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Five-flavor pentaquarks and other light- and heavy-flavor symmetry partners of the LHCb hidden-charm pentaquarks

Fang-Zheng Peng [a], Ming-Zhu Liu [a,b], Ya-Wen Pan [a], Mario Sánchez Sánchez [c], Manuel Pavon Valderrama [a,*]

[a] *School of Physics, Beihang University, Beijing 100191, China*
[b] *School of Space and Environment, Beihang University, Beijing 100191, China*
[c] *Centre d'Études Nucléaires, CNRS/IN2P3, Université de Bordeaux, 33175 Gradignan, France*

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Abstract

The discovery of three pentaquark peaks — the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ — by the LHCb collaboration has a series of interesting consequences for hadron spectroscopy. If these hidden-charm objects are indeed hadronic molecules, as suspected, they will be constrained by heavy-flavor and SU(3)-flavor symmetries. The combination of these two symmetries will imply the existence of a series of five-flavor pentaquarks with quark content $\bar{b}csdu$ and $b\bar{c}sdu$, that is, pentaquarks that contain each of the five quark flavors that hadronize. In addition, from SU(3)-flavor symmetry alone we expect the existence of light-flavor partners of the three P_c pentaquarks with strangeness $S = -1$ and $S = -2$. The resulting structure for the molecular pentaquarks is analogous to the light-baryon octet — we can label the pentaquarks as $P^N_{Q'\bar{Q}}$, $P^\Lambda_{Q'\bar{Q}}$, $P^\Sigma_{Q'\bar{Q}}$, $P^\Xi_{Q'\bar{Q}}$ depending on their heavy- and light-quark content (with N, Λ, Σ, Ξ the member of the light-baryon octet to which the light-quark structure resembles and Q', $\bar{Q}$ the heavy quark-antiquark pair). In total we predict 45 new pentaquarks from heavy- and light-flavor symmetries alone, which extend up to 109 undiscovered states if we also consider heavy-quark spin symmetry. If an isoquartet ($I = 3/2$) hidden-charm pentaquark is ever observed, this will in turn imply a second multiplet structure resembling the light-baryon decuplet: $P^\Delta_{Q'\bar{Q}}$, $P^{\Sigma^*}_{Q'\bar{Q}}$, $P^{\Xi^*}_{Q'\bar{Q}}$, $P^\Omega_{Q'\bar{Q}}$.

* Corresponding author.
E-mail address: mpavon@buaa.edu.cn (M. Pavon Valderrama).

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1. Introduction

The discovery by the LHCb collaboration of three hidden-charm pentaquarks [1] — the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ — extends the previous observation of the $P_c(4450)$ peak in 2015 [2]. Their masses and widths (in MeV) are

$$m_{P_{c1}} = 4311.9 \pm 0.7^{+6.8}_{-0.6}\,, \quad \Gamma_{P_{c1}} = 9.8 \pm 2.7^{+3.7}_{-4.5}\,, \tag{1}$$

$$m_{P_{c2}} = 4440.3 \pm 1.3^{+4.1}_{-4.7}\,, \quad \Gamma_{P_{c2}} = 20.6 \pm 4.9^{+8.7}_{-10.1}\,, \tag{2}$$

$$m_{P_{c3}} = 4457.3 \pm 0.6^{+4.1}_{-1.7}\,, \quad \Gamma_{P_{c3}} = 6.4 \pm 2.0^{+5.7}_{-1.9}\,, \tag{3}$$

where from now on we will use the notation P_{c1}, P_{c2} and P_{c3} for these three pentaquarks. The P_{c1} is 8.9 MeV below the $\bar{D}\Sigma_c$ threshold, while the P_{c2} and P_{c3} are 21.8 and 4.8 MeV below the $\bar{D}^*\Sigma_c$ threshold, respectively (where we have considered these thresholds in the isospin-symmetric limit). This, together with the existence of hidden-charm pentaquark predictions in the molecular picture before their experimental observation [3–9], suggests a molecular interpretation of these pentaquarks, i.e. that they are bound states of a charmed antimeson and a charmed baryon [10–15], though this is not the only explanation that has been considered by theoreticians [16–18].

Heavy-hadron molecules are highly symmetrical: their light- and heavy-quark content implies that they are constrained both by SU(3)-flavor symmetry [19,20] and heavy-quark symmetry [21,22]. Heavy-quark symmetry has in turn different manifestations, namely heavy-quark spin symmetry (HQSS), heavy-flavor symmetry (HFS) and heavy-antiquark-diquark symmetry (HADS) [23], which altogether provide deep insights into the molecular spectrum [24–32]. The application of HQSS to the particular case of the LHCb pentaquarks implies that the P_{c1}, P_{c2} and P_{c3} actually belong to a multiplet composed of seven members [12,14,33,34], four of which have not been observed yet. Before knowing that the $P_c(4450)$ peak contained two peaks, HQSS was already used to predict a $J^P = 5/2^-$ $\bar{D}^*\Sigma_c^*$ molecular pentaquark and other partner states [6,35–38]. In the past HFS and HADS have been applied to heavy meson-antimeson molecules to explain spectroscopic relations among known molecular states [30] or to deduce the existence of new states [31]. In this manuscript we will explore what are the consequences of SU(3)-flavor symmetry and HFS if the hidden-charm pentaquarks are indeed molecular.

2. Symmetries

First, we will consider the constraints that HFS and SU(3)-flavor symmetry impose on the potential between a heavy antimeson and a heavy baryon. HFS refers to the fact that the structure of a heavy-light hadron (i.e. the "brown muck" around the heavy quark) is independent of the flavor of the heavy quark. As applied to heavy-hadron molecules, HFS implies that the potential among heavy hadrons is independent of the flavor of the heavy quarks inside the heavy hadrons. The clearest example of this symmetry in molecular states are the Z_c's and Z_b's resonances [30], which are repeated in the charm and bottom sectors and are conjectured to be $D^{(*)}\bar{D}^*$ and $B^{(*)}\bar{B}^*$

Table 1
The SU(3)-flavor structure of the potential for heavy meson-baryon molecules, where the heavy meson belongs to a SU(3)-flavor triplet and the heavy baryon to a sextet. The heavy meson-baryon potential can be decomposed into an octet and decuplet component, from which the octet piece corresponds to the potential for the hidden-charm molecular candidates. As a consequence other molecular pentaquarks belonging to the octet representation are also expected to bind. In addition to the SU(3)-flavor decomposition, the S-wave potential can be further decomposed into its light-quark structure, which is not explicitly shown here.

Molecule	I	S	V	V_{eigen}
$\bar{P}\Sigma_Q$	$\frac{1}{2}$	0	V^O	–
$\bar{P}\Sigma_Q$	$\frac{3}{2}$	0	V^D	–
$\bar{P}\Xi'_Q$	0	-1	V^O	–
$\bar{P}\Xi'_Q - \bar{P}_s\Sigma_Q$	1	-1	$\begin{pmatrix} \frac{1}{3}V^O + \frac{2}{3}V^D & -\frac{\sqrt{2}}{3}(V^O - V^D) \\ -\frac{\sqrt{2}}{3}(V^O - V^D) & \frac{2}{3}V^O + \frac{1}{3}V^D \end{pmatrix}$	$\{V^O, V^D\}$
$\bar{P}\Omega_Q - \bar{P}_s\Xi'_Q$	$\frac{1}{2}$	-2	$\begin{pmatrix} \frac{1}{3}V^O + \frac{2}{3}V^D & -\frac{\sqrt{2}}{3}(V^O - V^D) \\ -\frac{\sqrt{2}}{3}(V^O - V^D) & \frac{2}{3}V^O + \frac{1}{3}V^D \end{pmatrix}$	$\{V^O, V^D\}$
$\bar{P}_s\Omega_Q$	0	-3	V^D	–

bound states, respectively. If applied to the molecular pentaquarks, from HFS we expect the potentials in the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_b$, $B\Sigma_c$ and $B\Sigma_b$ two-body systems to be identical (plus similar relations for the $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Sigma_c^*$ family of molecules). For simplicity we will often use the generic notation P and P^* for the $J^P = 0^-, 1^-$ heavy mesons and Σ_Q and Σ_Q^* for the $J^P = 1/2^+$ and $3/2^+$ heavy baryons, irrespective of whether they are their charm or bottom versions. In addition we will use the notation P_s, P_s^* for the heavy mesons with $S = 1$ and Ξ'_Q, Ξ_Q^* (Ω_Q, Ω_Q^*) for the heavy baryons with $S = -1$ ($S = -2$).

If we now consider SU(3)-flavor symmetry instead, it happens that the $\bar{P}$, $\bar{P}_s$ heavy antimesons and the Σ_Q, Ξ'_Q and Ω_Q heavy baryons belong to the 3 and 6 representation of the SU(3)-flavor group, respectively.[1] Two-body heavy antimeson-baryon states can be decomposed into $3 \otimes 6 = 8 \oplus 10$, i.e. into the octet and decuplet representations, where the SU(3) Clebsch-Gordan coefficients can be consulted in Ref. [39]. This octet and decuplet decomposition is not dependent on the nature of the pentaquarks, but on their light-quark content, and it has indeed been previously pointed out for compact pentaquarks [40]. Within the molecular explanation, this decomposition specifically implies that the heavy antimeson-baryon potential can be decomposed into a linear combination of an octet and decuplet contribution

$$V = \lambda^O V^O + \lambda^D V^D \,, \tag{4}$$

with V^O and V^D the octet and decuplet pieces and λ^O, λ^D numerical coefficients. We show the full decomposition in Table 1, which happens to be surprisingly simple: for most heavy antimeson-baryon molecules, the potential is a pure octet or decuplet contribution. In turn, this is easily explained from the observation that the resulting pentaquarks have the same quantum numbers as the corresponding octet or decuplet light baryons. Even for the $\bar{P}\Xi'_Q - \bar{P}_s\Sigma_Q$ and $\bar{P}\Omega_Q - \bar{P}_s\Xi'_Q$ molecules (where the dash indicates that these channels couple), for which the potential is a 2×2 matrix, when we look at the eigenvalues we recover

[1] We will not consider explicitly the difference between ground- and excited-state heavy hadrons, as it does not affect their light-flavor structure.

$$V = \{V^O, V^D\}\,, \tag{5}$$

depending on the linear combination of the two channels, with the octet eigenvalue corresponding to

$$|8\rangle = -\sqrt{\frac{1}{3}}\,|\bar{P}\,\Xi'_Q(I=1)\rangle + \sqrt{\frac{2}{3}}\,|\bar{P}_s\,\Sigma_Q\rangle\,, \tag{6}$$

$$|8\rangle = -\sqrt{\frac{1}{3}}\,|\bar{P}\Omega_Q\rangle + \sqrt{\frac{2}{3}}\,|\bar{P}_s\,\Xi'_Q\rangle\,, \tag{7}$$

and the decuplet eigenvalue to

$$|10\rangle = \sqrt{\frac{2}{3}}\,|\bar{P}\,\Xi'_Q(I=1)\rangle + \sqrt{\frac{1}{3}}\,|\bar{P}_s\,\Sigma_Q\rangle\,, \tag{8}$$

$$|10\rangle = \sqrt{\frac{2}{3}}\,|\bar{P}\Omega_Q\rangle + \sqrt{\frac{1}{3}}\,|\bar{P}_s\,\Xi'_Q\rangle\,. \tag{9}$$

These two molecular systems, $\bar{P}\,\Xi'_Q - \bar{P}_s\,\Sigma_Q$ and $\bar{P}\Omega_Q - \bar{P}_s\,\Xi'_Q$, will adopt the lowest-energy configuration, be it either the octet or decuplet one. In the absence of additional experimental information and knowing that the P_{c1}, P_{c2} and P_{c3} hidden-charm pentaquarks most probably belong to the octet, we naively expect the lowest-energy configuration to be the octet.[2]

Owing to heavy-flavor symmetry, the potential is expected to be independent of the flavor of the heavy quarks. This implies in particular that the octet configurations

$$\bar{D}\,\Xi'_b(I=0)\,, \quad \bar{D}\,\Xi'_b(I=1) - \bar{D}_s\,\Sigma_b\,, \tag{10}$$

$$B\,\Xi'_c(I=0)\,, \quad B\,\Xi'_c(I=1) - B_s\,\Sigma_c\,, \tag{11}$$

which contain the five quark flavors that hadronize, will display as much attraction as the hidden-charm pentaquarks. Out of the four five-flavor configurations, the strange-isoscalar molecules $[\bar{D}\,\Xi'_b(0),\ B\,\Xi'_c(0)]$ are relatively easy to deal with (they are single-channel systems). For the strange-isovector molecules $[\bar{D}\,\Xi'_b(I=1) - \bar{D}_s\,\Sigma_b,\ B\,\Xi'_c(I=1) - B_s\,\Sigma_c]$ we have a two-channel problem where the thresholds are separated by about 20 MeV and 40 MeV for the isovector $\bar{b}csqq$ and $b\bar{c}sqq$ pentaquark configurations, respectively. The question is whether this energy gap will prevent a predominantly octet molecular state to form or not. The answer depends on the comparison of the momentum scales of the binding mechanism and the coupled-channel dynamics. The typical momentum scale of the coupled channels[3] in the previous cases is about 250 MeV and 350 MeV for the $\bar{b}csud$ and $b\bar{c}sud$ pentaquarks, while the binding mechanism is expected to be short-ranged (e.g. vector-meson exchange), with a momentum scale of the order of $(0.5 - 1.0)$ GeV give or take. As a consequence, we expect the isovector five-flavor pentaquarks to bind (a conjecture which we confirm by means of concrete calculations in what follows).

[2] We notice that a recent work [41] has predicted a series of $c\bar{c}sss$ (P_c^Ω) pentaquarks (but compact, instead of molecular). This suggests that a few of the decuplet configurations might be attractive as well.

[3] This momentum scale is defined as $\sqrt{2\mu\Delta}$, with μ the reduced mass of the system and Δ the mass gap between the channels.

3. Effective field theory description

To explicitly check the effects of the previous symmetries, we will describe the pentaquarks as non-relativistic meson-baryon bound states interacting by means of a contact-range potential that is heavy- and SU(3)-flavor symmetric.

This choice is not arbitrary, but corresponds with the lowest or leading order (LO) effective field theory (EFT) description of the heavy antimeson and heavy baryon two-body system. EFTs exploit the existence of a separation of scales to formulate generic low energy descriptions of physical systems. The idea is to identify characteristic low and high energy scales Q and M such that $Q/M \ll 1$ and then express every physical quantity as a power series in terms of the ratio Q/M. The first term in this series is the LO, the second is the next-to-leading order (NLO), and so on.

For molecular pentaquarks the required scale separation manifest itself as follows: the typical low energy scale Q is of the order of $(100-200)\,\mathrm{MeV}$ and can be identified with the pion mass or the binding momentum of the pentaquarks. At this scale the meson-baryon dynamics is well known and involves the exchanges of pions and other pseudoscalar mesons. The high energy scale M is in the $(0.5-1.0)\,\mathrm{GeV}$ range and can be identified with the rho meson mass or the momentum scale at which the internal structure of the hadrons becomes evident. This part of the interaction is less well-known and might very well involve non-molecular components of the pentaquark wave function. EFT parametrizes it as a series of contact-range operators.

Our LO description of the pentaquarks only involves the contact-range potential. This choice is justified (i) from a well-known EFT observation that indicates that the existence of shallow bound states (e.g. the deuteron or near-threshold states such as hadronic molecules) increases the importance of contact-range interactions at low energies [42,43] and (ii) from concrete EFT calculations for the LHCb pentaquarks that suggest that pion exchanges are NLO and thus a perturbative correction to the LO results [44].

From the previous, the LO S-wave interaction binding the P_{c1}, P_{c2} and P_{c3} molecular pentaquarks will be given by the Lagrangian

$$\mathcal{L}_{\text{contact}} = C_i^O \sum_{IS} (o_{IS}^{abc} M_a P_i^J B_{bc})^\dagger (o_{IS}^{abc} M_a P_i^J B_{bc})\,, \tag{12}$$

where C_i^O is the (octet) coupling constant, $i = 1, 2, 3$ is the index with which we label the hidden-charm pentaquarks, M_a is a triplet heavy meson with the quark content $|\bar{Q} q_a\rangle$, where $q_a = u, d, s$ depending on the flavor index a, B_{bc} a sextet heavy baryon with quark content $|Q \frac{1}{\sqrt{2}}(q_b q_c + q_c q_b)\rangle$ (i.e. symmetric in the flavor indices), o_{IS}^{abc} is a tensor in flavor space that projects the heavy antimeson-baryon system in an octet state with given isospin I and strangeness S (the exact form of this tensor can be deduced from Table 1), and P_i^J is a projector into the corresponding spin channel J if there is more than one.[4] For molecular pentaquarks, the spin of the P_{c1} will be $J = \frac{1}{2}$, while for the P_{c2} and P_{c3} it will be either $J = \frac{1}{2}$ or $\frac{3}{2}$, though we do not know which of these two pentaquarks corresponds to each of the two possible spin configurations. We are

[4] The form of this projector is trivial ($P_1 = 1$) for the P_{c1} pentaquark, while P_2 and P_3 depend on the spin of the P_{c2} and P_{c3} pentaquarks, which is either $J = \frac{1}{2}$ or $\frac{3}{2}$, where the projector for the $|JM\rangle$ spin configuration in the $\bar{D}^* \Sigma_c$ system takes the form $\langle 1 m_1 | P_{JM} | \frac{1}{2} m_2 \rangle = \langle 1 m_1 \frac{1}{2} m_2 | JM \rangle$, i.e. it coincides with the Clebsch-Gordan coefficients coupling a $\bar{D}^*$ meson and Σ_c baryon with spin wave functions $|1 m_1\rangle$ and $|\frac{1}{2} m_2\rangle$ to total spin $|JM\rangle$.

also assuming that the decuplet contact-range interaction is subleading, which is why it is not included in the Lagrangian above.

The previous Lagrangian generates a simple contact-range potential of the type

$$\langle p'|V|p\rangle = C_i^O(\Lambda)\, f(\frac{p}{\Lambda})\, f(\frac{p'}{\Lambda})\,, \tag{13}$$

where we have regularized the potential, originally a Dirac delta in momentum space, with the Gaussian regulator $f(x) = e^{-x^2}$ and a cutoff Λ. For the cutoff we choose the range $\Lambda = (0.5 - 1.0)\,\text{GeV}$, i.e. around the ρ meson mass. With this potential we solve a coupled-channel Lippmann-Schwinger equation of the type

$$\phi_A(k) + \sum_B \int \frac{d^3p}{(2\pi)^3}\, \langle k|V_{AB}|p\rangle\, \frac{\phi_B(p)}{M_B + p^2/(2\mu_B) - M_P} = 0, \tag{14}$$

where A, B are indices for the channels we are considering, ϕ_A the vertex function (i.e. the wave function Ψ_A times the propagator, $\phi_A(p) = [M_A + p^2/(2\mu_A) - M_P]\,\Psi_A(p)$), V_{AB} the potential between channels A and B, M_B the total mass of the heavy antimeson and baryon comprising channel B, μ_B their reduced mass and M_P the mass of the molecular pentaquark we are predicting. We notice that the only configurations with more than one channel are the $(I, S) = (1, -1)$ and $(\frac{1}{2}, -2)$, see Table 1. For illustrative purposes we consider the bound-state equation for a Gaussian regulator in the single-channel case, in which it reduces to

$$1 + C_i^O(\Lambda)\, \frac{\mu_A}{4\pi^2}\, I_0(\gamma_A, \Lambda) = 0\,, \tag{15}$$

with $\gamma_A = \sqrt{2\mu_A(M_A - M_P)}$ the wave number of the molecular pentaquark and where I_0 is given by

$$I_0(\gamma_A, \Lambda) = \sqrt{2\pi}\,\Lambda - 2\, e^{2\gamma_A^2/\Lambda^2}\, \pi\, \gamma_A\, \text{erfc}\left(\frac{\sqrt{2}\gamma_A}{\Lambda}\right), \tag{16}$$

where $\text{erfc}\,(x)$ is the complementary error function.

If we determine the C_i^O couplings from reproducing the masses of the $i = 1, 2, 3$ P_{ci} pentaquark, for $\Lambda = 0.75\,\text{GeV}$ we obtain the couplings

$$C_1^O = -1.19\,(-(2.17 - 0.80))\,\text{fm}^2\,, \tag{17}$$
$$C_2^O = -1.44\,(-(2.88 - 0.93))\,\text{fm}^2\,, \tag{18}$$
$$C_3^O = -1.02\,(-(1.80 - 0.71))\,\text{fm}^2\,, \tag{19}$$

where the values in parentheses correspond to varying the cutoff in the $(0.5 - 1.0)\,\text{GeV}$ window.[5] With these couplings, for $\Lambda = 0.75\,\text{GeV}$ we predict the location of the $\bar{c}b$ five-flavor pentaquarks to be

$$m(P_{\bar{c}b}^{\Lambda}) = 7783^{+6}_{-5}\,,\ 7907 \pm 7\,,\ 7930^{+5}_{-4}\,\text{MeV}\,, \tag{20}$$
$$m(P_{\bar{c}b}^{\Sigma}) = 7765^{+6}_{-5}\,,\ 7892^{+8}_{-9}\,,\ 7914^{+5}_{-4}\,\text{MeV}\,, \tag{21}$$

[5] For simplicity, we have not considered the errors esteeming from the uncertainties in the pentaquark masses, see Eqs. (1)-(3), nor from the further dependence of these masses on the resonance profile, check for instance Ref. [45] in which the $P_c(4312)$ is found to be a virtual (instead of a bound) state.

where the uncertainty comes from varying the cutoff (i.e. taking $\Lambda = (0.5 - 1.0)\,\text{GeV}$), but does not include the SU(3) symmetry breaking effects, which we discuss later. For the $c\bar{b}$ five-flavor pentaquarks we predict instead

$$m(P_{c\bar{b}}^{\Lambda}) = 7829^{+10}_{-9}\,,\ 7858^{+12}_{-10}\,,\ 7883^{+8}_{-7}\,\text{MeV}\,, \tag{22}$$

$$m(P_{c\bar{b}}^{\Sigma}) = 7804^{+6}_{-5}\,,\ 7835^{+8}_{-7}\,,\ 7858^{+5}_{-4}\,\text{MeV}\,. \tag{23}$$

The complete list of predictions (including not only cutoff but also SU(3)-flavor uncertainties) can be consulted in Table 2.

The spectrum of Table 2 implies that each of the observed hidden-charm pentaquarks belongs to a light/heavy-flavor multiplet with 16 members. As three hidden-charm pentaquarks have been observed, this means a total of 48 states (of which 45 are so far unobserved). The experimental observation of these pentaquarks could be achieved by means of the SU(3)-flavor and HFS analogues of the $J/\Psi N$ decay channel that has been used in the discovery of the P_{c1}, P_{c2} and P_{c3}. For instance, the five-flavor pentaquarks $P_{c\bar{b}}^{\Lambda}$ and $P_{c\bar{b}}^{\Sigma}$ could be detected by means of their $B_c^+\Lambda$ and $B_c^+\Sigma$ decays.

Even though for the moment we have not considered HQSS explicitly, it is easy to figure out its consequences: from HQSS we expect the hidden-charm pentaquarks to come in multiplets of up to seven members [6,35–37]. Within the scope of contact-range EFTs incorporating HQSS [38], the observation of the P_{c1}, P_{c2} and P_{c3} pentaquarks suggests that the aforementioned septuplet is probably complete [12,48], meaning that there are 4 unobserved states. This result is reproduced in most schemes that include HQSS, e.g. models with a compact core coupled to the molecular degrees of freedom [34], indicating that it depends on HQSS instead of the specific dynamics generating the pentaquarks. The bottom-line is that if we compound the HQSS multiplets with the SU(3)-flavor and HFS ones, the heavy molecular pentaquark family could contain a total of 112 states (3 observed, 109 to be discovered), as we will discuss later.

Among the results in Table 2 it is interesting to notice the strange-isoscalar P_c^{Λ} partners of the three LHCb pentaquarks, which were predicted (together with the pentaquarks) nearly a decade ago [3,4]. This prediction has been recently updated in Ref. [49], which uses a contact-range theory where the couplings are saturated by vector-meson exchange and the regularization is set as to reproduce the $P_c(4312)$ pentaquark. The prediction of Ref. [49] for the mass of the $\bar{D}\Xi_c'$ molecule is 4436.7 MeV, which happens to be pretty close to ours (check Table 2). Refs. [50,51] have also made a series of molecular pentaquark predictions which closely match ours.

On the experimental side it is worth mentioning that a P_c^{Λ} pentaquark — the $P_{cs}(4459)$ — has been observed by the LHCb collaboration [52], but owing to its mass it is probably a $\bar{D}^*\Xi_c$ molecule [53–56]. As such it involves a $\bar{3}$ charmed baryon (Λ_c, Ξ_c) instead of a sextet one (Σ_c, Ξ_c', Ω_c and their excited states), which means that this pentaquark is not expected to be one of the SU(3)-flavor partners of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ that we predict here. Nonetheless, the $P_{cs}(4459)$ will prove useful as a phenomenological cross-check of the size of SU(3)-flavor violations, as we will argue later. Regarding the possible five-flavor partners of the $P_{cs}(4459)$, there is a recent exploration in Ref. [57].

4. Uncertainties

We are predicting the molecular pentaquarks within a contact-range EFT, which entails that they are amenable to systematic error estimations. A conventional way to estimate these theoretical errors is to vary the predictions within a sensible cutoff window (which is what we

Table 2
The heavy- and light-flavor symmetry partners of the LHCb pentaquark trio, the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ (or P_{c1}, P_{c2}, P_{c3} for short). This includes the five-flavor pentaquarks with quark content $\bar{b}csdu$ and $b\bar{c}sdu$. The column "Molecule" displays the two-hadron system under consideration, I the isospin, S the strangeness, B_P the binding energy, M_P the mass (where $M_P = M_{\rm th} - B_P$, with $M_{\rm th}$ the mass of the corresponding heavy antimeson-baryon threshold, for which we take the isospin symmetric limit of the masses listed in the Review of Particle Physics (RPP) [46]) and "Partner" represents which hidden-charm pentaquark (P_{ci}, $i = 1, 2, 3$) is the partner of the predicted state. In the coupled-channel cases, the binding energy is calculated relative to the channel with the lowest mass. For the calculations we use a contact-range EFT, with the potential of Eq. (13) and a Gaussian regulator with a cutoff $\Lambda = 0.75$ GeV. The error comes from two different sources, which are added in quadrature: (i) varying the cutoff in the $\Lambda = (0.5 - 1.0)$ GeV range and (ii) assuming a 20% uncertainty in SU(3)-flavor symmetry as applied to the contact-range couplings (this second error only pertains pentaquarks with strangeness). In general the SU(3)-flavor uncertainty dominates in the $c\bar{c}$, $c\bar{b}$, $b\bar{c}$ sectors, while for the $b\bar{b}$ pentaquarks the bulk of the errors come from the cutoff variation (in agreement with theoretical expectations [47]).

Molecule	I	S	B_P	M_P	Partner	Molecule	I	S	B_P	M_P	Partner
$\bar{D}\Sigma_c$	$\frac{1}{2}$	0	Input	Input	P_{c1}	$B\Sigma_c$	$\frac{1}{2}$	0	$27.5^{+9.5}_{-8.0}$	$7710.5^{+8.0}_{-9.5}$	P_{c1}
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	0	Input	Input	P_{c2}	$B^*\Sigma_c$	$\frac{1}{2}$	0	$43.6^{+10.6}_{-9.3}$	$7734.6^{+9.3}_{-10.6}$	P_{c2}
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	0	Input	Input	P_{c3}	$B^*\Sigma_c$	$\frac{1}{2}$	0	$18.6^{+7.6}_{-6.0}$	$7759.7^{+6.0}_{-7.6}$	P_{c3}
$\bar{D}\Xi_c'$	0	−1	$9.6^{+10.4}_{-7.3}$	$4436.3^{+7.3}_{-10.4}$	P_{c1}	$B\Xi_c'$	0	−1	29^{+18}_{-16}	7829^{+16}_{-18}	P_{c1}
$\bar{D}^*\Xi_c'$	0	−1	23^{+16}_{-13}	4565^{+13}_{-16}	P_{c2}	$B^*\Xi_c'$	0	−1	45^{+23}_{-21}	7858^{+21}_{-23}	P_{c2}
$\bar{D}^*\Xi_c'$	0	−1	$5.4^{+7.7}_{-4.7}$	$4581.8^{+4.7}_{-7.7}$	P_{c3}	$B^*\Xi_c'$	0	−1	20^{+15}_{-12}	7883^{+12}_{-15}	P_{c3}
$\bar{D}\Xi_c' - \bar{D}_s\Sigma_c$	1	−1	$5.2^{+9.4}_{-5.0}$	$4416.7^{+5.0}_{-9.4}$	P_{c1}	$B\Xi_c' - B_s\Sigma_c$	1	−1	20^{+17}_{-14}	7801^{+14}_{-17}	P_{c1}
$\bar{D}^*\Xi_c' - \bar{D}_s^*\Sigma_c$	1	−1	18^{+16}_{-12}	4548^{+12}_{-16}	P_{c2}	$B^*\Xi_c' - B_s^*\Sigma_c$	1	−1	36^{+22}_{-19}	7833^{+19}_{-22}	P_{c2}
$\bar{D}^*\Xi_c' - \bar{D}_s^*\Sigma_c$	1	−1	$2.0^{+6.5}_{-2.0}$	$4563.7^{+2.0}_{-6.5}$	P_{c3}	$B^*\Xi_c' - B_s^*\Sigma_c$	1	−1	12^{+13}_{-10}	7857^{+10}_{-13}	P_{c3}
$\bar{D}\Omega_c - \bar{D}_s\Xi_c'$	$\frac{1}{2}$	−2	$2.6^{+9.4}_{-2.6}$	$4544.2^{+2.6}_{-9.4}$	P_{c1}	$B\Omega_c - B_s\Xi_c'$	$\frac{1}{2}$	−2	14^{+17}_{-13}	7931^{+13}_{-17}	P_{c1}
$\bar{D}^*\Omega_c - \bar{D}_s^*\Xi_c'$	$\frac{1}{2}$	−2	16^{+16}_{-13}	4675^{+13}_{-15}	P_{c2}	$B^*\Omega_c - B_s^*\Xi_c'$	$\frac{1}{2}$	−2	31^{+23}_{-20}	7963^{+23}_{-20}	P_{c2}
$\bar{D}^*\Omega_c - \bar{D}_s^*\Xi_c'$	$\frac{1}{2}$	−2	$0.4^{+6.2}_{-0.4}$	$4690.3^{+0.4}_{-6.2}$	P_{c3}	$B^*\Omega_c - B_s^*\Xi_c'$	$\frac{1}{2}$	−2	$7.3^{+13.0}_{-8.2}$	$7986.5^{+8.2}_{-13.0}$	P_{c3}
$\bar{D}\Sigma_b$	$\frac{1}{2}$	0	$20.2^{+5.3}_{-4.7}$	$7660.1^{+4.7}_{-5.3}$	P_{c1}	$B\Sigma_b$	$\frac{1}{2}$	0	48^{+23}_{-18}	11044^{+18}_{-23}	P_{c1}
$\bar{D}^*\Sigma_b$	$\frac{1}{2}$	0	$37.5^{+7.3}_{-6.5}$	$7784.2^{+6.5}_{-7.3}$	P_{c2}	$B^*\Sigma_b$	$\frac{1}{2}$	0	68^{+25}_{-28}	11070^{+28}_{-25}	P_{c2}
$\bar{D}^*\Sigma_b$	$\frac{1}{2}$	0	$14.3^{+4.7}_{-4.0}$	$7807.4^{+4.0}_{-4.7}$	P_{c3}	$B^*\Sigma_b$	$\frac{1}{2}$	0	37^{+19}_{-15}	11101^{+15}_{-19}	P_{c3}
$\bar{D}\Xi_b'$	0	−1	20^{+15}_{-12}	7783^{+12}_{-15}	P_{c1}	$B\Xi_b'$	0	−1	49^{+29}_{-25}	11166^{+25}_{-29}	P_{c1}
$\bar{D}^*\Xi_b'$	0	−1	38^{+20}_{-18}	7907^{+18}_{-20}	P_{c2}	$B^*\Xi_b'$	0	−1	68^{+34}_{-30}	11192^{+34}_{-30}	P_{c2}
$\bar{D}^*\Xi_b'$	0	−1	15^{+12}_{-10}	7930^{+10}_{-12}	P_{c3}	$B^*\Xi_b'$	0	−1	38^{+25}_{-21}	11222^{+21}_{-25}	P_{c3}
$\bar{D}\Xi_b' - \bar{D}_s\Sigma_b$	1	−1	16^{+14}_{-12}	7765^{+12}_{-14}	P_{c1}	$B\Xi_b' - B_s\Sigma_b$	1	−1	40^{+28}_{-24}	11140^{+24}_{-28}	P_{c1}
$\bar{D}^*\Xi_b' - \bar{D}_s^*\Sigma_b$	1	−1	34^{+20}_{-18}	7892^{+18}_{-20}	P_{c2}	$B^*\Xi_b' - B_s^*\Sigma_b$	1	−1	59^{+33}_{-29}	11161^{+29}_{-33}	P_{c2}
$\bar{D}^*\Xi_b' - \bar{D}_s^*\Sigma_b$	1	−1	11^{+12}_{-10}	7914^{+12}_{-10}	P_{c3}	$B^*\Xi_b' - B_s^*\Sigma_b$	1	−1	30^{+26}_{-19}	11199^{+26}_{-19}	P_{c3}
$\bar{D}\Omega_b - \bar{D}_s\Xi_b'$	$\frac{1}{2}$	−2	15^{+15}_{-12}	7888^{+12}_{-15}	P_{c1}	$B\Omega_b - B_s\Xi_b'$	$\frac{1}{2}$	−2	35^{+29}_{-24}	11267^{+24}_{-29}	P_{c1}
$\bar{D}^*\Omega_b - \bar{D}_s^*\Xi_b'$	$\frac{1}{2}$	−2	34^{+20}_{-18}	8013^{+18}_{-20}	P_{c2}	$B^*\Omega_b - B_s^*\Xi_b'$	$\frac{1}{2}$	−2	56^{+34}_{-29}	11295^{+30}_{-34}	P_{c2}
$\bar{D}^*\Omega_b - \bar{D}_s^*\Xi_b'$	$\frac{1}{2}$	−2	11^{+12}_{-9}	8037^{+9}_{-12}	P_{c3}	$B^*\Omega_b - B_s^*\Xi_b'$	$\frac{1}{2}$	−2	26^{+24}_{-19}	11325^{+19}_{-24}	P_{c3}

have done for the five-flavor pentaquarks in Eqs. (20)-(23)). Here the cutoff floats from 0.5 to 1 GeV, which can be either identified with the mass of the vector mesons or with the momenta at which the internal structure of the hadrons starts to be resolved. For the $c\bar{c}$ family of pentaquarks this translates into a systematic error of less than 1 MeV, which explains why the predictions of other theoretical works [49–51] are basically identical to ours. Yet this uncertainty is calcu-

lated under the assumption that SU(3)-flavor symmetry is perfectly preserved, which is not the case. Violations of SU(3)-flavor symmetry relations are usually of the order of 20%, as estimated from the difference between the pion and kaon weak decay constants ($f_\pi \simeq 130\,\text{MeV}$ and $f_K \simeq 160\,\text{MeV}$). From this, within the EFT we are using we can easily take into account the SU(3)-flavor symmetry breaking effects by randomly varying the C_i^O couplings by 20% around their central values. For $\Lambda = 0.75\,\text{GeV}$, this translates into an uncertainty of $2-15\,\text{MeV}$ depending on the specific $c\bar{c}$ pentaquark, where the largest uncertainties correspond to the states with the largest binding energies.

For the $c\bar{b}$, $\bar{c}b$ and $b\bar{b}$ molecular pentaquarks the situation is different owing to the considerably larger cutoff dependence (about 5, 10 and $20-30\,\text{MeV}$ respectively), which we will discuss in the next paragraph. The SU(3)-flavor uncertainties in these cases will be $10-20$ and $15-25\,\text{MeV}$ for the $c\bar{b}/\bar{c}b$ and $b\bar{b}$ cases, respectively. That is, while for the $c\bar{c}$, $c\bar{b}$, $\bar{c}b$ the uncertainties are dominated by flavor symmetry breaking effects, for the $b\bar{b}$ pentaquarks cutoff variation tends to be the largest source of uncertainty.

However, the application of SU(3)-flavor symmetry remains theoretical in the sense that we do not really have a clear molecular example from where we can determine how well this symmetry works at the quantitative level. Two qualitative examples are already known:

(i) The $Z_c(3900)$ [58] and $Z_{cs}(3985)$ [59] (Z_c and Z_{cs} from now on), which have been theorized to be $I=1$ $D^*\bar{D}$ [30,60,61] and $I=\frac{1}{2}$ $D_s^*\bar{D}-D_s\bar{D}^*$ [62,63] molecules, respectively.
(ii) The $P_{cs}(4459)$ pentaquark [52], which has been theorized to be an $I=0$ $\bar{D}^*\Xi_c$ bound state [53–56].

In the first case, the SU(3) decomposition of heavy meson-antimeson states is $3\otimes\bar{3}=1\oplus 8$, i.e. a singlet and an octet representation, where the Z_c and Z_{cs} both belong to the octet and thus their potential is expected to be the same [29,62]. But it happens that the masses of the Z_c and Z_{cs} resonances are above their corresponding meson-antimeson thresholds, which means that they are not necessarily bound states but more probably resonances (or even virtual states if we take into account that their Breit-Wigner masses might not correspond to their physical masses). If this happens to be the case, they will require a different contact-range EFT description than the one we employ here for the pentaquarks (or the direct extraction of the couplings from the data instead of the masses, as done in Refs. [61,62]), which renders it difficult to make direct comparisons between the Z_c's and the P_c's.

In the second case, as pointed out previously, the Ξ_c charmed baryon is a flavor antitriplet and the $\bar{D}^*\Xi_c$ system will essentially belong to a different and independent representation of SU(3). That is, the $\bar{D}^*\Xi_c$ potential can be described with a new coupling constant $D(\Lambda)$, i.e.

$$\langle p'|V|p\rangle = D(\Lambda)\, f(\frac{p}{\Lambda})\, f(\frac{p'}{\Lambda})\,, \tag{24}$$

the value of which is in principle unrelated to the $C_i^O(\Lambda)$ couplings we have used to reproduce the three P_c pentaquarks. However, phenomenological models based on vector-meson exchanges predict that $D=C_1^O$ [3,4], i.e. the $I=0$ $\bar{D}^*\Xi_c$ and $I=\frac{1}{2}$ $\bar{D}\Sigma_c$ potentials are expected to be similar. Concrete calculations with the same type of EFT, regulator and cutoff range we have used for the P_{c1}, P_{c2} and P_{c3} yield $D=1.17\,C_1^O$ when calibrating $D(\Lambda)$ to the $P_{cs}(4459)$ mass, showing a 17% discrepancy from $D=C_1^O$. The more complete analysis of Ref. [54] (which includes a series of effects not considered here, like coupled channel dynamics or the double-peak solution considered in the experimental analysis of Ref. [52]) provides a compatible figure of

$D = (0.90 - 1.11)\, C_1^O$, which deviates a merely 10% away from the phenomenological relation $D = C_1^O$. The previous numbers are well within the 20% SU(3) uncertainty estimated from the f_π and f_K difference. This is despite the fact that the $D = C_1^O$ relation is based on phenomenology, from which further uncertainties (beyond SU(3) symmetry breaking) should be expected.

Regarding HFS, as already pointed out, its application beyond the $c\bar{c}$ sector has a serious limitation in terms of model dependence within the contact-range EFT framework. The cutoff dependence of the predictions becomes larger as the reduced mass of the system is increased, from merely 1 MeV at most in the hidden-charm sector to a couple of tens of MeV in the hidden-bottom sector. This limitation was already pointed out in Ref. [47], where here we merely confirm the impossibility of making model independent predictions with HFS. Yet we notice that there is *systematicity* in this model dependence, as increasing the cutoff Λ invariably leans towards more binding. This is important, as it implies that the conclusion that the $c\bar{b}$, $\bar{c}b$ and $b\bar{b}$ molecular pentaquarks bind is indeed model independent, with the model dependence limited to how much they bind. In fact it can be shown that for two-body molecular systems where the potential respects HFS (i.e. the potential is independent of the heavy-quark mass), the binding energy B_2 increases monotonically with the reduced mass μ, $\partial B_2/\partial \mu \geqslant 0$ (check Appendix A for further details). That is, though the specific masses of the $c\bar{b}$, $\bar{c}b$ and $b\bar{b}$ pentaquarks are model dependent to a certain extent, the fact that these systems bind is a model independent outcome of the calculations.

5. Including heavy-quark spin symmetry

Previously we have made the simplifying assumption that the potentials binding the P_{c1}, P_{c2} and P_{c3} pentaquarks are unrelated. However, HQSS connects the potentials of these three configurations and allows for a common description of the $\bar{P}\Sigma_Q$, $\bar{P}^*\Sigma_Q$ and $\bar{P}^*\Sigma_Q^*$ molecules [6,35–37] (where here we will concentrate on the consequences of HQSS for the type of contact-range EFTs we are using). The disadvantage though is that we do not know which of the P_{c2} and P_{c3} pentaquarks corresponds to the $J = \frac{1}{2}$ and $\frac{3}{2}$ $\bar{D}^*\Sigma_c$ configurations. As a consequence there are two possible set of predictions for the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ family of molecules, depending on which spin identification we propose for the P_{c2} and P_{c3} pentaquarks.

HQSS indicates that the $|\bar{Q}q\rangle$ and $|Qqq\rangle$ family of heavy hadrons are related by means of rotations of the spin of the heavy quark. Indeed, we can group the ground and excited states of a heavy hadron in a single superfield, which for the S-wave heavy mesons and baryons are defined as

$$H = \frac{1}{\sqrt{2}}\left[P + \vec{\sigma}\cdot\vec{P}^*\right], \tag{25}$$

$$\vec{S} = \frac{1}{\sqrt{3}}\,\vec{\sigma}\,B_6 + \vec{B}_6^*\,, \tag{26}$$

where for simplicity we are ignoring the SU(3)-flavor indices and with P, P^* the $J = 0, 1$ heavy mesons, B_6, B_6^* the $J = \frac{1}{2}$, $\frac{3}{2}$ heavy baryons and $\vec{\sigma}$ the Pauli matrices. With the previous definitions, the lowest-order contact-range Lagrangian describing molecular pentaquarks reads [38]

$$\begin{aligned}\mathcal{L}_{\rm contact} = {} & C_a\,{\rm Tr}[H^\dagger H]\,\vec{S}^\dagger\cdot\vec{S} \\ & + C_b\sum_{i=1}^{3}{\rm Tr}[H^\dagger\sigma_i H]\,\vec{S}^\dagger\cdot(J_i\vec{S})\,,\end{aligned} \tag{27}$$

where J_i are the $i = 1, 2, 3$ spin-1 matrices. The terms proportional to the couplings C_a and C_b correspond to central and spin-spin contact-range interactions. Thus, the practical implication of the HQSS version of the contact-range Lagrangian is that the C_i^O couplings we previously defined in Eq. (12) can be decomposed in central and spin-spin components:

$$C_i^O \to C_a^O + \lambda_i \, C_b^O \,, \tag{28}$$

where the explicit decomposition for the three known molecular pentaquark candidates is

$$V_C(\bar{P}\Sigma_Q) = C_a^O \,, \tag{29}$$

$$V_C(\bar{P}^*\Sigma_Q, J = \tfrac{1}{2}) = C_a^O - \frac{4}{3} C_b^O \,, \tag{30}$$

$$V_C(\bar{P}^*\Sigma_Q, J = \tfrac{3}{2}) = C_a^O + \frac{2}{3} C_b^O \,, \tag{31}$$

while for the four potentially unobserved configurations we will have

$$V_C(\bar{P}\Sigma_Q^*) = C_a^O \,, \tag{32}$$

$$V_C(\bar{P}^*\Sigma_Q^*, J = \tfrac{1}{2}) = C_a^O - \frac{5}{3} C_b^O \,, \tag{33}$$

$$V_C(\bar{P}^*\Sigma_Q^*, J = \tfrac{3}{2}) = C_a^O - \frac{2}{3} C_b^O \,, \tag{34}$$

$$V_C(\bar{P}^*\Sigma_Q^*, J = \tfrac{5}{2}) = C_a^O + C_b^O \,. \tag{35}$$

Now, for the P_{c1} pentaquark the identification of its particle and spin channel is trivial: $J = \frac{1}{2}$ $\bar{D}\Sigma_c$. Meanwhile this is not the case for the P_{c2} and P_{c3} pentaquarks: both are expected to be $\bar{D}^*\Sigma_c$ molecules, but what is not clear is which one is the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ state, as their spins have not been experimentally determined yet. Thus there are two possibilities:

(i) that the P_{c2} and P_{c3} pentaquarks are $J = \frac{1}{2}$ and $\frac{3}{2}$ states, respectively, thus following the standard pattern of mass increasing with spin, which we will call scenario A, and
(ii) the opposite pattern, mass decreasing with spin, is scenario B.

These scenarios have been named following the convention found in Ref. [12]. Different theoretical works prefer scenario A [11,64], scenario B [34,48,65,66], do not find a strong preference [12,44] or explore alternative possibilities [67,68]. Scenario A has recently been explained as a consequence of the short-range interaction of the light-quarks within the heavy antimeson and heavy baryon composing the pentaquarks [69]. Scenario B appeared before the discovery of the pentaquark trio, for instance in Ref. [35], and has received explanations both in terms of pion [7] and vector meson exchanges [70].

Here, we will calibrate the C_a^O and C_b^O couplings to the masses of the P_{c1} and P_{c3} pentaquarks in scenarios A and B, leading to

$$C_a^O = -1.17\,(-(0.78 - 2.16))\,\text{fm}^2 \text{ (A)}\,, \tag{36}$$

$$C_b^O = +0.21\,(+(0.11 - 0.54))\,\text{fm}^2 \text{ (A)}\,, \tag{37}$$

$$C_a^O = -1.30\,(-(0.85 - 2.52))\,\text{fm}^2 \text{ (B)}\,, \tag{38}$$

$$C_b^O = -0.21\,(-(0.11 - 0.54))\,\text{fm}^2 \text{ (B)}\,, \tag{39}$$

depending on the scenario, where the intervals in parentheses refer to the cutoff variation (i.e. $(0.5-1.0)$ GeV). From this we can calculate the complete spectrum of the $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$ and their SU(3)- and heavy-flavor counterparts, where we show the results in Tables 3 ($c\bar{c}$ and $b\bar{c}$ sectors) and 4 ($c\bar{b}$ and $b\bar{b}$ sectors). We find that most pentaquark configurations (112 in total) bind within theoretical uncertainties (which are computed as before).

6. Compositeness of the pentaquarks

Here we have described the pentaquarks as meson-baryon bound states, which implicitly assumes that they are predominantly molecular or composite in nature. Yet, owing to the unspecified nature of the interaction binding the meson and the baryon (which could have its origin in elementary components, e.g. a five-quark compact core [34,36]) and the finite binding energy of these states, it is sensible to expect that they will not be purely molecular.

From the EFT point of view, our assumption that the wave function of a pentaquark only involves meson-baryon degrees of freedom is expected to be valid up to $\mathcal{O}(Q/M)$ corrections:

$$|P_{QQ'}\rangle = |\text{meson-baryon}\rangle \times \left(1-\mathcal{O}(\frac{Q}{M})\right) + \mathcal{O}(\frac{Q}{M})\,|\text{compact}\rangle\,. \tag{40}$$

Here a caveat is in place: the wave function is not an observable and as a consequence there will always remain a degree of ambiguity on whether a particular state is composite or not (or how composite it is). In fact, the EFT framework usually does not rely on including new degrees of freedom at subleading orders in the wave function to improve predictions. Instead, it includes new contact-range operators acting on the degrees of freedom already present, which means that compact components often manifest as energy dependence.

Be it as it may, EFT can be used to derive a dimensional estimation of the compositeness (X_{comp}, i.e. the probability of the meson-baryon component) of the pentaquarks

$$\begin{aligned} X^{\text{dim}}_{\text{comp}}(P_{QQ'}) &= 1-\mathcal{O}(\frac{Q}{M}) = \frac{1}{1+\mathcal{O}(\frac{Q}{M})} \\ &\approx \frac{1}{1+x_c\,\frac{\sqrt{2\mu B_2}}{m_\rho}} + \mathcal{O}(\frac{Q}{M})\,, \end{aligned} \tag{41}$$

where we have reordered the terms in order to obtain an expression that is suitable when Q/M is not small (i.e. when the binding energy is closer to the limit at which the EFT will fail, so we only have $Q/M<1$ but not $Q/M\ll 1$). In the second line we have particularized for the choice $Q=\gamma_2=\sqrt{2\mu B_2}$ and $M=m_\rho$, where x_c is a numerical constant of $\mathcal{O}(1)$ for which we will choose $x_c=1$. This yields a compositeness of around $X^{\text{dim}}_{\text{comp}}=(0.85,0.78,0.88)$ for the P_{c1}, P_{c2} and P_{c3} pentaquarks in the $c\bar{c}$ sector, $(0.76,0.70,0.79)$ and $(0.71,0.67,0.75)$ for $\bar{c}b$ and $c\bar{b}$, respectively, while merely a value of $(0.60,0.56,0.63)$ for their $b\bar{b}$ counterparts. As a comparison, for the deuteron ($\gamma_2=45$ MeV) we will obtain a compositeness of 0.94, compatible with a pure molecular interpretation. Yet, we remind that these estimates are purely based on a comparison of scales and are not very precise. This is illustrated by the numerical factor x_c in Eq. (41), where by taking $x_c=1/2$ or $x_c=2$ instead of $x_c=1$ (all of which are $\mathcal{O}(1)$), the compositeness will change by a factor of order Q/M.

Table 3
The heavy-quark spin, heavy-flavor and light-flavor symmetry partners of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ pentaquarks, where in this table we consider the configurations with heavy-quark content $c\bar{c}$ and $b\bar{c}$. The predictions depend on which are the assumptions made for the spin of the $P_c(4440)$ and $P_c(4457)$ pentaquarks: scenario A refers to the $P_c(4440)$ and $P_c(4457)$ having spin $J = \frac{1}{2}$ and $\frac{3}{2}$, while scenario B considers the opposite identification. The columns "Molecule", I, S, B_P and M_P have the same meaning as in Table 2, while J refers to the spin of the molecular configuration and "Scenario" to the two aforementioned possibilities (A & B). In the coupled-channel cases, the binding energy is calculated relative to the channel with the lowest mass. The calculations use the contact-range EFT of Eq. (13) and a Gaussian regulator with a cutoff $\Lambda = 0.75\,\text{GeV}$. The uncertainties are obtained from two sources (and then summed in quadrature): the error coming from varying the cutoff in the $(0.5 - 1.0)\,\text{GeV}$ window and an expected violation of SU(3)-flavor symmetry of 20% in the contact-range couplings (this later error only applies to configurations containing strangeness). The notation $-(B_P/M_P)$ indicates a configuration that does not bind for the central estimation of the parameters, but could have binding energy B_P / mass M_P within uncertainties. For the mass of the Ω_b^* (which has not been experimentally observed yet), we simply assume $m(\Omega_b^*) - m(\Omega_b) \simeq m(\Xi_b^*) - m(\Xi_b') \simeq m(\Sigma_b^*) - m(\Sigma_b') \simeq 20\,\text{MeV}$; the effect of the Ω_b^* mass on the predictions of the $\bar{P}^*\Omega_b^*$-$\bar{P}_s^*\Xi_b^*$ pentaquarks is minimal though because the lowest mass threshold corresponds to the $\bar{P}_s^*\Xi_b^*$ two-hadron system.

Molecule	I	S	B_P	M_P	J	Scenario	Molecule	I	S	B_P	M_P	J	Scenario
$\bar{D}\Sigma_c^*$	$\frac{1}{2}$	0	$8.4^{+0.5}_{-0.4}$	$4376.9^{+0.4}_{-0.5}$	$\frac{3}{2}$	A	$\bar{D}\Sigma_c^*$	$\frac{1}{2}$	0	$14.0^{+0.6}_{-0.6}$	$4371.4^{+0.6}_{-0.6}$	$\frac{3}{2}$	B
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$25.9^{+0.3}_{-0.4}$	$4500.7^{+0.4}_{-0.3}$	$\frac{1}{2}$	A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$3.2^{+0.2}_{-0.2}$	$4523.5^{+0.2}_{-0.2}$	$\frac{1}{2}$	B
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$15.8^{+0.1}_{-0.0}$	$4510.9^{+0.0}_{-0.1}$	$\frac{3}{2}$	A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$9.9^{+0.1}_{-0.0}$	$4516.8^{+0.0}_{-0.0}$	$\frac{3}{2}$	B
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$3.2^{+0.1}_{-0.2}$	$4523.5^{+0.2}_{-0.1}$	$\frac{5}{2}$	A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	0	$25.9^{+0.3}_{-0.4}$	$4500.7^{+0.4}_{-0.3}$	$\frac{5}{2}$	B
$\bar{D}\Xi_c^*$	0	−1	$9.2^{+10.1}_{-7.1}$	$4503.7^{+7.1}_{-10.1}$	$\frac{3}{2}$	A	$\bar{D}\Xi_c^*$	0	−1	15^{+13}_{-10}	4498^{+10}_{-13}	$\frac{3}{2}$	B
$\bar{D}^*\Xi_c^*$	0	−1	27^{+17}_{-15}	4627^{+15}_{-17}	$\frac{1}{2}$	A	$\bar{D}^*\Xi_c^*$	0	−1	$3.6^{+6.5}_{-3.5}$	$4650.5^{+3.5}_{-6.5}$	$\frac{1}{2}$	B
$\bar{D}^*\Xi_c^*$	0	−1	17^{+13}_{-11}	4638^{+11}_{-13}	$\frac{3}{2}$	A	$\bar{D}^*\Xi_c^*$	0	−1	$10.7^{+6.5}_{-7.8}$	$4643.5^{+7.8}_{-6.5}$	$\frac{3}{2}$	B
$\bar{D}^*\Xi_c^*$	0	−1	$3.5^{+6.5}_{-3.4}$	$4650.5^{+3.4}_{-6.5}$	$\frac{5}{2}$	A	$\bar{D}^*\Xi_c^*$	0	−1	27^{+17}_{-15}	4627^{+15}_{-17}	$\frac{5}{2}$	B
$\bar{D}\Xi_c^* - \bar{D}_s\Sigma_c^*$	1	−1	$4.5^{+8.7}_{-4.4}$	$4481.9^{+4.4}_{-8.7}$	$\frac{3}{2}$	A	$\bar{D}\Xi_c^* - \bar{D}_s\Sigma_c^*$	1	−1	$9.6^{+12.0}_{-8.0}$	$4477.0^{+12.0}_{-8.0}$	$\frac{3}{2}$	B
$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	21^{+17}_{-14}	4609^{+14}_{-17}	$\frac{1}{2}$	A	$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	$0.8^{+4.8}_{-0.8}$	$4629.5^{+0.8}_{-4.8}$	$\frac{1}{2}$	B
$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	$11.4^{+12.7}_{-9.0}$	$4618.9^{+9.0}_{-12.7}$	$\frac{3}{2}$	A	$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	$6.1^{+9.7}_{-5.6}$	$4624.3^{+5.6}_{-9.7}$	$\frac{3}{2}$	B
$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	$0.8^{+4.8}_{-0.8}$	$4629.5^{+0.8}_{-4.8}$	$\frac{5}{2}$	A	$\bar{D}^*\Xi_c^* - \bar{D}_s^*\Sigma_c^*$	1	−1	21^{+17}_{-14}	4609^{+14}_{-17}	$\frac{5}{2}$	B
$\bar{D}\Omega_c^* - \bar{D}_s\Xi_c^*$	$\frac{1}{2}$	−2	$1.4^{+8.4}_{-1.4}$	$4612.6^{+1.4}_{-8.4}$	$\frac{1}{2}$	A	$\bar{D}\Omega_c^* - \bar{D}_s\Xi_c^*$	$\frac{1}{2}$	−2	$5.7^{+12.0}_{-5.7}$	$4608.2^{+5.7}_{-12.0}$	$\frac{1}{2}$	B
$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	18^{+17}_{-14}	4739^{+14}_{-17}	$\frac{1}{2}$	A	$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	−(2.7)	−(4755.1)	$\frac{1}{2}$	B
$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	$8.4^{+12.9}_{-8.2}$	$4749.4^{+8.2}_{-12.9}$	$\frac{3}{2}$	A	$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	$3.1^{+9.7}_{-3.1}$	$4754.7^{+3.1}_{-9.7}$	$\frac{3}{2}$	B
$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	−(2.7)	−(4755.1)	$\frac{5}{2}$	A	$\bar{D}^*\Omega_c^* - \bar{D}_s^*\Xi_c^*$	$\frac{1}{2}$	−2	18^{+17}_{-14}	4739^{+14}_{-17}	$\frac{5}{2}$	B
$\bar{D}\Sigma_b^*$	$\frac{1}{2}$	0	$19.1^{+4.3}_{-3.7}$	$7680.7^{+3.7}_{-4.3}$	$\frac{3}{2}$	A	$\bar{D}\Sigma_b^*$	$\frac{1}{2}$	0	$26.6^{+5.0}_{-3.6}$	$7673.2^{+3.6}_{-5.0}$	$\frac{3}{2}$	B
$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$41.9^{+7.8}_{-7.2}$	$7799.2^{+7.2}_{-7.8}$	$\frac{1}{2}$	A	$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$11.1^{+4.3}_{-3.6}$	$7830.0^{+3.6}_{-4.3}$	$\frac{1}{2}$	B
$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$29.2^{+6.3}_{-5.6}$	$7811.9^{+5.6}_{-6.3}$	$\frac{3}{2}$	A	$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$21.4^{+5.5}_{-4.7}$	$7819.7^{+4.7}_{-5.5}$	$\frac{3}{2}$	B
$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$11.1^{+4.3}_{-3.6}$	$7830.0^{+3.6}_{-4.3}$	$\frac{5}{2}$	A	$\bar{D}^*\Sigma_b^*$	$\frac{1}{2}$	0	$41.9^{+7.8}_{-7.1}$	$7799.2^{+7.1}_{-7.8}$	$\frac{5}{2}$	B
$\bar{D}\Xi_b^*$	0	−1	22^{+18}_{-10}	7800^{+10}_{-17}	$\frac{3}{2}$	A	$\bar{D}\Xi_b^*$	0	−1	29^{+21}_{-12}	7792^{+12}_{-21}	$\frac{3}{2}$	B
$\bar{D}^*\Xi_b^*$	0	−1	42^{+22}_{-20}	7920^{+20}_{-22}	$\frac{1}{2}$	A	$\bar{D}^*\Xi_b^*$	0	−1	$11.3^{+10.6}_{-8.3}$	$7951.1^{+8.3}_{-10.6}$	$\frac{1}{2}$	B
$\bar{D}^*\Xi_b^*$	0	−1	29^{+17}_{-15}	7933^{+15}_{-17}	$\frac{3}{2}$	A	$\bar{D}^*\Xi_b^*$	0	−1	22^{+15}_{-13}	7941^{+13}_{-15}	$\frac{3}{2}$	B
$\bar{D}^*\Xi_b^*$	0	−1	$11.3^{+10.5}_{-8.3}$	$7951.1^{+8.3}_{-10.5}$	$\frac{5}{2}$	A	$\bar{D}^*\Xi_b^*$	0	−1	42^{+22}_{-20}	7920^{+20}_{-22}	$\frac{5}{2}$	B
$\bar{D}\Xi_b^* - \bar{D}_s\Sigma_b^*$	1	−1	15^{+14}_{-11}	7786^{+11}_{-14}	$\frac{3}{2}$	A	$\bar{D}\Xi_b^* - \bar{D}_s\Sigma_b^*$	1	−1	23^{+17}_{-14}	7778^{+14}_{-17}	$\frac{3}{2}$	B
$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	38^{+22}_{-20}	7906^{+20}_{-22}	$\frac{1}{2}$	A	$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	$8.0^{+10.2}_{-7.2}$	$7936.7^{+7.2}_{-10.2}$	$\frac{1}{2}$	B

(*continued on next page*)

Table 3 (*continued*)

Molecule	I	S	B_P	M_P	J	Scenario	Molecule	I	S	B_P	M_P	J	Scenario
$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	28^{+18}_{-15}	7919^{+15}_{-18}	$\frac{3}{2}$	A	$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	18^{+15}_{-12}	7927^{+12}_{-15}	$\frac{3}{2}$	B
$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	$8.0^{+10.2}_{-7.2}$	$7936.7^{+7.2}_{-10.2}$	$\frac{5}{2}$	A	$\bar{D}^*\Xi_b^* - \bar{D}_s^*\Sigma_b^*$	1	−1	38^{+22}_{-20}	7906^{+20}_{-22}	$\frac{5}{2}$	B
$\bar{D}\Omega_b^* - \bar{D}_s\Xi_b^*$	$\frac{1}{2}$	−2	13^{+14}_{-11}	7909^{+11}_{-14}	$\frac{3}{2}$	A	$\bar{D}\Omega_b^* - \bar{D}_s\Xi_b^*$	$\frac{1}{2}$	−2	21^{+17}_{-14}	7901^{+14}_{-17}	$\frac{3}{2}$	B
$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{1}{2}$	−2	38^{+22}_{-20}	8029^{+20}_{-22}	$\frac{1}{2}$	A	$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{1}{2}$	−2	$6.8^{+10.4}_{-7.3}$	$8059.2^{+7.3}_{-10.4}$	$\frac{1}{2}$	B
$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{1}{2}$	−2	25^{+18}_{-15}	8041^{+15}_{-18}	$\frac{3}{2}$	A	$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{1}{2}$	−2	17^{+15}_{-12}	8049^{+12}_{-15}	$\frac{3}{2}$	B
$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{3}{2}$	−2	$6.8^{+10.4}_{-7.3}$	$8059.2^{+7.3}_{-10.4}$	$\frac{5}{2}$	A	$\bar{D}^*\Omega_b^* - \bar{D}_s^*\Xi_b^*$	$\frac{3}{2}$	−2	38^{+22}_{-20}	8029^{+20}_{-22}	$\frac{5}{2}$	B

Actually, there is a rich literature dealing with ways of quantifying the compositeness of a state [71–82], which we can use to obtain a refined estimation of $X_{\rm comp}$. They began with the compositeness criterion proposed by Weinberg [71–73], which can be written as

$$X^{\rm W}_{\rm comp} = \sqrt{\frac{1}{1-2\frac{r_0}{a_0}}}\,, \tag{42}$$

where a_0 and r_0 are the scattering length[6] and effective range and which showed in a model-independent way that the deuteron is probably composite. It actually returns $X_{\rm comp} > 1$ for the deuteron, which indicates we are using the previous formula beyond its domain of validity ($r_0 < 0$ for obtaining $X_{\rm comp} < 1$ for a bound state, not to mention that there will be corrections coming from the range of the interaction, as already pointed in [73]), but this result is usually interpreted as molecular. The bottom-line though is that the Weinberg criterion relies heavily on the sign of the effective range of the purported components of the state: if positive (negative) the state will be predominantly composite (elementary). As a consequence the application of this criterion will lead to the conclusion that the pentaquarks we are dealing with here are mostly molecular. This however will be an artifact of the formalism we are using: our LO calculation automatically generates a positive effective range, which is a consequence of the dynamics we are using.[7] Besides, even though it is evident that the energy dependence of a compact core coupled to a two-hadron system is such that it will generate a negative effective range, a sufficiently short-ranged potential combined with a large binding energy implies a sizable superposition of the hadrons and, owing to their finite size, also a degree of non-compositeness. From this and other arguments, extensions of the Weinberg criterion have been proposed that apply to situations different from a bound state with negative effective range [74–82].

A recent proposal of a model-independent estimation of the compositeness of a state is the following [80]

$$\tilde{X}_{\rm comp} = \sqrt{\frac{1}{1+2|\frac{r_0}{a_0}|}}\,, \tag{43}$$

[6] In our convention, for attractive potentials $a_0 < 0$ in the absence of bound states and $a_0 > 0$ when there is one bound state.

[7] Only at NLO will we be able to obtain a negative effective range, as it is at this order that energy and momentum dependent corrections to the contact-range potential enter. Unfortunately this calculation implies new couplings, the calibration of which require meson-baryon scattering data that are not available at the moment.

Table 4
Same as Table 3 but for the $c\bar{b}$ and $b\bar{b}$ sectors.

Molecule	I	S	B_P	M_P	J	Scenario	Molecule	I	S	B_P	M_P	J	Scenario
$B\Sigma_c^*$	$\frac{1}{2}$	0	$27.0^{+8.6}_{-7.3}$	$7770.6^{+7.3}_{-8.6}$	$\frac{3}{2}$	A	$B\Sigma_c^*$	$\frac{1}{2}$	0	$35.5^{+9.8}_{-8.4}$	$7762.2^{+8.4}_{-9.8}$	$\frac{3}{2}$	B
$B^*\Sigma_c^*$	$\frac{1}{2}$	0	49^{+12}_{-10}	7794^{+10}_{-12}	$\frac{1}{2}$	A	$B^*\Sigma_c^*$	$\frac{1}{2}$	0	$15.6^{+7.0}_{-5.8}$	$7827.3^{+5.8}_{-7.0}$	$\frac{1}{2}$	B
$B^*\Sigma_c^*$	$\frac{1}{2}$	0	$35.6^{+9.8}_{-8.5}$	$7807.3^{+8.5}_{-9.8}$	$\frac{3}{2}$	A	$B^*\Sigma_c^*$	$\frac{1}{2}$	0	$27.1^{+8.7}_{-7.3}$	$7815.7^{+7.3}_{-8.7}$	$\frac{3}{2}$	B
$B^*\Sigma_c^*$	$\frac{1}{2}$	0	$15.6^{+7.0}_{-5.8}$	$7827.3^{+5.8}_{-7.0}$	$\frac{5}{2}$	A	$B^*\Sigma_c^*$	$\frac{1}{2}$	0	49^{+12}_{-10}	7794^{+10}_{-12}	$\frac{5}{2}$	B
$B\Xi_c^*$	0	-1	28^{+18}_{-15}	7897^{+15}_{-18}	$\frac{3}{2}$	A	$B\Xi_c^*$	0	-1	37^{+21}_{-18}	7888^{+18}_{-21}	$\frac{3}{2}$	B
$B^*\Xi_c^*$	0	-1	51^{+25}_{-23}	7919^{+23}_{-25}	$\frac{1}{2}$	A	$B^*\Xi_c^*$	0	-1	17^{+13}_{-11}	7954^{+11}_{-13}	$\frac{1}{2}$	B
$B^*\Xi_c^*$	0	-1	37^{+21}_{-18}	7933^{+18}_{-21}	$\frac{3}{2}$	A	$B^*\Xi_c^*$	0	-1	29^{+18}_{-15}	7942^{+15}_{-18}	$\frac{3}{2}$	B
$B^*\Xi_c^*$	0	-1	17^{+13}_{-11}	7954^{+11}_{-13}	$\frac{5}{2}$	A	$B^*\Xi_c^*$	0	-1	51^{+25}_{-23}	7919^{+23}_{-25}	$\frac{5}{2}$	B
$B\Xi_c^* - B_s\Sigma_c^*$	1	-1	19^{+16}_{-13}	7866^{+13}_{-16}	$\frac{3}{2}$	A	$B\Xi_c^* - B_s\Sigma_c^*$	1	-1	27^{+19}_{-16}	7858^{+17}_{-19}	$\frac{3}{2}$	B
$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	40^{+24}_{-21}	7893^{+21}_{-24}	$\frac{1}{2}$	A	$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	$9.2^{+11.2}_{-8.1}$	$7924.4^{+8.1}_{-11.2}$	$\frac{1}{2}$	B
$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	27^{+19}_{-16}	7906^{+19}_{-16}	$\frac{3}{2}$	A	$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	20^{+16}_{-13}	7914^{+13}_{-16}	$\frac{3}{2}$	B
$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	$9.2^{+11.2}_{-8.1}$	$7924.4^{+8.1}_{-11.2}$	$\frac{5}{2}$	A	$B^*\Xi_c^* - B_s^*\Sigma_c^*$	1	-1	40^{+24}_{-21}	7893^{+21}_{-24}	$\frac{5}{2}$	B
$B\Omega_c^* - B_s\Xi_c^*$	$\frac{1}{2}$	-2	12^{+16}_{-11}	8000^{+11}_{-16}	$\frac{3}{2}$	A	$B\Omega_c^* - B_s\Xi_c^*$	$\frac{1}{2}$	-2	20^{+19}_{-16}	7993^{+16}_{-19}	$\frac{3}{2}$	B
$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{1}{2}$	-2	35^{+25}_{-22}	8026^{+22}_{-25}	$\frac{1}{2}$	A	$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{1}{2}$	-2	$3.9^{+10.7}_{-3.9}$	$8057.1^{+3.9}_{-10.7}$	$\frac{1}{2}$	B
$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{1}{2}$	-2	22^{+20}_{-16}	8040^{+16}_{-20}	$\frac{3}{2}$	A	$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{1}{2}$	-2	14^{+16}_{-12}	8048^{+12}_{-16}	$\frac{3}{2}$	B
$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{3}{2}$	-2	$3.9^{+10.7}_{3.9}$	$8057.1^{+3.9}_{-10.7}$	$\frac{5}{2}$	A	$B^*\Omega_c^* - B_s^*\Xi_c^*$	$\frac{5}{2}$	-2	35^{+25}_{-22}	8026^{+22}_{-25}	$\frac{5}{2}$	B
$B\Sigma_b^*$	$\frac{1}{2}$	0	46^{+21}_{-17}	11065^{+17}_{-21}	$\frac{3}{2}$	A	$B\Sigma_b^*$	$\frac{1}{2}$	0	57^{+23}_{-9}	11055^{+9}_{-23}	$\frac{3}{2}$	B
$B^*\Sigma_b^*$	$\frac{1}{2}$	0	73^{+26}_{-22}	11084^{+22}_{-26}	$\frac{1}{2}$	A	$B^*\Sigma_b^*$	$\frac{1}{2}$	0	32^{+18}_{-14}	11125^{+14}_{-18}	$\frac{1}{2}$	B
$B^*\Sigma_b^*$	$\frac{1}{2}$	0	57^{+23}_{-19}	11100^{+19}_{-23}	$\frac{3}{2}$	A	$B^*\Sigma_b^*$	$\frac{1}{2}$	0	47^{+21}_{-17}	11110^{+17}_{-21}	$\frac{3}{2}$	B
$B^*\Sigma_b^*$	$\frac{1}{2}$	0	32^{+18}_{-14}	11125^{+14}_{-18}	$\frac{5}{2}$	A	$B^*\Sigma_b^*$	$\frac{1}{2}$	0	73^{+26}_{-22}	11084^{+22}_{-26}	$\frac{5}{2}$	B
$B\Xi_b^*$	0	-1	47^{+28}_{-24}	11186^{+24}_{-28}	$\frac{3}{2}$	A	$B\Xi_b^*$	0	-1	54^{+30}_{-27}	11176^{+27}_{-30}	$\frac{3}{2}$	B
$B^*\Xi_b^*$	0	-1	73^{+35}_{-32}	11205^{+32}_{-35}	$\frac{1}{2}$	A	$B^*\Xi_b^*$	0	-1	33^{+23}_{-19}	11246^{+19}_{-23}	$\frac{1}{2}$	B
$B^*\Xi_b^*$	0	-1	58^{+31}_{-27}	11221^{+27}_{-31}	$\frac{3}{2}$	A	$B^*\Xi_b^*$	0	-1	47^{+28}_{-24}	11231^{+24}_{-28}	$\frac{3}{2}$	B
$B^*\Xi_b^*$	0	-1	33^{+23}_{-19}	11246^{+19}_{-23}	$\frac{5}{2}$	A	$B^*\Xi_b^*$	0	-1	73^{+35}_{-32}	11205^{+32}_{-35}	$\frac{5}{2}$	B
$B\Xi_b^* - B_s\Sigma_b^*$	1	-1	39^{+27}_{-22}	11161^{+22}_{-27}	$\frac{3}{2}$	A	$B\Xi_b^* - B_s\Sigma_b^*$	1	-1	49^{+30}_{-26}	11151^{+26}_{-30}	$\frac{3}{2}$	B
$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	65^{+35}_{-31}	11183^{+31}_{-35}	$\frac{1}{2}$	A	$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	26^{+22}_{-17}	11223^{+17}_{-22}	$\frac{1}{2}$	B
$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	49^{+30}_{-26}	11199^{+26}_{-30}	$\frac{3}{2}$	A	$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	40^{+27}_{-23}	11208^{+23}_{-27}	$\frac{3}{2}$	B
$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	26^{+22}_{-17}	11223^{+17}_{-22}	$\frac{5}{2}$	A	$B^*\Xi_b^* - B_s^*\Sigma_b^*$	1	-1	65^{+35}_{-31}	11183^{+31}_{-35}	$\frac{5}{2}$	B
$B\Omega_b^* - B_s\Xi_b^*$	$\frac{1}{2}$	-2	33^{+27}_{-22}	11288^{+22}_{-27}	$\frac{3}{2}$	A	$B\Omega_b^* - B_s\Xi_b^*$	$\frac{1}{2}$	-2	43^{+30}_{-26}	11278^{+26}_{-30}	$\frac{3}{2}$	B
$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{1}{2}$	-2	60^{+35}_{-31}	11309^{+31}_{-35}	$\frac{1}{2}$	A	$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{1}{2}$	-2	21^{+22}_{-17}	11349^{+17}_{-22}	$\frac{1}{2}$	B
$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{1}{2}$	-2	45^{+30}_{-26}	11324^{+26}_{-30}	$\frac{3}{2}$	A	$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{1}{2}$	-2	35^{+27}_{-23}	11334^{+23}_{-27}	$\frac{3}{2}$	B
$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{3}{2}$	-2	21^{+22}_{-17}	11349^{+17}_{-22}	$\frac{5}{2}$	A	$B^*\Omega_b^* - B_s^*\Xi_b^*$	$\frac{5}{2}$	-2	60^{+35}_{-31}	11309^{+31}_{-35}	$\frac{5}{2}$	B

which returns $\tilde{X}_{\rm comp} < 1$, where the calculation of a_0 and r_0 for our contact-range theory is explained in Appendix B. This criterion would provide a compositeness of $\tilde{X}_{\rm comp} = (0.73, 0.67, 0.76)$ for each of the three LHCb pentaquarks (i.e. P_{c1}, P_{c2}, P_{c3}), $(0.66, 0.62, 0.68)$ and $(0.63, 0.61, 0.65)$ for the $\bar{c}b$ and $c\bar{b}$ ones and $(0.59, 0.57, 0.60)$ for the hidden-bottom P_{b1}, P_{b2} and P_{b3} pentaquarks. However, the problem here is that we are using a LO EFT description with only one parameter (the binding energy), which means that the value of the effective range

thus obtained is only a dimensional estimation within our EFT. For comparison the compositeness of the deuteron ($a_0 = 5.419\,\text{fm}$, $r_0 = 1.753\,\text{fm}$ [83]) with this criterion will be 0.78, but in this case there is plenty of experimental information available about neutron-proton scattering, i.e. a_0 and r_0 are well-known.

Regardless of the specific criterion used to estimate compositeness (after all, the wave function is not an observable), it seems that in general the hidden-charm pentaquarks are less composite than the deuteron, and as we move into heavier flavor sectors their compositeness reduces further. This is in turn compatible with the observation that the EFT description is less convergent and has larger uncertainties for two-body systems with larger binding energies. Thus, as binding increases with the reduced mass, we expect compositeness to decrease accordingly.

7. Flavor symmetry and non-molecular explanations

The present predictions have been done under the assumption that the hidden-charm pentaquarks are molecular. But, as a matter of fact, the light- and heavy-flavor symmetries we have used here are expected to apply to other light-heavy hadrons as well, independently of their nature (though the uncertainties stemming from the violations of these symmetries could be very different). For instance, the existence of this type of pentaquark multiplets has been predicted in the compact [40] and hadroquarkonium pictures [84,85]. Theoretical explorations in the previous pictures have been mostly concentrated in the hidden-charm sector, where the mass splittings of the octet [$m(P_c^\Lambda) - m(P_c^N)$, $m(P_c^\Sigma) - m(P_c^N)$ and $m(P_c^\Xi) - m(P_c^N)$] are 141, 205 and 315 MeV for compacts pentaquarks [40] and 150, 217 and 327 MeV for hadrocharmonia [86]. These mass splittings happen to be larger than for molecular pentaquarks (125, 105 and 232 MeV) and might provide a way to distinguish their nature if they are observed. For the hidden-bottom sector there are indeed predictions of P_b^N pentaquarks in the local hidden-gauge approach of Ref. [87] and in models considering a five-quark core and pion exchanges [36]. It is plausible that other theoretical models of $Q'\bar{Q}$ pentaquarks will lead to analogous predictions for their flavor partners, as these predictions are constrained by symmetry principles (instead of the details of the dynamics, which will matter for how the spectrum is organized in terms of quantum numbers, spin-spin splitting, etc.). Recent calculations of $qqsQ'\bar{Q}$ pentaquarks in the hadroquarkonium [85] and chiral quark models [88] provide further support for this conjecture.

8. Summary

The observation of the LHCb hidden-charm pentaquarks in combination with SU(3)- and heavy-flavor symmetries leads to the prediction of a series of flavor partners. In particular, pentaquarks (molecular and non-molecular [40] alike) are expected to form a light-flavor octet reminiscent of the light-baryon octet and are also expected to appear in the $c\bar{b}$, $\bar{c}b$ and $b\bar{b}$ sectors as well as in the original hidden-charm sector where they have been discovered. We denote these pentaquarks as $P^N_{Q'\bar{Q}}$, $P^\Lambda_{Q'\bar{Q}}$, $P^\Sigma_{Q'\bar{Q}}$, $P^\Xi_{Q'\bar{Q}}$, with the superscript and subscript referring to their light- and heavy-quark structure, respectively (which we shorten to P^N_Q, P^Λ_Q, P^Σ_Q and P^Ξ_Q when the heavy flavors coincide $Q' = Q$, i.e. for hidden-flavor). For predicting their masses, we have made use of a contact-range theory with a natural cutoff in the range $\Lambda = (0.5 - 1.0)\,\text{GeV}$. Among the predictions, it is worth noticing the existence of five-flavor pentaquarks, i.e. pentaquarks containing all the five flavors that hadronize ($P^\Lambda_{c\bar{b}}$, $P^\Lambda_{\bar{c}b}$, $P^\Sigma_{c\bar{b}}$, $P^\Sigma_{\bar{c}b}$ in our notation) in the $7770 - 7910\,\text{MeV}$ region. The five-flavor pentaquarks could be detected via their $B_c^\pm\Lambda$ and $B_c^\pm\Sigma$ decays.

The predictions made in this work assume the LHCb pentaquarks to be meson-baryon bound states the dynamics of which can be described in terms of a contact-range theory. It is worth noticing that the applicability of this description decreases with increasing binding energy, as this implies pentaquarks that are less composite, and with heavier reduced masses owing to the model-dependent nature of HFS [47]. This is reflected in the larger uncertainties, particularly in the hidden-bottom sector. Yet, it is sensible to expect these predictions to be more dependent on the general symmetry principles we have applied than on the details of the dynamics generating the pentaquarks, e.g. models with a compact five-quark core coupled to the meson-baryon degrees of freedom do reproduce the hidden-charm pentaquarks [34] and also predict the hidden-bottom ones [36], giving credence to the aforementioned conjecture. Thus it might be the case that the light- and heavy-flavor symmetry partners of the hidden-charm pentaquarks exist irrespective of the binding mechanism, though the details of the spectrum will be different than in the molecular case.

CRediT authorship contribution statement

The authors contributed equally in the completion of this manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data used in this manuscript comes from analyses published by experimental collaborations.

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Appendix A. Heavy-quark mass dependence of the binding energy

Here we consider the variation of the binding energy of a heavy hadron molecule with respect to the heavy-quark mass. If the potential between two heavy hadrons does not depend on the heavy-quark mass, it can be shown that the binding energy increases with the heavy-quark mass (in agreement with naive expectations).

At leading order in the $1/m_Q$ expansion, we can write the Schrödinger equation for a heavy hadron molecule as follows

$$-\nabla^2 \Psi_Q(\vec{r}) + 2\,\mu_Q\, V_Q(\vec{r})\, \Psi_Q(\vec{r}) = -2\,\mu_Q B_Q \Psi_Q(\vec{r})\,, \tag{A.1}$$

where the subindex $_Q$ indicates the dependence (explicit and implicit) on the heavy-quark mass, Ψ_Q is the wave function, μ_Q the reduced mass of the molecule, V_Q the potential and B_Q the

two-body binding energy. We can construct a Wronskian identity for the Schrödinger equation at two different heavy-quark masses as follows

$$\begin{aligned} &-\left(\Psi_{Q'}\nabla^2\Psi_Q - \Psi_Q\nabla^2\Psi_{Q'}\right) \\ &+2\left(\mu_Q V_Q - \mu_{Q'} V_{Q'}\right)\Psi_Q\,\Psi_{Q'} \\ &= -2\left(\mu_Q B_Q - \mu_{Q'} B_{Q'}\right)\Psi_Q\,\Psi_{Q'}\,, \end{aligned} \tag{A.2}$$

where, again, $_Q$ and $_{Q'}$ represent the different quantities we are considering at m_Q and $m_{Q'}$, respectively. The Wronskian identity can be integrated, leading to

$$\begin{aligned} 2\int d^3\vec{r}\,\left(\mu_Q V_Q - \mu_{Q'} V_{Q'}\right)\Psi_Q(\vec{r})\,\Psi_{Q'}(\vec{r}) = \\ -2\left(\mu_Q B_Q - \mu_{Q'} B_{Q'}\right)\int d^3\vec{r}\,\Psi_Q(\vec{r})\,\Psi_{Q'}(\vec{r}) \end{aligned} \tag{A.3}$$

where the kinetic term disappears because it is exactly differentiable and can be rewritten as a surface term, which vanishes if we consider bound state solutions. Now we will consider a small change in the heavy-quark mass, which we can symbolically indicate by

$$Q' = Q + \delta\,Q\,. \tag{A.4}$$

We can deduce that

$$\int d^3\vec{r}\,\Psi_Q(\vec{r})\,\Psi_{Q'}(\vec{r}) = 1 + (\delta\,Q)^2\,, \tag{A.5}$$

which is a consequence of the normalization of the wave function (i.e. $\langle\Psi_Q|\Psi_Q\rangle = \langle\Psi_{Q'}|\Psi_{Q'}\rangle = 1$, which is why the δQ term vanishes). If we assume that the potential does not depend on the heavy-quark mass, i.e. $V_Q = V_{Q'}$, we can use the previous result to prove that

$$2\,\delta\mu_Q\,\langle V_Q\rangle = -2\,\delta(\mu_Q B_Q)\,, \tag{A.6}$$

which we can differentiate to obtain

$$\langle V_Q\rangle = -B_Q - \mu_Q\,\frac{\partial B_Q}{\partial\mu_Q}\,. \tag{A.7}$$

If we take into account

$$\langle T_Q\rangle + \langle V_Q\rangle = -B_Q\,, \tag{A.8}$$

where $\langle T_Q\rangle \geq 0$ is the kinetic energy of the heavy molecule, we can rewrite the binding energy dependence on the reduced mass as

$$\langle T_Q\rangle = \mu_Q\,\frac{\partial B_Q}{\partial\mu_Q} \tag{A.9}$$

or, equivalently

$$\frac{\partial B_Q}{\partial\mu_Q} \geq 0\,, \tag{A.10}$$

as a consequence of the fact that the kinetic energy is positive. That is, the system will become more bound the heavier the mesons (this is a model-independent result). What is difficult (and model-dependent) is to determine by what amount. Finally, we notice that including a heavy-quark mass dependence of the type $V_Q = V_0 + \frac{1}{m_Q}\,V_1 + \ldots$ in the potential does only induce $1/m_Q$ corrections to the previous relation, which can be safely neglected in the heavy-quark mass limit.

Appendix B. Calculation of the effective range expansion parameters

The evaluation of the different compositeness conditions available in the literature usually require the effective range parameters as input. Here we briefly explain how to calculate them. We begin by writing down the relation between the effective range expansion and the on-shell T-matrix ($T_{\rm os}$):

$$-\frac{2\pi}{\mu}\,\mathrm{Re}\left[\frac{1}{T_{\rm os}(k)}\right]=-\frac{1}{a_0}+\frac{1}{2}\,r_0\,k^2+\sum_{n=2}^{\infty}v_n k^{2n}\,, \tag{B.1}$$

where a_0 is the scattering length, r_0 the effective range, v_n the shape parameters, k the center-of-mass momentum and μ refers to the reduced mass of the two-body system. For attractive potentials, the previous convention implies $a_0<0$ if there is no bound state (or an even number of bound states) and $a_0>0$ if there is an odd number of bound states. The on-shell T-matrix corresponds to the following matrix element of the full T-matrix

$$T_{\rm os}(k)=\langle k|T(k)|k\rangle\,, \tag{B.2}$$

where T obeys the Lippmann-Schwinger equation, which for scattering states takes the form

$$T=V+VG_0(E+i\epsilon)T\,, \tag{B.3}$$

with $G_0(E)=1/(E-H_0)$ the resolvent operator and $E=k^2/2\mu$ the center-of-mass energy of the system. If we consider a regularized contact-range of the type

$$\langle p'|V_C|p\rangle=C(\Lambda)\,f(\frac{p'}{\Lambda})\,f(\frac{p}{\Lambda})\,, \tag{B.4}$$

then the explicit solution of the Lippmann-Schwinger equation for the on-shell T-matrix reads

$$\mathrm{Re}\left[\frac{1}{T_{\rm os}(k)}\right]=\frac{1}{C(\Lambda)}-\frac{\mu}{\pi^2}\,\mathcal{P}\int_0^{\infty}\frac{p^2dp}{k^2-p^2}\,f^2(\frac{p}{\Lambda})\,, \tag{B.5}$$

where $\mathcal{P}$ denotes the principal value of the integral. By expanding in powers of the center-of-mass momentum, we arrive at

$$\frac{1}{a_0}=\frac{2\pi}{\mu}\,\frac{1}{C(\Lambda)}+\frac{2}{\pi}\int_0^{\infty}dp\,f^2(\frac{p}{\Lambda})\,, \tag{B.6}$$

$$r_0=-\frac{4}{\pi}\int_0^{\infty}\frac{dp}{p^2}\left(f^2(\frac{p}{\Lambda})-f^2(0)\right)\,, \tag{B.7}$$

where we can appreciate that at LO in our contact-range theory r_0 depends solely on the regulator and cutoff, i.e. EFT merely provides a dimensional estimation of its size. If we particularize for our choices of regulator function, cutoff and couplings, we will obtain the values of a_0 and r_0 that we have used as input for Eq. (43).

References

[1] R. Aaij, et al., LHCb, Phys. Rev. Lett. 122 (2019) 222001, arXiv:1904.03947 [hep-ex].
[2] R. Aaij, et al., LHCb, Phys. Rev. Lett. 115 (2015) 072001, arXiv:1507.03414 [hep-ex].
[3] J.-J. Wu, R. Molina, E. Oset, B.S. Zou, Phys. Rev. Lett. 105 (2010) 232001, arXiv:1007.0573 [nucl-th].
[4] J.-J. Wu, R. Molina, E. Oset, B.S. Zou, Phys. Rev. C 84 (2011) 015202, arXiv:1011.2399 [nucl-th].
[5] J.-J. Wu, B.S. Zou, Phys. Lett. B 709 (2012) 70, arXiv:1011.5743 [hep-ph].
[6] C.W. Xiao, J. Nieves, E. Oset, Phys. Rev. D 88 (2013) 056012, arXiv:1304.5368 [hep-ph].
[7] M. Karliner, J.L. Rosner, Phys. Rev. Lett. 115 (2015) 122001, arXiv:1506.06386 [hep-ph].
[8] W.L. Wang, F. Huang, Z.Y. Zhang, B.S. Zou, Phys. Rev. C 84 (2011) 015203, arXiv:1101.0453 [nucl-th].
[9] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, S.-L. Zhu, Chin. Phys. C 36 (2012) 6, arXiv:1105.2901 [hep-ph].
[10] H.-X. Chen, W. Chen, S.-L. Zhu, Phys. Rev. D 100 (2019) 051501, arXiv:1903.11001 [hep-ph].
[11] R. Chen, Z.-F. Sun, X. Liu, S.-L. Zhu, Phys. Rev. D 100 (2019) 011502, arXiv:1903.11013 [hep-ph].
[12] M.-Z. Liu, Y.-W. Pan, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, A. Hosaka, M. Pavon Valderrama, Phys. Rev. Lett. 122 (2019) 242001, arXiv:1903.11560 [hep-ph].
[13] F.-K. Guo, H.-J. Jing, U.-G. Meißner, S. Sakai, Phys. Rev. D 99 (2019) 091501, arXiv:1903.11503 [hep-ph].
[14] C. Xiao, J. Nieves, E. Oset, Phys. Rev. D 100 (2019) 014021, arXiv:1904.01296 [hep-ph].
[15] Z.-H. Guo, J.A. Oller, Phys. Lett. B 793 (2019) 144, arXiv:1904.00851 [hep-ph].
[16] M.I. Eides, V.Y. Petrov, M.V. Polyakov, Mod. Phys. Lett. A 35 (2020) 2050151, arXiv:1904.11616 [hep-ph].
[17] Z.-G. Wang, Int. J. Mod. Phys. A 35 (2020) 2050003, arXiv:1905.02892 [hep-ph].
[18] J.-B. Cheng, Y.-R. Liu, Phys. Rev. D 100 (2019) 054002, arXiv:1905.08605 [hep-ph].
[19] M. Gell-Mann, Phys. Rev. 125 (1962) 1067.
[20] Y. Ne'eman, Nucl. Phys. 26 () (1961) 222, [34 (1961)].
[21] N. Isgur, M.B. Wise, Phys. Lett. B 232 (1989) 113.
[22] N. Isgur, M.B. Wise, Phys. Lett. B 237 (1990) 527.
[23] M.J. Savage, M.B. Wise, Phys. Lett. B 248 (1990) 177.
[24] M.T. AlFiky, F. Gabbiani, A.A. Petrov, Phys. Lett. B 640 (2006) 238, arXiv:hep-ph/0506141.
[25] M. Voloshin, Phys. Rev. D 84 (2011) 031502, arXiv:1105.5829 [hep-ph].
[26] T. Mehen, J.W. Powell, Phys. Rev. D 84 (2011) 114013, arXiv:1109.3479 [hep-ph].
[27] M.P. Valderrama, Phys. Rev. D 85 (2012) 114037, arXiv:1204.2400 [hep-ph].
[28] J. Nieves, M.P. Valderrama, Phys. Rev. D 86 (2012) 056004, arXiv:1204.2790 [hep-ph].
[29] C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. D 87 (2013) 076006, arXiv:1210.5431 [hep-ph].
[30] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. D 88 (2013) 054007, arXiv:1303.6608 [hep-ph].
[31] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. D 88 (2013) 054014, arXiv:1305.4052 [hep-ph].
[32] J.-X. Lu, L.-S. Geng, M.P. Valderrama, Phys. Rev. D 99 (2019) 074026, arXiv:1706.02588 [hep-ph].
[33] S. Sakai, H.-J. Jing, F.-K. Guo, Phys. Rev. D 100 (2019) 074007, arXiv:1907.03414 [hep-ph].
[34] Y. Yamaguchi, H. García-Tecocoatzi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, M. Takizawa, Phys. Rev. D 101 (2020) 091502, arXiv:1907.04684 [hep-ph].
[35] Y. Yamaguchi, E. Santopinto, Phys. Rev. D 96 (2017) 014018, arXiv:1606.08330 [hep-ph].
[36] Y. Yamaguchi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, M. Takizawa, Phys. Rev. D 96 (2017) 114031, arXiv:1709.00819 [hep-ph].
[37] Y. Shimizu, Y. Yamaguchi, M. Harada, Phys. Rev. D 98 (2018) 014021, arXiv:1805.05740 [hep-ph].
[38] M.-Z. Liu, F.-Z. Peng, M. Sánchez Sánchez, M.P. Valderrama, Phys. Rev. D 98 (2018) 114030, arXiv:1811.03992 [hep-ph].
[39] T.A. Kaeding, At. Data Nucl. Data Tables 61 (1995) 233, arXiv:nucl-th/9502037 [nucl-th].
[40] E. Santopinto, A. Giachino, Phys. Rev. D 96 (2017) 014014, arXiv:1604.03769 [hep-ph].
[41] Q. Meng, E. Hiyama, K.U. Can, P. Gubler, M. Oka, A. Hosaka, H. Zong, Phys. Lett. B 798 (2019) 135028, arXiv:1907.00144 [nucl-th].
[42] U. van Kolck, Nucl. Phys. A 645 (1999) 273, arXiv:nucl-th/9808007.
[43] J.-W. Chen, G. Rupak, M.J. Savage, Nucl. Phys. A 653 (1999) 386, arXiv:nucl-th/9902056.
[44] M. Pavon Valderrama, Phys. Rev. D 100 (2019) 094028, arXiv:1907.05294 [hep-ph].
[45] C. Fernández-Ramírez, A. Pilloni, M. Albaladejo, A. Jackura, V. Mathieu, M. Mikhasenko, J. Silva-Castro, A. Szczepaniak JPAC, Phys. Rev. Lett. 123 (2019) 092001, arXiv:1904.10021 [hep-ph].
[46] P. Zyla, et al., Particle Data Group, PTEP 2020 (2020) 083C01.

[47] V. Baru, E. Epelbaum, J. Gegelia, C. Hanhart, U.G. Meißner, A.V. Nefediev, Eur. Phys. J. C 79 (2019) 46, arXiv:1810.06921 [hep-ph].
[48] M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meißner, J.A. Oller, Q. Wang, Phys. Rev. Lett. 124 (2020) 072001, arXiv:1910.11846 [hep-ph].
[49] C. Xiao, J. Nieves, E. Oset, Phys. Lett. B 799 (2019) 135051, arXiv:1906.09010 [hep-ph].
[50] T. Gutsche, V.E. Lyubovitskij, Phys. Rev. D 100 (2019) 094031, arXiv:1910.03984 [hep-ph].
[51] B. Wang, L. Meng, S.-L. Zhu, Phys. Rev. D 101 (2020) 034018, arXiv:1912.12592 [hep-ph].
[52] R. Aaij, et al., LHCb, Sci. Bull. 66 (2021) 1391, arXiv:2012.10380 [hep-ex].
[53] H.-X. Chen, W. Chen, X. Liu, X.-H. Liu, Eur. Phys. J. C 81 (2021) 409, arXiv:2011.01079 [hep-ph].
[54] F.-Z. Peng, M.-J. Yan, M. Sánchez Sánchez, M.P. Valderrama, Eur. Phys. J. C 81 (2021) 666, arXiv:2011.01915 [hep-ph].
[55] R. Chen, Phys. Rev. D 103 (2021) 054007, arXiv:2011.07214 [hep-ph].
[56] M.-Z. Liu, Y.-W. Pan, L.-S. Geng, Phys. Rev. D 103 (2021) 034003, arXiv:2011.07935 [hep-ph].
[57] C.-W. Shen, U.-G. Meißner, arXiv:2203.09804 [hep-ph], 2022.
[58] M. Ablikim, et al., BESIII, Phys. Rev. Lett. 110 (2013) 252001, arXiv:1303.5949 [hep-ex].
[59] M. Ablikim, et al., BESIII, Phys. Rev. Lett. 126 (2021) 102001, arXiv:2011.07855 [hep-ex].
[60] Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111 (2013) 132003, arXiv:1303.6355 [hep-ph].
[61] M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque, J. Nieves, Phys. Lett. B 755 (2016) 337, arXiv:1512.03638 [hep-ph].
[62] Z. Yang, X. Cao, F.-K. Guo, J. Nieves, M.P. Valderrama, Phys. Rev. D 103 (2021) 074029, arXiv:2011.08725 [hep-ph].
[63] Z.-F. Sun, C.-W. Xiao, arXiv:2011.09404 [hep-ph], 2020.
[64] B. Wang, L. Meng, S.-L. Zhu, J. High Energy Phys. 11 (2019) 108, arXiv:1909.13054 [hep-ph].
[65] M.-Z. Liu, T.-W. Wu, M. Sánchez Sánchez, M.P. Valderrama, L.-S. Geng, J.-J. Xie, Phys. Rev. D 103 (2021) 054004, arXiv:1907.06093 [hep-ph].
[66] N. Yalikun, Y.-H. Lin, F.-K. Guo, Y. Kamiya, B.-S. Zou, Phys. Rev. D 104 (2021) 094039, arXiv:2109.03504 [hep-ph].
[67] T.J. Burns, E.S. Swanson, Phys. Rev. D 100 (2019) 114033, arXiv:1908.03528 [hep-ph].
[68] T.J. Burns, E.S. Swanson, arXiv:2112.11527 [hep-ph], 2021.
[69] K. Chen, R. Chen, L. Meng, B. Wang, S.-L. Zhu, arXiv:2109.13057 [hep-ph], 2021.
[70] F.-Z. Peng, M.-Z. Liu, M. Sánchez Sánchez, M. Pavon Valderrama, Phys. Rev. D 102 (2020) 114020, arXiv:2004.05658 [hep-ph].
[71] S. Weinberg, Phys. Rev. 130 (1963) 776.
[72] S. Weinberg, Phys. Rev. 131 (1963) 440.
[73] S. Weinberg, Phys. Rev. 137 (1965) B672.
[74] V. Baru, C. Hanhart, Y.S. Kalashnikova, A.E. Kudryavtsev, A.V. Nefediev, Eur. Phys. J. A 44 (2010) 93, arXiv:1001.0369 [hep-ph].
[75] T. Hyodo, Phys. Rev. Lett. 111 (2013) 132002, arXiv:1305.1999 [hep-ph].
[76] T. Sekihara, T. Hyodo, D. Jido, PTEP 2015 (2015) 063D04, arXiv:1411.2308 [hep-ph].
[77] Y. Kamiya, T. Hyodo, Phys. Rev. C 93 (2016) 035203, arXiv:1509.00146 [hep-ph].
[78] Y. Kamiya, T. Hyodo, PTEP 2017 (2017) 023D02, arXiv:1607.01899 [hep-ph].
[79] T. Sekihara, Phys. Rev. C 95 (2017) 025206, arXiv:1609.09496 [quant-ph].
[80] I. Matuschek, V. Baru, F.-K. Guo, C. Hanhart, Eur. Phys. J. A 57 (2021) 101, arXiv:2007.05329 [hep-ph].
[81] J. Song, L.R. Dai, E. Oset, arXiv:2201.04414 [hep-ph], 2022.
[82] M. Albaladejo, J. Nieves, arXiv:2203.04864 [hep-ph], 2022.
[83] J.J. de Swart, C.P.F. Terheggen, V.G.J. Stoks, in: 3rd International Symposium on Dubna Deuteron 95, 1995, arXiv:nucl-th/9509032.
[84] M.I. Eides, V.Yu. Petrov, M.V. Polyakov, Phys. Rev. D 93 (2016) 054039, arXiv:1512.00426 [hep-ph].
[85] J. Ferretti, E. Santopinto, J. High Energy Phys. 04 (2020) 119, arXiv:2001.01067 [hep-ph].
[86] M.I. Eides, V.Yu. Petrov, M.V. Polyakov, Eur. Phys. J. C 78 (2018) 36, arXiv:1709.09523 [hep-ph].
[87] C.W. Xiao, E. Oset, Eur. Phys. J. A 49 (2013) 139, arXiv:1305.0786 [hep-ph].
[88] Q. Zhang, B.-R. He, J.-L. Ping, arXiv:2006.01042 [hep-ph], 2020.