

POLARISATION AT NLO IN WZ PRODUCTION AT THE LHC

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The pair production of a W and a Z boson at the LHC is an important process to study the triple-gauge boson couplings as well as to probe new physics that could arise in the gauge sector. In particular the leptonic channel $pp \rightarrow W^\pm Z \rightarrow 3\ell + \nu + X$ is considered by ATLAS and CMS collaborations. Polarisation observables can help pinning down new physics and give information on the spin of the gauge bosons. Measuring them requires high statistics as well as precise theoretical predictions. We define in this contribution fiducial polarisation observables for the W and Z bosons and we present theoretical predictions in the Standard Model at next-to-leading order (NLO) including QCD as well as NLO electroweak corrections, the latter in the double-pole approximation. We also show that this approximation works remarkably well for $W^\pm Z$ production at the LHC by comparing to the full results.

1 Introduction

The production of a WZ pair at the LHC, in the fully leptonic channel $pp \rightarrow W^\pm Z \rightarrow 3\ell + \nu + X$, is an important part of the Large Hadron Collider (LHC) programme to test the electroweak (EW) sector of the Standard Model (SM). In particular, this process is sensitive already at leading order (LO) to the triple-gauge-boson couplings that can be modified by new physics. High statistics than can be collected at the LHC allows for precise measurements even in complex observables such as kinematical distributions and polarisation observables. This also requires precise predictions from the theory side. The next-to-leading order (NLO) QCD corrections were calculated for the first time in the nineties^{1,2}, while the next-to-next-to-leading order QCD corrections became available recently, including off-shell effects^{3,4}. The NLO EW corrections to the on-shell WZ production were first calculated in Refs.^{5,6} and the full NLO EW corrections to the process $pp \rightarrow 3\ell + \nu + X$ have been recently computed providing results for various kinematical distributions⁷.

Even if the initial proton beams at the LHC are unpolarised, the fundamental asymmetry in the Z and W boson couplings to left- and right-handed quarks means that the gauge bosons are produced in polarised states. This polarisation is reflected in angular asymmetries in the final-state-lepton distributions. First measurements of these polarisation observables at the 13 TeV LHC have been presented by ATLAS in 2019⁸.

We will present in the following polarisation observables built out from the polar-azimuthal angular distributions, that can be defined in the fiducial phase-space and that reflect the underlying spin structure of the W and Z bosons. These fiducial polarisation observables allow

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for direct comparisons with the experiment, without using template fitting. They will be calculated at NLO in QCD and also at NLO EW in the double-pole approximation. We will also show that the DPA for the EW corrections works remarkably well in this process and is a good approximation to perform the NLO calculation of polarisation. All the details of the analysis can be found in the original publication⁹.

2 EW corrections in the double-pole approximation

We calculate the cross section and the kinematical distributions, in particular the fiducial distribution $d\sigma/(\sigma d\cos\theta d\phi)$, at LO and NLO QCD using the full matrix elements. We use the VBFNLO program^{10,11} to generate the LO and NLO QCD results. The NLO EW corrections are obtained using the double pole approximation (DPA) in which the on-shell $W^\pm Z$ production and the on-shell decays $W \rightarrow e\nu_e$, $Z \rightarrow \mu^+\mu^-$ are combined. At LO, the partonic amplitude in the DPA for the process $ab \rightarrow e\nu_e\mu^+\mu^-$ reads

$$\mathcal{A}_{\text{LO, DPA}}^{ab \rightarrow WZ \rightarrow e\nu_e\mu^+\mu^-} = \sum_{\lambda_1, \lambda_2} \frac{\mathcal{A}_{\text{LO}}^{ab \rightarrow WZ} \mathcal{A}_{\text{LO}}^{W \rightarrow e\nu_e} \mathcal{A}_{\text{LO}}^{Z \rightarrow \mu^+\mu^-}}{(q_{W^*}^2 - M_W^2 + iM_W\Gamma_W)(q_{Z^*}^2 - M_Z^2 + iM_Z\Gamma_Z)}, \quad (1)$$

where q_{W^*} and q_{Z^*} are the intermediate off-shell W and Z momenta, λ_1 and λ_2 are the helicities of the intermediate on-shell W and Z bosons, respectively. At NLO EW we take over the corrections to the production part from Ref.⁶ and we calculate the NLO EW corrections to the decay parts. When compared to the full calculation of the EW corrections⁷, the virtual and real corrections of the production and of the decays parts are included, in particular the quark-photon induced $q\gamma \rightarrow W^\pm Z q' \rightarrow e\nu_e\mu^+\mu^- q'$ channel, while the non-factorisable contributions as well as the off-shell effects are neglected. A great advantage of the DPA is also the lesser time spent in the calculation, as we avoid calculating six-point integrals.

We display in Fig. 1 a comparison of our results in the DPA with the calculation of the full EW corrections, at the 13 TeV LHC using ATLAS fiducial cuts¹², using the p_T distribution of the positron in the W^+Z channel as an illustrative example. The agreement is very good, signalling that the DPA is an excellent approximation that is suitable for the calculation of the polarisation observables.

3 Fiducial polarisation observables

At LO in the DPA it is possible to parameterise the differential cross section as

$$\begin{aligned} \frac{1}{\sigma_{\text{LO}}^{\text{DPA}}} \frac{d\sigma_{\text{LO}}^{\text{DPA}}}{d\cos\theta d\phi} &= \frac{3}{16\pi} \left[(1 + \cos^2\theta) + A_0 \frac{1}{2} (1 - 3\cos^2\theta) + A_1 \sin(2\theta) \cos\phi \right. \\ &+ A_2 \frac{1}{2} \sin^2\theta \cos(2\phi) + A_3 \sin\theta \cos\phi + A_4 \cos\theta \\ &\left. + A_5 \sin^2\theta \sin(2\phi) + A_6 \sin(2\theta) \sin\phi + A_7 \sin\theta \sin\phi \right], \quad (2) \end{aligned}$$

where θ and ϕ are the angular variables of one of the final-state lepton in the gauge-boson rest frame, and A_i are called the angular coefficients. To study the W^\pm boson the electron (or positron) is taken, to study the Z boson we chose the μ^- lepton.

A polarised on-shell massive gauge boson can be described by a 3×3 spin-density matrix ρ_{ij} . This matrix is Hermitian and satisfies the normalisation condition of $\text{Tr}(\rho) = 1$, hence it can be parameterised by 8 real parameters that characterise the spin nature of the massive gauge boson. At LO in the DPA, the link between the angular coefficients and the spin-density matrix

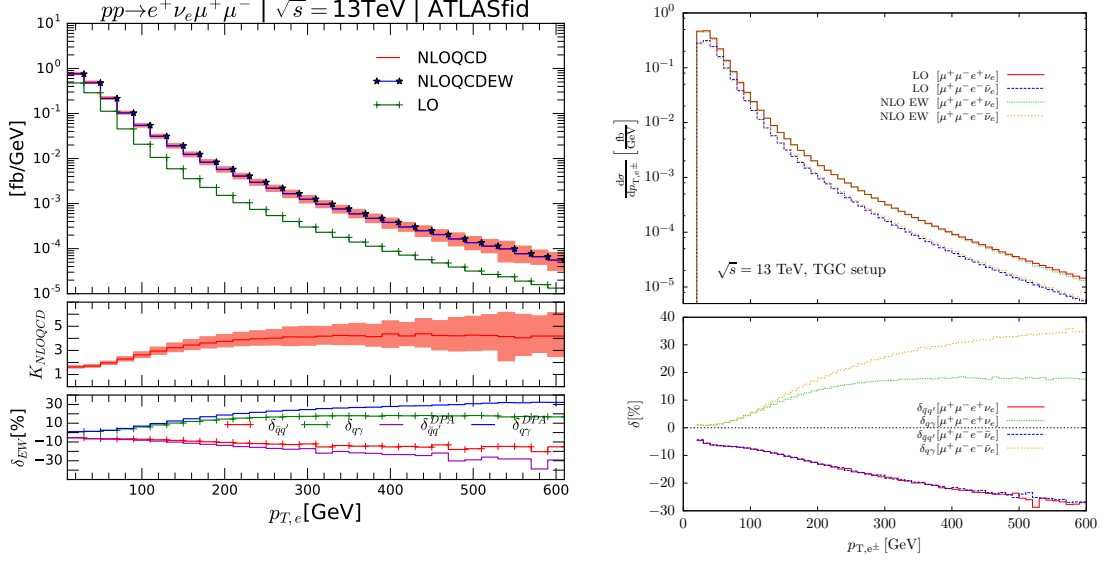


Figure 1 – Transverse momentum distributions of the positron in the process $pp \rightarrow e^+ \nu_e \mu^+ \mu^- + X$ at the 13 TeV LHC using ATLAS fiducial cuts. Left: Our calculation⁹ including full NLO QCD corrections only (in red) and their combination with EW corrections in the DPA (in blue) in the main panel, as well as the percent corrections of the EW corrections in the lower sub-panel. Right: The full NLO EW calculation taken from Ref. 7.

is given by^{13,14}

$$\begin{aligned}
A_0 &= 2\rho_{00}, \quad A_1 = \frac{1}{\sqrt{2}}(\rho_{+0} - \rho_{-0} + \rho_{0+} - \rho_{0-}), \\
A_2 &= 2(\rho_{+-} + \rho_{-+}), \quad A_3 = \sqrt{2}b(\rho_{+0} + \rho_{-0} + \rho_{0+} + \rho_{0-}), \\
A_4 &= 2b(\rho_{++} - \rho_{--}), \quad A_5 = \frac{1}{i}(\rho_{-+} - \rho_{+-}), \\
A_6 &= -\frac{1}{i\sqrt{2}}(\rho_{+0} + \rho_{-0} - \rho_{0+} - \rho_{0-}), \quad A_7 = \frac{\sqrt{2}b}{i}(\rho_{0+} - \rho_{0-} - \rho_{+0} + \rho_{-0}),
\end{aligned} \tag{3}$$

with $b = 1$ for the W^\pm bosons and $b = -c$ for the Z boson, and

$$c = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} = \frac{1 - 4s_W^2}{1 - 4s_W^2 + 8s_W^4}, \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \tag{4}$$

When looking at Eq. (3) it is expected that the coefficients $A_{5,6,7}$ are small as they are related to the imaginary part of the spin-density matrix. Furthermore, A_3^Z and A_4^Z are proportional to the coefficient c and are thus directly related to the left-right asymmetry in the decay $Z^* \rightarrow \mu^+ \mu^-$. It is possible to relate the angular coefficients to the angular projections of the two-dimensional distribution $d\sigma_{\text{LO}}^{\text{DPA}}/d\cos\theta d\phi$ ^{15,16},

$$\begin{aligned}
A_0 &= 4 - \langle 10 \cos^2 \theta \rangle, \quad A_1 = \langle 5 \sin 2\theta \cos \phi \rangle, \quad A_2 = \langle 10 \sin^2 \theta \cos 2\phi \rangle, \quad A_3 = \langle 4 \sin \theta \cos \phi \rangle, \\
A_4 &= \langle 4 \cos \theta \rangle, \quad A_5 = \langle 5 \sin^2 \theta \sin 2\phi \rangle, \quad A_6 = \langle 5 \sin 2\theta \sin \phi \rangle, \quad A_7 = \langle 4 \sin \theta \sin \phi \rangle,
\end{aligned} \tag{5}$$

with the following definition for angular projections

$$\langle g(\theta, \phi) \rangle = \int_{-1}^1 d\cos\theta \int_0^{2\pi} d\phi g(\theta, \phi) \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta d\phi}. \tag{6}$$

If off-shell and radiation effects as well as cuts on the individual leptons are considered, as in the full NLO calculation in the fiducial region, the parameterisation in Eq. (2) is not valid

anymore: It only contains information on the Z and W bosons and the eight coefficients A_i cannot reconstruct the full differential cross section anymore. The problem gets worse when considering the same-flavour final states of $\ell\nu_\ell\ell^+\ell^-$, where interference effects occur. Nevertheless it is always possible to use the two-dimensional distribution and the definitions given in Eq. (5). In this equation it is even possible to replace the cross section σ by a differential distribution such as $d\sigma/dp_{T,W/Z}$ ¹⁶. In this way we can define what we call fiducial polarisation observables that are totally equivalent to the inclusive polarisation observable in the DPA limit in the inclusive phase-space. The fiducial polarisation observables are calculated using the full matrix elements including off-shell, interference, and higher-order effects as well as arbitrary cuts on the individual final-state leptons, and they do contain the spin information of the underlying W and Z bosons. The fiducial polarisation observables are measurable and they do not require a template fit method, contrary to what is done in Ref.⁸.

It is also possible to define polarisation fractions as

$$f_L^V = -\frac{1}{2} + d\langle\cos\theta\rangle + \frac{5}{2}\langle\cos^2\theta\rangle, \quad f_R^V = -\frac{1}{2} - d\langle\cos\theta\rangle + \frac{5}{2}\langle\cos^2\theta\rangle, \quad f_0^V = 2 - 5\langle\cos^2\theta\rangle, \quad (7)$$

with $d = \mp 1$, $\theta = \theta_e$ for $V = W^\pm$ and $d = 1/c$, $\theta = \theta_{\mu^-}$ for $V = Z$. The fractions fulfil the relation $f_L + f_R + f_0 = 1$. Their value as well as the value of the angular coefficients A_i depend on the reference frame and on the coordinate system. Our study is made in the rest frame of the massive gauge boson under study, using either the helicity coordinate system (HE)¹⁵ in which the z -direction is aligned with the gauge boson momentum in the laboratory frame, or the Collins-Soper coordinate system (CS)¹⁷ in which the z -direction is the bisection of the momenta of the colliding protons in the gauge boson rest frame, see Ref.⁹ for more details.

4 Results

We now present the results for the angular coefficients for the W^+ and Z bosons at the 13 TeV LHC with the ATLAS fiducial cuts¹². The study in Ref.⁹ contains also results for the polarisation fractions and a study of the CMS cuts as well as a complete study for the W^-Z process.

Method	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
HE NLOQCD	1.016(1) $^{+3}_{-4}$	-0.326(2) $^{+2}_{-3}$	-1.413(2) $^{+10}_{-12}$	-0.229(1) $^{+2}_{-1}$	-0.295(7) $^{+11}_{-11}$	-0.001(1) $^{+0.1}_{-0.2}$	-0.0002(6) $^{+3}_{-2}$	0.003(1) $^{+1}_{-0.5}$
HE NLOQCDEW	1.017	-0.326	-1.420	-0.229	-0.287	-0.002	-0.002	0.007
CS NLOQCD	1.513(3) $^{+7}_{-7}$	0.192(1) $^{+2}_{-2}$	-0.918(3) $^{+2}_{-2}$	0.061(4) $^{+4}_{-4}$	-0.469(6) $^{+10}_{-10}$	-0.0001(11) $^{+0}_{-3}$	0.001(0.5) $^{+0.3}_{-0.2}$	-0.003(0.4) $^{+1}_{-1}$
CS NLOQCDEW	1.518	0.189	-0.921	0.065	-0.463	0.0004	0.003	-0.007

Table 1: (W boson) Fiducial angular coefficients of the e^+ distribution for the process $pp \rightarrow e^+\nu_e\mu^+\mu^- + X$ at the 13 TeV LHC with the ATLAS fiducial cuts. Results are presented in the helicity (HE) and Collins-Soper (CS) coordinate systems, at NLO QCD and NLO QCD+EW. The PDF uncertainties (in parenthesis) and the scale uncertainties are also provided on the last digit of the central prediction.

The angular coefficients of the W^+ boson A_i^W are presented in Table 1 and those of the Z boson A_i^Z are in Table 2. The results are given in both the HE and CS coordinate systems. As expected from the analysis in Section 3 for the inclusive polarisation observables (see Eq. (3)) the coefficients $A_{5,6,7}$ are very small and can be neglected in the SM. However, they could serve as a probe for new physics as any sizeable non-zero values for these coefficients constitute a striking signal for physics beyond the SM. The parton distribution function and the scale uncertainties are found to be very small in both coordinate systems and for both sets of angular coefficients, signalling an observable under very good theoretical control.

Method	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
HE NLOQCD	$0.985(2)_{-6}^{+5}$	$-0.306(1)_{-3}^{+4}$	$-0.734(1)_{-2}^{+2}$	$0.031(1)_{-2}^{+2}$	$0.003(1)_{-1}^{+1}$	$-0.004(1)_{-0.4}^{+0.3}$	$-0.004(1)_{-0.2}^{+0.3}$	$0.003(1)_{-0}^{+0.2}$
HE NLOQCDEW	0.986	-0.308	-0.742	0.023	0.001	-0.004	-0.004	0.003
CS NLOQCD	$1.267(2)_{-4}^{+4}$	$0.221(1)_{-1}^{+1}$	$-0.455(2)_{-2}^{+2}$	$-0.021(1)_{-3}^{+3}$	$0.023(1)_{-1}^{+1}$	$0.0004(6)_{-2}^{+2}$	$0.006(0.5)_{-0.4}^{+0.2}$	$-0.003(1)_{-0.1}^{+0}$
CS NLOQCDEW	1.273	0.218	-0.457	-0.016	0.016	0.001	0.007	-0.003

Table 2: Same as in Table 1 but for the (Z boson) fiducial angular coefficients of the μ^- distribution.

The NLO EW corrections are found negligible for all W coefficients in both coordinate systems, while they are significant for A_3^Z and A_4^Z . A more detailed study of the different DPA contributions to the EW corrections performed in the original study⁹ shows that these large EW corrections originate from the radiative corrections to the $Z \rightarrow \mu^- \mu^+$ decay, aligning with the remark in Section 3 about the proportionality of A_3^Z and A_4^Z to the left-right asymmetry in the $Z \rightarrow \mu^+ \mu^-$ decay.

We also display in Fig. 2 the $p_{T,W}$ distribution of the W^+ fiducial polarisation fractions calculated from Eq. (7), in the HE (left) and CS (right) coordinate systems. The two distributions have a very different behaviour: in the CS system the fraction f_R is negative at low $p_{T,W}$ and the longitudinal fraction f_L does not decrease at high $p_{T,W}$, while all fractions are positive in the HE coordinate system and the longitudinal fraction does decrease at high transverse momentum, in line with the equivalence theorem and closer to the behaviour of inclusive polarisation fractions. The same remark can be made for the Z polarisation observables⁹.

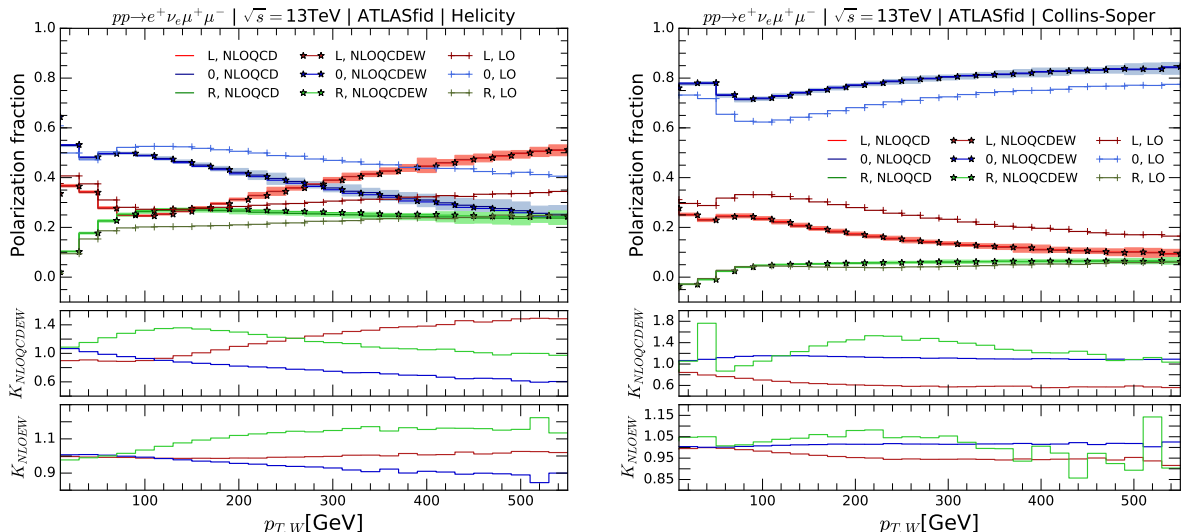


Figure 2 – Transverse momentum distributions of the W^+ boson fiducial polarisation fractions. The left-hand-side plot is for the helicity coordinate system, while the right-hand-side plot is for the Collins-Soper coordinate system. The bands include PDF and scale uncertainties calculated at NLO QCD using a linear summation. The K factors in the small panels are the ratios of the NLO results over the LO ones.

5 Conclusion

We have presented in this contribution our calculation of the W^\pm and Z bosons polarisation observables, inclusively and at the differential level, in the process $pp \rightarrow e^\pm \nu_e \mu^\pm \mu^\mp + X$ at the 13 TeV LHC including full NLO QCD corrections and NLO EW corrections calculated in the

double-pole approximation. This approximation has been found to be in a very good agreement with the calculation of the full EW corrections, while being much faster and easier to calculate.

We have proposed the study of fiducial polarisation observables that can be directly measured without using template fit methods and give information about the underlying spin structure of the W and Z bosons. The helicity coordinate system seems to be more suitable than the Collins-Soper coordinate system as the polarisation fractions are positive and decrease at high energies. EW corrections are found to be important especially for the coefficients A_3^Z and A_4^Z that are related to the Z boson radiative decay. We highly encourage the ATLAS and CMS experiment to perform the measurement of these fiducial observables.

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