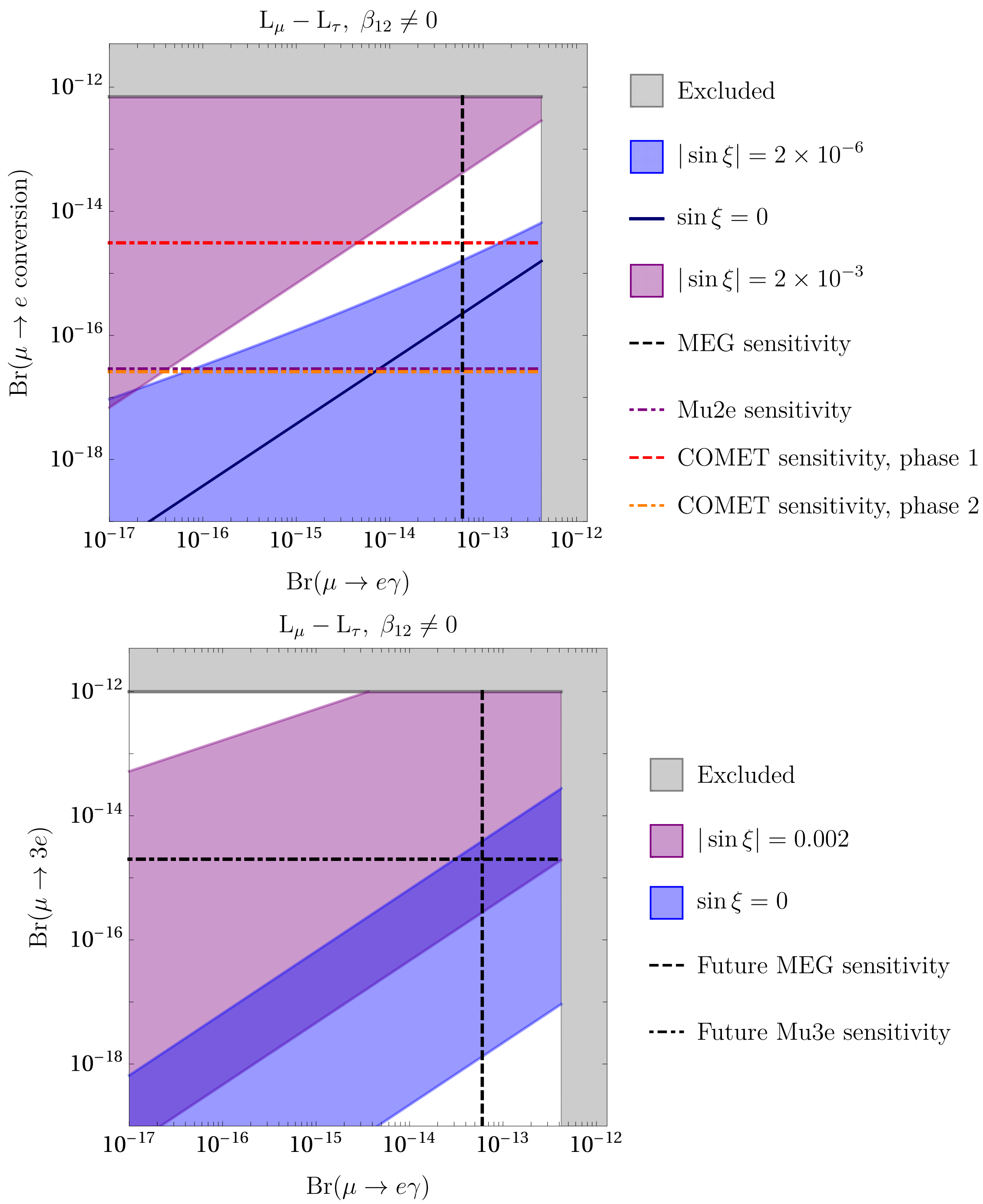

Figure 12. Correlations between different $\mu \to e$ transitions in the $L_\mu - L_\tau$ scenario with left-handed 1-2 rotations ($\beta_{12} \neq 0$).

A LEP-II constraints for general coupling structure

LEP-II measured with high precision the total cross section $\sigma_{\rm TOT} = \sigma_F + \sigma_B$ and the Forward-Backward assymetry $A_{FB} = (\sigma_F - \sigma_B)/\sigma$ for the process $e^+e^- \to \ell^+\ell^-$ (with

$\ell = e, \mu, \tau$) at various $\sqrt{s}$ between 130 and 207 GeV (see Tables 3.4 and 3.8 - 3.12 from Ref. [15]). One can extract bounds on our model parameters by computing the BSM contribution to $\sigma_F \pm \sigma_B$. Here we work within an EFT aproach with a Lagrangian defined as $\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{v^2} C_i O_i$ and we fierzed the operators giving an effect at the dimension six level into the basis:

$$\begin{aligned}
\left[O_{\ell\ell}\right]_{11jj} &= \left(\ell_1 \gamma_\mu P_L \ell_1\right)\left(\ell_j \gamma^\mu P_L \ell_j\right), \\
\left[O_{ee}\right]_{11jj} &= \left(\ell_1 \gamma_\mu P_R \ell_1\right)\left(\ell_j \gamma^\mu P_R \ell_j\right), \\
\left[O_{\ell e}\right]_{11jj} &= \left(\ell_1 \gamma_\mu P_L \ell_1\right)\left(\ell_j \gamma^\mu P_R \ell_j\right), \\
\left[O_{\ell e}\right]_{jj11} &= \left(\ell_j \gamma_\mu P_L \ell_j\right)\left(\ell_1 \gamma^\mu P_R \ell_1\right).
\end{aligned} \tag{A.1}$$

Assuming that the $Z - Z'$ mixing effects are sub-leading, and for vanishing lepton masses, the BSM contributions to $\sigma_{\rm TOT}$ and A_{FB} for $\ell = \mu$ are given by:

$$\begin{aligned}
\delta\left(\sigma_F + \sigma_B\right) = \frac{1}{24\pi v^2} \Bigg\{ & e^2 \left([c_{\ell\ell}]_{1122} + [c_{ee}]_{1122} + [c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \\
& + \frac{s(g_2^2 + g_1^2)}{s - m_Z^2} \left[\left(-\frac{1}{2} + s_W^2 \right)^2 [c_{\ell\ell}]_{1122} + s_W^4 [c_{ee}]_{1122} \right. \\
& \left. + \left(-\frac{1}{2} + s_W^2 \right) s_W^2 \left([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \right] \Bigg\},
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\delta\left(\sigma_F - \sigma_B\right) = \frac{1}{32\pi v^2} \Bigg\{ & e^2 \left([c_{\ell\ell}]_{1122} + [c_{ee}]_{1122} - [c_{\ell e}]_{1122} - [c_{\ell\ell}]_{2211} \right) \\
& + \frac{s(g_2^2 + g_1^2)}{s - m_Z^2} \left[\left(-\frac{1}{2} + s_W^2 \right)^2 [c_{\ell\ell}]_{1122} + s_W^4 [c_{ee}]_{1122} \right. \\
& \left. - \left(-\frac{1}{2} + s_W^2 \right) s_W^2 \left([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \right] \Bigg\}.
\end{aligned} \tag{A.3}$$

For the $\tau^+\tau^-$ channel we have the same expressions with the index exchange $2 \leftrightarrow 3$.

On the other hand for $e^+e^- \to e^+e^-$ Ref. [15] gives also the angular differential cross section, measured at different $\sqrt{s}$ and $\cos\theta$ ranges (see Tables 3.8 - 3.12 in Ref. [15]). In this case the BSM contribution to the differential cross section is given by:

$$\delta\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{8\pi s}\frac{1}{v^2}\Bigg\{2u^2\Bigg[e^2\left([c_{\ell\ell}]_{1111}+[c_{ee}]_{1111}\right)\left(\frac{1}{s}+\frac{1}{t}\right)$$
$$+\left(g_2^2+g_1^2\right)\left(\frac{1}{s-m_Z^2}+\frac{1}{t-m_Z^2}\right)\left(\left(\frac{1}{2}+s_W^2\right)^2[c_{\ell\ell}]_{1111}+s_W^4\,[c_{ee}]_{1111}\right)\Bigg]$$
$$+t^2\left[[c_{\ell e}]_{1111}\frac{e^2}{s}+[c_{\ell e}]_{1111}\frac{(g_2^2+g_1^2)\,(\frac{1}{2}+s_W^2)s_W^2}{s-m_Z^2}\right]$$
$$+s^2\left[[c_{\ell e}]_{1111}\frac{e^2}{t}+[c_{\ell e}]_{1111}\frac{(g_2^2+g_1^2)\,(\frac{1}{2}+s_W^2)s_W^2}{t-m_Z^2}\right]\Bigg\}, \tag{A.4}$$

where $t=-\frac{s}{2}\left(1-\cos\theta\right)$ and $u=-\frac{s}{2}\left(1+\cos\theta\right)$.

The relations between our model parameters and the SMEFT at dimension six level are as follows:

$$\begin{aligned}
\left[c_{\ell\ell}\right]_{1122} &= -\frac{v^2}{M_{Z'}^2}\,\left(g_{ee}^L g_{\mu\mu}^L + g_{e\mu}^L g_{\mu e}^L\right),\\
\left[c_{\ell e}\right]_{1122} &= -\frac{v^2}{M_{Z'}^2}\,\left(g_{ee}^L g_{\mu\mu}^R + g_{\mu e}^R g_{e\mu}^L\right),\\
\left[c_{\ell e}\right]_{2211} &= -\frac{v^2}{M_{Z'}^2}\,\left(g_{ee}^R g_{\mu\mu}^L + g_{\mu e}^L g_{e\mu}^R\right)\,,\\
\left[c_{ee}\right]_{1122} &= -\frac{v^2}{M_{Z'}^2}\,\left(g_{ee}^R g_{\mu\mu}^R + g_{\mu e}^R g_{e\mu}^R\right)\,,\\
\left[c_{\ell\ell}\right]_{1111} &= -\frac{v^2}{2M_{Z'}^2}g_{ee}^L g_{ee}^L\,,\\
\left[c_{ee}\right]_{1111} &= -\frac{v^2}{2M_{Z'}^2}g_{ee}^R g_{ee}^R\,,\\
\left[c_{\ell e}\right]_{1111} &= -\frac{v^2}{M_{Z'}^2}g_{ee}^R g_{\mu\mu}^L\,.
\end{aligned} \tag{A.5}$$

and similarly for $2\leftrightarrow 3$.

B QED Penguin Contributions

B.1 Hidden Operators

The following operators contained in Eq. (3.12)

$$\mathcal{O}^{AB}_{e\mu,\mu\mu},\qquad \mathcal{O}^{AB}_{e\mu,\tau\tau},\qquad \mathcal{O}^{AB}_{e\tau,\tau\mu},\qquad \mathcal{O}^{AB}_{e\tau,\tau\tau},\qquad \mathcal{O}^{AB}_{\mu\tau,\tau\tau}, \tag{B.1}$$

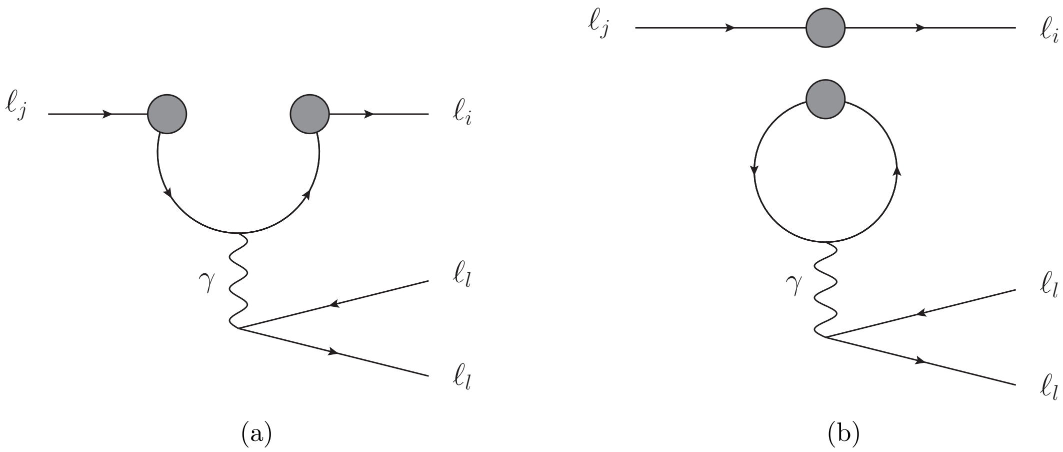

Figure 13. Feynman diagrams with two operator insertions corresponding to different Wick contractions in the EFT where the Z' is integrated out.

with $AB = LL, LR, RL, RR$, do not contribute to the flavour changing processes considered by us at tree level and could thus be considered as *hidden operators*. However, they contribute to these processes through QED penguin diagrams (as depicted in Fig. 13) [201–203].

In the formal language, these diagrams generate mixing of the operators in Eq. (B.1) into operators contributing already at tree-level to flavour changing processes. In this appendix we present the results for this additional effect.

This mixing can be found by means of standard methods [14]. That is by calculating the relevant one-loop anomalous dimensions obtained by inserting the operators in Eq. (B.1) into one-loop off-shell photon penguin diagrams [201–203]. As shown in Fig. 13, there are two possible operator insertions which result in the same contributions to the Wilson coefficients, unless they vanish. In the cases at hand one should note that for the inserted LL and RR operators with three electrons or three muons both penguin topologies contribute, which brings in a factor of two for this mixing relative to the remaining operator insertions. However, in the case of RL and LR operators, only the insertions into diagram (b) in Fig. 13 contribute, and this implies that the operator $\mathcal{O}^{V,RL}_{e\tau,\tau\mu}$ cannot contribute to this mixing.

The standard renormalization group evolution is then performed from the scale $M_{Z'}$ down to m_τ or m_μ for τ decays and μ decay, respectively. In the latter case one takes into acount that the τ-lepton is integrated out at m_τ, which implies two different logarithms in the final results. Keeping only the leading logarithms, we find the following results for the additional contributions to the Wilson coefficients of the operators in Eq. (3.12).

$$
\begin{aligned}
C_{e\mu}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\left(g_{e\mu}^L(2g_{\mu\mu}^L+g_{\mu\mu}^R)\ln\left(\frac{M_{Z'}^2}{m_\mu^2}\right)+\left(g_{e\tau}^L g_{\tau\mu}^L+g_{e\mu}^L(g_{\tau\tau}^L+g_{\tau\tau}^R)\right)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right)\right),\\
C_{e\mu}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\left(g_{e\mu}^R(2g_{\mu\mu}^R+g_{\mu\mu}^L)\ln\left(\frac{M_{Z'}^2}{m_\mu^2}\right)+\left(g_{e\tau}^R g_{\tau\mu}^R+g_{e\mu}^R(g_{\tau\tau}^L+g_{\tau\tau}^L)\right)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right)\right),\\
C_{e\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\,g_{e\tau}^L(2g_{\tau\tau}^L+g_{\tau\tau}^R)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right),\\
C_{e\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\,g_{e\tau}^R(g_{\tau\tau}^L+2g_{\tau\tau}^R)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right),\\
C_{\mu\tau}^{LL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\,g_{\mu\tau}^L(2g_{\tau\tau}^L+g_{\tau\tau}^R)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right),\\
C_{\mu\tau}^{RL} &= \frac{-e^2}{16\pi^2 M_{Z'}^2}\,\frac{2}{3}\,g_{\mu\tau}^R(g_{\tau\tau}^L+2g_{\tau\tau}^R)\ln\left(\frac{M_{Z'}^2}{m_\tau^2}\right).
\end{aligned}
\tag{B.2}
$$

The vectorial nature of the photon implies $C_{ij}^{LR} = C_{ij}^{LL}$ and $C_{ij}^{RR} = C_{ij}^{RL}$. It should be noted that these results apply universally to all operators with $j = k$ independent of the lepton flavour, as seen in the first term on the r.h.s in (3.14).

The Wilson coefficients of the operators contributing already at tree-level can also be affected by inserting them in penguin and one-loop current-current operators. But since these are already constrained by tree-level processes and loop-effects turn out to be subleading. Yet, for completeness we present expressions for these effects in the next appendix.

B.2 Running of Visible Operators

So far we have only included the contributions from hidden operators mixing into visible operators. But there are also contributions from the mixing of visible operators into visible operators, both through QED penguins and insertions in current-current topologies. The QED penguin diagrams are again given in Fig. 13, the current-current topologies in Fig. 14.

The QED penguin contributions imply shifts in the coefficients of the operators

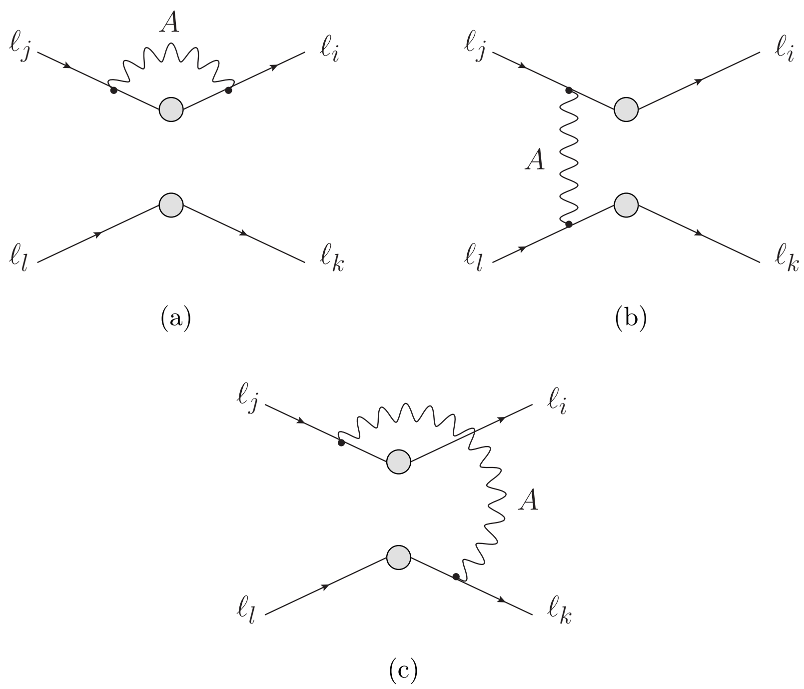

Figure 14. 1-loop QED corrections generating mixing among four-lepton operators.

in (B.2). We find

$$\Delta C^{LL}_{e\mu} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3}\, g^L_{e\mu} \left(2 g^L_{ee} + g^R_{ee}\right) \ln\left(\frac{M^2_{Z'}}{m^2_\mu}\right), \tag{B.3}$$

$$\Delta C^{RL}_{e\mu} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3}\, g^R_{e\mu} \left(g^L_{ee} + 2 g^R_{ee}\right) \ln\left(\frac{M^2_{Z'}}{m^2_\mu}\right),$$

$$\Delta C^{LL}_{e\tau} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3} \Big(g^L_{e\tau}\, \left(2\, g^L_{ee} + g^R_{ee}\right) + \left(g^L_{e\mu}\, g^L_{\mu\tau} + g^L_{e\tau}\, \left(g^L_{\mu\mu} + g^R_{\mu\mu}\right)\right) \Big) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right),$$

$$\Delta C^{RL}_{e\tau} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3} \Big(g^R_{e\tau}\, \left(2\, g^R_{ee} + g^L_{ee}\right) + \left(g^R_{e\mu}\, g^R_{\mu\tau} + g^R_{e\tau}\, \left(g^R_{\mu\mu} + g^L_{\mu\mu}\right)\right) \Big) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right),$$

$$\Delta C^{LL}_{\mu\tau} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3} \Big(g^L_{\mu\tau}\, \left(2\, g^L_{\mu\mu} + g^R_{\mu\mu}\right) + \left(g^L_{\mu e}\, g^L_{e\tau} + g^L_{\mu\tau}\, \left(g^L_{ee} + g^R_{ee}\right)\right) \Big) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right),$$

$$\Delta C^{RL}_{\mu\tau} = \frac{-e^2}{16\pi^2 M^2_{Z'}} \frac{2}{3} \Big(g^R_{\mu\tau} \left(2\, g^R_{\mu\mu} + g^L_{\mu\mu}\right) + \left(g^R_{\mu e}\, g^R_{e\tau} + g^R_{\mu\tau}\, \left(g^R_{ee} + g^L_{ee}\right)\right) \Big) \ln\left(\frac{M^2_{Z'}}{m^2_\tau}\right),$$

and for RR and LR with L and R interchanged.

On the other hand, the current-current contributions modify the coefficients $X^{AB}_{ij,kl}$ in (3.14). We find

$$\Delta X^{LL}_{ij,kl} = \frac{-e^2}{16\pi^2 M^2_{Z'}}\, 6\, g^L_{ij} g^L_{kl} \ln\left(\frac{M^2_{Z'}}{m^2_r}\right), \tag{B.4}$$

$$\Delta X^{RL}_{ij,kl} = \frac{e^2}{16\pi^2 M^2_{Z'}}\, 6\, g^R_{ij} g^L_{kl} \ln\left(\frac{M^2_{Z'}}{m^2_r}\right), \tag{B.5}$$

where $m_r = m_\tau$ and $m_r = m_\mu$ for τ and μ decays, respectively. For RR- and LR-coefficients, L and R should be interchanged.

The visible operators also lead to contributions to $\mu \to e$ conversion. The resulting shifts in the Wilson coefficients of Eq. (3.22) are given by

$$\begin{aligned}\Delta C^{LL}_{e\mu,qq} &= \frac{e^2 Q_q}{16\pi^2}\frac{2}{3}\frac{1}{M_{Z'}^2} g^L_{e\mu}\left(2\, g^L_{ee} + g^R_{ee}\right)\log\left(\frac{M_{Z'}^2}{m_\mu^2}\right),\\ \Delta C^{RL}_{e\mu,qq} &= \frac{e^2 Q_q}{16\pi^2}\frac{2}{3}\frac{1}{M_{Z'}^2} g^R_{e\mu}\left(2\, g^R_{ee} + g^L_{ee}\right)\log\left(\frac{M_{Z'}^2}{m_\mu^2}\right).\end{aligned} \tag{B.6}$$

where, as in Eq. (3.22), Q_q stands for the electric charge of the quarks.

C General LFV scenarios

Here we study the general cases 4, 5 and 6 from Section 4.3. Case 4, where $g^L_{ij} = g^R_{ij} = g^V_{ij}$, is shown in Fig 15. Similarly to previous cases in Sections 4.1 and 4.2, all couplings involving the first generation are strongly constrained. Also the flavour violating ones are strictly bounded while the flavour conserving ones involving muons and taus can be of order unity. Similarly, we present the results for the cases 5 and 6, where only g^L_{ij} and g^R_{ij} are non-zero, as shown in Fig. 16 and Fig. 17, respectively. Evidently, in these cases the couplings are allowed to be larger than for the case of vectorial couplings.

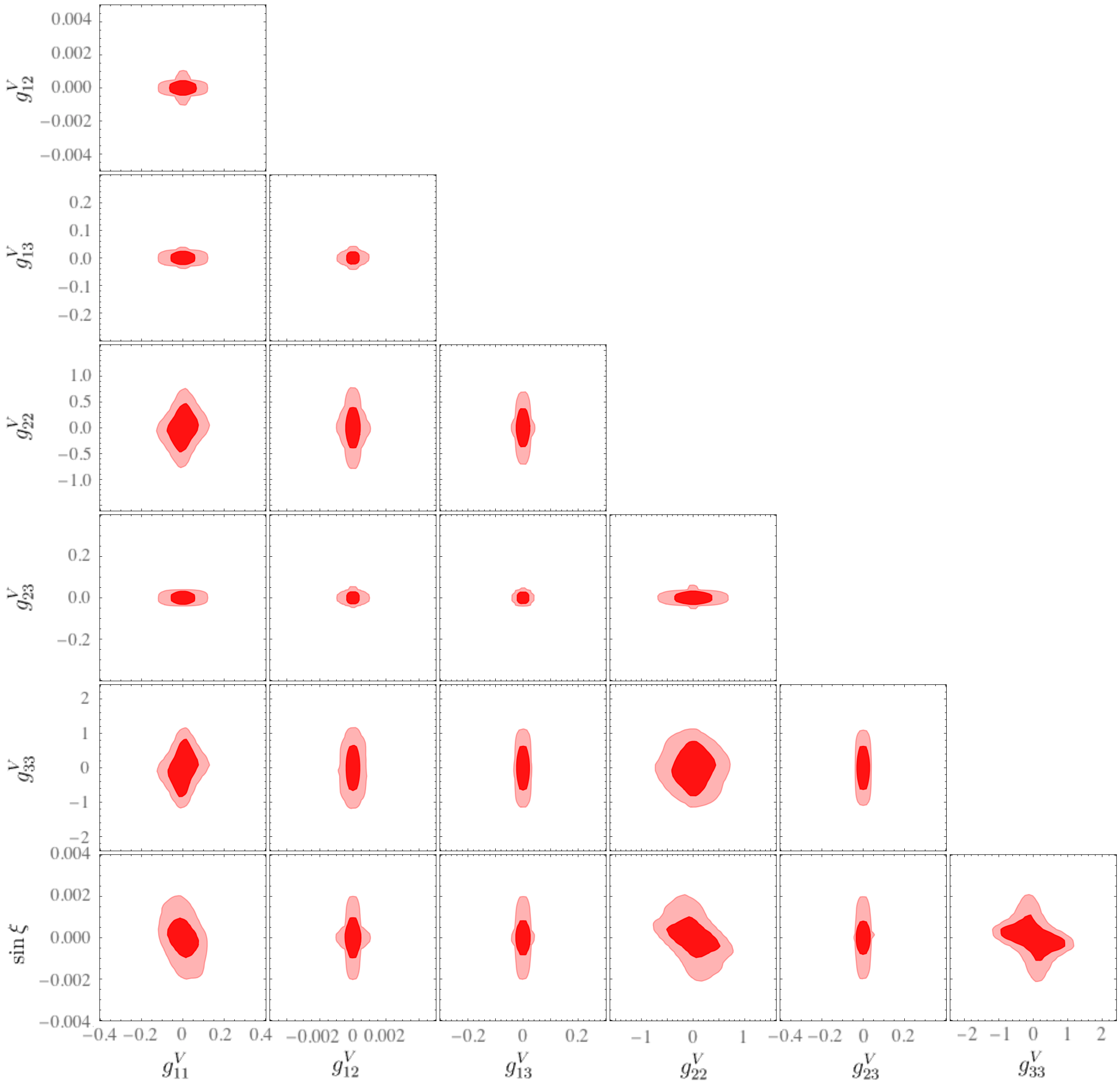

Figure 15. 68% and 95% CL regions for $g^V_{ij} = g^L_{ij} = g^R_{ij}$ and $\sin\xi$.

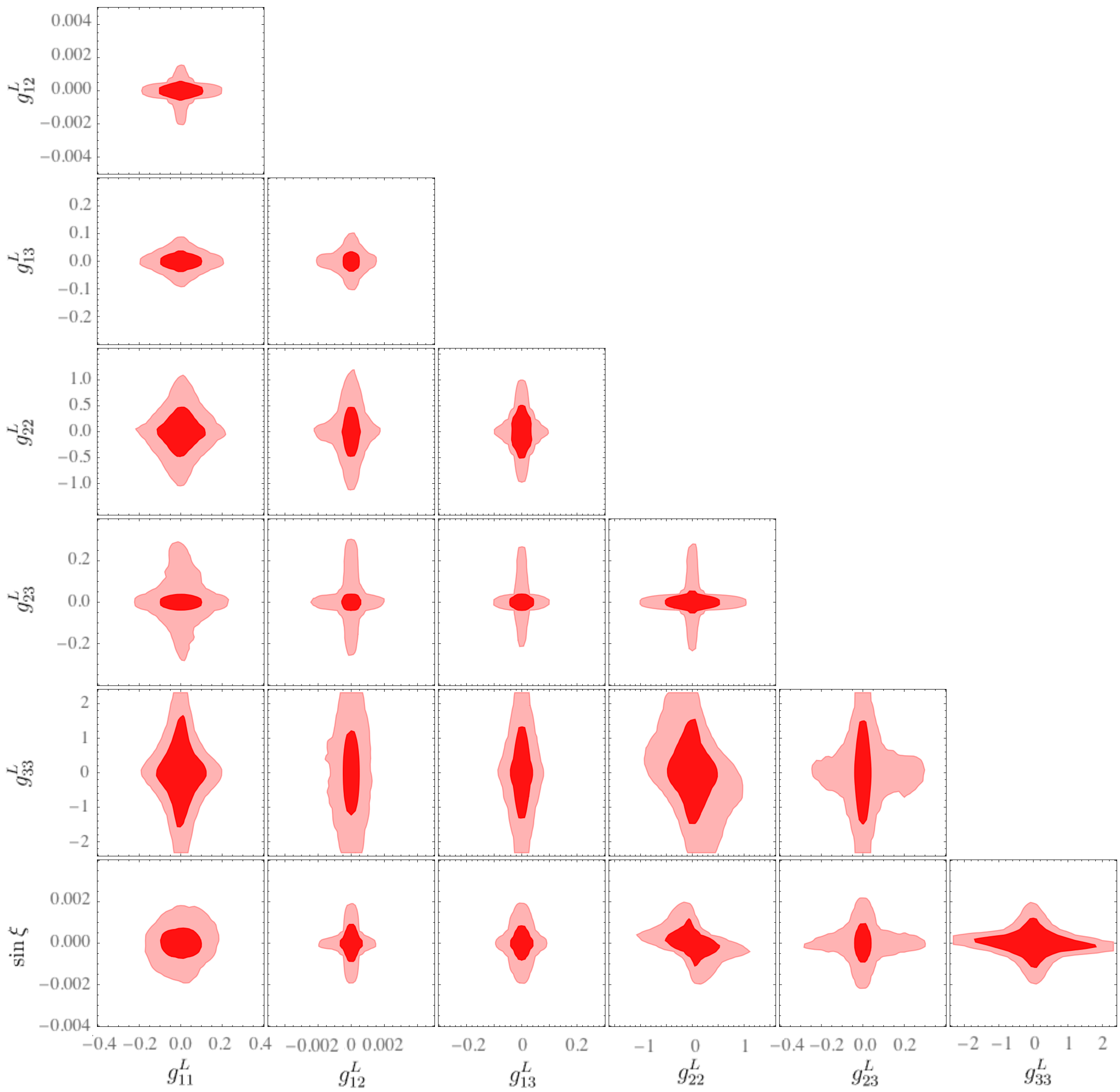

Figure 16. 68% and 95% CL regions for the LFV case where only g_{ij}^L is non-zero.

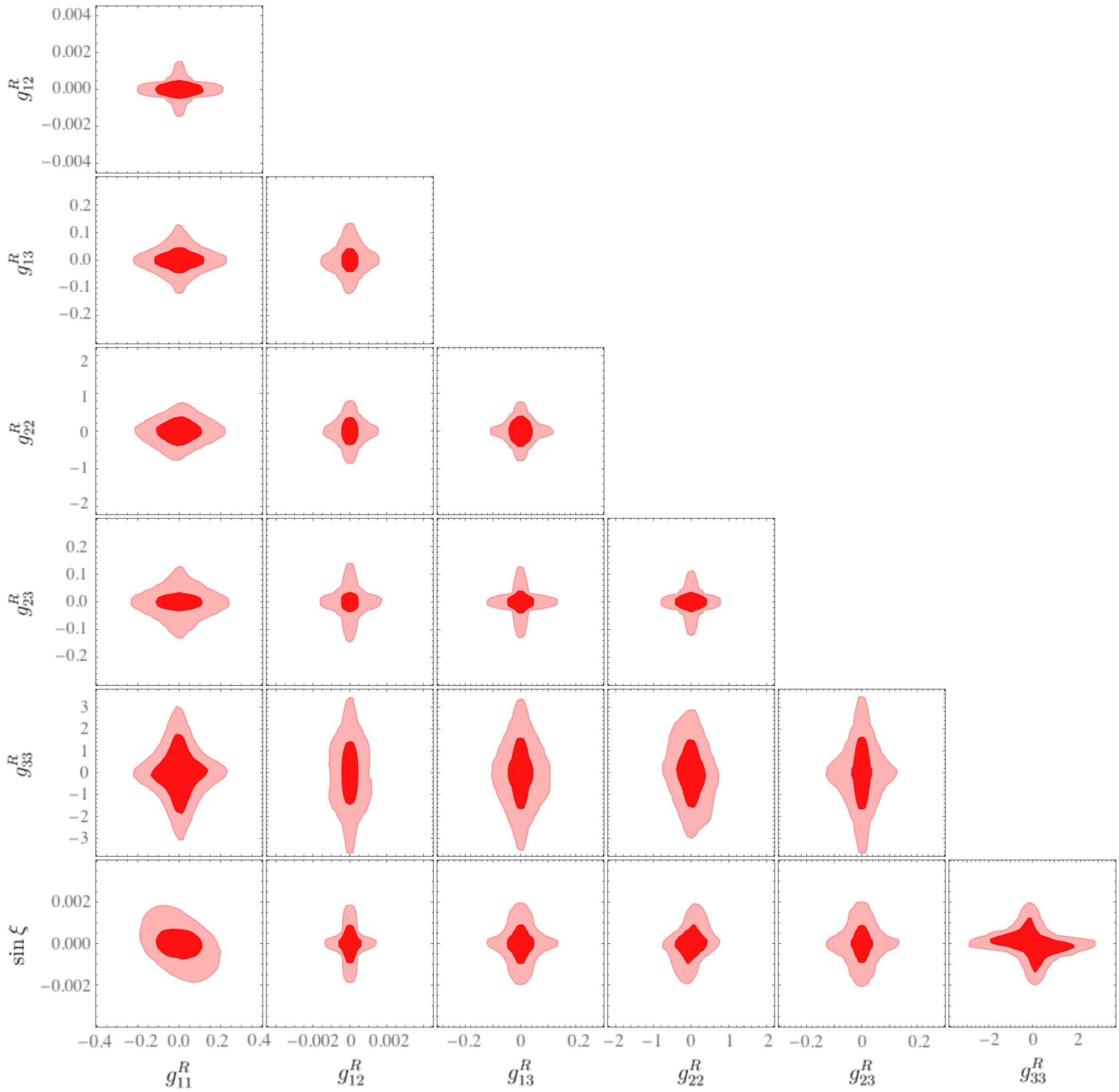

Figure 17. 68% and 95% CL regions for the LFV case where only g_{ij}^R is non-zero.

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