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Non-Oscillatory No-Scale Inflation

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ABSTRACT

We propose a non-oscillatory no-scale supergravity model of inflation (NO-NO inflation) in which the inflaton does not oscillate at the end of the inflationary era. Instead, the Universe is then dominated by the inflaton kinetic energy density (kination). During the transition from inflation to kination, the Universe preheats instantly through a coupling to Higgs-like fields. These rapidly annihilate and scatter into ultra-relativistic matter particles, which subsequently dominate the energy density, and reheating occurs at a temperature far above that of Big Bang Nucleosynthesis. After the electroweak transition, the inflaton enters a tracking phase as in some models of quintessential inflation. The model predictions for cosmic microwave background observables are consistent with Planck 2018 data, and the density of gravitational waves is below the upper bound from Big Bang Nucleosynthesis. We also find that the density of supersymmetric cold dark matter produced by gravitino decay is consistent with Planck 2018 data over the expected range of supersymmetric particle masses.

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1 Introduction

The theory of inflation is the most successful mechanism for explaining how the Universe became so homogeneous and isotropic on large scales [1]. The usual scenarios assume that, after the epoch of cosmic inflation, the scalar inflaton field rolled down to a minimum and started oscillating, ultimately decaying into elementary particles and reheating the Universe [2, 3].

However, one may also consider an alternative mechanism of reheating, caused by non-perturbative effects arising from a parametric resonance, known as preheating [4–7] (see [8] for reviews). If the inflaton field is coupled to another scalar field, h, the large effective mass of the h field changes non-adiabatically when the inflaton crosses the origin, causing explosive production of h particles.

The original models of preheating [4–7] discussed the exponential particle production that occurs non-perturbatively due to inflaton field oscillations about an effective minimum. However, it was later shown by Felder et al. [9] that preheating can occur instantaneously without inflaton oscillations or a parametric resonance, if the coupling between the inflaton and the additional scalar field is sufficiently large. This discovery stimulated interest in non-oscillatory (NO) models of inflation [10]. These have suffered from the paucity of efficient reheating mechanisms other than gravitational particle production induced by the changing space-time metric [11–13]. One alternative possible reheating mechanism for NO models of inflation was to introduce a curvaton field [14, 15], which decays into thermalized radiation and reheats the Universe. Models with curvaton reheating do not need an interaction term between the inflaton field and another scalar field.

In this paper, we focus our attention on NO models of inflation with instant preheating. Because the process of instant preheating is extremely efficient, only a small fraction of the inflaton energy is converted to the production of the heavy h particles, which in turn rapidly annihilate and scatter into ultra-relativistic particles. After the period of instant preheating, the Universe is dominated by the kinetic energy density of the inflaton, which scales as $\propto a^{-6}$ [16], until the energy density of ultra-relativistic products, which scales as $\propto a^{-4}$, starts dominating the Universe, resulting in reheating. A viable scenario of instant preheating requires that the Universe enters the radiation-dominated era significantly before the epoch of Big Bang Nucleosynthesis (BBN), which necessitates a reheating temperature $T_{\rm RH} \gg 1\,{\rm MeV}$. On the other hand, in supersymmetric models too large a value of $T_{\rm RH}$ would bring the risk of excessive production of gravitinos whose decays would overproduce dark matter.

The non-oscillatory model of instant preheating postulated in [9] assumed that, after the inflationary epoch, when the inflaton field crosses the zero point and becomes negative, the effective scalar potential vanishes and the energy density of the inflaton consists only of the kinetic energy contribution. It was later shown that the mechanism of instant preheating works very well in models incorporating quintessential inflation, as originally introduced by Peebles and Vilenkin [17], which assume that the inflaton field slowly rolls toward zero potential after inflation, and could explain the present-day dark energy. Quintessential inflation with instant preheating has been studied in several different contexts, including the production of gravitational waves [18, 19], α-attractor models of inflation [20], the primordial production of black holes [21], Gauss-Bonnet models [22], and UV freeze-in models [23]. In these models the coupling between the inflaton and another scalar field is generally put in by hand, and the coupling strength is adjusted to avoid the overproduction of radiation created by instant preheating, which would otherwise cause the reheating temperature $T_{\rm RH}$ to be too large.

In this paper we consider a new scenario for instant preheating, derived from no-scale supergravity [24–26]. In this scenario, immediately after instant preheating the effective mass of the non-perturbatively produced particle h, which we treat as a proxy for the Higgs boson of the Standard Model (SM), is similar to the inflationary scale. These superheavy particles then annihilate and scatter into ultra-relativistic products. The Universe enters a period dominated by the inflaton energy density (kination) until the ultra-relativistic particles start to dominate the energy density of the Universe and reheating occurs. The motion of the inflaton is slowed down during this radiation-dominated phase. Then, after the electroweak phase transition, when the radiation density drops to the level of the inflaton potential, the universe enters a tracking phase, as in quintessence models [27]. The residual inflaton potential energy later acts as dark energy in the present-day Universe.

We are motivated to consider models based on $\mathcal{N} = 1$ supergravity, because it provides a natural framework that incorporates a viable dark matter candidate and also connects the inflaton to the SM fields at high scale. The theory of supergravity is characterized by the geometric properties of a non-trivial Kähler manifold, which can incorporate naturally the required effective interaction terms between the inflaton and the proxy Higgs field h. Further, we are motivated to study scenarios based on no-scale supergravity [24–26] because it avoids undesirable anti-de Sitter states and emerges in the effective low-energy limit of string compactification [28]. For some previous models combining the SM with no-scale models of inflation, see [29–36].

We demonstrate in this paper the construction of a non-oscillatory, no-scale supergravity model of inflation (NO-NO inflation), whose predictions agree with the most recent cosmic microwave background (CMB) measurements [37, 38]. This model has a simple plateau inflationary potential, whose predictions are similar to those of the original Starobinsky model of inflation based on $R + R^2$ gravity [40] (see [41–51] for other Starobinsky-like models in the context of no-scale supergravity), and the general framework can be extended easily to models of no-scale attractors [52], which we leave for future work. We note that plateau models of inflation with a minimum have also been studied in [53, 54], where they were referred to as T-models.

The structure of this paper is as follows. In Section II we describe our NO-NO inflation scenario, first reviewing briefly relevant aspects of no-scale supergravity, then discussing the effective potential of the model and equations of motion before analyzing its predictions for CMB observables, which are compatible with the Planck 2018 data [37, 38]. Then post-inflationary preheating is studied in Section III. An estimate of the reheating temperature is given in Section IV, where the tracking regime and its conclusion is also discussed. The abundance of gravitational waves is calculated in Section V and found to be compatible with the upper limit from Big Bang Nucleosynthesis, and the abundance of gravitinos and the density of supersymmetric dark matter produced by their decays are considered in Section VI. Finally, Section VII summarizes our results and discusses our conclusions.

2 NO-NO Inflation

2.1 No-Scale Framework

In this Section we illustrate how to construct non-oscillatory (NO) models of inflation in the non-minimal $\mathcal{N} = 1$ no-scale supergravity framework. [1] In addition to the inflaton field, ϕ, we include a scalar field, h, corresponding to the Higgs boson. The two scalars are coupled only through interactions induced by the supergravity Lagrangian. During inflation, the inflaton field has a large field value of $\sim \mathcal{O}(M_P)$, and $h \simeq 0$. [2] When the inflaton field ϕ crosses the region near the null point, $\phi \simeq 0$, the field velocity, $\dot{\phi}$, is large and the mass of the Higgs boson changes nonadiabatically due to the effective interaction term between the ϕ and h fields, and heavy particles are produced explosively through the preheating process, as we discuss in detail in the next Section.

We introduce the following Kähler potential

$$K \;=\; -\alpha \log[1 - x\bar{x} - y\bar{y}] - \beta \log[1 - z\bar{z}]\,, \tag{1}$$

where x is associated with the inflaton field, y with the Higgs field, and z is an auxiliary field. The Kähler potential (1) parametrizes an $\frac{SU(2,1)}{SU(2)\times U(1)} \times \frac{SU(1,1)}{U(1)}$ coset manifold, and the

[1] For discussions of other $\mathcal{N} = 1$ no-scale supergravity models, see [24–26].

[2] We are using the reduced Planck mass, $M_P = 1/\sqrt{8\pi G_N} \simeq 2.4 \times 10^{18}$ GeV. With the exception of providing units for numerical quantities, we will work in units where $M_P = 1$, and thus factor of M_P will be omitted in most analytical expressions.

curvature parameters α, β describe the characteristic geometry of the manifold. [3] We recall that the Kähler curvature is given by $R_{i\bar{j}} \equiv \partial_i \partial_{\bar{j}} \ln K_{i\bar{j}}$, where the Kähler metric is defined as $K_{i\bar{j}} \equiv \partial^2 K / \partial \Phi^i \partial \bar{\Phi}^{\bar{j}}$ and where the Φ_i are the complex chiral fields, and the Ricci scalar curvature can be expressed as

$$R = R_{i\bar{j}} K^{i\bar{j}} , \tag{2}$$

where $K^{i\bar{j}}$ is the inverse Kähler metric.

The scalar curvature for the Kähler potential (1) is $R = 4/\alpha + 2/\beta$, where we use the convention that $R > 0$ for a hyperbolic manifold and $R < 0$ for a spherical manifold. In our further discussion, we assume that $\alpha, \beta > 0$ and hence we have a hyperbolic manifold with $R > 0$.

The scalar $\mathcal{N} = 1$ supergravity action for the chiral fields is given by

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_{\text{kin}} - V \right) , \tag{3}$$

where the scalar kinetic term takes the form

$$\mathcal{L}_{\text{kin}} = K_{i\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}} , \tag{4}$$

and the effective scalar potential is given by

$$V = e^G \left[\frac{\partial G}{\partial \Phi^i} K^{i\bar{j}} \frac{\partial G}{\partial \bar{\Phi}^{\bar{j}}} - 3 \right] \tag{5}$$

where, in order to include interactions, we have introduced an extended Kähler potential including a superpotential W:

$$G \equiv K + \ln W + \ln \overline{W} . \tag{6}$$

We assume in our discussion of inflationary dynamics in the supergravity framework a spatially-flat Universe, which is described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 . \tag{7}$$

The classical equations of motion for the scalar fields are given by

$$H^2 = \dot{N}^2 = \frac{1}{3} \left[K_{i\bar{j}} \dot{\Phi}^i \dot{\bar{\Phi}}^{\bar{j}} + V(\mathbf{\Phi}) \right] , \tag{8}$$

$$\dot{H} = -K_{i\bar{j}} \dot{\Phi}^i \dot{\bar{\Phi}}^{\bar{j}} , \tag{9}$$

$$\ddot{\Phi}^i + 3H\dot{\Phi}^i + \Gamma^i_{jk} \dot{\Phi}^j \dot{\Phi}^k + K^{i\bar{j}} \frac{\partial V}{\partial \bar{\Phi}^{\bar{j}}} = 0 , \tag{10}$$

where $\mathbf{\Phi} \equiv (x, y, z)$, $H = \dot{a}/a$ is the Hubble parameter, $\Gamma^i_{jk} = K^{i\bar{l}} \partial_j K_{k\bar{l}}$ are the Christoffel symbols, and N is the number of e-folds.

[3] One could also consider a Kähler potential of the form $K = -\alpha \log[1 - x\bar{x} - y\bar{y} - z\bar{z}]$, and construct similar non-oscillatory scalar potentials, albeit with a different superpotential. The relation between these forms and the more standard form $K = -\alpha \log[T + \bar{T} - \phi_i \bar{\phi}_i]$ is explained in Appendix A.

2.2 Non-Oscillatory Effective Potential

We now construct a non-oscillatory model of inflation starting with Kähler potential (1). We introduce a superpotential of the form

$$W = \frac{mz}{\sqrt{3}} \left(1 - x^2\right)^{b/2} (1 + x)^{c/2} + \mu y^2 \,, \tag{11}$$

where m is inflationary scale and the μ term is associated with the electroweak symmetry-breaking scale. Although we consider, for simplicity, a single field associated with the Higgs field, the general framework can easily be extended to more realistic supersymmetric scenarios with two Higgs doublets as in the minimal supersymmetric extension of the SM.

In order to stabilize the fields in the tachyonic directions, we introduce the following higher-order correction term in the Kähler potential (1): [30, 42, 55]

$$K \supset -\beta \log \left[1 - z\bar{z} + \frac{|z|^4}{\Lambda_z^2}\right] \,, \tag{12}$$

where Λ_z is a mass scale that is smaller than the Planck scale M_P. This higher-order correction term, which we view as a perturbation of the original no-scale structure, does not spoil the inflationary potential and ensures that other fields remain fixed during inflation. [4] The higher-order term in the Kähler potential (12) stabilizes the field z in the real and imaginary directions with $z = \bar{z} = 0$. Due to the symmetric structure of the superpotential (11), we find numerically that when $z = \bar{z} = 0$, the curvature in the imaginary field directions $\mathrm{Im}\, x$ and $\mathrm{Im}\, y$ is positive, with a minimum located at $\langle \mathrm{Im}\, x\rangle = \langle \mathrm{Im}\, y\rangle = 0$, and no additional stabilization terms in the Kähler potential (1) are necessary. This ensures that the potential is minimized along $y = \bar{y}$ and $x = \bar{x}$.

Combining the superpotential (11) with the Kähler potential (1), we find the following effective scalar potential

$$V = \frac{\frac{1}{3}m^2(1-x^2)^b(x+1)^c}{\beta(1-y^2-x^2)^\alpha} + \frac{\mu^2 y^2(4-4x^2+y^2(\alpha-8+(\alpha-2)^2x^2)+y^4(\alpha-2)^2)}{\alpha(1-y^2-x^2)^\alpha} \,. \tag{13}$$

If we choose the following illustrative parameter values: $c = 4$, $b = \alpha = 2$, and $\beta = 1$, we obtain a simpler expression

$$V = \frac{\frac{1}{3}m^2(x-1)^2(x+1)^6 + \mu^2 y^2(2-3y^2-2x^2)}{(y^2+x^2-1)^2} \,, \tag{14}$$

for which the scalar curvature $R = 4$. [5]

[4] For a related discussion of the flatness and stability conditions in supergravity, see [56, 57].

[5] If we used a Kähler potential parametrized by a $\frac{SU(3,1)}{SU(3)\times U(1)}$ coset manifold, the choices $c = 4$, $\alpha = 2$, and $b = \alpha - 1$ would yield the same scalar potential, with $m/\sqrt{3} \to m/\sqrt{3/2}$.

Because we expect $\mu \ll m$, inflation is driven by the first terms in Eq. (13) and the numerator of Eq. (14). The curvature parameter β is associated with the auxiliary field z, so its value does not affect the inflationary dynamics nor, in turn, the cosmological observables n_s and r. It does, however, scale the inflaton potential, so that the quantity m^2/β is fixed by the amplitude of the density fluctuations as discussed below. It should be noted that one can treat the parameters (α, β, b, c) in the effective scalar potential (13) as free variables and build alternative non-oscillatory models similar to no-scale attractors, whose different values of these parameters lead to different values of the scalar tilt n_s and tensor-to-scalar ratio r. However, not all parameter values are expected to accommodate sufficient preheating, because different values of the parameters change the steepness of the effective inflationary potential, which in turn affects the inflaton velocity that is directly related to the violation of adiabaticity. Here we focus on one particular relatively simple example as an existence proof, and we leave such considerations for future work.

At the end of inflation, the inflaton field value x keeps decreasing towards negative values and its energy density exhibits the typical kination scaling $\rho_x \propto a^{-6}$. After reheating, the Universe is dominated by radiation and the evolution of the inflaton slows, until the Universe enters a tracking period, where the radiation density, and the kinetic and potential energies of the inflaton evolve together. Eventually, the field x approaches a field value of -1.[6] At later times, cold dark matter dominates the expansion, until very late times when the Universe becomes dominated once again by the residual vacuum energy, with the slowly-evolving inflaton providing dark energy.

During inflation the Higgs field acquires a large mass (as mentioned above and discussed further below), and we can safely assume that $y \simeq 0$ and neglect the kinetic mixing between the fields in the Lagrangian (4). This assumption is valid during the inflation, kination and preheating eras, but fails to hold for a brief period when $x \simeq y \simeq 0$. We find from numerical calculations that neglecting the kinetic mixing between the two fields does not affect the preheating mechanism.

If we neglect the kinetic mixing terms in the Lagrangian (4), the canonical parametrization for the fields x and y is given by

$$x = \tanh\left(\frac{\phi}{2}\right), \tag{15}$$

$$y = \frac{h}{\sqrt{2\left(\cosh\phi + 1\right)}}, \tag{16}$$

where the canonically-normalized Higgs field, h, is coupled to the inflaton field, ϕ. If we

[6] This asymptotic value follows from the canonical field parametrization (15).

use the field redefinitions (15, 16), we can rewrite the scalar potential (14) as

$$V \simeq V_{\rm inf} + \frac{1}{2}\left(\mu^2 + V_{\rm inf}\right) h^2 + \cdots \,, \tag{17}$$

where

$$V_{\rm inf} = \frac{m^2}{3}\left(1 + \tanh\frac{\phi}{2}\right)^4 \,, \tag{18}$$

and we have omitted Planck-suppressed higher-order terms other than those contributing to the effective Higgs mass:

$$m^2_{\rm h,eff} = \frac{m^2}{3}\left(\tanh\left(\frac{\phi}{2}\right) + 1\right)^4 + \mu^2 \,. \tag{19}$$

For $\phi \simeq 0$, we find $m_{h,\rm eff} \simeq m/\sqrt{3}$, since $m \gg \mu$. The potential $V_{\rm inf}$ given by Eq. (18) with $m = 4 \times 10^{-6}\, M_P$ is shown in Fig. 1.

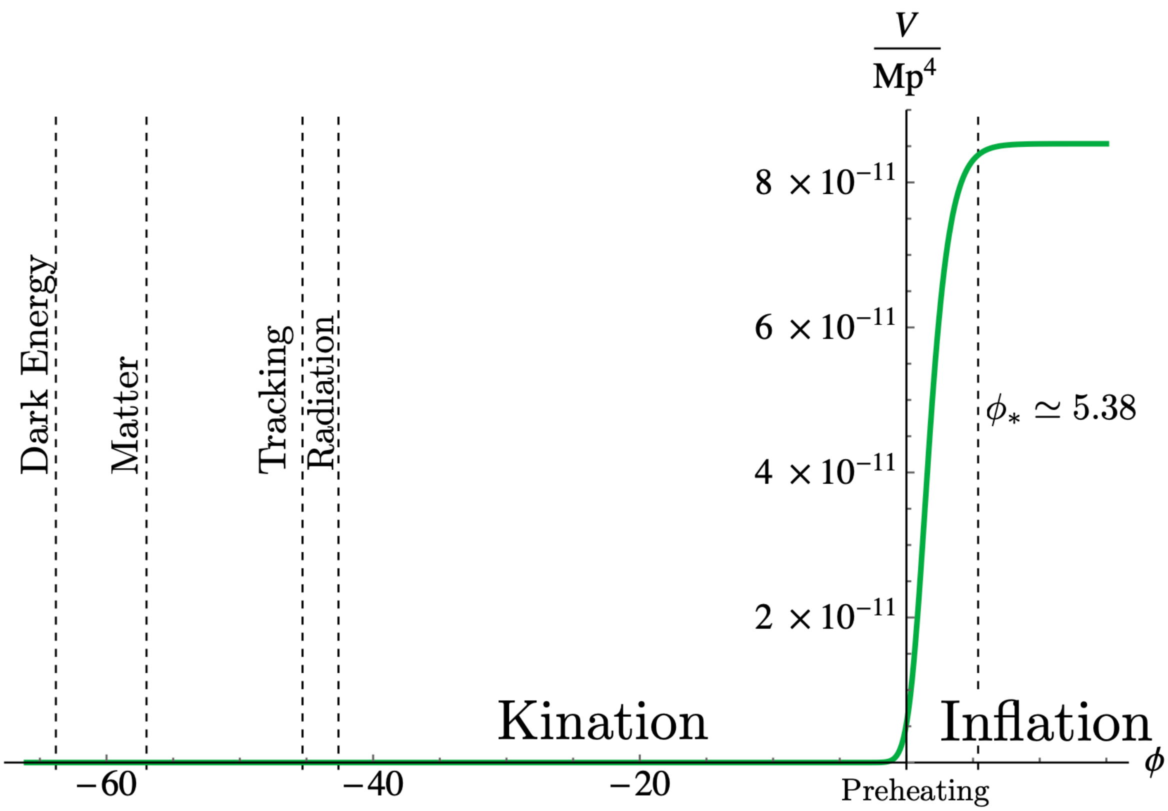

Figure 1: *The scalar potential, $V_{\rm inf}(\phi)$, in the NO-NO model of inflation given by Eq. (18) with $m = 4 \times 10^{-6}\, M_P$.*

In this approximation, the potential does not account for the VEV of h, as it does not include any effects due to supersymmetry breaking. In principle, the additional supersymmetry breaking sector may spoil the scalar potential at both early (inflationary) times and at late times. Indeed, even adding a simple constant to the superpotential greatly distorts the scalar potential in Eq. (18). In order to avoid these potential problems, we can follow the treatment presented in [31, 44, 52] and include an adjustable parameter for supersymmetry breaking and dark energy without invoking any additional fields. A full treatment of supersymmetry breaking and the electroweak transition lies beyond the scope of this paper and will be considered in future work. However, a specific example in the basis (1) is given in Appendix B. Here we assume that the full theory contains supersymmetry breaking and generates a soft mass term for the Higgs field that runs to a negative value at the electroweak scale [25]. When this and D-terms are included in the potential, a VEV for h is found.

2.3 Inflation

We use the slow-roll approximation in order to analyze the primary cosmological observables, namely the scalar tilt, n_s, and the tensor-to-scalar ratio, r. The slow-roll parameters

$$\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \qquad \eta \equiv \frac{V''}{V} \tag{20}$$

for the inflationary potential (17) are given by:

$$\epsilon = \frac{8}{(1+e^{\phi})^2} , \qquad \eta = \frac{4(4-e^{\phi})}{(1+e^{\phi})^2} . \tag{21}$$

The total cosmological expansion is characterized by the number of e-folds:

$$N_* \simeq -\int_{\phi_*}^{\phi_{\rm end}} \frac{1}{\sqrt{2\epsilon}} d\phi \, , \tag{22}$$

where ϕ_* denotes the inflaton field value at the Hubble horizon crossing. Using (21), we obtain

$$N_* \simeq \frac{(e^{\phi_*}+\phi_*)-(e^{\phi_{\rm end}}+\phi_{\rm end})}{4} . \tag{23}$$

We define the end of inflation to occur when $\epsilon = 1$, [7] and (21) yields $\phi_{\rm end} \simeq 0.6$. For the nominal choices $N_* = 50, 55, 60$, we find that $\phi_* \simeq 5.28, 5.38, 5.47$.

In order to connect the NO-NO model of inflation to the most recent CMB 2018 data [37, 38], we use the following relations for the scalar tilt and scalar-to-tensor ratio in terms of the slow-roll parameters:

$$n_s \simeq 1 - 6\epsilon + 2\eta, \qquad r \simeq 16\epsilon \, . \tag{24}$$

[7] A more precise definition for the end of inflation would be $\epsilon_H = 1$, where $\epsilon_H = -\dot{H}/H^2$, and in this case we find $\phi_{\rm end} \simeq 0.14$, corresponding to an insignificant difference in ϕ_*.

For our inflationary potential (18) and the choices $N_* = 50, 55, 60$, we find $n_s \simeq 0.9594$, 0.9631, 0.9660 and $r \simeq 0.0032, 0.0027, 0.0023$, which are consistent with the most recent CMB data [37,38].

We determine the inflationary scale m from the amplitude of the scalar density fluctuations [37], given by

$$A_s = \frac{V}{24\pi^2 \epsilon}, \tag{25}$$

where $A_s \simeq 2.1 \times 10^{-9}$. For the nominal values of $N_* = 50$, 55, 60, we find $m \simeq 4.4 \times 10^{-6}\, M_P$, $4.0 \times 10^{-6}\, M_P$, $3.7 \times 10^{-6}\, M_P$. The key results for our model of NO-NO inflation are summarized in Table 1 below. In our further analysis we choose the representative value $N_* = 55$, and the corresponding values $\phi_* \simeq 5.38\, M_P$ and $m \simeq 4 \times 10^{-6}\, M_P$.

N_*	50	55	60
ϕ_*	5.28	5.38	5.47
m	$4.4 \times 10^{-6}\, M_P$	$4.0 \times 10^{-6}\, M_P$	$3.7 \times 10^{-6}\, M_P$
n_s	0.9594	0.9631	0.9660
r	0.0032	0.0027	0.0023

Table 1: *Parameters of the NO-NO inflationary potential (18) and CMB predictions for the specific choices* $N_* = 50, 55, 60$.

Since the effective mass of the Higgs field is similar to the inflationary scale during inflation, $m_{\mathrm{h,eff}} \simeq m$ as can be readily seen from (17), the scalar potential in the h direction is very steep, which leads to $h \simeq 0$. Therefore, in our model we do not have large Higgs field fluctuations, nor production of isocurvature perturbations [10,39].

Next, we use the classical equations of motion (8 - 10) for the scalar fields to find numerical results for the time evolution of the model, which we use when discussing preheating in the next Section. In our numerical solutions and figures, time is measured in units M_P^{-1}, unless mentioned explicitly.

We first show in Fig. 2 the evolution of the inflaton field ϕ as a function of time. The inflationary period ends when $\epsilon = 1$, or $\phi_{\mathrm{end}} \simeq 0.6\, M_P$, and instant preheating begins. We then show in Fig. 3 the evolution of the inflaton velocity $\dot{\phi}$, plotted as a function of the field value ϕ. Our numerical calculations show that the inflaton velocity peaks right at the end of inflation, when $\phi \simeq 0.6\, M_P$ and $|\dot{\phi}| \simeq 0.7 m\, M_P$, and that the kinetic energy of the inflaton becomes equal to its potential energy when $\phi \simeq -0.5\, M_P$, when kination begins.

After the inflaton crosses this point, the inflaton energy density becomes dominated by its kinetic energy contribution. This is shown as the red curve in Fig. 4, which traces the history of the relevant energy densities as functions of time and the inflaton field value.

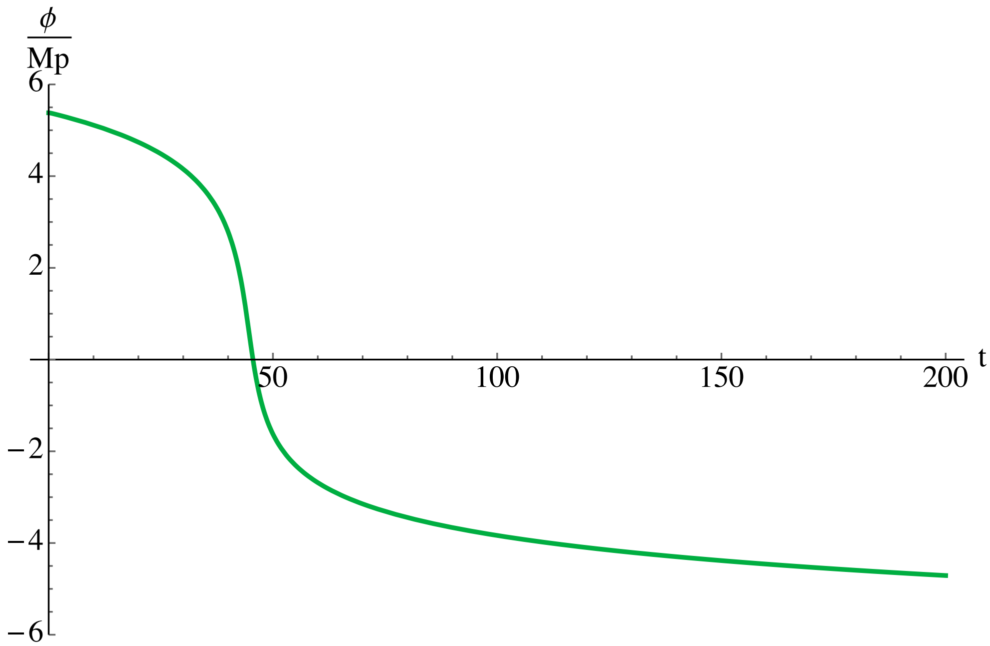

Figure 2: *Evolution of the NO-NO inflaton field ϕ as a function of time in the units of m^{-1}.*

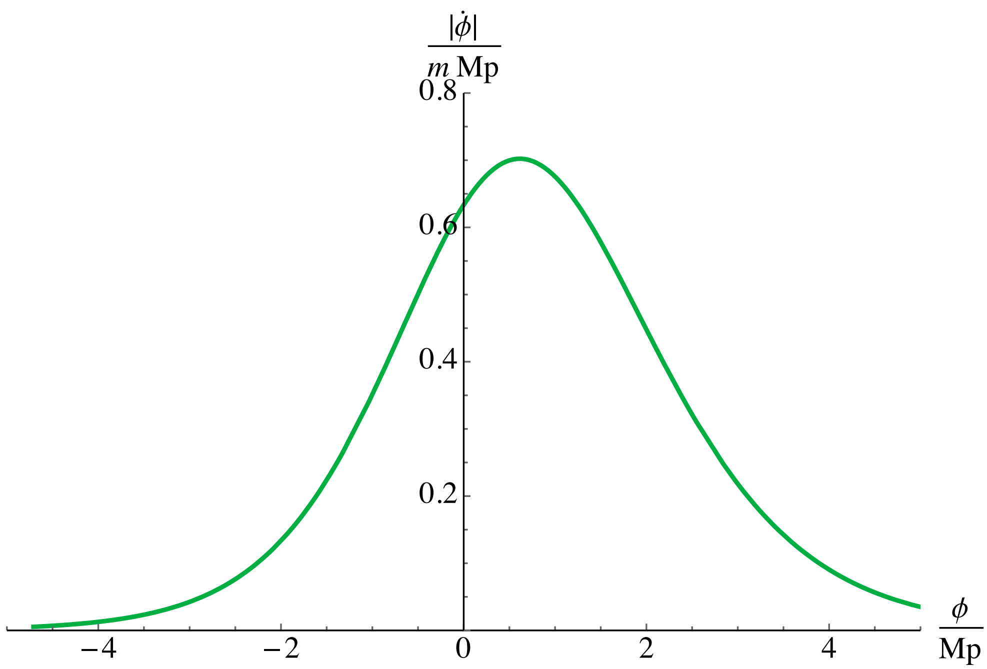

Figure 3: *Evolution of the NO-NO inflaton velocity $|\dot{\phi}|$ as a function of inflaton field value.*

When $\phi \simeq 0$, $t \simeq 10^7\, M_P^{-1}$ (corresponding to $t \simeq 45\, m^{-1}$ in Fig. 2) and the Universe becomes dominated by the inflaton kinetic energy. The inflaton potential energy density (blue curve) then remains a subdominant contribution until the Universe enters a tracking stage (discussed further below) at $t \simeq 10^{44}\, M_P^{-1}$. This result can be understood intuitively from the expression (17) for the effective scalar potential where, as $\tanh\frac{\phi}{2}$ approaches -1, a small value in the parentheses is raised to the fourth power, leading to a very small value of V. Therefore, when the inflaton passes the point $\phi \simeq -0.5\,M_P$, we can use an approximation for the classical equations of motion $\ddot{\phi} + 3H\dot{\phi} \simeq 0$ and, as expected, the energy density of the inflaton field scales $\propto a^{-6}$.

The value of ϕ at each evolutionary stage is indicated on the upper horizontal axis in Fig. 4. As $\phi(t)$ is highly non-linear, we show an extended version of Fig. 2 in Fig. 5. There we see, for example, that during inflation the inflaton field value is relatively constant until the end of inflation (as seen in Fig. 2) and then drops rapidly, moving to negative values during the period of kination.

3 Preheating

Because of its coupling (17, 18) to the inflaton field, ϕ, the effective mass of the Higgs-like boson, h, starts changing non-adiabatically when the inflaton field passes near the point $\phi \simeq 0$, removing a small fraction of the inflaton kinetic energy and producing a large quantity of heavy Higgs particles through the instant preheating mechanism [9]. This mechanism is extremely efficient and, for non-oscillatory models of inflation, the Universe reheats without inflaton decay and is instead reheated by the annihilations and scatterings of the effective heavy Higgs boson whose mass is $m_{\rm h,eff} \simeq \frac{m}{\sqrt{3}} \simeq 5.5 \times 10^{12}\,{\rm GeV}$, via the usual Yukawa interaction $\lambda_\psi h\bar{\psi}\psi$, where ψ represents Standard Model fermionic fields. These massive Higgs particles would scatter mainly into third-generation fermions, with a rate $\propto n_h/m_h^2 \gg H$, as will be seen below when we derive and expression for the Higgs number density, n_h.

After the rapid scattering of Higgs bosons, the energy density of ultra-relativistic particles scales as radiation (the green curve in Fig. 4), which scales as $\rho_r \sim a^{-4}$, and the inflaton field energy density is dominated by its kinetic energy contribution (the red curve), which scales as $\rho_\phi \sim a^{-6}$. We define the reheating temperature, $T_{\rm RH}$, as the temperature when $\rho_r \simeq \rho_\phi$, and we assume that the radiation products are thermalized by that point. As we show later, reheating happens significantly before Big Bang Nucleosynthesis, $T_{\rm RH} \gg 1\,{\rm MeV}$.

It is important to note that the effective Higgs mass decreases rapidly as the inflaton field value decreases. Examining Eqs. (17) and (18), we find that the m-dependent term becomes subdominant when $\phi \lesssim -11.9\,M_P$. This happens significantly before electroweak symmetry breaking. Therefore, the first term is subdominant at the present day and makes a negligible contribution to the physical Higgs boson mass.

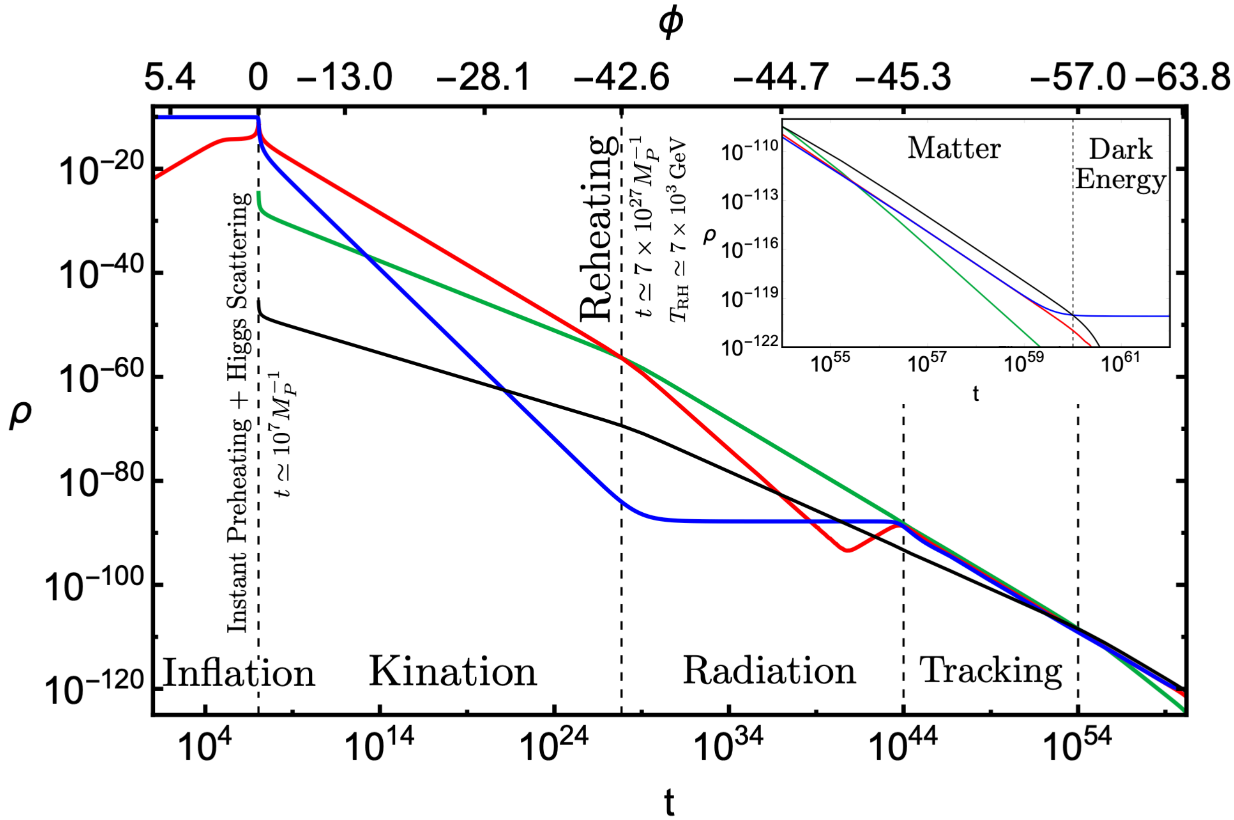

Figure 4: *Evolution of the NO-NO inflaton and radiation energy densities as functions of time. The energy stored in the potential is depicted by the blue curve, the red curve shows the contribution of the inflaton kinetic energy, the green and black curves corresponds to the radiation and matter energy densities respectively. Time is shown on the lower horizontal axis in units of* M_P^{-1}, *and some specific corresponding inflaton field values are given on the upper horizontal axis, in Planck units. The energy densities,* ρ, *are given in units of* M_P^4. *The Figure shows six distinct phases of evolution. At early times,* $t \leq 10^7\, M_P^{-1}$, *the Universe is inflating, and the potential energy is relatively constant with a value given by* $m^2 M_P^2/3$. *As* ϕ *approaches* 0, *its kinetic energy increases and dominates the energy density as the Universe enters a period of kination. When* $\phi \simeq 0$, *instant preheating occurs, the radiation bath is produced and begins to dominate at* $t \simeq 7 \times 10^{27}\, M_P^{-1}$, *which is defined as the moment of reheating. In the radiation-dominated phase, the inflaton slows to a near halt and the potential energy density becomes constant again until the radiation density, which continues to fall like* $\rho_r \sim a^{-4} \sim t^{-2}$, *becomes close to the inflaton potential and kinetic energy densities. At this time,* $t \simeq 10^{44}\, M_P^{-1}$, *the Universe enters a tracking phase that lasts until matter domination at* $t \simeq 10^{54}\, M_P^{-1}$. *At the slightly later time of* $t \simeq 10^{60}\, M_P^{-1}$, *the vacuum energy which has been tuned to its current value* $\sim 10^{-120}\, M_P^4$, *begins to dominate as dark energy.*

The equation of motion for the Higgs field h in Fourier space is

$$\ddot{h}_k + 3\frac{\dot{a}}{a}\dot{h}_k + \left(\frac{k^2}{a^2} + m^2_{\text{h,eff}}\right) h_k = 0\,, \tag{26}$$

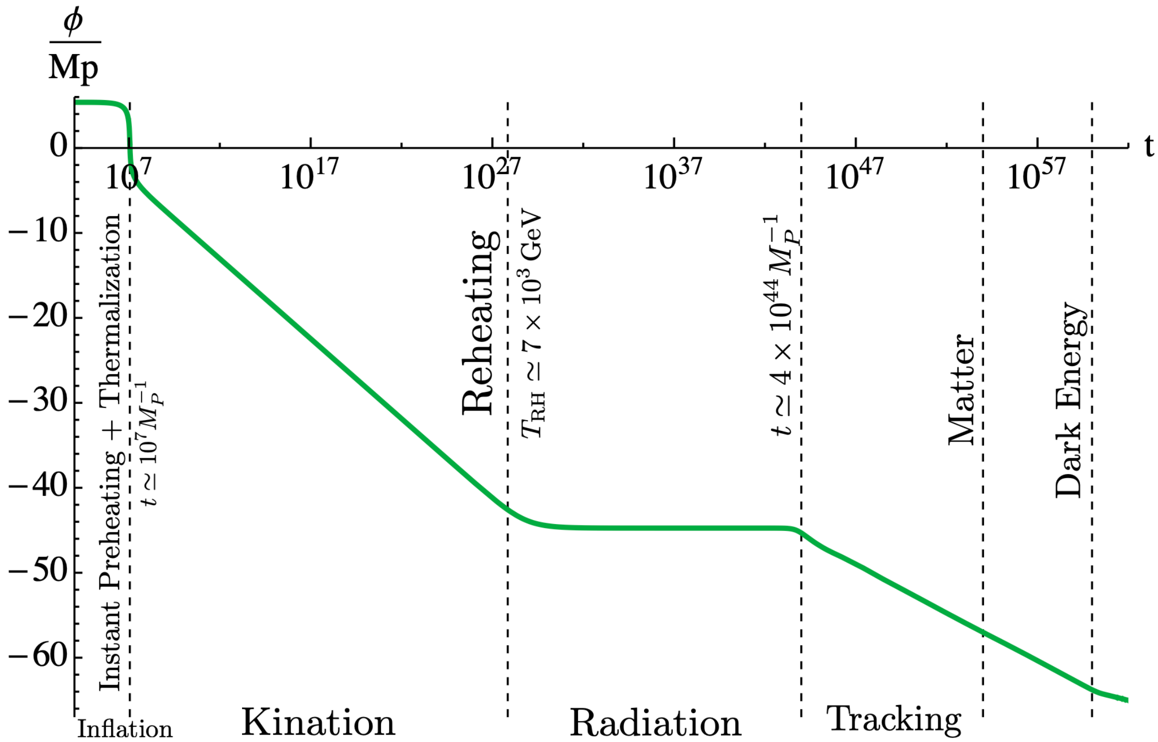

Figure 5: *Evolution of the NO-NO inflaton field ϕ as a function of time in units of M_P^{-1}. During inflation, which lasts until $t \simeq 10^7 M_P^{-1}$, the inflaton field value is relatively constant. When kination begins, the inflaton quickly moves to large negative values until the universe becomes radiation-dominated, and the inflaton effectively ceases to evolve when $\phi \simeq -45 M_P$. When the tracking regime begins, the inflaton starts moving again and at present $\phi \simeq -62 M_P$.*

where k is its momentum, and the angular frequency $\omega_k(t)$ is defined as

$$\omega_k^2(t) = \frac{k^2}{a(t)^2} + m_{\rm h,eff}^2(t)\,. \tag{27}$$

When considering a broad parametric resonance in an expanding Universe, it is convenient to introduce a new variable $H_k = a^{3/2} h_k$, which incorporates the Hubble friction term in (26), so that the equation of motion becomes

$$\ddot{H}_k + \omega_k^2 H_k = 0\,, \tag{28}$$

where

$$\omega_k^2 = \frac{k^2}{a(t)^2} + m_{\rm h,eff}^2 - \frac{3}{4}\left(\frac{\dot{a}(t)}{a(t)}\right)^2 - \frac{3}{2}\frac{\ddot{a}(t)}{a(t)}\,, \tag{29}$$

and the last two terms are responsible for gravitational particle production. In the case of instant preheating their contribution is negligible, and we can approximate the time-dependent harmonic oscillator frequency as $\omega_k^2 \simeq \frac{k^2}{a(t)^2} + m_{\rm h,eff}^2$.

We take as the initial condition for the equation of motion (28) the Bunch-Davies vacuum $H_k(t) \simeq e^{-i\omega_k t}/\sqrt{2\omega_k}$. Since the parametric resonance occurs rapidly, we can safely ignore the expansion of the Universe and approximate the frequency as $\omega_k^2 \simeq k^2 + m_{\rm h,eff}^2$ during this period.

The harmonic-oscillator equation of motion (28) can be used to construct the invariant particle occupation number, n_k, for the Fourier momentum mode k, which is independent of the scale factor, and is given by

$$n_k = \frac{\omega_k}{2}\left(\frac{\left|\dot{H}_k\right|^2}{\omega_k^2} + |H_k|^2\right) - \frac{1}{2}\,. \tag{30}$$

The dominant contribution to particle production occurs when the standard adiabaticity condition is violated, i.e.,

$$\dot{\omega}_k \gtrsim \omega_k^2\,, \tag{31}$$

and we discuss two different modes of production: when $k \simeq 0$ and $k \sim m_{\rm h,eff}$.[8]

If we assume that the momentum values are negligibly small, the adiabaticity violation condition (31) becomes $|\dot{m}_{\rm h,eff}| \gtrsim m_{\rm h,eff}^2$ and, using (19), we find that

$$\frac{32m^2e^{4\phi}}{3(e^\phi+1)^5}\frac{|\dot{\phi}|}{m_{\rm h,eff}^{3/2}} \gtrsim 1\,. \tag{32}$$

From numerical approximations, we find that the adiabaticity condition (32) is violated when $0.1\,M_P \gtrsim \phi \gtrsim -13.8\,M_P$. However, because the physical Higgs particle density is $n_h \propto \int_0^\infty k^2 n_k$, modes with $k \simeq 0$ make a negligible contribution to the total particle number density, and total particle production can be approximated as instantaneous.

We consider next the case when $k \sim m_{\rm h,eff}$. Now the adiabaticity violation condition (31) becomes

$$\frac{32m^2e^{4\phi}}{3(e^\phi+1)^5}\frac{|\dot{\phi}|}{(k^2+m_{\rm h,eff}^2)^{\frac{3}{2}}} \gtrsim 1\,, \tag{33}$$

where we find from numerical calculations that the dominant particle production occurs when $\phi \simeq 0$. We see from Fig. 3 that the magnitude of the velocity of the inflaton field $\dot{\phi}$ is maximized at the end of inflation, when $\phi \simeq 0.6\,M_P$ and $|\dot{\phi}| \simeq 0.7\,m\,M_P$. Near $\phi \simeq 0$, when the dominant particle production occurs, the inflaton velocity is approximately $|\dot{\phi}| \simeq 0.6\,m\,M_P$.

It should be noted that our model is quite different from the model considered in [4], where the authors coupled the two boson fields through the interaction term $-\frac{1}{2}g^2\phi^2\chi^2$,

[8] Particle production becomes exponentially suppressed when $\dot{\omega}_k < \omega_k^2$.

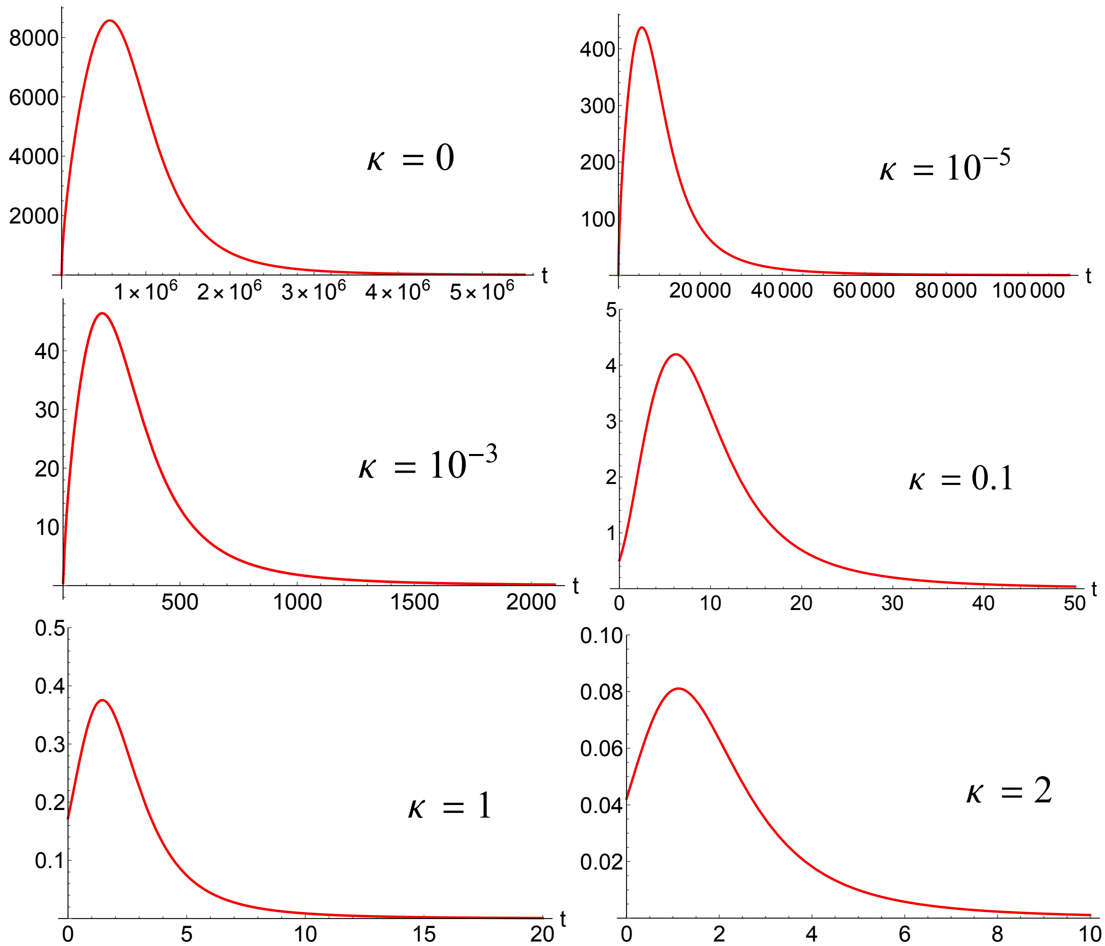

Figure 6: *The left-hand side of the condition (33) that corresponds to the violation of the adiabaticity condition (31) as a function of time in units of m^{-1} for different values of $\kappa =$* $0,\, 10^{-5},\, 10^{-3},\, 0.1,\, 1,$ and 2. *For $\kappa = 1, 2$ the condition (33) is not satisfied, and particle production is exponentially suppressed.*

where χ corresponds to another scalar field. In this case, the effective mass of the χ particles is given by $m_\chi = g|\phi|$. As the inflaton field rolls to infinity and its field value increases, the effective mass m_χ increases as well, until the inflaton field ϕ reaches its maximal value and the χ particles decay to two fermions. In contrast, in our model the instant preheating mechanism produces heavy Higgs bosons whose effective mass $m_{\rm h,eff}$, given by (19), *decreases* with increasing inflaton field value, which then rapidly scatter into ultra-relativistic light particles.

In order to understand better the instant preheating mechanism and adiabaticity violation condition (33), it is convenient to introduce a dimensionless parameter $\kappa \equiv \frac{k}{m_{\rm h,eff}}$. We illustrate in Fig. 6 the violation of the adiabaticity condition (33) as a function of time for different values of κ. For these numerical calculations, we set the parameters at the end of inflation to $\phi \simeq 0.6\, M_P$, $|\dot{\phi}| \simeq 0.7\, m\, M_P$, $t = 0$, and $a(0) = 1$. Therefore, the condition is violated for a relatively long time when $\kappa \simeq 0$, but particle production is relatively instantaneous when $\kappa \sim \mathcal{O}(1)$.

In order to calculate numerically the produced Higgs boson density, we use the following expression

$$n_h = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2\, n_k\,. \tag{34}$$

In Fig. 7 we show the particle occupation number n_k and the integrand $k^2 n_k$ as a function of κ. We see that the dominant Higgs particle production occurs in the range $0 \lesssim k \lesssim 2$, and particle production peaks for the momentum modes $k \simeq m_{\rm h,eff}$.

We find from numerical calculations that the Higgs boson number density after instant preheating, when $\phi \simeq -0.5\, M_P$ and $|\dot{\phi}| \simeq 0.5\, m\, M_p$, is

$$n_h \simeq 4 \times 10^{-4}\, m^3 \simeq 3.5 \times 10^{35}\, \text{GeV}^3\,, \tag{35}$$

where at $\phi \simeq -0.5\, M_P$, $\Gamma_{sc} \sim n_h/m_{h,\rm eff}^2 > H$, and the newly-created Higgs bosons thermalize. Because the Higgs fields are produced at low momentum and are very heavy initially, we can treat them as non-relativistic particles with an energy density given by

$$\rho_h \simeq n_h\, m_{\rm h,eff} \simeq 1.3 \times 10^{-4}\, m^4 \simeq 1.1 \times 10^{48}\, \text{GeV}^4\,, \tag{36}$$

where we use $m_{h,\rm eff} \simeq m/3$ from Eq. (19) for $\phi \simeq -0.5\, M_P$. This is a small fraction ($\sim 10^{-15}$) of the energy density in the inflaton potential and kinetic energy which is of order $m^2 M_P^2$. Shortly after they are produced, the heavy Higgs particles annihilate and scatter into ultra-relativistic products, r, and $\rho_h \simeq \rho_r$, which scales as $\propto a^{-4}$. Note that although instant preheating occurs at a time similar to the onset of kination, the Universe still expands quite rapidly before $a \sim t^{1/3}$ becomes a good approximation which only occurs at a time $t \sim 10^8\, M_P^{-1}$, and a significant amount of dilution occurs as seen by the dips in the energy densities shown in Fig. 4 at $\phi \simeq 0$. This will result in a lower reheating temperature than was found in many models in the literature, as discussed in the next Section.

We need to check that back-reaction effects are negligible during preheating, and do not affect the particle production. In order to take into account the back-reaction effects of h quantum fluctuations, we use the Hartree approximation [4, 10], and write the equation of motion for ϕ as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{64 m^2 e^{4\phi}}{3(e^\phi + 1)^5} + \frac{32 m^2 e^{4\phi}}{3(e^\phi + 1)^5} \langle h \rangle^2 = 0\,, \tag{37}$$

where the vacuum expectation value for h^2 is given by

$$\langle h^2 \rangle = \frac{1}{2\pi^2 a^3} \int_0^\infty \frac{k^2 n_k}{\omega} dk\,. \tag{38}$$

Using the angular frequency (27), numerical calculations show that

$$\langle h\rangle^2 \simeq 5.6 \times 10^{-4} \left(\frac{m}{M_P}\right)^2 \simeq 9 \times 10^{-15} \,. \tag{39}$$

When instant preheating occurs we have $\phi \simeq 0$, and the equation of motion (37) becomes

$$\ddot{\phi} + 3H\dot{\phi} + \frac{2}{3}m^2 + \frac{m^2}{3}\langle h\rangle^2 = 0 \,. \tag{40}$$

One can use the approximation that the back-reaction effects are negligible if the effective inflationary scale $m_{\phi,\,\mathrm{eff}} = m^2 + \frac{m^2}{2}\langle h\rangle^2$ does not change significantly and the term in (40) arising from the quantum fluctuations of h is significantly smaller than the first term. We conclude from Eq. (39) that back-reaction effects are negligible in our model.

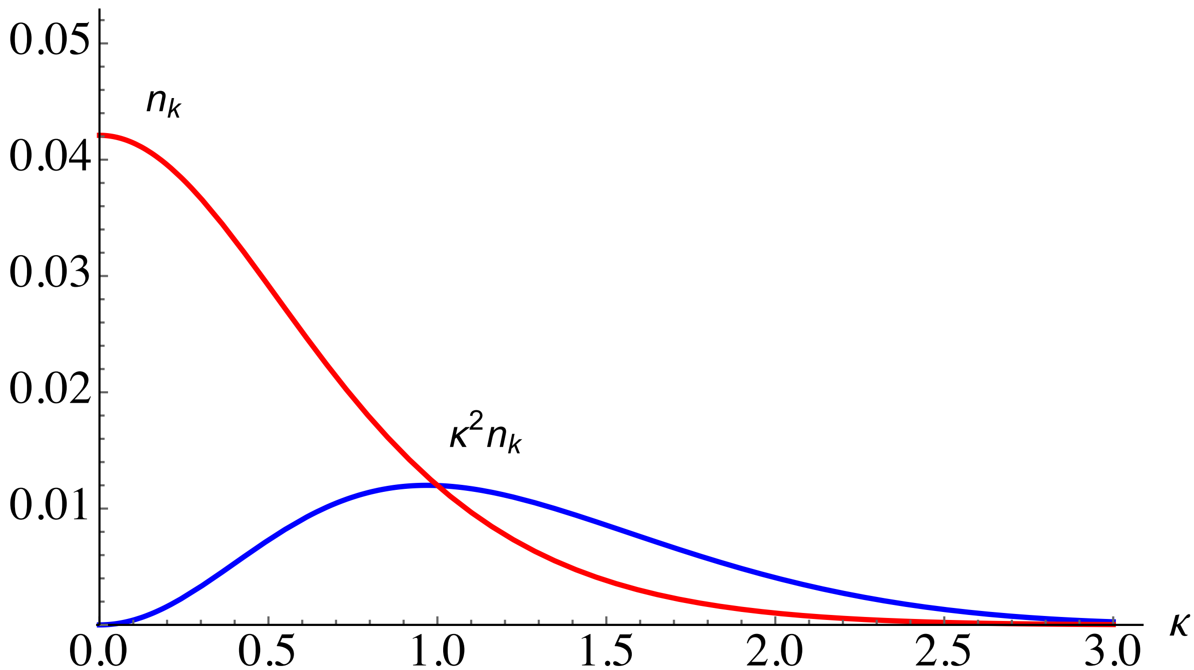

Figure 7: *The particle occupation number n_k (red) and $\kappa^2 n_k$ (blue) in units of m^3 as functions of $\kappa = k/m_{\mathrm{h,eff}}$.*

4 Reheating and the Electroweak Phase Transition

The inflaton field loses only a small fraction of its energy while producing the Higgs bosons non-perturbatively. At the end of inflation, the inflaton field has an energy density $\rho_\phi \simeq 1.1 \times 10^{62}\,\mathrm{GeV}^4$, and we find the following energy density ratio

$$\Delta(t_{\mathrm{end}}) \equiv \frac{\rho_h(t_{\mathrm{end}})}{\rho_\phi(t_{\mathrm{end}})} \simeq 2 \times 10^{-15} \,, \tag{41}$$

where $t_{\rm end} \simeq 1.1 \times 10^7\, M_P^{-1}$ is the time at the end of inflation. When inflation ends, the inflaton potential energy density still dominates the total energy density until the time $t_i \simeq 1.2 \times 10^7\, M_P^{-1}$ when kination begins (see Fig. 4). Between $t_{\rm end}$ and t_i, the Universe still expands rather rapidly and ϕ evolves from $\sim 0.6\, M_P$ to $-0.5\, M_P$. As noted earlier, even though the inflaton kinetic energy dominates the total energy density, the Universe continues to expand rapidly until $t \sim 10^8\, M_P^{-1}$, when $H = 1/3t$ becomes a good approximation.

The radiation bath is quickly formed after preheating. The Higgs bosons annihilate and scatter with a rate $\Gamma_{sc} \sim n_h/m_{h,\rm eff}^2$. As the inflaton evolves to more negative field values, the effective mass of the Higgs boson drops rapidly and the scattering rate quickly overtakes the expansion rate, leading to the formation of the thermal bath. We estimate that this occurs at $t_{sc} \sim 1.25 \times 10^7\, M_P^{-1}$ when $\phi \simeq -1.65\, M_P$. The energy density in radiation at this time is $\rho_h \simeq 3.6 \times 10^{46}\,{\rm GeV}^4$ having been diluted by a factor of ~ 5 relative to the naive estimate which assumes $H \sim 1/3t$ at t_i. This corresponds to a maximum temperature

$$T_{\rm max} = \left(\frac{30\rho_h(t_{\rm end})}{\pi^2 g_*}\right)^{1/4} \simeq 1.5 \times 10^{11}\,{\rm GeV}\,, \tag{42}$$

where $g_* = 915/4$ is the number of relativistic degrees of freedom, assuming instantaneous thermalization at t_{sc}. The temperature, however, quickly drops (by a factor of ~ 30 over that due to kination) as the Universe begins its kination phase as seen in Fig. 4.

The thermal bath does not dominate the energy density of the Universe when it is formed, and initially the radiation density drops faster than ρ_ϕ. At t_{sc}, Δ is similar to that in Eq. (41). However, during kination, $\rho_\phi \propto a^{-6}$ and $\rho_r \propto a^{-4}$. While the Universe is dominated by the kinetic energy of the inflaton, we have $a \sim t^{1/3}$, and $\Delta \sim a^2 \sim t^{2/3}$.

We define the reheating temperature $T_{\rm RH}$ as when $\rho_\phi = \rho_r$, so that reheating occurs when $\Delta(t_{\rm RH}) = 1$, which occurs when $t_{\rm RH} \simeq 7 \times 10^{27}\, M_P^{-1}$. The radiation energy density at reheating is

$$\rho_r(t_{\rm RH}) \ \simeq \ \rho_r(t_{sc})\left(\frac{t_{sc}}{t_{\rm RH}}\right)^{4/3} \ \simeq \ 1.8 \times 10^{17}\,{\rm GeV}^4 \tag{43}$$

according to our numerical calculations. This corresponds to a reheating temperature of

$$T_{\rm RH} = \left(\frac{30\rho_r(t_{\rm RH})}{\pi^2 g_*}\right)^{1/4} \simeq 7\,{\rm TeV}\,. \tag{44}$$

During the radiation-dominated period, the inflaton kinetic energy decreases rapidly, while the inflaton field value and its potential remain roughly constant, as seen in Figs. 4 and 5. Radiation domination continues until ρ_r (which drops as a^{-4}) is similar to ρ_ϕ, whereupon the Universe enters a tracking regime [27]. It is easy to verify that, for the inflaton potential (18), the two tracking criteria

$$\Gamma \equiv \frac{V''V}{(V')^2} \geq 1 \quad \text{and} \quad \left| \frac{d\ln(\Gamma - 1)}{d\ln a} \right| \ll 1, \tag{45}$$

are both satisfied, since $\Gamma = 1 - \frac{e^{\phi}}{4}$ and is very nearly constant when $\phi \lesssim -8\, M_P$. Tracking begins when $t \simeq 10^{44}\, M_P^{-1}$, corresponding to a temperature $T = 70$ keV. During tracking, the radiation energy density and the kinetic and potential energy densities of ϕ track each other, as seen in Fig. 4. As a result, the inflaton field value evolves as well, as seen in Fig. 5. At this time, the energy density in the inflaton is about 1/3 the total energy density. While tracking begins after BBN has begun and therefore does not affect weak interaction freeze-out or the deuterium bottleneck, there may be some residual effects on BBN that merit further study.

Tracking continues until the cold dark matter density begins to dominate the energy density at $t \simeq 10^{54}\, M_P^{-1}$, and shortly thereafter vacuum energy density takes over. Tuning is required to give the present observed values of the fractions of the critical density in dark energy, matter, and radiation Ω_Λ, Ω_m, and Ω_r, respectively.

Before concluding this Section, we comment on the probable effect of including additional degrees of freedom on reheating. In all of the preceding discussion, we have restricted our attention to a simplified model with three fields, x, y, z. We assumed that y is a proxy for the Higgs field, and its coupling to the inflaton field x in Eq. (17) was derived from an expansion of the Kähler potential. In a more realistic model, there would be $N \sim 50$ chiral superfields with similar couplings. We expect Eq. (35) to hold for each of these, and therefore the total number density of fields would scale with N. Since the scattering rate would also scale as N, $\Gamma_{sc} \sim N^2\, n_h / m_{\text{h,eff}}^2 > H(t_{\text{end}})$, thermalization would occur immediately after preheating, and we would expect a higher maximal temperature $T_{\max} \simeq 10^{12}$ GeV. The increased radiation density would also lead to a higher reheating temperature, which we estimate would be roughly $T_{\text{RH}} \sim 4 \times 10^5$ GeV. In this case, tracking would begin earlier, and constraints from BBN may become relevant. However, in this paper we present only the model with a single field associated with the Higgs field, and leave a complete treatment in the minimal supersymmetric extension of the SM (MSSM) for study in future work.

5 Gravitational Wave Spectrum

The fractional energy density of primordial gravitational waves (GWs) relative to the critical energy density in a flat Universe is given by [58]

$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\,\rho_{\rm GW}}{d\ln k} \propto k^{2\left(\frac{3w-1}{3w+1}\right)} \propto a^{3w-1}\,, \tag{46}$$

where $\rho_c \equiv 3H_0^2 M_P^2 \simeq 1.05\,h^2 \times 10^{-5}\,\mathrm{GeV\,cm^{-3}}$ is the critical density of the Universe, $\rho_{\rm GW}$ is the energy density of the gravitational waves, $k = aH$ is the momentum mode at the Hubble horizon crossing, and $w = p/\rho$ is the effective equation of state parameter. When the Universe becomes dominated by the radiation density, $w = 1/3$, and the gravitational wave spectrum remains flat because $\Omega_{\rm GW} \sim$ const.

However, during the kination epoch, $w = 1$, and the fractional energy density of gravitational waves scales as $\Omega_{\rm GW} \sim k \sim a^2$. Therefore, a prolonged period of the inflaton kinetic energy regime could lead to a blue-tilted gravitational wave spectrum capable of contributing significantly to the total radiation density of the Universe and, in turn, affecting the effective number of neutrino species $N_{\rm eff}$ at the time of BBN.

One can use the current limit on $N_{\rm eff} < 3.17$ [59] to set an upper limit on the energy density of gravitational waves today [60],

$$\Omega_{\rm GW} < \left(\frac{4}{11}\right)^{4/3}\frac{7}{8}(N_{\rm eff}-3)\Omega_\gamma\,, \tag{47}$$

where $\Omega_\gamma = (\pi^2/15)T_0^4/\rho_c \simeq 2.47 \times 10^{-5}\,h^{-2}$ and $T_0 = 2.73$ K is the present temperature of the microwave background. In order to ensure that higher-frequency gravitational waves do not affect BBN, we impose the bound [60]

$$I = h^2 \int_{k_{\rm BBN}}^{k_{\rm i}} \Omega_{\rm GW}\, d\ln k \lesssim 10^{-6}\,, \tag{48}$$

where $k_{\rm BBN}$ and k_i are the momentum modes associated with BBN and the onset of kination, respectively. We find numerically that the dominant contribution to the integral (48) comes from the momentum modes when $0.9 \lesssim w \lesssim 1$, which is the case in the range $k_{\rm rad} \leq k \leq k_{\rm kin}$, where $k_{\rm rad}$ is the momentum mode when the radiation energy density becomes significant, and $k_{\rm kin}$ is the momentum mode when the kinetic energy of the inflaton is significantly larger than its potential energy. [9]

[9] A common approximation is that contributions to the integral (48) come from the entire range $[k_i, k_{\rm RH}]$, where the latter corresponds to the momentum mode when radiation dominates. However, this overestimates the contributions to the integral from the ranges $[k_i, k_{\rm kin}]$ and $[k_{\rm rad}, k_{\rm RH}]$ where $w < 0.9$, e.g., $w \simeq 0.1$ at k_i.

During the kination epoch, the fractional energy density of the gravitational waves is given by [19]

$$\Omega_{\rm GW} \simeq \varepsilon \Omega_\gamma h_{\rm GW}^2 \left(\frac{k}{k_{\rm rad}}\right) \left[\ln\left(\frac{k}{k_{\rm kin}}\right)\right]^2 , \tag{49}$$

where $\varepsilon = 2R_i \left(\frac{3.36}{g_*}\right)^{1/3}$ with $R_i = \frac{81}{32\pi^3}$, which takes into account the contribution of the massless scalar degrees of freedom, and $h_{\rm GW}^2 = \frac{1}{8\pi}\left(\frac{H(t_{\rm end})}{M_P}\right)^2$ is the dimensionless gravitational wave amplitude.

Using the expression (49) as the integrand in (48), we find

$$I \simeq 2\varepsilon h_{\rm GW}^2 \Omega_\gamma h^2 \left(\frac{k_{\rm kin}}{k_{\rm rad}}\right) , \tag{50}$$

where we have neglected the subdominant logarithmic contribution to the integral because $k_{\rm kin} \gg k_{\rm rad}$. Next, we rewrite the ratio $k_{\rm kin}/k_{\rm rad}$ as

$$\frac{k_{\rm kin}}{k_{\rm rad}} = \frac{a_{\rm kin}}{a_{\rm rad}}\frac{H_{\rm kin}}{H_{\rm rad}} \simeq 2.2 \times 10^{13}, \tag{51}$$

and we find

$$I \simeq 0.3\, \varepsilon\, \Omega_\gamma\, h^2. \tag{52}$$

Inserting the numerical values, we find

$$I \simeq 6 \times 10^{-7} , \tag{53}$$

which is consistent with the bound (48), and would be reduced if additional chiral superfields couple to the inflaton.

6 Gravitino Production

Gravitinos can be produced after inflation either by the direct decay of the inflaton or through thermal production in the newly-created radiation bath [3, 61–81]. In more conventional oscillatory models, inflaton decay products thermalize rapidly to a temperature $T_{\rm max}$. Then, while the Universe is dominated by the inflaton oscillations and undergoes effective matter-dominated expansion, the temperature of the radiation falls as $T \sim a^{-3/8}$ until the Universe becomes radiation-dominated at the scale defined as $T_{\rm RH}$ [23, 80–85]. In this case, the relevant Boltzmann equation can be written as

$$\frac{dY}{dT} = -\frac{8}{3}\frac{R}{HT^9} , \tag{54}$$

where $Y \equiv n_{3/2}/T^8$, $R = \langle\sigma v\rangle\zeta(3)^2T^6/\pi^4$ is the production rate per unit volume, and $\langle\sigma v\rangle$ is the gravitino production cross section [80, 81]. While dominated by inflaton oscillations, the Hubble parameter may be written as $H = \sqrt{\alpha/3}T^4/T_{\rm RH}^2 M_P$. Integrating the equations of motion from $T_{\rm max}$ to $T_{\rm RH}$, one finds that the abundance of gravitinos is linear in the reheating temperature with a negligible dependence on $T_{\rm max}$,

$$\begin{aligned}\frac{n_{3/2}}{s} &\simeq \frac{\zeta(3)^2}{\pi^4\sqrt{3\alpha^3}}\left(M_P^2\langle\sigma v\rangle\right)\frac{T_{\rm RH}}{M_P} \\ &\simeq 1.6\times10^{-18}\left(1+0.89\frac{m_{1/2}^2}{m_{3/2}^2}\right)\frac{T_{\rm RH}}{10^4\ \text{GeV}}\,, \end{aligned} \tag{55}$$

where $\alpha = \pi^2 g_*/30$, $m_{1/2}$ is a universal high-scale gaugino mass and the terms in the parentheses correspond to the production of the transverse and longitudinal gravitino components, respectively.

However, unlike oscillatory models of inflation, in our NO-NO model reheating is not produced by inflaton decay. Therefore, gravitino production occurs solely through scattering in the thermal plasma produced by Higgs decays. As noted earlier, while the Universe is in the epoch of kination, a radiation bath is produced with initial temperature $T_{\rm max}$ and then cools as $T \sim a^{-1}$. In this case, the relevant Boltzmann equation can be written as

$$\frac{dY}{dT} = -\frac{R}{HT^4}\,, \tag{56}$$

where now $Y \equiv n_{3/2}/T^3$, and the Hubble parameter may be written as $H = \sqrt{\alpha/3}T^3/T_{\rm RH}M_P$ during the kination-dominated period. Integration of (56) yields

$$\begin{aligned}\frac{n_{3/2}}{s} &\simeq \frac{3\sqrt{3}\zeta(3)^2}{4\pi^4\alpha^{3/2}}\left(M_P^2\langle\sigma v\rangle\right)\frac{T_{\rm RH}}{M_P}\ln\frac{T_{\rm max}}{T_{\rm RH}} \\ &\simeq 3.7\times10^{-18}\left(1+0.89\frac{m_{1/2}^2}{m_{3/2}^2}\right)\frac{T_{\rm RH}}{10^4\ \text{GeV}}\ln\frac{T_{\rm max}}{T_{\rm RH}}\,. \end{aligned} \tag{57}$$

We see that in addition to a factor of ~ 2 enhancement of the numerical prefactor, there is a substantial logarithmic boost factor $\ln T_{\rm max}/T_{\rm RH}$ due to production during the kination-dominated era. [10]

We assume that the gravitino is not the lightest supersymmetric particle (LSP), in which case it is expected to be unstable with a lifetime $\Gamma_{3/2} \sim m_{3/2}^2/M_P^2$. Since this rate is relatively small for $m_{3/2} \ll M_P$, a light gravitino may decay so late as to disturb BBN. We assume that the gravitino is sufficiently heavy, $\gtrsim \mathcal{O}(10)$ TeV, for this not to be an issue. However, the decays of the gravitinos eventually produce lighter supersymmetric

[10] Particle production during kination was also considered in [23], though the case applicable to gravitino production was not considered there.

particles, in particular the lightest neutralino, χ, and the inflatino, which is much lighter than χ in the simplified NO-NO model presented here, in which we have not discussed how supersymmetry breaking is communicated to the supersymmetric partners of SM particles. In this simplified model χ is a Higgsino with a lifetime for decays to inflatinos that vastly exceeds the age of the Universe, in which case it is a candidate for cold dark matter. Requiring $\Omega_\chi h^2 \lesssim 0.12$ [37] and assuming conservatively that most gravitino decays produce χ particles, we obtain an upper limit on the gravitino density relative to the entropy density:

$$\frac{n_{3/2}}{s} \lesssim 4.4 \times 10^{-12} \left(\frac{100\,\mathrm{GeV}}{m_\chi}\right) . \tag{58}$$

For sufficiently heavy gravitinos ($m_{3/2} \gtrsim 10$ TeV), this bound is more important than that from BBN (see, e.g., [86]). We can combine the expected abundance from scattering in Eq. (57) and the limit in Eq. (58) to find the constraint

$$\Omega_{\mathrm{DM}} h^2 \simeq 10^{-7} \left(\frac{m_\chi}{100\,\mathrm{GeV}}\right) \left(\frac{T_{\mathrm{RH}}}{10^4\,\mathrm{GeV}}\right) \ln \frac{T_{\max}}{T_{\mathrm{RH}}} \lesssim 0.12\,, \tag{59}$$

where we have assumed in Eq. (59) that $m_{1/2} \ll m_{3/2}$, so that the production of the longitudinal components can be ignored. Using the values of $T_{\max}$ (42) and T_{RH} (44) estimated earlier, we find $m_\chi \lesssim 10\,\mathrm{PeV}$, which is compatible with generic supersymmetric models of dark matter. Even in a more complete NO-NO scenario with $N \sim 50$ chiral fields, the increased reheating temperature would still allow $m_\chi \lesssim 200\,\mathrm{TeV}$. These limits do not include the additional thermal freeze-out production of neutralinos. Consequently, the right-hand side of (59) should be reduced by the value of $\Omega_\chi h^2$ left over from thermal freeze-out [63], strengthening the limit on the direct production of gravitinos. However, these limits could be relaxed if there were late-time entropy production (see, e.g., [31,34,35,87]).

However, this discussion is over-simplified, since it neglects supersymmetry breaking in the visible sector. Moreover, it is also possible that the LSP is in some hidden sector [88], or that R-parity is broken. We plan to return to these issues in a future paper, but conclude provisionally that LSP dark matter production in gravitino decays is not a showstopper for the NO-NO model.

7 Results and Discussion

We have presented in this paper a novel model of no-scale supergravity inflation in which there are no inflaton oscillations at the end of the inflationary era. Instead, the inflaton preheats instantly through a coupling to a Higgs-like field that subsequently decays into relativistic matter particles. The density of the Universe is then dominated by the inflaton kinetic energy density (kination) until the density of ultra-relativistic particles starts to dominate. Reheating occurs at a temperature $\mathcal{O}(10^3 - 10^5)$ GeV, far above that of Big Bang Nucleosynthesis. Subsequently, the inflaton field then rolls to infinity while losing energy density. This continues until the radiation density falls to a level near the inflaton potential energy, and tracking begins where the inflaton kinetic and potential energy density are similar to that of radiation. Eventually the Universe is dominated by cold dark matter, and finally dark energy.

We have estimated the production of gravitational waves and gravitinos in this non-oscillatory scenario. We have found that the density of relic gravitational waves is below the present upper limit on the effective number of light neutral particle species imposed by the success of conventional BBN calculations. We have also estimated the density of LSPs produced by gravitino decays, finding that this is below the upper limit on the dark matter density from Planck 2018 data [37] throughout the expected range of LSP masses. One issue that we have not addressed in this paper is baryogenesis, but we anticipate that leptogenesis [89], the Affleck-Dine mechanism [90] and electroweak-scale baryogenesis should all be possible in this NO-NO scenario.

We conclude that this no-oscillation model that invokes preheating provides an interesting alternative to no-scale supergravity models in which inflaton oscillations reheat the Universe, and warrants further study.

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A Kähler Transformations and the Choice of Basis

Field transformations relate equivalent representations of the same no-scale model in different bases. We give some examples in this Appendix.

As a first example, consider the following form for the SU(3, 1)/SU(3)×U(1) Kähler potential

$$K = -\alpha \log \left(T + \bar{T} - |\phi|^2 - |S|^2 \right) , \tag{A1}$$

where the complex field T is associated with the volume modulus, and the complex fields ϕ and S are matter-like fields. In order to express the Kähler potential (A1) in the symmetric form [42, 43, 91]

$$K = -\alpha \log\left(1 - x\bar{x} - y\bar{y} - z\bar{z}\right) , \tag{A2}$$

we relate the complex scalar fields T, ϕ, and S to the complex fields x, y, z appearing in the symmetric Kähler potential form (A2) by using the transformations

$$x = \frac{1-2T}{1+2T}; \qquad y = \frac{2\phi}{1+2T}; \qquad z = \frac{2S}{1+2T}, \tag{A3}$$

whose inverse transformations are

$$T = \frac{1}{2}\left(\frac{1-x}{1+x}\right); \qquad \phi = \frac{y}{1+x}; \qquad S = \frac{z}{1+x}. \tag{A4}$$

When the fields are transformed according to Eqs. (A3, A4), the effective superpotential in the (T, ϕ, S) basis transforms as

$$W(T, \phi, S) \longrightarrow \widetilde{W}(x, y, z) = (1+x)^{\alpha}\, W(T, \phi, S)\,, \tag{A5}$$

which leaves invariant the extended Kähler potential G.

Similarly, the Kähler potential that parametrizes an (SU(2, 1)/SU(2)×U(1))×(SU(1, 1)/U(1) coset manifold

$$K = -\alpha \log\left(S + \bar{S} - |\phi|^2\right) - \beta \log\left(T + \bar{T}\right) , \tag{A6}$$

can be transformed to a symmetric form for the Kähler potential by applying the following transformations

$$x = \frac{1-2S}{1+2S}; \qquad y = \frac{2\phi}{1+2S}; \qquad z = \frac{1-2T}{1+2T}, \tag{A7}$$

whose inverse relations are

$$T = \frac{1}{2}\left(\frac{1-z}{1+z}\right); \qquad \phi = \frac{y}{1+x}; \qquad S = \frac{1}{2}\left(\frac{1-x}{1+x}\right). \tag{A8}$$

If we transform the coordinates using the relations (A7)-(A8), the effective superpotential transforms as

$$W(T, \phi, S) \longrightarrow \widetilde{W}(x, y, z) = (1+x)^{\alpha}\,(1+z)^{\beta}\, W(T, \phi, S)\,, \tag{A9}$$

again leaving invariant the extended Kähler potential G.

B Supersymmetry Breaking

In this Appendix we follow the treatment presented in [31, 44, 52] and introduce an adjustable parameter for supersymmetry breaking without invoking additional fields. We introduce the following superpotential in the (T, ϕ, S) basis:

$$W(T, \phi, S) = W_{\rm inf} + W_{\rm SUSY} \;, , \tag{B1}$$

where $W_{\rm inf}$ is the inflationary superpotential part and $W_{\rm SUSY}$ is responsible for supersymmetry breaking, and is given by

$$W_{\rm SUSY}(T, \phi, S) \;=\; \lambda \left(2S - \frac{\phi^2}{3}\right)^{\frac{1}{2}(\alpha + \sqrt{3\alpha})} (2T)^{\beta/2} \;, \tag{B2}$$

where λ is an adjustable constant. It is important to note that the supersymmetry-breaking superpotential $W_{\rm SUSY}$ does not affect the inflationary potential and also does not spoil its non-oscillatory behavior.

If we use the transformations (A8) together with the superpotential transformation (A9), we find that the supersymmetry-breaking superpotential contribution in the symmetric (x, y, z) basis is given by

$$W_{\rm SUSY}(x, y, z) = \lambda \left(1 - x^2 - y^2\right)^{\frac{1}{2}(\alpha + \sqrt{3\alpha})} (1 + x)^{-\sqrt{3\alpha}} \left(1 - z^2\right)^{\beta/2} \;, \tag{B3}$$

where we dropped the tilde over the left-hand side. In this case, supersymmetry breaking is generated through an F-term, which is given by

$$F^i = -e^{G/2} K^{i\bar{j}} G_{\bar{j}} \,. \tag{B4}$$

Combining the superpotential (B3) with the inflationary superpotential (11) and setting the model parameters to $c = 4$, $b = \alpha = 2$, and $\beta = 1$, we find that supersymmetry is broken through an F-term for the inflaton field, x, which is given by

$$|F^x|^2 \;=\; \frac{6\lambda^2 e^{-\sqrt{6}\phi}}{(1 + \cosh\phi)^2} \neq 0 \,, \quad \text{when} \quad \lambda \neq 0 \,, \tag{B5}$$

where we used the canonical normalization of the field x and safely ignored the contribution arising from the field $\langle y \rangle \ll \langle x \rangle$, which is associated with the Higgs field. We find numerically that supersymmetry breaking at the present day, when $\phi \simeq -62$, is given by

$$|F_x|^2 \;\simeq\; 3 \times 10^{13}\, \lambda^2 \,. \tag{B6}$$

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