TMCI, WHY IS THE HORIZONTAL PLANE SO DIFFERENT FROM THE VERTICAL ONE ?

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Abstract

Based on the recent work of R. Lindberg on transverse collective instabilities [1] it was observed that if the ratio of quadrupolar to dipolar impedance ρ is equal to -1 there is no TMC-instability. This relationship is actually fulfilled by resistive wall (RW)-impedance on the horizontal plane in case of a flat vacuum chamber. HEADTAIL [2]-simulations were carried out to check if this observation can be confirmed. Additionally the effect of radial modes on the TMC-instability was studied.

INTRODUCTION

The motion of particles in a single bunch can be described by the Vlasov equation as it was found by [3]. The linearisation of the Vlasov equation was solved by several authors [4–7]. In particular under the effect of dipolar impedance the transverse motion of particles of a bunch was described by [13] by decomposition into azimuthal and radial modes. In the meantime it was found that quadrupolar impedance is significant in many synchrotrons and has a sensible effect on the transverse motion [8–10]. Shortly after the discovery of its importance its effect was just superimposed on the dipolar mode detuning. However, R. Lindberg showed [1] that its effect has to be fully included into the dynamics of the bunch motion. Therefore the main purpose of this work is to demonstrate the difference between the Lindberg's description and the more naive descriptions in the past [11]. Whereas on the vertical plane the naive superposition of the dipolar and quadrupolar detuning corresponds quite well to Lindberg's result this is no longer true for the horizontal plane: a naive superposition of the mode detuning caused by dipolar and quadrupolar impedance indeed leads to a zero slope of mode 0 as expected, but mode 0 would still couple with mode -1, but in Lindberg's description the coupling is not compulsory. In order to support this observation HEADTAIL simulations were applied.

SUMMARY OF LINDBERG'S MODE EVOLUTION THEORY

In [1] the Vlasov equation is linearized with the Planck-Fokker terms included but truncated to a matrix equation. In the following it is assumed that the TMCI is strong enough for the disregard of the Planck-Fokker terms. This leads to the following equation:

$$\Delta\omega^m a_p^m + \sum_{n,q}(D+Q)_{p,q}^{m,n} a_q^n = 0$$

with $\Delta\omega^m = \Delta\Omega + m\omega_s$ (with ω_s as synchrotron ang. frequency), with dipolar and quadrupolar matrix elements (with $C_{p,q}^{m,n} = \sqrt{(p+|m|)!(q+|n|)!}$, $\epsilon_m = (-1)^{m(1-\delta_{|m|}^m)}$, $\Lambda = \frac{I}{4\pi(E/e)}$ as intensity parameter, and σ_τ as bunch length):

$$D_{p,q}^{m,n} = \frac{\Lambda i^{m-n+1}\epsilon_m\epsilon_n}{\sqrt{p!q!}C_{p,q}^{m,n}}\int_{-\infty}^{\infty} Z_D^\beta(\omega) e^{-(\omega\sigma_\tau)^2}\left[\frac{\omega\sigma_\tau}{\sqrt{2}}\right]^{2p+|m|}\left[\frac{\omega\sigma_\tau}{\sqrt{2}}\right]^{2q+|n|} d\omega \tag{1}$$

with $Z_D^\beta(\omega)$ as β-weighted dipolar impedance and

$$Q_{p,q}^{m,n} = \frac{\Lambda i^{m-n+1}\sqrt{p!q!}}{C_{p,q}^{m,n}}\int_{-\infty}^{\infty} Z_Q^\beta(\omega) e^{\frac{-(\omega\sigma_\tau)^2}{2}} I_{p,q}^{m,n}(\omega) d\omega$$

with $Z_Q^\beta(\omega)$ as β-weighted quadrupolar impedance and the following abreviation:

$$I_{p,q}^{m,n}(\omega) = \int_0^\infty dr e^{-r} r^{\frac{-(|n|+|m|)}{2}} J_{m-n}(\omega\sigma_\tau\sqrt{2r}) L_p^{|m|}(r) L_q^{|n|}(r)$$

with $L_p^n(x)$ as general Laguerre-polynomials.[1] This formalism can be applied to any type of transverse impedance. In the first part we will focus on RW-impedance as it fulfills the requirement $\rho = -1$ in case of horizontal RW-impedance of a horizontally flat parallel-plate beam pipe geometry which is at least approximately very common in many synchrotrons. The parameters used in the simulations can be looked up in table 1.

Table 1: simulation parameters used

parameter	value	unit
E/e	3	GV
ω_s	59.39	kHz
σ_τ	0.0154	ns
$\kappa_\perp^{RW}$	4-11	$\frac{kV}{pC}$
f^{bbr}	1.5, 3, 5	GHz
Q^{bbr}	2.3	
$\beta_\perp \cdot R_\perp^{bbr}$	52.9, 100, 41.1, 17.3	MΩ

RW-impedance

In order to study the evolution of the headtail-modes, the linearized and truncated Vlasov-equation is solved for a 2-mode system of two modes m=-1 and m=0 with radial mode number $r = 0$ (Higher radial modes are only discussed in the conclusions.). This was already done in the past by MOSES [13] for a pure dipolar impedance. In order to include the quadrupolar impedance the detuning slope related to it was added to the dipolar mode detuning computed

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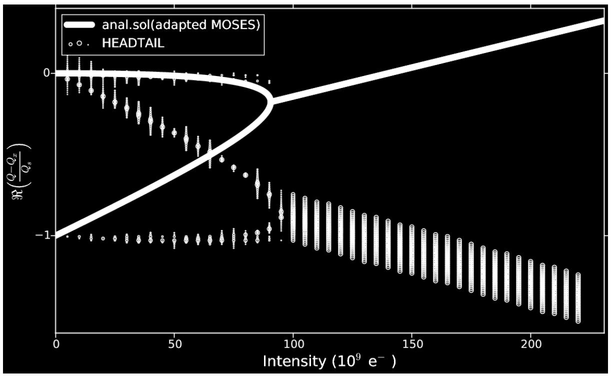

Figure 1: Imposing quadrupolar tune shift on the horizontal mode detuning from dipolar RW-impedance: For $\rho = -1$ mode 0 detuning is $\Delta\Omega \approx 0$, whereas the TMCI threshold is maintained.

by MOSES (this procedure is called adapted MOSES). All modes were correspondingly shifted, but the onset of the TMC-instability did not change (Fig. 1). So in case of horizontal impedance generated in a flat parallel-plate like beam pipe geometry the dipolar detuning of mode $m = 0$ was compensated by the quadrupolar detuning resulting in zero detuning of the mode. It seemed that mode m=0 still hit an instability if it met the mode m=-1, now with a strong positive slope. This was supported by measurements at the ESRF [11]. It had also the advantage that the measured threshold current allowed an estimation of the effective horizontal impedance even in the case of the zero slope. The mode evolution can be found from Vlasov's equation by the solution of the secular equation here demonstrated for the 2-mode system:

$$\begin{vmatrix} \Delta\Omega + \overbrace{A_H + A_Q}^{=0} & \alpha A \\ -\alpha A & \Delta\Omega + \omega_s + \frac{\tilde{\beta}}{2} A_H + A_Q \end{vmatrix} = 0 \qquad (2)$$

where $A \equiv A_H = -A_Q = \frac{I}{2(E/e)}\beta_\perp\kappa_\perp$ and $\alpha = \frac{\Gamma(3/4)}{\Gamma(1/4)\sqrt{2}}$ the coupling parameter and $\tilde{\beta} = 1/4$. $\kappa_\perp$ is the horizontal dipolar respectively quadrupolar RW-impedance's kick factor of the beam pipe. To account for the quadrupolar detuning, the term A_Q was introduced which is the same as the dipolar detuning A_H apart from the sign. Including A_Q does not change the threshold which can be found by solving the secular equation and searching for the detuning $\Delta\Omega$ where it becomes complex. But this description was obviously not complete as HEADTAIL-simulations cannot reproduce this behaviour (Fig. 1). If, however, for the consideration of the quadrupolar impedance Lindberg's formalism is used the secular equation for the 2-mode system looks differently:

$$\begin{vmatrix} \Delta\Omega + \overbrace{A(1+\rho)}^{=0} & \alpha A(1-\rho) \\ -\alpha \overbrace{A(1+\rho)}^{=0} & \Delta\Omega + \omega_s + A\frac{1+7\rho}{8} \end{vmatrix} = 0 \qquad (3)$$

As one of the off-diagonal terms cancels out the coupling disappears. Both modes still approach and meet, but do not couple, they just pass through each other (Fig. 2). This is qualitatively an important change. A couple of questions pop up: Will there be no TMCI-threshold anymore on the horizontal plane ? How will it be possible to estimate the effective horizontal impedance from single-bunch detuning? Some answers can be found in the next section.

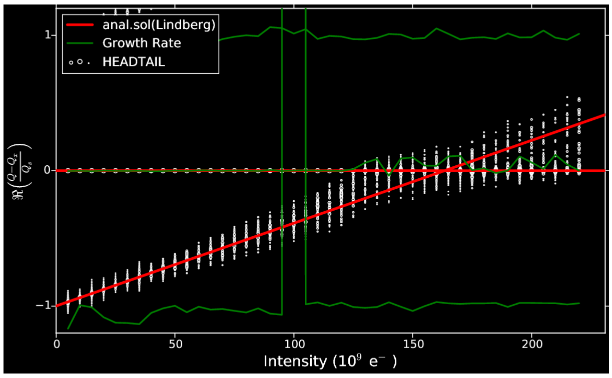

Figure 2: Applying Lindberg's full theory on horizontal RW-impedance leads to very good agreement with HEADTAIL. The growth rate (green) is not excited at the meeting point of the modes.

BBR-impedance

In case of Broad Band Resonator (BBR) impedance (with $(R_\perp, Q, \omega_r)$ and $Q' = \sqrt{Q^2 - 0.25}$) the quadrupolar impedance is also of importance when the cross section changing beam pipes generating it are not circular. It will be shown that the modes principally behave the same as they do in case of horizontal RW-impedance if the rule $\rho = -1$ is imposed. So initially the spectral distribution of dipolar and quadrupolar impedance are assumed to be the same in order to demonstrate that qualitatively there is no difference to RW-impedance (Fig. 3). The secular equation for this case turns out to be very similar:

$$\begin{vmatrix} \Delta\Omega + \overbrace{B(1+\rho)}^{=0} & \alpha B(1-\rho) \\ -\alpha \overbrace{B(1+\rho)}^{=0} & \Delta\Omega + \omega_s + \rho B + B\frac{\tilde{\beta}}{2}(1-\rho) \end{vmatrix} = 0 \qquad (4)$$

where $\alpha = \frac{Re[sw(s)]}{\sqrt{2}Im[w(s)]}$, $\tilde{\beta} = \frac{Im[s^2 w(s)]}{Im[w(s)]} + \frac{\omega_r\sigma_\tau Q'}{Im[w(s)]Q\sqrt{\pi}}$ with $s = \frac{\omega_r\sigma_\tau}{Q}\left(\frac{i}{2} - Q'\right)$ and $w(s) = \frac{i}{\pi}\int_{-\infty}^{\infty} dt \frac{e^{-t^2}}{s+t}$. Finally $B = \frac{I}{2(E/e)}\beta_\perp\kappa_\perp$.

[1] Actually we stick to the mode expansion of [12].

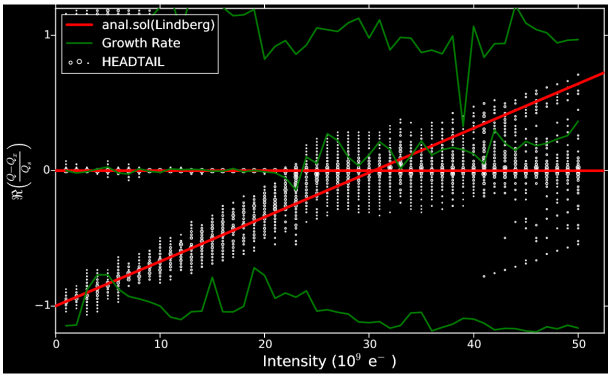

Figure 3: Modes under the effect on the common dipolar and quadrupolar BBR-impedance essentially show the same behaviour as in the RW-impedance case [14].

Before we discuss more involved cases for completeness the vertical mode detuning will be touched upon. As for vertical impedance $\rho \sim 0.5$, we are far away from the intriguing case $\rho = -1$. So in Lindberg's theory the 2 azimuthal modes m=-1 and m=0 couple (Fig. 4) as they do in adapted MOSES including the naively superimposed quadrupolar impedance. However, the threshold current can be different. But the difference between adapted MOSES (with naive superimposed quadrupolar detuning) and Lindberg's theory is rather small and above all does not go necessarily in the desired direction. So at this level of study Lindberg's theory does not give an explanation for the notorious failure of matching the measured vertical impedance in single bunch with the computed one found in electron synchrotrons [15]. The picture becomes a bit more complicated for higher frequency and with the consideration of radial modes, but this is out of scope of this work.

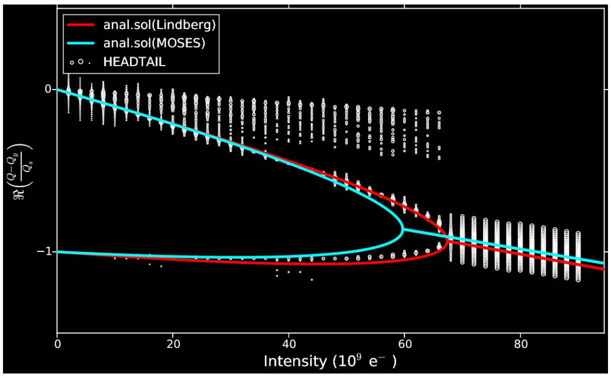

Figure 4: A low-frequency BBR-impedance HEADTAIL simulation agrees well with 2-mode case of Lindberg's theory (red), even better than with the adapted MOSES(cyan). For higher frequency the radial modes have to be considered which change the picture slightly.

Finally we assume that the spectral distribution of the dipolar impedance is different from the quadrupolar one since it is much more realistic but with still agreeing the kick factors. This is actually easy to achieve as BBR-impedance is described by 3 parameters to play with. So instead of requiring $\rho = \frac{Z_Q(\omega)}{Z_D(\omega)} = -1$ we only require

$$\rho = \frac{Z_Q^{eff}(\omega)}{Z_D^{eff}(\omega)} = -1 \tag{5}$$

We keep on studying the horizontal plane. In this case (at least) 2 BBR-models (here indexed with H for horizontal and Q for quadrupolar) are needed, one for the dipolar part and another one for the quadrupolar part. Including both in the formalism the secular equation for the eigenvalues amounts to ($B :\equiv B_H = -B_Q$):

$$\begin{vmatrix} \Delta\Omega + \overbrace{B_H + B_Q}^{=0} & B(\alpha_H + \alpha_Q) \\ B(-\alpha_H + \alpha_Q) & \Delta\Omega + \omega_s + B_Q + B_H\frac{\tilde{\beta}_H}{2} + B_Q\frac{\tilde{\beta}_Q}{2} \end{vmatrix} = 0 \tag{6}$$

It yields essentially two different cases, one where $\alpha_Q - \alpha_H < 0$ and the other one where $\alpha_Q - \alpha_H > 0$. In the

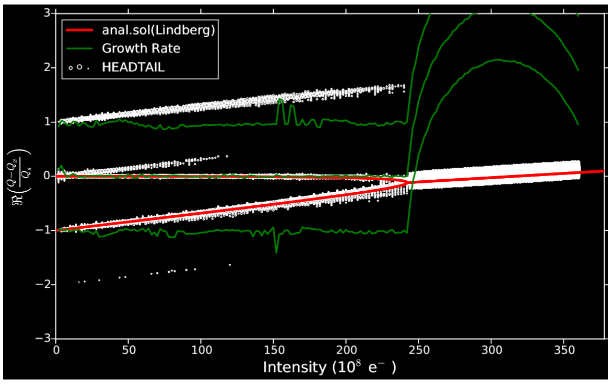

Figure 5: If the same effective dipolar and quadrupolar BBR-impedance of opposite sign but with different resonance frequencies are assumed modes can still couple as in this example ($\alpha_H > \alpha_Q$) confirmed by HEADTAIL modes (white) and their growth rate (green).

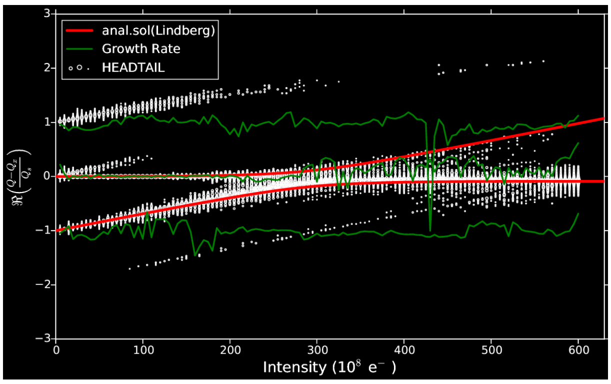

Figure 6: Essentially the same case as in the precedent figure, but different coupling coefficients of dipolar and quadrupolar BBR-impedance. Mode coupling no longer occurs as confirmed by the low growth rate (green).

first case $\alpha_Q < \alpha_H$, there is coupling (Fig. 5), whereas in the second case $\alpha_H < \alpha_Q$ (Fig. 6), there is no coupling anymore. One of most important consequences is that it is indeed possible that mode 0 and -1 meet without coupling. This seems also to be possible in more complex impedance models.

CONCLUSIONS

Now impedance budgeting on the horizontal plane is rather different from the vertical plane. It cannot be relied upon the horizontal threshold anymore for the measurement of the effective impedance. There might be even no horizontal threshold at all.

Even the threshold on the vertical plane changes with respect to the results of MOSES. However, the change is much smaller than on the horizontal plane.

In this report only examples with low BB-resonance frequency are studied. At higher frequency there might be deviations between Lindberg's mode theory and HEADTAIL.

In the future the 2-mode example will be extended to larger number of modes including also radial modes. It was already observed that higher radial modes of m=-1 do not couple with m=0 in the pure case $\rho = -1$.

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