Chapter 1
Accelerators, Colliders and Their Application

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1.1 Why Build Accelerators?

Accelerators are modern, high precision tools with applications in a broad spectrum that ranges from material treatment, isotope production for nuclear physics and medicine, probe analysis in industry and research, to the production of high energy particle beams in physics and astronomy. At present about 35,000 accelerators exist world-wide, the majority of them being used for industrial and medical applications. Originally however the design of accelerators arose from the request in basic physics research, namely to study the basic constituents of matter.

The first accelerators were inspired by the early experiments in nuclear physics. In the early years of the twentieth century Rutherford discovered that by using alpha particles from radioactive disintegration and detecting the pattern of particles scattered by atoms one might deduce that the nucleus was a tiny but massive central element in the atom. Alpha particles from disintegration can only be of energies of 10 MeV; comparable with the nuclear binding forces. Higher energies were needed and a more reliable and steady supply to ease the tedium of counting occasional flashes of light on the scintillation screen that was Rutherford's detector. De Broglie had shown that there was an inverse relationship between the momentum, and hence the energy of a particle and the wavelength of its representation in quantum mechanics.

$$\lambda = \frac{h}{p}$$

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where h represents Planck's constant and p the particles' momentum which relates to its energy via the well-known equation of special relativity, $E^2 = p^2c^2 + m^2c^4$. The limit to the scale of detail that experiments can reveal is set by the length of the wave which is scattered: rather as the wave breaking in the beach can only be deflected by islands larger than itself. It was argued correctly that higher energy particle, having the property of shorter wavelengths could better reveal the structure of the nuclei that Rutherford has detected.

Such arguments led to the invention of the first accelerators and have sustained the development of particle accelerators of higher and higher energy over the best part of the last 100 years. At first, physicists used accelerators to probe the structure of the nucleus, but went on to use higher energy accelerators to search for structure in the "fundamental" particles—protons, neutrons and electrons they discovered. Inevitably higher energies implied larger accelerators, for it was quickly discovered that the best way to accelerate repetitively was to keep particle in a circular path whose radius was itself proportional to energy, limited by the strength of the magnetic field one might use to do the bending.

As energies were raised physicists found new and interesting particles to fit into the pattern of those that their theories might predict. Einstein's

$$E = mc^2$$

tells us that only high energies will create the more massive particles. The latest and largest accelerator, LHC, flagship of the whole community, was designed to search for the Higgs Boson and the successful discovery of this missing puzzle piece in 2013 allowed us to complete the Standard Model of Particle Physics.

As we write, this machine is carrying on the search for physics beyond the standard model, seeking to disclose the nature of dark matter and dark energy.

As more powerful accelerators have been developed for high energy particle physics, advances in the field have been exploited in a whole range of smaller accelerators for other applications. From the time of the first cyclotrons they have been used for producing isotopes and for treating cancer. The development of compact high-frequency linac structures triggered the manufacture of hundreds of small electron linacs producing X-rays for cancer treatment in hospitals around the developed and, latterly, the developing world. Electron rings of a few GeV, specially designed to produce beams of synchrotron radiation have become popular. Each facility serves scores of experiments to investigate the structure of complex molecules—particularly the proteins of today's biomedical studies. Proton accelerators of about 1 GeV produce pulsed beams of neutrons by spallation which are used principally to study the structure of materials. In addition thousands of lower energy accelerators are used in industry for sterilisation and ion implantation in the fabrication of sophisticated CPU chips for computers.

1.2 Types and Evolution of Accelerators

The development of accelerators to ever higher energy is marked by a number of milestones. Each of these marks the invention of a new type of accelerator or the invention of a new principle of transverse or longitudinal focusing which enables a higher energy to be reached for a lower unit cost. The best way to describe this evolution and introduce the different types of accelerator is to follow the road charted by these milestones. Each is described in one of the sections which follow.

1.2.1 Early Accelerators

The nineteenth century had produced a number of electrostatic high-voltage generators. They were unpredictable in performance and electrical breakdown became a serious problem above a few tens of kV. Early accelerators were simply two electrodes enclosed in an evacuated tube with external connections to such high voltage source. A proton or electron source close to one electrode at a potential of V (or $-V$ for electrons) provided the particles which were then accelerated towards the second electrode at earth potential. They emerged or were observed through a small hole in the earthed electrode. The energy acquired by each particle with charge, e Coulombs, was just $e_{*}V$ Joules or, in the units commonly used for accelerated beams, V electron-Volts. An electron Volt is then just 1.6×10^{-19} Joules. If the particle is a fully stripped ion of an atom with atomic number A and charge Z then the energy is ZV/A electron Volts per nucleon.

The first high-voltage generator to approach 1 MeV was built by Cockcroft and Walton [1–3] in the 1930s to accelerate particles for their fission experiments. Their combination of diodes and capacitors, also known as rectifier circuit, is still used today to apply high voltage to the ion or proton source at the beginning of many linacs and synchrotrons although these are gradually being replaced by radio frequency quadrupoles.

The early 1930s also saw the invention by R.J. Van der Graaf [4] of an electrostatic generator which used a moving belt to carry charge into the high voltage terminal until it reaches a potential of several MV (Fig. 1.1). Van der Graaf accelerators have proved a useful source of low energy particles to this day but are inevitably limited by problems of voltage breakdown. Voltages up to 27 MV have been reached, putting the device in a discharge suppressing gas atmosphere (e.g. SF_6). Although it is possible in theory to chain together several electrostatic accelerators, each with its cathode connected to the anode of the next, each stage increases the potential between the ends of the device and between the ends and ground and eventually electrical breakdown discharges the high voltage terminals.

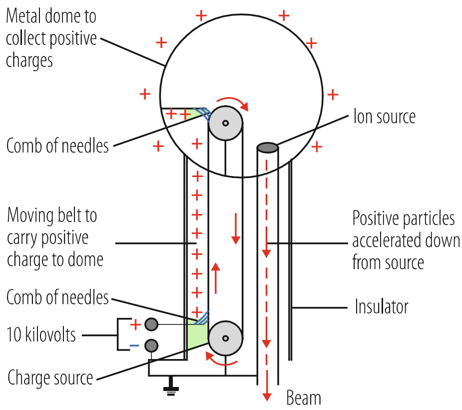

Fig. 1.1 Van der Graaf accelerator

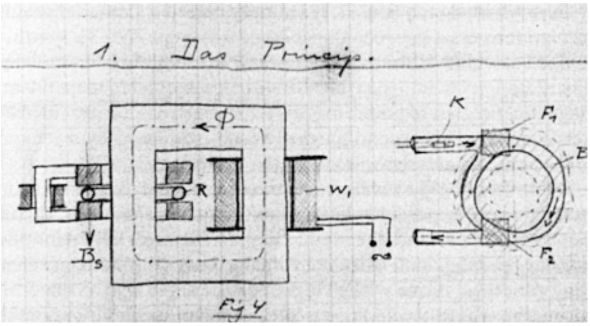

Fig. 1.2 Wideröe's sketch of the ray transformer

1.2.2 The Ray Transformer

The earliest idea of how to overcome the limitations of electrostatic acceleration involved using the time varying property of magnetic fields and came from the inventive mind of Rolf Wideröe.

Beginning his studies at Karlsruhe Technical University in 1923, he wondered if electrons in an evacuated ring would flow in the same way as the electrons in copper if they replaced the secondary winding of a transformer· His notebooks of that time contain sketches of a device he called a "ray transformer"; the first circular accelerator and the precursor of the "betatron" [5].

These sketches show a beam tube, in the form of an annulus, *R*, placed in the gap between the parallel poles or faces of a small electromagnet (on the left in Fig. 1.2). This magnet is in the form of a "C" and the field between the poles, B_z, guides particles in a circular orbit in the mid plane between the poles. A circular hole is cut in each pole through which the yoke of the transformer passes linking the beam tube. The primary winding of the transformer, labelled W_1, is powered with alternative voltage from the mains. The beam tube is placed where one would normally expect the secondary winding. The beam within it carries the induced secondary current.

Unlike almost all accelerators that followed, the ray transformer relied entirely upon the inductive effect of a varying magnetic field. It is the rate of change of flux, ϕ, in the yoke which establishes an accelerating potential difference around the beam's path. The windings, that of the C-magnet and of the primary of the transformer, W_1, give independent control of the guide field and accelerating flux.

Wideröe calculated that electrons circulating in a ring of only 10 or 20 cm diameter could reach several MeV within one quarter wave of the AC excitation of the transformer. He had to use Einstein's newly discovered theory of special relativity to correctly describe the motion of particles close to the speed of light. He also found an important principle which ensures that the beam radius does not change as it accelerates. To ensure constant radius during acceleration the total flux linking the beam including that generated by both sets of the coils, $B_{a,}$ must be twice that generated by the left hand coil pair which produces the field keeping the beam in a circular orbit, B_g.

$$\dot{B}_a = 2\dot{B}_g$$

Unfortunately, Wideröe was dissuaded from building the ray transformer by difficulties with surface fields and by his professor, who wrongly assumed the beam would be lost because of gas scattering. However, his Ray Transformer and the 2 to 1 ratio, now known as the Wideröe principle, were important discoveries which were put into practice 15 years later when D.W. Kerst and R. Serber [6] built a series of betatrons.

Wideröe went on to develop a second basic acceleration method to overcome the electrostatic limitation: the drift tube linac.

1.2.3 Repetitive Acceleration

There are two broad classes of accelerator characterized by the way they achieve repetitive acceleration and which overcome the insulation problems of the electrostatic machines. The simplest concept is that of the linear accelerator. Particles pass though cavities excited by radio frequency generators. They arrive on the threshold of each cavity with the energy they have already received and gain a further increment in energy from the electric field in the cavity which points in their direction of motion. Each cavity performs the function of the gap between the anode and cathode of an electrostatic accelerator but, unlike the electrostatic case, the increments of energy may be added together without developing a huge voltage to ground in any part of the apparatus. Of course, there is a limit to the voltage (energy increment) each cavity can apply and the length of the device becomes very long for energies above 1 GeV. Nevertheless, a linac has become the only way of accelerating highly relativistic electrons which radiate a large fraction of their energy when bent into a circular path.

As alternative concept, circular machines, like cyclotrons and synchrotrons use the same set of accelerating cavities over and over again as the particles make complete turns around the accelerator, being guided and focused by the magnet structure of the ring which is thus constraining their orbit. On each turn an increment of energy is added and, once accelerated, particles may be allowed to circulate indefinitely at their top energy. Two circulating beams of say protons and antiprotons or electrons and positrons can be sustained in the same ring and, colliding at experiments around the circumference, create new particles up to a mass (centre of mass energy as it is called) that is the sum of the two energies. Colliders are today the preferred configuration for a high-energy machine. Earlier, new particles were sought in the debris from a particles collision with a nucleon in a fixed target but such collisions are limited to a smaller centre of mass energy—which rises only as the square root of the accelerated beam energy.

1.2.4 Linear Accelerators

Although disappointed by the rejection of his ray-transformer as a subject for his PhD, Wideröe was led to the idea of a linear accelerator by a paper by G. Ising [7] who tried to overcome the voltage breakdown problem of a single stage of acceleration by placing a series of hollow cylindrical electrodes one after another in a straight line to form what today we would call a 'drift tube linac' or linear accelerator. Wideröe realised that an oscillating potential applied to one drift tube flanked by two others which are earthed, accelerates at both gaps provided the oscillator's phase changes by 180° during the flight time between gaps.

In 1927 he built a three-tube model which accelerated sodium ions. At the wavelengths that radio transmitters generated at that time a particle travelling near the velocity of light would travel hundreds of meters in the time it would take for the r.f. to swing by half a sine wave. This would make the length of a drift tube impractically large. Sodium ions, being rather heavy compared with protons or electrons, travelled much slower than the velocity of light and this helped keep the apparatus down to table-top proportions. Although he realised that one might extend such a series of tubes indefinitely he did not take the idea any further as he was due to start his professional employment designing high voltage circuit breakers. Between 1931 and 1934, D. Sloan and E.O. Lawrence at Berkeley took up Wideröe's idea and constructed linacs with as many as 30 drift tubes to accelerate mercury ions but, these were never used for research.

Much later, in the mid-1940s, and when suitable high-power high-frequency oscillators had become available to meet the needs of war-time radar, L.W. Alvarez (1946) started to build the first serious proton linac at the Radiation Laboratory of the University of California. Figure 1.3 shows an Alvarez linac. A series of drift tubes are mounted within a copper-lined cylinder excited by a radio transmitter. As in Wideröe's linac, particles gain energy from the accelerating potential differences between the ends of the drift tube, but now the phase shift between drift tube

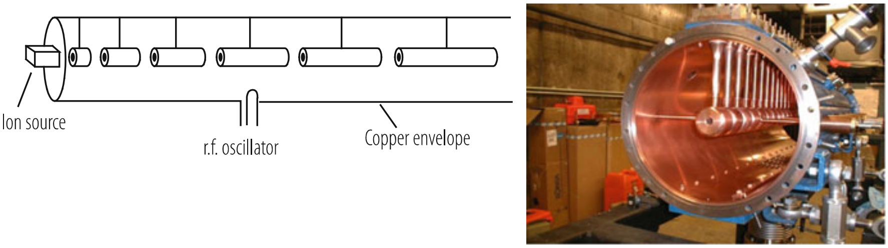

Fig. 1.3 left: The concept of the drift tube linac (from [8]); right: CERN's Linac 1

gaps is 360°. Each gap appears to the particle to be an identical field gradient which accelerates particles from left to right. The particles are protected from the decelerating phase while inside the metallic drift tubes. Although the particle gains energy steadily as it passes each gap, the total voltage between parts of the assembly and ground does not become larger along the length of the device as it would for an electrostatic machine.

The distances between gaps, or the lengths of the tubes, increase as the particle is accelerated since it travels an ever increasing distance during one swing of the radio frequency oscillation. At low energy, we would expect this distance to increase with the velocity or the root of the kinetic energy but when the energy is large we find the length of the drift tubes and their spacing no longer increases—a practical demonstration of special relativity. The Alvarez structure is still widely used, especially for non-relativistic proton and ion beams.

It was well known at the time that waves might be propagated along a much simpler smooth waveguide and that some of the modes have an accelerating electric field in the direction of propagation. Closer examination however shows that the stumbling block is that the phase velocity of these modes in a wave guide is always greater than that of light and hence the particle sees a field which sometimes accelerates and then decelerates as the wave overtakes the particle. It was later found that the phase velocity could be reduced by a series of iris diaphragms in the pipe. Such a structure (Fig. 1.4) is very popular in electron linacs and also in storage rings in which the particle is close to the velocity of light and cavities need not be tuned to follow the acceleration cycle.

These diaphragm-loaded linac structures have been commonly used as injectors for circular accelerators to accelerate electrons and protons to energies in the range 10 to 1000 MeV. As compact high frequency structures they have also been widely used to accelerate electrons to, typically 10 MeV, as a source of X-rays for cancer therapy. An early and very successful adventure in the electron linac development was the "two-mile long" Stanford Linear Accelerator at SLAC in California which has been the work horse for a number of ground breaking fixed target experiments and circulating beam storage ring projects at 20 to 50 GeV. With the help of two semi-circular arcs it was used to bring beams of electrons and positrons into head-on collision in the Stanford Linear Collider Project. This project, is forerunner for

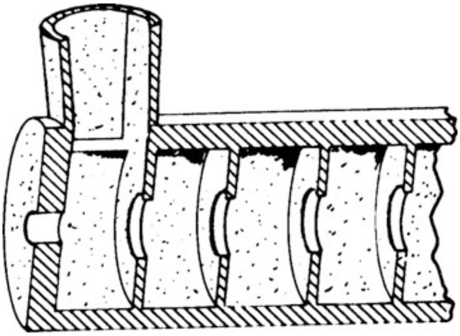

Fig. 1.4 Iris loaded structure (from [9]). The 'chimney' is the input waveguide

today's projected Linear Colliders in which linear accelerators accelerate positrons and electrons to energies approaching 1 TeV to collide them head on in a bid to overcome the very considerable energy lost by an electron to synchrotron radiation in circular lepton rings at high energy.

1.2.5 Cyclotrons

Unlike a linac, whose length must be extended to reach a higher energy, the cyclotron, as it is called, is a relatively compact accelerator in which the energy is only limited by the diameter and field strength of the magnet. The cyclotron idea first occurred to E.O. Lawrence who, reading through Wideröe's thesis, ruminated on the possibility of using a magnetic field to recirculate the beam through two of drift tubes. The cyclotron idea was published in 1930 [10] and another colleague, M.S. Livingston, who was also later to contribute much to the field, was given the job of making a working model as his doctoral thesis.

In Fig. 1.5 we see the two 'Dee's' which comprise the positive and negative electrodes of the accelerating system between the poles of the magnet. These are like two halves of a closed cylinder divided along its diameter. A radio-frequency generator excites them with an alternating field of constant frequency. The potential difference between the 'Dee's' accelerates the ions as they pass the gap between the two halves of the structure. The fundamental trick is that the field oscillates at the particle's circulation frequency and hence the sign of the potential difference at each gap is always in the accelerating direction.

As long as cyclotrons accelerate ions to modest energies, classical rather than relativistic mechanics still applies. In Fig. 1.6 we see the balance between centripetal acceleration of motion in a circle and the force exerted by the vertical magnetic field,

$$evB = \frac{mv^2}{\rho}, \text{ if } v \ll c, \tag{1.1}$$

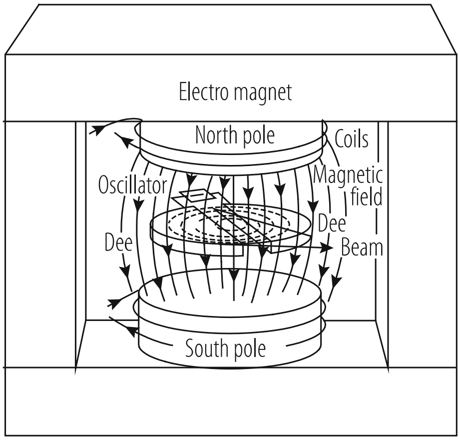

Fig. 1.5 The principle of the cyclotron

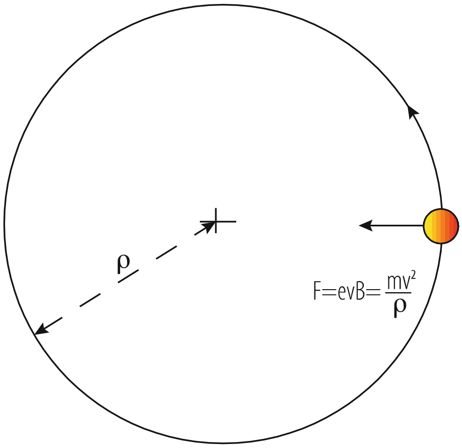

Fig. 1.6 Balance of forces in a cyclotron

and, rearranging, we can define the magnetic rigidity—the reluctance of the beam to be bent in a curve:

$$B\varrho = \frac{mv}{e}, \text{ if } v \ll c. \tag{1.2}$$

In the relativistic regime if we replace the classical momentum, *mv,* by the relativistic momentum, $p = \gamma mv$, with γ being the Lorentz factor, we obtain the equation, valid in the relativistic regime:

$$B\varrho = \frac{p}{e}. \tag{1.3}$$

By good fortune the radius of the orbit in a cyclotron is proportional to the velocity and the frequency of revolution this being the inverse of the time of revolution—just the length of one turn divided by the particle's velocity

$$f = \frac{v}{2\pi\varrho} = \frac{v}{2\pi} \cdot \frac{eB}{mv}. \tag{1.4}$$

has a numerator and denominator which are both proportional to v. This frequency remains constant as the particle is accelerated in the low energy, classical, regime. Thus, the circulating particles stay in synchronisation with the oscillating RF field and a continuous stream of ions injected in the centre will follow a spiral path to reach their highest energy at the rim of the poles.

Unfortunately, the synchronism between r.f. voltage and revolution frequency breaks down as the particles velocity begins to approach that of light and the relativistic mass in the above equation is no longer constant. This happens over 30 MeV for protons and at double this energy for deuterons. Electrons are much too light and relativistic to be accelerated in a cyclotron to any significant energy. For them other acceleration concepts are more adequate, like the disk loaded travelling wave linac or the betatron that both were described before.

The possible remedy of making the field stronger at the edge of the poles would have preserved synchronism and continuous beams but, as we shall see, was in conflict with the need to have a negative radial gradient to the field to provide vertical weak focusing. As a consequence a more powerful concept had to be developed to achieve highest particle beam energies: The synchrotron.

1.2.6 The Synchrotron

Meanwhile, in the 1940s, still higher energies were needed to pursue the aims of physics and the stage was set for the discovery of the synchrotron principle which opened the way to the series of circular accelerators and storage rings which have served particles physics up to the present day. It was Australian physicist Mark Oliphant who synthesized three old ideas into a new concept—the synchrotron. The ideas were: accelerating between the gaps of resonators, varying the frequency, and pulsing the magnet. In 1943 he described his invention in a memo to the UK Atomic Energy Directorate (see [11]).

> Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field ... which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations by an alternating electric field applied between coaxial hollow electrodes.

Unlike the cyclotron, the synchrotron accelerates the beam as a series of discrete pulses or "bunches" as they are called. Each short pulse is injected at low field and then the field rises in proportion to the momentum of particles as they are accelerated. This ensures that the radius of the orbit remains constant. In contrast to cyclotrons and betatrons, the synchrotron needs no massive poles to support a

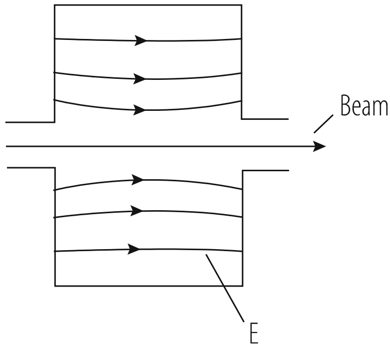

Fig. 1.7 A simple accelerating cavity

magnetic field within the beam's circular orbit. The guide field is instead provided by a slender ring of individual magnets. The fact that the machine is pulsed and the frequency must be controlled to track the increasing speed of particles is a complication, but it solves the difficulty that isochronous cyclotron builders had encountered in accelerating relativistic particles.

Instead of the Dees of a cyclotron acceleration is provided in a synchrotron by fields within a hollow cylindrical resonator or "pillbox" cavity, Fig. 1.7, excited by a radio transmitter. A particle passes from left to right as it completes each turn of the synchrotron receiving another increment in energy at each revolution.

The early synchrotrons, like the cyclotron before them, relied on a slight negative radial gradient in the vertical magnet field to produce field lines which belly outwards from the magnet gap. A small radial field component deflects any particles which head off towards the poles back to the median plane. Unfortunately, this field shape has the opposite (defocusing) effect horizontally but, up to a certain, rather weak, gradient strength focusing is assured by a slight imbalance between the central force and the centrifugal acceleration. The gradient cannot be too large—hence the term "weak focusing".

Oliphant was the first to start building a proton synchrotron (at Birmingham University) but he was overtaken by Stan Livingston's 3 GeV Cosmotron at Brookhaven National Laboratory and later by the 6 GeV Bevatron at Berkeley.

Due to the weak focusing forces in these first synchrotrons, the particles' excursions, both horizontally and vertically are large and the magnet pole width and gap correspondingly so. Strong focusing changed this. It was invented at the Cosmotron, which was actually the first proton synchrotron to operate, whose weak focusing 'C' shaped magnet was open to the outside. The top energy of the Cosmotron was limited by the extra fall-off in field caused by the effect of saturation. Stan Livingston and E.D. Courant wanted to compensate this by re-installing some of the C magnets with their return yokes towards the outside. They were afraid of the variations in gradient around the ring but were surprised to calculate that the focusing seemed to improve as the strength of the alternating component of the gradient increased. Courant, Livingston, and H.S. Snyder [12, 13] were able to explain this retrospectively with an optical analogy of alternating focusing by equal

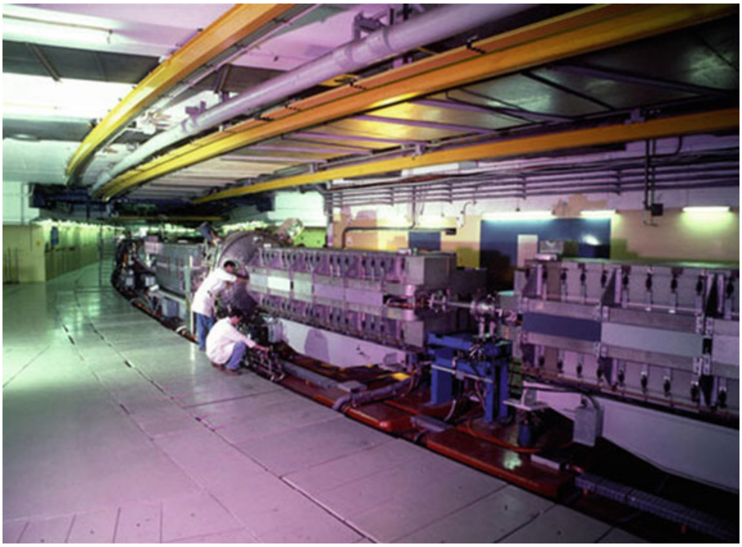

Fig. 1.8 The CERN 25 GeV proton synchrotron

convex and concave lenses which will transport rays which pass through the centres of defocusing lenses.

Alternating gradient or strong focusing greatly reduces the beam's excursions and so the cross section of the magnet gap by more than an order of magnitude. Its discovery enabled Brookhaven and CERN to build the next generation of proton synchrotrons, AGS and PS, to reach 30 GeV—five times the energy of the Bevatron—yet use beam pipes of only a few centimetres height and width.

This was to lead to huge economies in the cost per unit length of the magnet system. Figure 1.8 shows how this was applied to the first of the two synchrotrons, AGS and PS that used this focusing system. From then on all synchrotrons and, later, storage ring colliders use this scheme. The history of synchrotrons has been always to seek methods of improving focusing and economizing on magnet aperture. The only other step function in their development to higher energies has been the use of superconducting magnets whose higher fields reduce the circumference of the machine by a factor between 3 and 5.

1.2.7 Phase Stability

When the first synchrotrons were built it was by no means obvious that the circulating beam and the accelerating voltage would remain in step. There were those who thought that any slight mistiming of the sine wave of accelerating voltage in the cavity might build up over many turns until particles would begin to arrive within the negative, decelerating, phase of the sine wave and be left behind. Even if one succeeded in achieving synchronism for the ideal, *synchronous particle*, others of slightly different energy would not have the same velocity and take a different time to circulate around the machine. Would not these particles gradually get out of step until they were lost? After all, particles had to make many hundred thousand turns before reaching full energy and while transverse focusing was understood there was no apparent focusing available in the longitudinal direction. Fortunately the comforting principle of phase stability, which prevents this happening, was soon to be independently discovered by V. I. Veksler in Moscow in 1944 [14] and McMillan in Berkeley in 1945 [15], opening the way to the construction of the first synchrotrons. We shall return to this later.

When it came to the next generation of synchrotrons, interest focused on colliding two opposing beams of particles. It had been known for some time that the energy available in the centre of mass from a collision of particles, one in the beam with energy E and the other of mass m_0 in a fixed target, only increased with the square root of the accelerators energy, $\sqrt{m_0 E}$. Two particles of the same mass and energy E colliding head on made available all their energy in the centre of mass, $2E$. The difficulty was making the two bunches of particles of sufficient density to have a significant probability of collision or, in technical jargon a high enough luminosity. Once this problem was solved a series of colliders: ISR, $\mathrm{Sp\bar{p}S}$, LEP, Tevatron, HERA and finally LHC followed. Some of these (ISR, HERA and LHC) were two separate rings which intersected to collide particles at several points around the circumference. Others ($\mathrm{Sp\bar{p}S}$ and LEP) collided protons with antiprotons and electrons with their anti-particles: positrons. These exploited the fact that beams of particles and antiparticles will circulate on identical trajectories, but in opposite directions, in a single ring of bending and focusing magnets.

At present several studies are ongoing, to pave the way to even higher energies, mainly increasing the size of the machine and using super conducting magnets with higher critical field, to gain more bending and focusing fields in the lattice. One example, the Future Circular Collider study, FCC, under the guidance of CERN, is studying a 100 km proton storage ring to achieve centre of mass energies of up to 100 TeV. The R & D effort of accelerators of this dimension and complexity, in any case, has to be done by a truly international, in other words worldwide effort.

References

1. J.D. Cockcroft, E.T.S. Walton: Proc. Roy. Soc. A 129 (1930) 477-489.
2. J.D. Cockcroft, E.T.S. Walton: Proc. Roy. Soc. A136 (1932) 619-630.
3. J.D. Cockcroft, E.T.S. Walton: Proc. Roy. Soc. A 137 (1932) 229-242.
4. R.J. Van der Graaf: Phys. Rev. 38 (1931) 1919.
5. R. Wideröe.: ETH Library, Zürich, Hs 903 (1923-28) 633-638.
6. D.W. Kerst, R. Serber: Phys. Rev. 60 (1941) 53-58.
7. G. Ising: Arkiv för matematik o. fysik 18 (1924) 1-4.
8. J.J. Livingood: *Principles of cyclic particle accelerators*, van Nostrand (1961).

9. P. Lapostolle, A. Septier: *Linear Accelerators*, North Holland (1971).
10. E.O. Lawrence, N.E. Edelfsen: Science 72 (1930) 376-7.
11. M. Oliphant: *The genesis of the Nuffield Cyclotron and the Proton Synchroton*, Publ. Department of Physics, University of Birmingham.
12. E.D. Courant, M.S. Livingston, H.S. Snyder: Phys. Rev. 88 (1952) 1190-1196.
13. E.D. Courant, H.S. Snyder: Annals of Physics 3 (1958) 1-48.
14. V.I. Veksler: Comptes rendues (Doklady) de l'Academie des Sciences de l'URSS 43 (1944) 329-341.
15. E.M. MacMillan: Phys. Rev. 68 (1945) 143.