Frascati Physics Series Vol. 69 (2019)
LFC19: Strong dynamics for physics within and beyond the Standard Model at LHC and Future Colliders
September 9-13, 2019

MODELING DOUBLE PARTON SCATTERING AT LHC

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Abstract

We examine present data for double parton scattering at LHC and discuss their energy dependence from its earliest measurements at the ISR. Different models for the effective cross-section are considered and their behavior studied for a variety of selected final states. We point out that data for $pp- > 4\ jets$ or $pp \to\ quarkonium\ pair$ indicate σ_{eff} to increase with energy. We compare this set of data with different models, including one inspired by our soft gluon resummation model for the impact parameter distribution of partons.

1 Introduction

Double parton scattering in hadron collisions has been searched for and measured for more than 30 years. Recently, the ATLAS collaboration [1] has examined all existing data for Double Parton Scattering events, from ISR to LHC 13 TeV, and a value for the the effective cross-section has been extracted. For a process of the type $pp \to A + B + X$ the following expression was used

$$\sigma^{AB}_{DPS} = \frac{k}{2}\frac{\sigma^{A}_{SPS}\sigma^{B}_{SPS}}{\sigma_{eff}} \qquad (1)$$

with k a symmetry factor to indicate identical or different final states, and σ_{eff} interpreted as the overlap area (in the transverse plane) between the interacting partons.

In this note, the energy dependence of σ_{eff} will be discussed in light of a few models and a rather general theorem. We shall start by presenting in Sect. 2 the general framework for multi parton scattering as recently presented by D'Enterria in [2] and then apply this formalism to show that, in general, σ_{eff} cannot be asymptotically a constant.

In Sect. 3 and Sect. 4 we consider various strategies for the calculation of σ_{eff}, a geometrical one in which σ_{eff} is interpreted as the area occupied by the partons involved in the collision and thus obtain it from modelling the impact parameter distribution of partons, another one in which the area is directly obtained as the Fourier transform of the scattering amplitude. These different strategies may lead to different energy dependence, as we shall see.

2 Matter distribution in a hadron

Theoretically multi-parton scattering (MPS) has been of great interest [3, 4, 5, 6, 7, 8, 9]. A key element in an analysis of an n-parton process (NPS) with final particle states $(a_1, a_2, ...a_n)$ in terms of the single-parton processes (SPS) is the role played by an *effective parton cross-section* defined as follows:

$$\sigma^{NPS}_{h_1 h_2 \to a_1, a_2, ...a_n} = \Big[\frac{m}{\Gamma(n+1)} \frac{\sigma^{SPS}_{h_1 h_2 \to a_1} \sigma^{SPS}_{h_1 h_2 \to a_2} .....\sigma^{SPS}_{h_1 h_2 \to a_n}}{[\sigma_{eff,NPS}]^{n-1}}\Big]. \tag{2}$$

As Eq. (2) deals with *probabilities* rather than *probability amplitudes*, it is clear that the description is *semi-classical* and ignores any correlation between production of particles. On the other hand, the degeneracy factor m in these equations, to be defined momentarily, does distinguish between identical and non-identical particle states and thus must be thought as of quantum mechanical origin. For a two parton process (DPS) (say, a_1, a_2), $m = 1$ if the two particle states are identical ($a_1 = a_2$) and $m = 2$ if they are different ($a_1 \neq a_2$). For a three particle process (TPS), $m = 1$ if $a_1 = a_2 = a_3$; $m = 3$ if $a_1 = a_2$ and $m = 6$ if $a_1 \neq a_2 \neq a_3$. Etc.

Under a set of *reasonable* hypothesis of factorization of parallel and transverse momenta, the quantity of interest σ^{NPS}_{eff} is approximated in terms of the normalized *single* parton distribution or, generally a matter distribution $T(\mathbf{b})$ inside a hadron in impact parameter space, as follows

$$\int (d^2\mathbf{b}) T(\mathbf{b}) = 1; \quad \Sigma^{(n)} \equiv \int (d^2\mathbf{b}) T^n(\mathbf{b}); \quad \sigma^{NPS}_{eff} = [\Sigma^{(n)}]^{-1/(n-1)} \tag{3}$$

Before turning our attention to the crucial input of the single parton *overlap function* we present here an argument as to why σ_{eff} cannot -in general i.e., for all types of final states in DPS or MPS scattering- be a constant.

In particular, we shall now show that if $\sigma_{eff}(s)$ approaches a constant as $s \to \infty$, then *all*, multi-parton cross-sections $\sigma^n_{a_1;....a_n}(s)$ must also approach constants asymptotically under the very mild hypothesis that $\sigma^{n+1}_{a_1;....a_{n+1}}(s) < \sigma^n_{a_1;....a_n}(s)$ for $a_i \neq a_j$. Consider in fact Eq.(9) of [2]

$$\sigma^{(2)}_{a_1;a_2}(s) = (\frac{m}{2}) \frac{\sigma^{(1)}_{a_1}(s) \sigma^{(1)}_{a_2}(s)}{\sigma_{eff}(s)}; m = 2 \text{ if } a_1 \neq a_2; m = 1 \text{ if } a_1 = a_2; \tag{4}$$

in an obvious notation. Let

$$(i)\ \sigma_{a_i}(s) \to L_i(s); \text{where } L_i(s) \text{ increase with } s; \qquad (ii)\ \sigma_{eff}(s) \to \text{a constant}; \tag{5}$$

Then, it follows from (i) and (ii) that

$$\text{for } a_1 \neq a_2 : \sigma^{(2)}_{a_1;a_2}(s) \propto L_1(s) L_2(s) \text{ but then } \Rightarrow \Big[\frac{\sigma^{(2)}_{a_1;a_2}(s)}{\sigma^{(1)}_{a_1}(s)}\Big] \propto L_2(s) \text{ increases with } s; \tag{6}$$

and thus not bounded by a constant thereby violating the initial hypothesis. Hence, $L_2(s)$ can not increase with s but must be bounded by a constant. We can repeat the proof by exchanging $a_1 \leftrightarrow a_2$

and show that also $L_1(s)$ must be a constant. Ergo, also $\sigma^{(2)}_{a_1;a_2}(s)$ must go to a constant as $s \to \infty$. Extension of the above to the identical case ($a_1 = a_2$) and for $n = 3, 4, ...$ are left as exercises to the reader. The proof is specially easy if Eqs.(3) & (7) of [2] are recalled. In the next section, we turn our attention to $T(\mathbf{b})$.

3 The BN model for σ_{eff}

In this section we examine a model for σ_{eff}, in which the impact parameter distribution of partons is obtained from soft gluon resummation. As we shall see later, this model reproduces the order of magnitude of σ_{eff} but bears different energy trend depending on the PDF used. A suitable model for a normalized $T(\mathbf{b})$ -albeit with a different name $A(\mathbf{b})$- has been an object of our attention for over two decades and detailed references can be found in our review [10]. We start with a model in which the area occupied by the partons involved in parton scattering can be related to soft gluon resummation. In this model for the total cross-section, the energy behaviour of the total and inelastic cross-sections are obtained in the eikonal formalism, with *mini-jets*, partons with $p_t > p_{tmin} \approx 1.1 - 1.5$ GeV, to drive the rise and soft gluon resummation to tame it. The impact parameter distribution is determined by the Fourier transform of the k_t distribution of soft gluons emitted during semi-hard parton scattering. Namely the normalized matter distribution in impact parameter space, $T(\mathbf{b})$ in this model, is

$$A(b,s) = N(s)\mathcal{F}[\Pi(\mathbf{K}_t)] = N(s)\int d^2\mathbf{K}_t \int d^2\mathbf{b} e^{i\mathbf{K}_t\cdot\mathbf{b}} e^{-h(b,s)}; \quad h(b,s) = \int_0^{qmax} d^3\bar{n}(k)[1 - e^{-i\mathbf{k_t}\cdot\mathbf{b}}] \quad (7)$$

where the overall distribution $\Pi(\mathbf{K}_t)$ is obtained by resummation of soft gluons emitted with average number $\bar{n}(\mathbf{k})$. The above expressions are semi-classical and can be obtained by summing all the gluons emitted with momentum $\mathbf{k}_t$ in a Poisson like distribution. The effect of imposing energy-momentum conservation to all possible distributions results in the factor among square brackets in Eq. (7). Such factor allows to integrate in k_t down to zero, if $\bar{n}(k)$ is no more singular than an inverse power. While this is true in QED, for gluons this is not possible. In our model for the total cross-section, which is related to large distance behaviour of the interaction, the impact parameter distribution is related to very small k_t values. This implies including very small k_t values, lower than Λ_{QCD}, values usually not included in the resummation or "lumped" into an intrinsic transverse momentum. In order to evaluate $h(b,s)$ down to such low values, we proposed a phenomenological approximation for $\alpha_s(k_t \to 0)$, namely our phenomenological choice is

$$\alpha_s(k_t \to 0) \propto [\frac{k_t}{\Lambda_{QCD}}]^{-2p}; \qquad \alpha_s(k_t >> QCDscale) = \alpha_s^{asym-free}(k_t) = \frac{1}{b_0 \ln \frac{k_t^2}{\Lambda^2_{QCD}}} \quad (8)$$

with $1/2 < p < 1$. Our model for σ_{eff}, using $A(b,s)$ from Eq. (7) is given as

$$\sigma_{eff}(s) = \frac{[\int d^2\mathbf{b} e^{-h(b,s)}]^2}{\int d^2\mathbf{b} e^{-2h(b,s)}} \quad (9)$$

We have indicated that the function $h(b,s)$ depends upon the c.m.s energy of the collision, so will then be true also for $A(b,s)$. Because of the minimum transverse momentum p_{tmin} allowed to the minijet cross-section, q_{max} will depend also on p_{tmin}. Through an average procedure [11], one can obtain $< q_{max}) >$ as a function of $\sqrt{s}, PDF, p_{tmin}$. The results from this resummation can then be used to model the eikonal function and calculate inclusive quantities such as total and inelastic cross-section. In our model, soft and semi-hard gluons contribute to the observed rise of the total cross-section with soft gluons tempering the

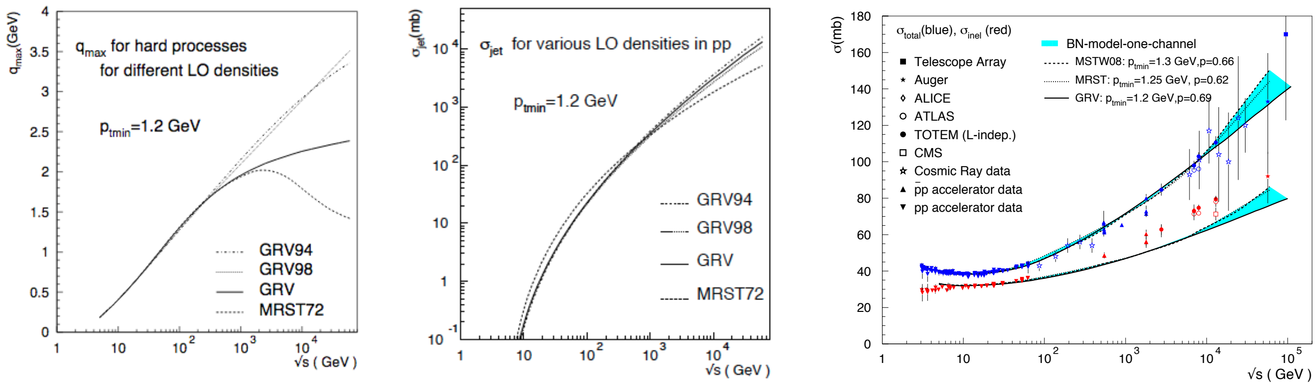

Figure 1: For a given set of PDFs with corresponding p_{tmin}, left and center plots respectively show the maximum transverse momentum allowed to soft gluons emitted by partons participating to a semi-hard collision and the behaviour of the minijet cross-section used in the model from [12]. The figure at right shows the corresponding description of total and Non Single Diffractive inelastic cross-section with appropriate choices of the singularity parameter p.

fast rise (with energy) due to the mini-jet cross section. In Fig. 1 results for $< q_{max} >$ and $\sigma_{jet}(\sqrt{s}, p_{tmin})$ are shown four different LO PDFs, together with the total or inelastic cross-section corresponding to the indicated for parameter choice, including updated PDFs, such as MSTW. One should notice that the energy behaviour of $< q_{max} >$ is different for different densities, as does the one from σ_{jet}, but that they compensate in the predicted behaviour of the total cross-sections, which both smoothly rise in accordance with the Froissart bound, as shown in [13]. This will not be true for σ_{eff}, as the model produces an energy dependence of σ_{eff} which correlates only with the energy dependence of q_{max}, i.e. the upper limit of integration over soft gluon spectrum, so that if $q_{max} \uparrow \sqrt{s}$, then $\sigma_{eff} \downarrow \sqrt{s}$ and vice-versa.

4 The elastic amplitude and σ_{eff} energy dependence

According to [2] and following the summary shown in Sec. 2,

$$\int (d^2\mathbf{b}) T(\mathbf{b}) = 1; \quad \Sigma^{(2)} \equiv \int (d^2\mathbf{b}) T^2(\mathbf{b}); \quad \sigma_{eff}^{DPS} = \frac{1}{\Sigma^{(2)}} \tag{10}$$

The above arrives upon considering factorization between the hard jet cross-sections and the impact parameter distribution of the involved partons, whose $\mathcal{F}$-transform gives the transverse momentum of partons involved in the hard cross-section. This model has a theoretical basis, but one needs an expression for $T(\mathbf{b})$ to use. The derivation in D'Enterria gives the following expression for T($\mathbf{b}$):

$$T(\mathbf{b}) = \int d^2\mathbf{b_1} f(\mathbf{b_1}) f(\mathbf{b} - \mathbf{b_1}) \tag{11}$$

where $f(\mathbf{b})$ describes the transverse parton density of the hadron.
Apart from phenomenological fits of the type $e^{-(b/scale)^m}$, which have problems with analyticity if $m < 1$ [13] consider what is at the root of the formalism being considered regarding the transverse spatial (in short, the b)-distribution adopted in Eq.(11). Also, we can recall the lessons learnt from analyticity of hadronic form factors and the elastic amplitudes.

One begins with $f(\mathbf{b})$, a normalized b-density function and its Fourier transform, the transverse momentum distribution $\hat{f}(\mathbf{q})$ for a single parton, as follows:

$$f(\mathbf{b}) = \int [\frac{d^2\mathbf{q}}{(2\pi)^2}] e^{i\mathbf{b}\cdot\mathbf{q}}\ \hat{f}(\mathbf{q}); \quad \hat{f}(\mathbf{q}) = \int (d^2\mathbf{b}) e^{-i\mathbf{b}\cdot\mathbf{q}}\ f(\mathbf{b}); \quad \hat{f}(\mathbf{q}=\mathbf{0}) = \int (d^2\mathbf{b}) f(\mathbf{b}) = 1 \tag{12}$$

Let us consider this parton distribution first in momentum space and then in b-space. The simplest case to start with is that of collinear partons. The probability density that two-partons are at the same momentum transfer is given by

$$\hat{T}(\mathbf{q}) \equiv\ [\hat{f}(\mathbf{q}]^2; \qquad with \qquad \hat{T}(\mathbf{q}=\mathbf{0}) = 1 \tag{13}$$

whose Fourier transform $T(\mathbf{b})$ reads

$$T(\mathbf{b}) \equiv\ \int [\frac{d^2\mathbf{q}}{(2\pi)^2}] e^{i\mathbf{b}\cdot\mathbf{q}} \hat{T}(\mathbf{q}) = \int [\frac{d^2\mathbf{q}}{(2\pi)^2}] e^{i\mathbf{b}\cdot\mathbf{q}} [\hat{f}(\mathbf{q}]^2 = \int (d^2\mathbf{b}_1) f(\mathbf{b}_1) f(\mathbf{b}-\mathbf{b}_1) \tag{14}$$

which exactly reproduces Eq.(11). Also, by virtue of Eq.(12;13), $T(\mathbf{b})$ is properly normalized, viz.,

$$\int (d^2\mathbf{b}) T(\mathbf{b}) = \int d^2(\mathbf{b}) \int (d^2\mathbf{b}_1) f(\mathbf{b}_1) f(\mathbf{b}-\mathbf{b}_1) = [\int (d^2\mathbf{b}) f(\mathbf{b})]^2 = 1; \quad \hat{T}(\mathbf{q}=\mathbf{0}) = \int (d^2\mathbf{b}) T(\mathbf{b}) = 1. \tag{15}$$

Now to some considerations about the effective cross-section $\sigma_{eff}(s)$, which for this simple identical parton model shall be taken to be (with a factor of a 1/2)

$$2\sigma_{eff}(s) = \Big[\frac{1}{\int (d^2\mathbf{b}) T^2(\mathbf{b})}\Big]; \tag{16}$$

but, by virtue of Eqs.(13) *et sec*, it follows that

$$\int (d^2\mathbf{b}) T^2(\mathbf{b}) = \int [\frac{d^2\mathbf{q}}{(2\pi)^2}] \hat{f}(\mathbf{q}]^2 \hat{f}(-\mathbf{q}]^2 = \int [\frac{d^2\mathbf{q}}{(2\pi)^2}] |\hat{f}(\mathbf{q}|]^4;$$

Since, $\hat{f}(\mathbf{q}=\mathbf{0}) = 1$, at first sight, it may appear reasonable to assume that it is the elastic form factor. So, for this form factor assuming the dipole form, we have

$$\hat{f}(\mathbf{q}) = \frac{1}{[1 + (q^2/t_o(s))]^2};$$
$$\sigma_{eff}^{(el)}(s) = \frac{1}{\int [\frac{d^2\mathbf{q}}{(2\pi)^2}] \frac{1}{[1+(q^2/t_o(s))]^8}} = \Big[\frac{14\pi}{t_o(s)}\Big]. \tag{17}$$

To get a simple estimate, we can employ the result from a fit to the elastic differential cross-section, discussed in [14]. At 13 TeV, our estimate for the elastic scattering form-factor value (work in preparation) is $t_o(13\ TeV) \approx\ 0.6\ GeV^2$, leading to

$$\sigma_{eff}^{(el)}(13\ TeV) \approx\ 28.6\ milli-barns. \tag{18}$$

We notice that the value predicted for σ_{eff} appears large compared to present data [1]. Of course, what the above naive calculation might be telling us is that $\hat{f}(\mathbf{q})$ is related not so much to the elastic but to an "inelastic form factor". Counting 4 protons being present in elastic events whereas only two (initial) protons being present in a true "break up" inelastic event, we expect only the second power and not the fourth power of the elastic form factor to appear in Eq.(17). If so,

$$\sigma_{eff}^{(inel)}(s) = \frac{1}{\int [\frac{d^2\mathbf{q}}{(2\pi)^2}] \frac{1}{[1+(q^2/t_o(s))]^4}} = \Big[\frac{6\pi}{t_o(s)}\Big]; \quad \sigma_{eff}^{inel}(13\ TeV) \approx\ 12.3\ milli-barns, \tag{19}$$

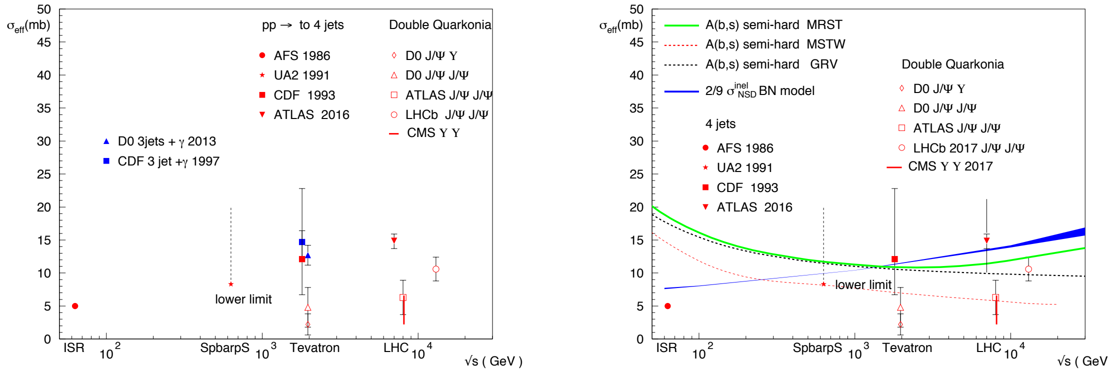

Figure 2: In the left panel, existing DPS data as described in the text from Axial Field Spectrometer (AFS), [15], UA2 [16], CDF 1993 [17] for $\bar{p}p \to 4\ jets$ and ATLAS at $\sqrt{s} = 7$ TeV [18] for $pp \to 4\ jets$. We also have plotted CDF 1997 [19] and D0 [20] for $\bar{p}p \to \gamma\ 3\ jets$, D0 [21] for $\bar{p}p \to J/\Psi\ J/\Psi$, ATLAS [22] and LHCb for $pp \to J/\Psi\ J/\Psi$ [23], CMS for $pp \to \Upsilon\ \Upsilon$ [24], D0 for $\bar{p}p \to J/\Psi\ \Upsilon$ [25]. At right, comparison of $\bar{p}/p\ p \to 4\ jets$ or with $\bar{p}/p\ p \to quarkonia\ pair$ with two models described in the text.

a bit closer to the phenomenological value estimated by exploration of ATLAS compilation [1].
In this model the energy dependence of σ_{eff} proceeds from that of the parameter $t_0(s)$. We notice here that in [14] we have shown that the presently available data for the differential elastic cross-section as well as the total cross-section, i.e. the imaginary part of the forward elastic amplitude, can be described rather accurately through an expression which includes an energy dependent form factor. In this model, $t_0(s)$ decreases with energy, hence this model would predict $\sigma_{eff}(s) \uparrow \sqrt{s}$. We now turn to a discussion of the data and a comparison with the models we have just illustrated.

5 About data and models

Available data not only span a very large energy range, but, as compiled by ATLAS, refer to very different types of final states. This may indeed generate confusion since parton distributions, hence the calculation of σ_{eff}, differ according to whether the initial state be mostly driven by gluon-gluon scattering or implicating valence quarks as well. Thus we have focused on similarly homogenous final states and show them in the left panel of Fig. 2. The figure may indicate the following trends:

- for processes dominated by gluon gluon scattering, such as $\bar{p}/p\ p \to 4jets$ and $\bar{p}/p\ p \to J/\Psi\ J/\Psi$, $\sigma_{eff}(\sqrt{s}) \uparrow \sqrt{s}$, although the scale is different, with $\sigma_{eff}^{singlet}(\sqrt{s}) \approx \frac{1}{3}\sigma_{eff}(\sqrt{s})^{all}$
- for processes in which at least one of the final state particles must originate from a valence quark, as in $3jets + \gamma$, the effective cross-section appears to be decreasing, as seen in the left panel of Fig.2 by the full blue symbols.

In the right hand panel we have compared the selected sets of data *vs.* two models: the BN-inspired soft gluon resummation model described in Sect. 3, and a model based on the ansatz that all inclusive cross-sections rise. This model would be adequate to describe the case of gluon initiated processes, less so when valence quarks initiate the process, as it is likely to be the case for the 3 $jets + \gamma$ final state.

Our ansatz, to describe σ_{eff} for $\bar{p}/p\ p \to 4\ jets$, is

$$\sigma_{eff} \propto \sigma_{inel}^{NSD} \tag{20}$$

We then use the description of σ_{inel}^{NSD} from the model of [14] and plot it as as blue band in the right hand panel of Fig. 2, with an arbitrarily chosen factor 2/9 for normalization to the data.. We consider the two different cases of GRV or MSTW densities (MRST densities for total and inelastic cross-section are in good agreement with results from MSTW, as shown in the right hand panel Fig. 1).
For the model which uses $A(b)$ from soft gluon resummation, Sect.3, we see that at LHC energies the model gives good agreement with data, but the trend with energy is different.
In summary for $pp \to 4\ jets$:

- the impact parameter description as from Sect. 3 (green, red and dotted curves in Fig. 2) gives an absolute overall normalization of LHC data in a good agreement with the plotted data, but is inconclusive as far as the energy dependence is concerned,
- the scattering amplitude *cum* form factor model from Sect. 4 would also reproduce the correct order of magnitude at LHC, and may indicate a rising σ_{eff} from ISR to LHC,
- an empirical description from the NSD inelastic cross-section of [26] would reproduce a rising energy trend from ISR to LHC.

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