BEAM TRANSFER FUNCTION AND STABILITY DIAGRAM IN THE LARGE HADRON COLLIDER*

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Abstract

Predictions of transverse instability thresholds in the LHC are based on the computation of Landau damping by calculating the Stability Diagram (SD). However any modification of the tune spread and/or particle distribution modifies the expected Landau damping in the beams. The Beam Transfer Function (BTF) provides direct measurements of the Landau damping and can be used to understand the limitations of the models.

INTRODUCTION

The instabilities driven by the beam coupling impedance can be mitigated by chromaticity, transverse feedback and/or Landau damping mechanisms. As long as the different modes of oscillations can be treated independently, the Landau damping is quantified by the dispersion integral for a given detuning with amplitude $q_{x,y}$ and particle distribution $\psi(J_x, J_y)$ as a function of the transverse actions J_x and J_y in each plane [1,2]

$$SD^{-1}_{(x,y)} = \frac{-1}{\Delta Q_{x,y}} = \int_0^\infty \frac{J_{x,y}}{Q_{x,y} - q_{x,y}(J_x, J_y)} \frac{d\psi}{dJ_{x,y}} dJ_x dJ_y, \quad (1)$$

where $\Delta Q_{x,y}$ are the complex tune shifts at the stability limit for each coherent tune $Q_{x,y}$. The transverse amplitude detuning (or tune spread) is generated in the beams by any non-linearities (including beam-beam interactions when beams are in collisions [3]).The dispersion integral is the inverse of the SD that defines the stability limit in the complex plane of the coherent tune shifts. In order to be stabilized, the coherent impedance modes must lie inside the SD. The BTF is proportional to the dispersion integral

$$BTF_{(x,y)} = A_{x,y} \int_0^\infty \frac{J_{x,y}}{Q_{x,y} - q_{x,y}(J_x, J_y)} \frac{d\psi}{dJ_{x,y}} dJ_x dJ_y \quad (2)$$

and therefore the BTF is proportional to the inverse of the SD

$$SD^{-1}_{(x,y)} = \frac{-1}{\Delta Q_{x,y}} = \frac{BTF_{(x,y)}}{A_{x,y}}. \quad (3)$$

The proportionality constant $A_{x,y}$ depends on the excitation amplitude of the BTF and on the beam conditions themselves. The BTFs are sensitive to the tune spread in the beams, as well as to particle distribution changes. In the LHC, the tune spread is mainly provided by the so-called

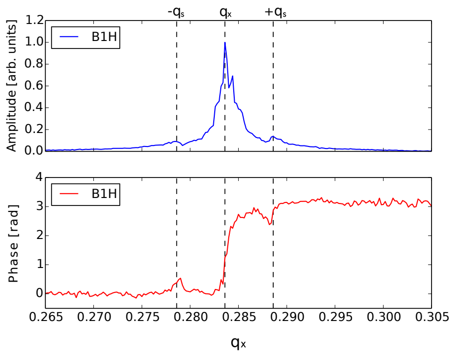

Figure 1: Example of a BTF measurement: amplitude and phase response for B1 in the horizontal plane at the LHC injection energy. Synchrotron sidebands are visible in the amplitude, with the corresponding phase jumps at $q_x \pm Q_s$ (where $Q_s = 5 \times 10^{-3}$).

Landau octupoles [2] and beam-beam interactions [3, 4] (long range and head-on) when present. In case of diffusive mechanisms and/or reduced dynamic aperture with particle losses or redistributions, the expected Landau damping for a Gaussian distribution of the beams is modified. The BTF provides measurements of Landau damping when in the presence of such effects. Therefore, in 2015 a transverse BTF system was installed in the LHC in order to measure the Landau damping of the beams and compare the measurements with the models that did not fully reproduce the instability observed [4]. For this purpose, a new method for the data analysis has been developed and applied to the measurements acquired in different configurations allowing quantitative comparisons with models.

THE LHC BTF SYSTEM

During a BTF acquisition the beam is excited (without causing beam losses or emittance blow-up) at a frequency close to the tune. The response of the beams itself is of course real, but the BTF is complex as we combine the amplitude and the phase separately [5]. An example of BTF measurement is shown in Fig. 1 for Beam 1 (B1) in the horizontal plane at injection energy. The blue line is the amplitude response while the red line is the phase response. The maximum amplitude corresponds to the fractional part of the horizontal betatron coherent tune ($q_x \approx 0.284$). At this frequency the phase assumes a value of $\pi/2$. The first synchrotron sidebands are also visible in the amplitude response, with the corresponding phase jumps, occurring at $q_x \pm q_s$, where $q_s = 5 \times 10^{-3}$ is the fractional longitudinal tune at injection energy. The synchrotron sidebands are

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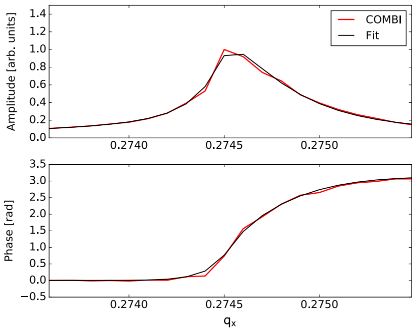


Figure 2: Amplitude and phase BTF responses simulated by using the COMBI code (red line) in the presence of linear detuning with amplitude. The black line represents the results of the parametric fit. The same tune spread has been applied for both cases and, as expected, the fit gives a tune spread factor close to 1 ($p_1 = 1.02$).

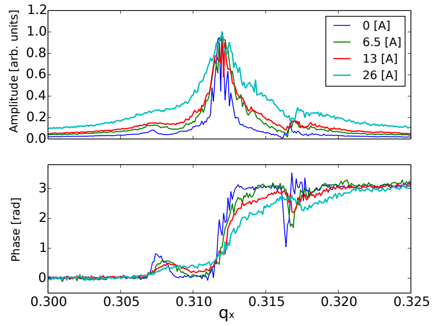
visible due to non-zero chromaticity $Q' \approx 5.0$ during the acquisition of the BTF measurements.

The fitting method

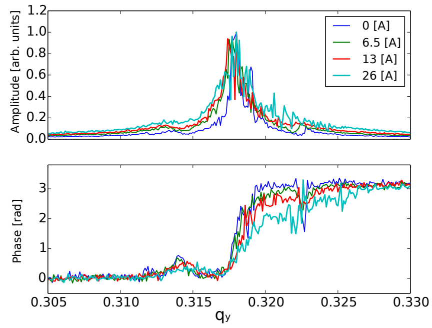
In order to quantitatively compare the BTF measurements with the expected Landau damping from the models, a fitting method has been developed. The amplitude of the beam oscillation driven by the BTF excitations is not calibrated, therefore, the amplitude response of the beam can be expressed in arbitrary units only. In addition, a full calibration of the system cannot be accomplished since the proportionality constant $A_{x,y}$ in Eq.(2) cannot be known a priori. Indeed, it depends not only on the calibration factor of the BTF excitation amplitude, but also on the tune spread introduced by the machine non-linearities. To overcome this problem, the following fitting method is applied to the BTF measurements. The fitting function takes as input the amplitude A_{model} and the phase φ_{model} of the computed dispersion integral with the PySSD code [3], by using the following parameterization

$$\begin{cases} \varphi(Q_{meas}) = \varphi_{model}\left[p_0 + p_1 \cdot (Q_{model} - Q_0)\right] \\ A(Q_{meas}) = p_2/p_1 \cdot A_{model}(Q_{model}) \end{cases} \tag{4}$$

where both A_{model} and φ_{model} depend on the Q_{model}. The parameter p_0 gives the tune shifts with respect to the frequency of the analytical detuning ($Q_{model} - Q_0$) with Q_0 the transverse coherent tune, p_1 is the factor related to the tune spread with respect to the expected one and it is called *tune spread factor*. For example, if the tune spread factor $p_1 > 1$ it means that the measured tune spread is bigger than the expected one. An example of the application of the fitting method to simulations for a well known case of linear detuning with amplitude the tune spread parameter $p_1 \approx 1$. This is shown in Fig.2 where the simulated BTF response (the red line) is compared to the fitting function (the black line).



(a) Horizontal beam response.



(b) Vertical beam response.

Figure 3: Measured BTF amplitude and phase responses for Beam 1 for different octupole currents at injection energy.

MEASUREMENTS ON SINGLE BEAM

Measurements of Landau damping with octupole magnets at injection energy

An octupole current scan has been performed at injection energy with collisions tunes (64.31, 59.32). The acquired BTF responses are presented in Fig. 3 for various Landau octupole currents (0 A, 6.5 A, 13 A, 26 A). The measured tune spread is therefore provided by the Landau octupole magnets and lattice non-linearities. As visible the tune spread increases as a function of the octupole current. However for the largest octupole strength (26 A) a larger tune spread is measured in the horizontal plane than in the vertical plane. The fitting method has been applied to compare the measurements with the expectations from the model. Figure 4 shows the results of the application of the fitting method. The measured tune spread factor is plotted as a function of the octupole current in the horizontal (blue dots) and in the vertical (green dots) planes. The solid black line represents the factors expected from the model with respect to a current $I_{oct} = 6.5$ A for which no asymmetry in the two transverse planes has been observed. As expected for such a current the tune spread factor of the model it is equal one. The red shadow is given by the model expectations including the initial non-zero tune spread corresponding to ≈ 5.5 A and considering an uncertainty of $\pm 10\%$ on the measured emittance. As expected the tune spread factor linearly increases as a function of the octupole current [2]. However in the ver-

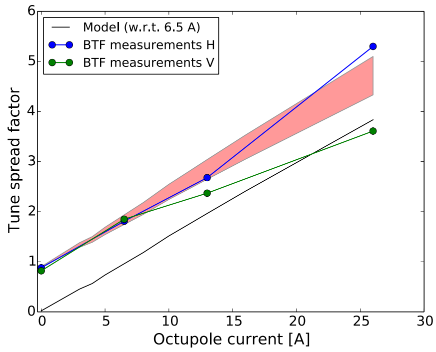


Figure 4: Measured tune spread factors as a function of the octupole current at injection energy in the horizontal (the blue dots) and in the vertical (the green dots) planes. The solid black line represents the factors expected from the model for a current $I_{oct} = 6.5\,$A. The red shadow represent the model expectations including the initial non-zero tune spread corresponding to $\approx 5.5\,$A with an uncertainty of $\pm 10\%$ on the measured emittance.

tical plane a deviation from the linear trend is observed for an octupole current of 26 A. Beam losses were observed for high octupole current at the moment of the data acquisition, due to a reduction of the Dynamic Aperture at injection with collisions tunes. The results of particle tracking simulations by using the SixTrack [6] code are presented in Fig. 5 where the surviving particle ratio is plotted as a function of the integral lower bound (l) used for the evaluation of the particle integral. The ratio is defined as the integral of the final distribution (H_{Fin}) over the integral of the initial distribution (H_{Ini})

$$R_{surv} = \frac{\int_l^{6\sigma} H_{Fin}(x,y)dxdy}{\int_l^{6\sigma} H_{Ini}(x,y)dxdy} \tag{5}$$

where l, expressed in units of transverse rms beam size σ, is the integral lower bound and varies from 0 to 5.5 σ. The upper bound of the particle distribution integral is fixed to 6 σ. As visible, particles are lost more and more from the

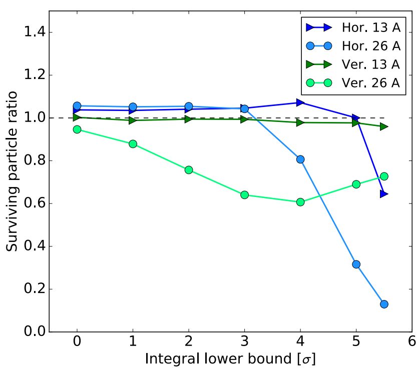
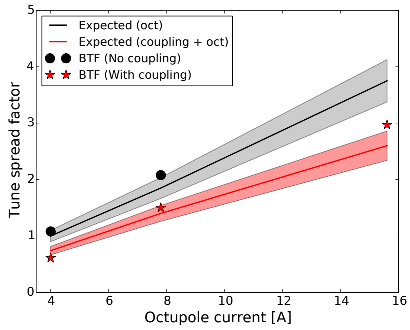
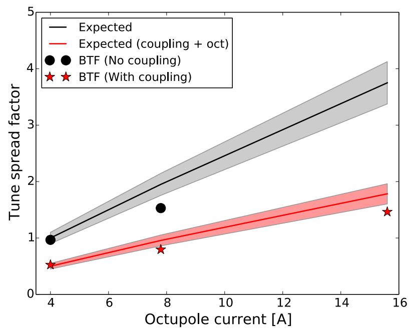


Figure 5: Surviving particle ratio as a function of the integral lower bound. The dark blue line with dark blue triangles and the light blue dotted line represent the horizontal plane for an octupole current of 13 A and 26 A respectively. The dark green line with dark green triangles and the light green dotted line represent the vertical plane for an octupole current of 13 A and 26 A respectively.



(a) Tune spread factors measured in the horizontal plane.



(b) Tune spread factors measured in the vertical plane.

Figure 6: Measured tune spread factors with respect to the analytical reference case of 4 A without linear coupling. The black dots are the measurements without linear coupling while the red stars are the measurements. The red and the black shadows represent the analytical expectations with an uncertainty of $\pm 10\%$ on the measured emittance for the case with and without linear coupling respectively.

tails to the core while increasing the octupole current. For the case with an octupole current of 26 A, a reduction up to the 40 % is observed in the core of the beam for the vertical plane (light green dotted line). This is in agreement with the reduction of the tune spread observed in the BTF measurements in the vertical plane. Indeed, the particles that mostly contribute to the BTF response are those in the beam core and not the ones in the tails [7]. If the particle lost are the ones in the core of the beam, a reduction of Landau damping is expected with respect to the unperturbed Gaussian distribution.

Measurements of Landau damping in the presence of linear coupling

In the presence of transverse linear coupling a reduction of the Landau damping is expected [8–10]. This has been measured by means of BTFs in the LHC. Measurements have been acquired at injection energy for global linear coupling coefficient (defined as the closest-tune approach) $|C^-| = 0.006$. The measurements were acquired for an octupole current of 4 A, 8 A and 15.6 A. The measured tune spread factors are shown in Fig. 6 as a function of the octupole current. The red line represents the model expectations including both octupoles and linear coupling in the MAD-X lattice [11], while the black line only includes octupoles. The red and the black shadows represent the model uncertainty of $\pm 10\%$ on the measured emittance value. The black dots are the measurements without coupling while the red stars are the measurements in the presence of linear coupling. As expected, an overall reduction of the tune spread is measured in both planes. However, the reduction is more important in the vertical plane for which the tune spread is reduced by a factor 2 with respect to the horizontal plane for the largest octupole current of 15.6 A value. The experimental data reproduce well the expectations for both cases.

Measurements of impedance tune shifts with BTFs

During a dedicated BTF MD in the 2018, measurements were acquired at flat top energy (6.5 TeV). The measurements were acquired after the correction of the linear coupling in the LHC for an octupole current of 546 A and a chromaticity of 2.5 units in order to minimize the synchrotron sidebands in the BTF response. An example of the acquired BTF response at flat top energy is shown in Fig. 7 where various horizontal BTF measurements (different colors) for Beam 1 were acquired. The solid black dashed lines corresponds to synchrotron sidebands at $q_{x0} \pm q_s$ due to the non zero chromaticity during the measurements, with q_{x0} the incoherent bare tune in the horizontal plane. As visible, the measured coherent tune is shifted with respect to the incoherent bare tune with a coherent tune shift $\Delta Q_{coh} \approx -3.4 \times 10^{-4}$. This observation suggested that the impedance was not negligible and it was modifying the BTF response [5, 12, 13]. This was also confirmed by the application of the fitting method. By including only the octupole magnets in the model the fitting function was not reproducing the measurements meaning that other effects need to be considered. In order to study the impact of the impedance in the BTF response, simulations using the COMBI code [14] have been carried out. Figure 8 shows the coherent tune shifts, as a function of the bunch intensity, evaluated from the simulated BTF response including the 2018 wake field model [15] (the light blue line) and the wake field evaluated from the collimator settings as during the MD (the orange line). As visible, in order to reproduce the observed tune shift in the measured BTF ($\Delta Q_{coh} \approx -3.4 \times 10^{-4}$) at flat top energy, one has to rescale the bunch intensity to 1.2×10^{11} p/bunch (the bunch intensity during the experiment was $\approx 0.8 \times 10^{11}$ p/bunch). This translates into a factor 1.5 on the impedance with respect to measurements. This value is in agreement with independent impedance measurements in the LHC for the horizontal plane of Beam 1 [16]. A first attempt to directly compare the measured BTF response to simulations is shown in Fig. 9 where the measured BTF response at flat top energy (red line) is compared to the simulated BTF response (the light blue line) including in the model the impedance and the Landau octupoles powered with a current of 546 A (as during the measurements). For completeness the analytical case without impedance is also plotted (black line) in the same picture. A factor 1.5 stronger impedance has been used as measured in the LHC.

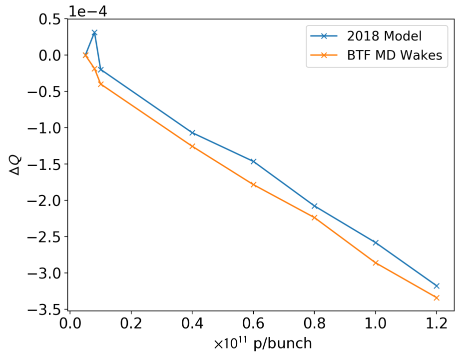


Figure 8: Coherent tune shifts as a function of bunch intensity. The tune shifts have been evaluated from the simulated BTF response including the 2018 wake field model (light blue line) and the wake field evaluated from the collimator settings as during the MD (orange line).

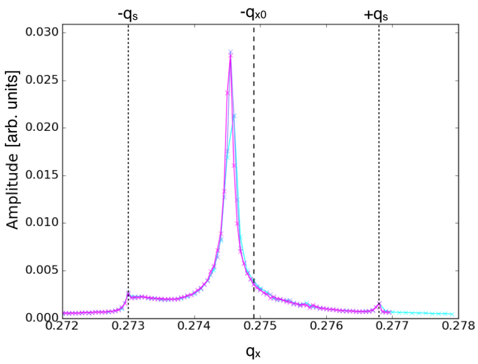


Figure 7: Various horizontal BTF acquisitions for Beam 1 at flat top energy. The chromaticity was reduced to 2.5 units during the measurements. The black dashed lines corresponds to synchrotron sidebands at $q_{x0} \pm q_s$ with q_{x0} the incoherent bare tune in the horizontal plane.

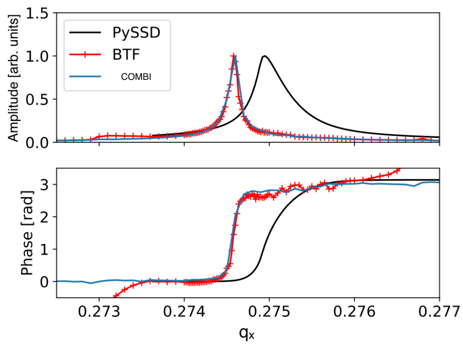


Figure 9: Measured BTF response at flat top energy (red line) compared to simulated BTF response (the blue line) including in the model the impedance and an octupole current of 546 A (as during the measurements). The analytical case without impedance is the black line.

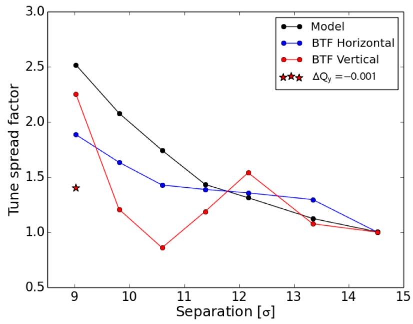


Figure 10: Measured tune spread factor as a function of the long range encounter separation at the interaction points in units of the rms beam size. Measurements were acquired at the end of the betatron squeeze. The red star represents the measured tune spread in the vertical plane with a reduced tune of $\Delta Q_y = -0.001$.

As visible the coherent tune shift is also fully reproduced. The shape of the measured BTF is fully recovered for a chromaticity of 1.0 units (the light blue line).

MEASUREMENTS IN THE PRESENCE OF BEAM-BEAM INTERACTION

The LHC beams collide in the interaction points (IPs) with a certain crossing angle to avoid multiple head-on collisions causing the so-called beam-beam long range interactions. A crossing angle scan was performed at the end of the betatron squeeze (with positive octupole polarity) in order to measure the modifications of Landau damping due to the beam-beam long range interactions. The fitting method has been applied to the BTF measurements with respect to the case with nominal crossing angle and the measured tune spread factor is plotted as a function of the long range separation at the first encounter (in units of the transverse rms beam size) in Fig.10. An unexpected behavior was found with respect to models (the black line): a larger tune spread was measured in the horizontal plane (the blue line), but still smaller than prediction below 11.5 σ, while a smaller one in the vertical plane (the red line) except for the measurement at 12 σ and for the last measurement at 9 σ separation for which a strong dependency on the working point was observed. The red star represents the measured tune spread in the vertical plane with a reduced tune of $\Delta Q_y = -0.001$. The tune spread reduction was not expected from the models unless the transverse linear coupling is considered. A parallel separation scan (from 0 to 6 σ in units of transverse beam size) was performed with beams in head-on collisions. The BTF measurements were acquired as a function of the parallel separation at the IPs. Figure 11 shows the measured BTF amplitude responses in the presence of head-on collisions in IP1 and IP5 for various beam offsets at the IPs. As visibile in fully head-on collisions the amplitude response is wider, meaning that the tune spread is the largest one, while at 1.45 σ the amplitude response is qualitatively the narrowest. Therefore, the tune spread results to be reduced, confirming the presence of a minimum of Landau damping

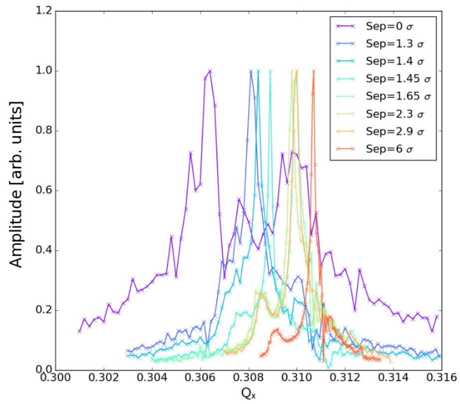


Figure 11: Measured BTF amplitude responses with head-on collisions in IP1 and IP5 for different offsets at the IPs.

at this separation as expected [3, 4]. Tune shifts due to the head-on interaction were also observed while separating the beams. The measured tune shifts were compared to MAD-X expectations [17] with a good agreement for separations below 2 σ.

CONCLUSIONS

The transverse BTF system was installed in the LHC in order to measure the Landau damping of the beams. A fitting method was successfully applied to the data for quantitative comparison with expectations. The effects of the linear coupling resonance on the Landau damping of the beams was measured at injection energy with a good agreement with models. It was observed that beam losses, due to a reduced dynamic aperture at injection energy for high octupole current, reduce the expected Landau damping of the beams. In the presence of long range beam-beam interaction unexpected behaviors were observed showing that other mechanisms should play a role such as linear coupling and/or particle redistributions in the beams [7]. The minimum of Landau damping expected at 1.5 σ beam to beam separation at the IP was observed in the width of the BTF amplitude response, confirming the presence of a minimum of Landau damping during the collapse of the separation bumps at such separation. The tune shifts due to the coupling impedance were quantified confirming a factor 1.5 on the effective imaginary part of the impedance expectations. The BTF measurements provided a good reconstruction of Landau damping especially at injection energy and for low chromaticity values helping to understand mechanisms responsible for reduction of the expected Landau damping for instance due to beam particle losses or linear coupling. However the BTF system and the fitting method present some limitations: when the chromaticity is not negligible it causes distortion in the reconstruction of the SD. It is not possible to apply the fitting method either when the impedance is too strong or when the beam is too close to the stability limit, the BTF excitation may cause coherent instability as observed in the 2017 due to an increase of the impedance [18].

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REFERENCES

[1] J. Berg, F. Ruggero, "Landau Damping with two-dimensional betatron tune spread", CERN, Geneva, Switzerland, CERN-SL-96-071-AP, Dec. 1996.

[2] J. Gareyte *et al.*, "Landau Damping, Dynamic Aperture and Octupoles in LHC", CERN, Geneva, Switzerland, LHC Project Report 91, Apr. 1997.

[3] X. Buffat *et al.*, "Stability Diagrams of colliding beams", Phys. Rev. ST Accel. Beams 17 111002, 2014.

[4] X. Buffat, Transverse beams stability studies at the Large Hadron collider, École Polytechnique Fédérale de Lausanne, CERN-THESIS-2014-246, September 2014.

[5] D. Mohl and A. M. Sessler, The use of the RF-knockout for determination of the characteristics of the transverse coherent instability of an intense beam, in Proceedings of The 8th International Conference on High-Energy Accelerators, p.334, Lawrence Berkeley Laboratory, University of California, Berkeley, USA, 1971 Accelerators, HEACC 1971: CERN, Geneva, Switzerland, September 20–24, 1971.

[6] SixTrack, `http://sixtrack.web.cern.ch/SixTrack/`.

[7] C. Tambasco, Beam Transfer Function measurements and transverse beam stability studies for the Large Hadron Collider and its High Luminosity upgrade, École Polytechnique Fédérale de Lausanne, EPFL-THESIS-7867, August 2017.

[8] L. R. Carver *et al.*, "Transverse beam instabilities in the presence of linear coupling in the Large Hadron Collider", Phys. Rev. Accel. Beams 21 044401, 2018.

[9] E. Métral, "Coupled Landau damping of transverse coherent instabilities in particle accelerators", Université Joseph Fourrier, CERN-THESIS-1999-074, 1999.

[10] E. Métral *et al.*, "Destabilising Effect of Linear Coupling in the HERA Proton Ring", CERN/PS 2002-021 (AE), CERN, Geneva, Switzerland.

[11] MAD-X, `http://mad.web.cern.ch/mad/`

[12] A. Hoffman and B. Zotter, Measurements of beam stability and coupling impedance by RF excitation,CERN ISR-TH/77-12, Geneva, Switzerland, 1977

[13] A. W. Chao, Physics of collective beam instabilities in high-energy accelerators, Wiley (1993) 371 p, New York, USA, 1993.

[14] T. Pieloni and W. Herr, "Coherent beam-beam modes in the Cern Large Hadron Collider (LHC) for multiple bunches, different collision schemes and machine symmetries", in *Proc. PAC'2005*, Knoxville, Tennessee, Oct. 2011, paper TUP014, pp. 89–91.

[15] `http://impedance.web.cern.ch/LHC_impedance/LHC_summary_2018.htm`

[16] D. Amorim *et al.*, Comparison of LHC impedance model predictions to beam based measurements, 29th Impedance Working Group, February 29$^{\text{th}}$ 2019, `https://indico.cern.ch/event/797962`, CERN, Geneva, Switzerland.

[17] C. Tambasco *et al.*, "MD 3292: Measurements of Landau damping by means of BTF measurements", CERN-ACC-NOTE-2019-0045, 22-10-2019.

[18] C. Tambasco *et al.*, Triggering of instability by BTF measurements, LBOC Meeting 27/03/2018, Geneva, Switzerland, `https://indico.cern.ch/event/715467/`.