Fully - heavy tetraquark production by $\gamma\gamma$ interactions in hadronic collisions at the LHC

Victor P. Gonçalves

High and Medium Energy Group, Instituto de Física e Matemática, Universidade Federal de Pelotas,
Caixa Postal 354, 96010-900, Pelotas, RS, Brazil.

Bruno D. Moreira

Departamento de Física, Universidade do Estado de Santa Catarina, 89219-710, Joinville, SC, Brazil.
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We investigate the production of the fully - heavy tetraquark states T_{4Q} in the $\gamma\gamma$ interactions present in proton-proton, proton-nucleus and nucleus-nucleus collisions at the CERN Large Hadron Collider (LHC). We focus on the $\gamma\gamma \rightarrow \mathcal{QQ}$ ($\mathcal{Q} = J/\psi, \Upsilon$) subprocess, mediated by the T_{4Q} resonance in the s - channel, and present predictions for the hadronic cross sections considering the kinematical ranges probed by the ALICE and LHCb Collaborations. Our results demonstrate that the experimental study of this process is feasible and can be used to investigate the existence and properties of the $T_{4c}(6900)$ and $T_{4b}(19000)$ states.

Over the last years the existence of exotic hadrons, which are a class of hadrons that cannot be easily accommodated in the remaining unfilled $q\bar{q}$ and qqq states, has been established and a large number of candidates have been proposed (For recent reviews, see e.g. Refs. [1–3]). In particular, the LHCb Collaboration has recently observed [4] a sharp peak in the di - J/ψ channel at $M = 6.9$ GeV, which suggests the presence of a fully - charm tetraquark state. Such a result has motivated a series of studies about the description of the fully - heavy tetraquark states T_{4Q}, composed by charm and bottom quarks, which propose the existence of a large number of new exotic states (See e.g. Refs. [5–20]). In order to improve our understanding of these resonances, it is fundamental to have theoretical control of the mechanism in which they are produced. In recent years, different authors have proposed distinct production mechanisms of the T_{4Q} in hadronic colliders (See e.g. Refs. [21–31]). In particular, in Ref. [23], the authors have proposed a model for the fully - charm tetraquark production in which the double $c\bar{c}$ pair is produced by the double scattering process and the cross section for T_{4c} state is estimated within the framework of the color evaporation model [32]. Recently, such an idea was elaborated in more detail in Ref. [27], which confirmed that this mechanism is one of the more promising ways to probe the T_{4c} state. However, even this mechanism still has limitations due to the current theoretical uncertainties present in the description of the hadronization and double scattering processes. An important alternative to probe exotic hadrons, proposed and developed in recent years [33–39], is the study of photon induced interactions at the LHC, which became a reality in the last decade (For a recent review see e.g. Ref. [40]). Our goal in this letter is to extend these previous studies for the T_{4Q} production.

One has that for ultra-relativistic collisions, the incident charged hadrons are an intense source of photons and in a collision at large impact parameters ($b > R_{h_1} + R_{h_2}$, with R_i being the hadron radius), denoted hereafter ultra - peripheral collisions (UPCs), photon – photon and photon – hadron interactions become dominant over the strong hadron – hadron one [41]. In the particular case of the T_{4Q} production by photon - photon interactions, represented in Fig. 1, the total cross section can be factorized in terms of the equivalent flux of photons of the incident hadrons and the photon-photon production cross section, which can be expressed in terms the two-photon decay width $\Gamma_{T_{4Q} \rightarrow \gamma\gamma}$. As the photon flux is well – known, one has that this process is sensitive to the description of annihilation process, $T_{4Q} \rightarrow \gamma\gamma$, i.e. to the description of the T_{4Q} wave function. Therefore, the study of the production in photon – induced interactions allows us to directly test the modeling of the fully - heavy tetraquark states. Another advantage of the T_{4Q} production by $\gamma\gamma$ interactions in hadronic colliders, is that the experimental separation of the associated events is relatively easy. As photon emission is coherent over the hadron and the photon is colorless, the events will be characterized by two intact recoiled hadrons (tagged hadrons) and the presence of two rapidity gaps, i.e., empty regions in pseudo-rapidity that separate the intact very forward hadrons from the T_{4Q} state, which we will assume to decay into a pair of vector mesons. Such characteristics can be used to separate the events in a clean environment with a small background. In this letter we will perform an exploratory study, which will analyze the possibility of producing the exotic T_{4c} and T_{4b} states by two-photon interactions in pp, pPb and $PbPb$ collisions at the LHC. As we will show the resulting cross sections are large, which implies that a future experimental analysis is, in principle, feasible. Therefore, the study of this process can be used to confirm (or not) the existence of these states as well it
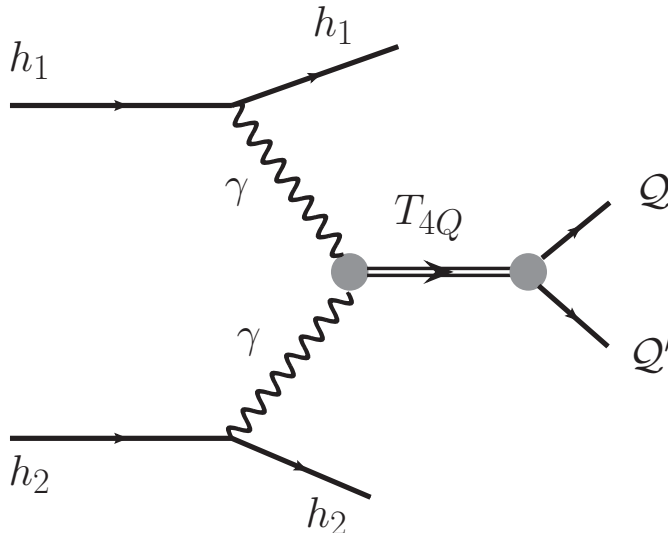
FIG. 1: Typical diagram for the production of a fully - heavy tetraquark T_{4Q} state by $\gamma\gamma$ interactions in a hadronic collision. The $T_{4Q} \rightarrow \mathcal{Q}\mathcal{Q}'$ decay, where $\mathcal{Q}$ is a quarkonium state, is also represented.

will allow us to investigate its properties.

Initially, let's present a brief review of the formalism needed to describe the T_{4Q} tetraquark production by $\gamma\gamma$ interactions in hadronic collisions. Using the equivalent photon approximation [41, 42], one has that the cross section for the collision between two hadrons, h_1 and h_2, is given by

$$\sigma\left(h_1 h_2 \rightarrow h_1 \otimes T_{4Q} \otimes h_2; s\right) = \int \hat{\sigma}\left(\gamma\gamma \rightarrow T_{4Q}; W\right) N\left(\omega_1, \mathbf{b}_1\right) N\left(\omega_2, \mathbf{b}_2\right) S_{abs}^2(\mathbf{b}) \frac{W}{2} \mathrm{d}^2\mathbf{b}_1 \mathrm{d}^2\mathbf{b}_2 \mathrm{d}W \mathrm{d}Y , \qquad (1)$$

where $\sqrt{s}$ is center-of-mass energy for the $h_1 h_2$ collision ($h_i = \mathrm{p}, \mathrm{Pb}$), $\otimes$ characterizes a rapidity gap in the final state, $W = \sqrt{4\omega_1\omega_2}$ is the invariant mass of the $\gamma\gamma$ system and ω_i are the photon energies. Moreover, Y is the rapidity of the outgoing resonance T_{4Q}, $N(\omega_i, b_i)$ is the equivalent photon spectrum generated by hadron (nucleus) i at a distance b_i from h_i and the factor $S_{abs}^2(\mathbf{b})$ is the absorption factor, which excludes the overlap between the colliding hadrons and allows to take into account only ultraperipheral collisions, where the impact parameter $\mathbf{b}$ is larger than the sum of the hadron radius. Finally, $\hat{\sigma}_{\gamma\gamma \rightarrow T_{4Q}}(\omega_1, \omega_2)$ is the cross section for the production of a state T_{4Q} from two real photons with energies ω_1 and ω_2. Using the Low formula [43], the cross section for the production of the T_{4Q} state due to the two-photon fusion can be written in terms of the two-photon decay width $\Gamma_{T_{4Q} \rightarrow \gamma\gamma}$ as follows

$$\hat{\sigma}_{\gamma\gamma \rightarrow T_{4Q}}(\omega_1, \omega_2) = 8\pi^2(2J+1)\frac{\Gamma_{T_{4Q} \rightarrow \gamma\gamma}}{M_{T_{4Q}}}\delta(4\omega_1\omega_2 - M_{T_{4Q}}^2), \qquad (2)$$

where $M_{T_{4Q}}$ and J are, respectively, the mass and spin of the produced state. As in Ref. [37], we will estimate the photon flux assuming that nucleus (proton) can be described by a monopole (dipole) form factor and that $S_{abs}^2(\mathbf{b}) = \Theta\left(|\mathbf{b}| - R_{h_1} - R_{h_2}\right) = \Theta\left(|\mathbf{b}_1 - \mathbf{b}_2| - R_{h_1} - R_{h_2}\right)$, where R_{h_i} is the radius of the hadron h_i ($i = 1, 2$), with $R_p = 0.7$ fm and $R_A = 1.2 A^{1/3}$ fm. A detailed discussion about the theoretical uncertainty associated to these choices is presented in Ref. [44].

One has that the cross section is directly dependent on the values for the decay width $\Gamma_{T_{4Q} \rightarrow \gamma\gamma}$, mass and spin of the resonance. Such quantities can be taken from experiment or can be theoretically estimated. In our analysis we will consider that the resonance decays into a $\mathcal{Q}\mathcal{Q}'$ final state, where $\mathcal{Q}$ is a quarkonium state. As a consequence, the cross section will be proportional to $\Gamma_{T_{4Q} \rightarrow \gamma\gamma} \times \mathcal{B}(T_{4Q} \rightarrow \mathcal{Q}\mathcal{Q}')$, where $\mathcal{B}(T_{4Q} \rightarrow \mathcal{Q}\mathcal{Q}')$ is the associated branching fraction. In principle, such a product can be measured precisely in future e^+e^- colliders, which will make the predictions for the LHC will be parameter free. However, as these quantities are currently unknown, we will assume some naive approximations in order to derive an estimate of the associated cross sections. We will focus on the case in that $\mathcal{Q} = \mathcal{Q}'$ and $\mathcal{Q} = J/\psi$ or Υ, which are expected to be present in the final state when the T_{4c} and T_{4b} are produced, respectively. Moreover, we will assume that the fully - charm tetraquark T_{4c} is the $X(6900)$ state, recently observed by the LHCb Collaboration [4]. On the other hand, for the T_{4b} case, we will assume that it corresponds to the $X(19000)$

Collision	Resonance	LHC Full rapidity range	LHCb $2.0 \leq Y \leq 4.5$	ALICE $-1.0 \leq Y \leq 1.0$
pp ($\sqrt{s}$ =13 TeV)	$X(6900)$, 0^{++}	26.3 fb	5.53 fb	6.34 fb
	$X(6900)$, 2^{++}	31.9 fb	6.71 fb	7.71 fb
pPb ($\sqrt{s}$ =8.1 TeV)	$X(6900)$, 0^{++}	76.3 pb	21.6 pb	22.4 pb
	$X(6900)$, 2^{++}	92.4 pb	26.2 pb	27.2 pb
$PbPb$ ($\sqrt{s}$ =5.02 TeV)	$X(6900)$, 0^{++}	171.0 nb	22.3 nb	70.0 nb
	$X(6900)$, 2^{++}	206.0 nb	26.7 nb	84.7 nb

TABLE I: Total cross sections for the $X(6900)[J^P] \to J/\psi J/\psi$ production by $\gamma\gamma$ interactions in pp, pPb and $PbPb$ collisions for different center - of - mass energies considering the full LHC rapidity range as well as the rapidity ranges covered by the ALICE and LHCb detectors.
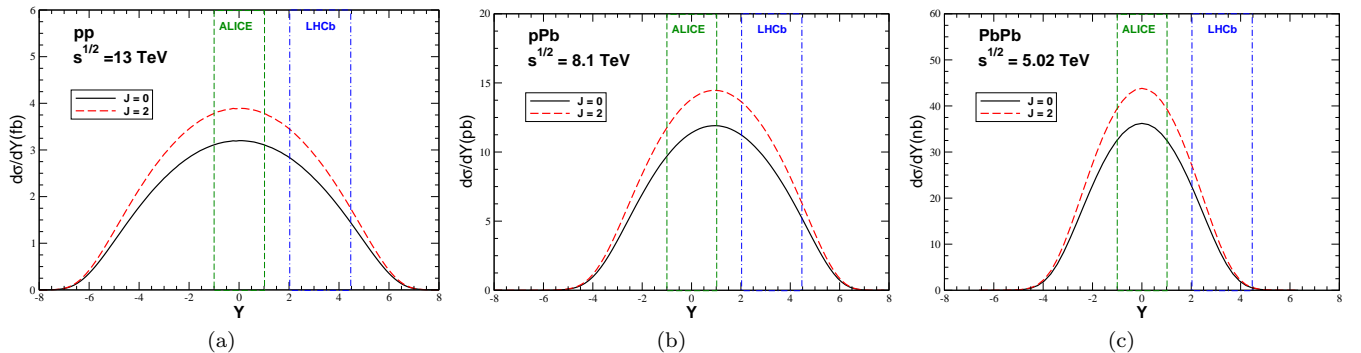

FIG. 2: Rapidity distributions for the $X(6900) \to J/\psi J/\psi$ production by $\gamma\gamma$ interactions in (a) pp ($\sqrt{s} = 13$ TeV), (b) pPb ($\sqrt{s} = 8.1$ TeV) and (c) $PbPb$ ($\sqrt{s} = 5.02$ TeV) collisions at the LHC.

tetraquark state, predicted by different phenomenological models (See e.g. Refs. [10, 12, 15, 16, 24, 25]). As the quantum numbers of these resonances are still unknown, we will estimate the cross sections assuming that $J^P = 0^+$ or 2^+. Motivated by the LHCb results [4] and following Ref. [26], we will take $\mathcal{B}(T_{4Q} \to \mathcal{Q}\mathcal{Q}) = 2\%$ (For a recent calculation of the T_{4c} branching ratio see, e.g., Ref. [8]). Moreover, as in our case the T_{4Q} state is expected to have spin - parity identical to that from the χ_Q quarkonium family, we will assume that $\Gamma_{T_{4Q} \to \gamma\gamma} \simeq \Gamma_{\chi_Q \to \gamma\gamma}$. Therefore, our predictions for the $X(6900)$ production will be derived using the values for $\Gamma_{\chi_c \to \gamma\gamma}$ presented in the latest Particle Data Group [45]. On the other hand, as the two - photon width $\Gamma_{\chi_b \to \gamma\gamma}$ was not still measured, we will estimate the $X(19000)$ production assuming the values derived in Ref. [46] using the covariant light - front framework. It is important to emphasize that our results can be easily generalized to other values of the branching ratio and two - photon width by a simple rescaling of our predictions, since they are linearly dependent on these quantities.

In Table I we present our predictions for the total cross sections for the $X(6900) \to J/\psi J/\psi$ production in $pp/pPb/PbPb$ collisions at the LHC energies considering the full LHC rapidity range as well as the rapidity ranges covered by the ALICE and LHCb detectors. We consider the two possible values for J^P and, following Ref. [16], we will assume that the mass of the resonance is equal to 6871.0 MeV for $J = 0$ and 6967.0 MeV for $J = 2$. Due to the Z^2 dependence of the photon spectra, we have that the following hierarchy is approximately valid for the $X(6900)$ production induced by $\gamma\gamma$ interactions: $\sigma_{PbPb} = Z^2 \cdot \sigma_{pPb} = Z^4 \cdot \sigma_{pp}$, with $Z = 82$. In Fig. 2 the corresponding rapidity distributions for the $X(6900) \to J/\psi J/\psi$ production are presented. One has that the predictions for the $J = 2$ resonance are larger than those for the $J = 0$ one, as expected from the Low formula. Due to the asymmetry in the proton and nuclear photon fluxes present in the initial state, we predict an asymmetric rapidity distribution in the case of pPb collisions. In Fig. 2 we also indicate the kinematical rapidity ranges probed by the ALICE ($-1 \leq Y \leq +1$) and LHCb ($+2 \leq Y \leq +4.5$) detectors. The resulting predictions for the total cross sections in the ALICE and LHCb rapidity ranges are also presented in Table I. In comparison with the results for the full LHC rapidity range, one has that the predictions are reduced by a factor between 3.0 and 8.0 depending on the initial state and the rapidity range covered by the detector, with the larger reduction being for $PbPb$ collisions at the LHCb. Although this reduction is nonnegligible, the final values are still large and imply a significant number of events if we consider that the expected integrated luminosity for the high luminosity run of the LHC is 50/fb (10/nb) for pp ($PbPb$) collisions [47]. In particular, we predict that the number of events per year in pp ($PbPb$) collisions will be $\approx$ 276 (335) at LHCb and 385

Collision	Resonance	LHC Full rapidity range	LHCb $2.0 \leq Y \leq 4.5$	ALICE $-1.0 \leq Y \leq 1.0$
pp ($\sqrt{s}$ =13 TeV)	$X(19000)$, 0^{++}	2.40×10^{-3} fb	4.90×10^{-4} fb	6.88×10^{-4} fb
	$X(19000)$, 2^{++}	5.91×10^{-3} fb	1.21×10^{-3} fb	1.70×10^{-3} fb
pPb ($\sqrt{s}$ =8.1 TeV)	$X(19000)$, 0^{++}	5.60 fb	1.62 fb	1.96 fb
	$X(19000)$, 2^{++}	13.80 fb	3.99 fb	4.83 fb
$PbPb$ ($\sqrt{s}$ =5.02 TeV)	$X(19000)$, 0^{++}	8.33 pb	0.564 pb	4.32 pb
	$X(19000)$, 2^{++}	20.5 pb	1.38 pb	10.6 pb

TABLE II: Total cross sections for the $X(19000)[J^P] \to \Upsilon\Upsilon$ production by $\gamma\gamma$ interactions in pp, pPb and $PbPb$ collisions for different center - of - mass energies considering the full LHC rapidity range as well as the rapidity ranges covered by the ALICE and LHCb detectors.

(840) at ALICE, which implies that the experimental analysis of this process is, in principle, feasible.

The analysis performed above can be directly extended for the production of fully - bottom tetraquark states. In particular, we will provide predictions for the $X(19000)$ production, assuming that M = 19434.0 MeV for J=0 and $M = 19481.0$ MeV for $J = 2$ as predicted in Ref. [16]. Our results for the cross sections are presented in Table II. In comparison to the results for the $X(6900)$, the cross sections for the $X(19000)[J^P] \to \Upsilon\Upsilon$ production are smaller by a factor $\gtrsim 10^3$. As a consequence, the associated number of events per year will be very small, making the experimental analysis of this exotic state in the next run of the LHC a hard task. However, one has verified that in $PbPb$ collisions at $\sqrt{s} = 39$ TeV, which is the expected center - of - mass energy of the Future Circular Collider (FCC) [48], the cross sections are increased by a factor ≈ 3. As the integrated luminosity for this future collider is expected to be $\approx$ 110/nb, one has that the analysis of the $X(19000)[J^P] \to \Upsilon\Upsilon$ production in ultraperipheral $PbPb$ collisions becomes feasible at the FCC.

Some comments are in order. As discussed before, the T_{4Q} production by $\gamma\gamma$ interactions in $pp/pPb/PbPb$ collisions will be characterized by two intact hadrons that can be detected by forward detectors and two rapidity gaps in the final state. This final state is also generated by the single and double scattering mechanisms discussed in Refs. [49–52]. The comparison of our results for the $X(6900) \to J/\psi J/\psi$ case with those derived in Refs. [51, 52] for the double J/ψ production indicates that both processes have similar cross sections before the inclusion of additional kinematical cuts. Such a result strongly motivates a more detailed analysis, including the cuts usually considered by the experimental collaborations. Another important comment is that during the development of this analysis we were aware that a similar study of the $X(6900)$ production in UPHICs is being independently performed by Y. Xie and collaborators [53]. These authors have estimated the decay width of $X(6900)$ to two photons using the effective Lagrangian method and derived slightly smaller values for the cross sections. Such result indicates that the naive assumptions assumed in our analysis are a good first approximation to estimate the fully - heavy tetraquark production by $\gamma\gamma$ interactions in hadronic collisions.

Finally, let's summarize our main results and conclusions. Motivated by the observation of a sharp peak in the di - J/ψ channel at $M = 6.9$ GeV by the LHCb Collaboration, which suggest the presence of a fully - charm tetraquark state, we have developed in this letter the treatment of the fully - heavy tetraquark production by $\gamma\gamma$ interactions in hadronic collisions. We have focused on the $X(6900)$ and $X(19000)$ states, and estimated the cross sections assuming that they decay into a $J/\psi J/\psi$ and $\Upsilon\Upsilon$ final state, respectively. Predictions for the kinematical ranges probed by the ALICE and LHCb detectors were presented, which indicate that the experimental analysis of the $X(6900)$ production is, in principle, feasible at the LHC. Such conclusion strongly motivates a more detailed analysis, taking into account of the experimental cuts assumed by the distinct experimental collaborations, which we intend to perform in a near future.

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