Impact parameter dependence of the collinearly improved Balitsky-Kovchegov evolution equation

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We have solved the Balistky-Kovchegov evolution equation including the impact-parameter dependence and obtained solutions which are not spoiled by the emergence of non-perturbative effects, dubbed Coulomb tails. This has been achieved due to the fact that using the collinearly-improved kernel to the BK equation suppresses heavily the part of the phase-space of the equation from which the Coulomb tails originate. This, in conjunction with an appropriate initial condition, allows for a correct description of existing data as well as to produce predictions of processes that are feasible for measurement at future facilities such as at the EIC or LHeC.

1 Introduction

The high-energy limit of QCD has been intensively studied in the past years due to the properties of the strong coupling and the applicability of perturbative expansions. This limit is reached from the experimental side by collider experiments and from the theoretical side by evolution equations. The evolution in energy (identified in this approach as rapidity) can be described by the Balitsky-Kuraev-Fadin-Lipatov (BFKL) equation [1, 2] that incorporates gluon branching processes. A non-linear contribution originating from gluon recombination is taken into account in the Balitsky-Kovchegov (BK) evolution equation [3–5]. This equation has been solved in the impact-parameter independent frame with great success in the past [6]. In this proceedings, we report our findings from [7, 8], namely a suppression of the Coulomb tails in the impact-parameter dependent computation in the collinearly improved framework. In this case, the Coulomb tails, that violate the Martin-Froissart bound and make data description impossible [9], are suppressed by the implementation of the recently proposed collinearly improved kernel [10]. The collinear resummation suppresses the contribution of the large daughter dipoles to the evolution, which are also sensitive to the non-perturbative region where Coulomb tails are the strongest. This in turn restores phenomenological predictive power of this equation for future and past experiments as shown in [7, 8].

2 The Balitsky-Kovchegov equation

The BK equation with the impact parameter dependence reads

$$\frac{\partial N(r,b;Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)\Big(N(r_1,b_1;Y) + N(r_2,b_2;Y) - N(r,b;Y) - N(r_1,b_1,Y)N(r_2,b_2;Y)\Big), \tag{2.1}$$

where $\vec{r_2} = \vec{r} - \vec{r_1}$ and $|\vec{r_i}| \equiv r_i$. The vectors $\vec{r_i}$ describe the size and orientation of the dipoles. The variables b_i denote the magnitudes of the impact parameters of the daughter dipoles.

The collinearly improved kernel suppresses the part of the phase space of the equation where large daughter dipoles are dominant [10–13] and is written as

$$K(r,r_1,r_2) = \frac{\overline{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2}\left[\frac{r^2}{\min(r_1^2,r_2^2)}\right]^{\pm\overline{\alpha}_s A_1} \frac{J_1(2\sqrt{\overline{\alpha}_s \rho^2})}{\sqrt{\overline{\alpha}_s}\rho}. \tag{2.2}$$

The value of A_1 is 11/12 and the sign in the third factor is chosen positive when $r^2 < \min(r_1^2, r_2^2)$ and negative otherwise. $\rho \equiv \sqrt{L_{r_1 r} L_{r_2 r}}$, J_1 is the Bessel function and $L_{r_i r} \equiv \ln(r_i^2/r^2)$. The smallest dipole prescription was chosen for the running coupling: $\alpha_s = \alpha_s(r_{\min})$, where $r_{\min} = \min(r_1, r_2, r)$ as in [11].

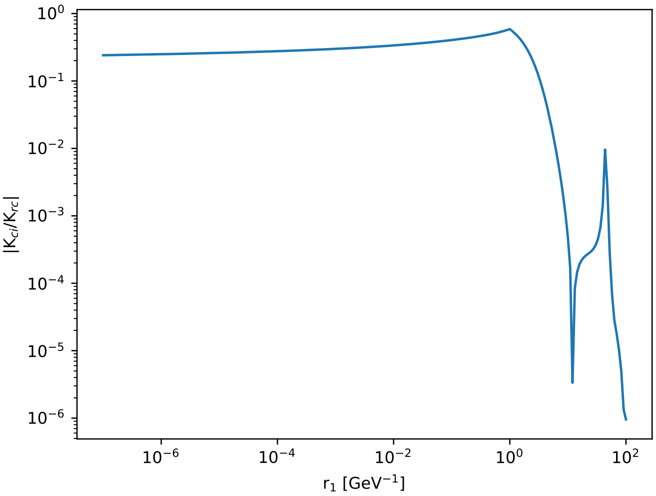

Figure 1: Absolute value of the ratio $K_{\text{ci}}/K_{\text{rc}}$ at a fixed dipole size $r = 1\,\text{GeV}^{-1}$ and orientation with respect to the daughter dipole $\theta_{rr_1} = \pi/2$ as a function of the daughter dipole size. Figure taken from [7].

The region, where Coulomb tails enter the evolution is the one where large daughter dipoles are emitted due to the fact that those regions allow for a sufficiently small impact-parameter of a daughter dipole even when the mother dipole is far from the target center [7, 9]. As discussed earlier, this region is suppressed in the collinearly improved kernel w.r.t. the running coupling kernel [14] (shown in Fig. 1). We proposed a new prescription for the initial condition used for impact-parameter dependent computations that is motivated by the physical size of the proton target

$$N(r,b,Y=0) = 1 - \exp\left(-\frac{1}{2}\frac{Q_s^2}{4} r^2 T(b_{q_1}, b_{q_2})\right), \tag{2.3}$$

where b_{q_i} are the impact parameters of the individual quark and antiquark of the initial bare dipole and

$$T(b_{q_1}, b_{q_2}) = \left[\exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right)\right]. \tag{2.4}$$

This initial condition combines the approach of the GBW model [15] for the dipole-size dependence and an exponential fall-off for the proton profile in the impact parameter space [16–20]. More details and the value of the parameters can be found in [7]. The geometry of the target-dipole interaction is taken into account by the fact that we consider the contribution of the two quarks separately to the initial condition [7].

3 Results

Fig. 2 shows the computed dipole scattering amplitude as a function of rapidity, impact parameter and transverse dipole size. Coulomb tails in the large-b regions are strongly suppressed [7] due to the nature of the collinear resummation. We have also used the obtained scattering amplitude to predict various observables that have been measured in the past years to take use of the fact, that these solutions are no longer spoiled by the presence of non-perturbative effects to such extent that it would spoil its predictive abilities (see Figs 3 and 4).

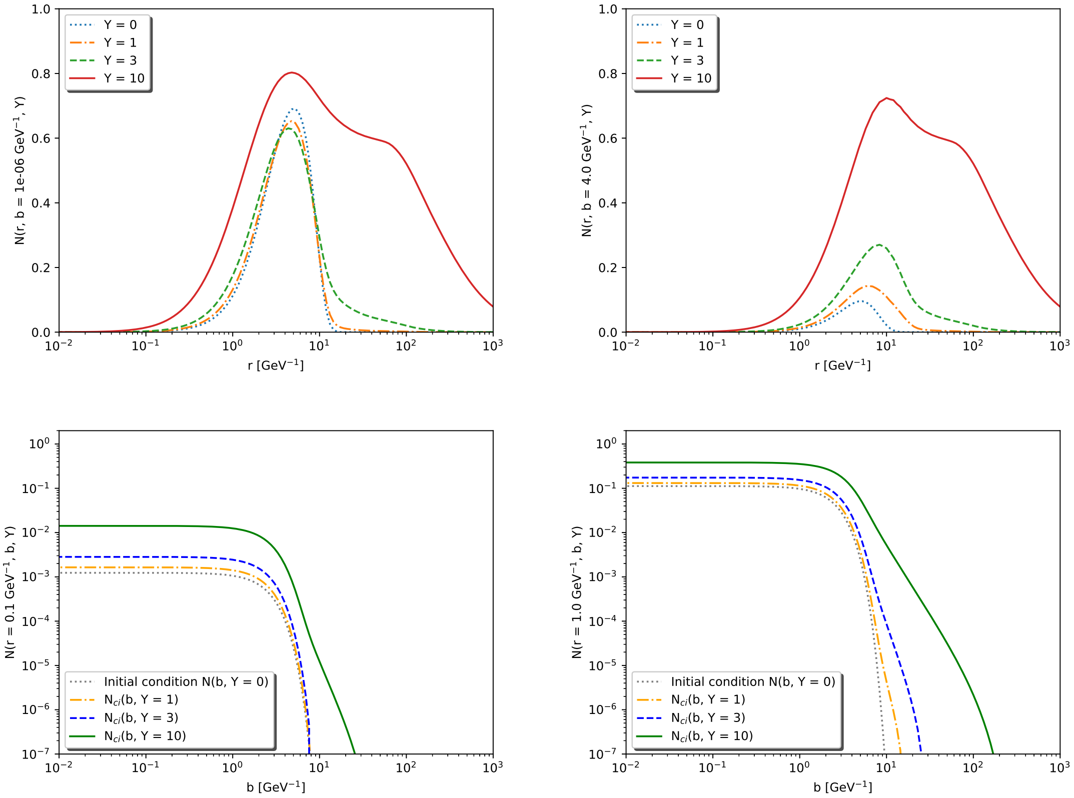

Figure 2: The dipole scattering amplitude as a solution to the BK equation with the collinearly improved kernel as a function of r for $b = 10^{-6}\,\text{GeV}^{-1}$ (upper left) and $b = 4\,\text{GeV}^{-1}$ (upper right), and as a function of b at $r = 0.1\,\text{GeV}^{-1}$ (lower left) and at $r = 1\,\text{GeV}^{-1}$ (lower right). Figure taken from [7].

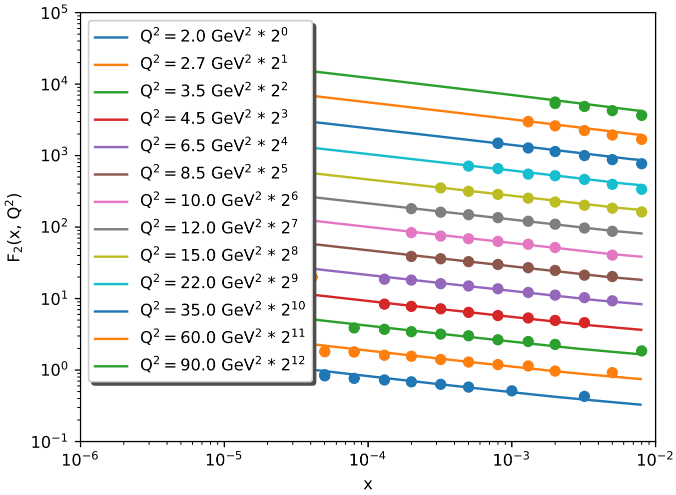

Figure 3: Comparison of the structure function data from HERA [21] (solid circles) to the prediction of the impact-parameter dependent BK equation with the collinearly improved kernel (lines). Figure taken from [7].

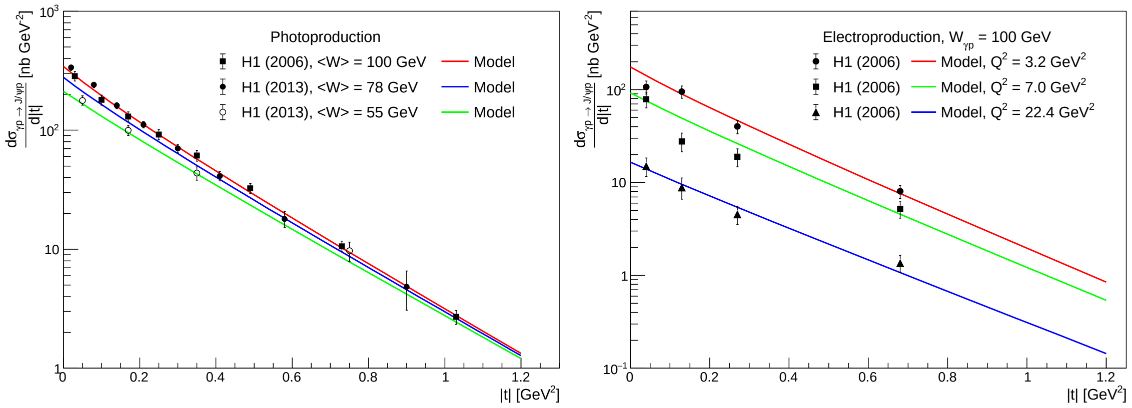

Figure 4: Comparison of the predictions of the model (solid lines) with HERA data from H1 [22, 23] for the $|t|$ dependence of the exclusive photoproduction (left) and electroproduction (right) cross sections of the J/ψ meson. Figure taken from [7].

4 Summary

The collinearly improved kernel along with the impact-parameter dependent BK equation has been used to demonstrate, that the previously established problem of Coulomb tails can be highly suppressed and the new solutions allow for a correct description of data, restoring thus the predictive capabilities of the equation when including the impact-parameter dependence. This is due to the fact, that the time-ordered gluon emissions that are embedded in the collinear resummation [11] suppress the region of large daughter dipoles [7]. This is useful for phenomenological applications in QCD namely for the future planned facilities such as LHeC and the EIC [24, 25].

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