Effective Lagrangians for BCS superconductors at $T = 0$

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We show that the low-frequency, long-wavelength dynamics of the phase of the pair field for a BCS-type s-wave superconductor at $T = 0$ is equivalent to that of a time-dependent nonlinear Schrödinger Lagrangian (TDNLSL), when terms required by Galilean invariance are included. If the modulus of the pair field is also allowed to vary, the system is equivalent to two coupled TDNLSL's.

The classic Ginzburg-Landau (GL) theory[1] is very successful[2,3] in describing a large class of static superconducting phenomena near the critical temperature T_c, and its form was established by Gorkov[4] shortly after the microscopic BCS theory.[5] Subsequently, a number of attempts[2,3,6,7] were made to obtain a generalized GL theory for time-dependent phenomena, and for temperatures well below T_c, but a consensus has still not been reached on the form of such a theory at $T = 0$. In this paper we shall show that the low-frequency, long-wavelength dynamics of the phase of the pair field for a BCS-type s-wave superconductor at $T = 0$ is equivalent to a time-dependent nonlinear Schrödinger Lagrangian (TDNLSL). At first sight, this result might seem almost obvious: after all, the energy density in GL theory looks formally like that of a nonlinear Schrödinger theory so that it seems natural to extend it to the corresponding time-dependent theory as, indeed, Feynman assumed[8] in his discussion of the dynamics of superconductors and of the Josephson effects. Yet neither the earlier discussions,[2,3,6] nor recent work based on the effective action formalism of quantum field theory,[7,9] appears to lead to this conclusion. This is in contrast to the case of a Bose superfluid, such as ^{4}He, which is well described by a TDNLSL near $T = 0$.[10] Indeed, there is considerable current interest in probing the relationship and "crossover" between BCS and Bose superfluidity.[11] Our result implies that both are fundamentally the same, at least near $T = 0$ in the clean limit; in particular, the existence of the Magnus force for a vortex line in a superconductor follows naturally. The last point is pertinent to the discussion of vortex dynamics in superconductors within the effective theory formulation.[12]

Three of the present authors have, in fact, recently shown[13] that the motion of the condensate is described by a nonlinear Schrödinger equation at $T = 0$, using a density matrix approach and the Born-Oppenheimer approximation. But this left open the question how this could be reconciled with the earlier work,[2,3,6,7,9] which was generally based on field theory (or Green function) techniques, and apparently led to a quite different result. The solution of this problem is contained in the present paper, and it is essentially very simple. In Ref. 13 the further approximation was made that the modulus of the energy gap (or pair field) is constant. If this approximation is made in the field theory approach, one can derive[9] from BCS theory an effective action for the phase $\theta(x)$ of the pair field (i.e., the Goldstone mode), which one then expands up to a certain order of derivatives. The resulting Lagrangian $L_{\rm eff}(\theta)$ is the same as that previously proposed on symmetry grounds,[14] and also corresponds precisely to the early results of Kemoklidze and Pitaevskii,[15] who started from Gorkov's equations.[5] We shall show that the dynamics of $\theta(x)$ as given by $L_{\rm eff}(\theta)$ can be rewritten in terms of a TDNLSL, which is equivalent to a particular case of the general nonlinear Schrödinger theory derived in Ref. 13 under the same approximation.

We also extend this to include variations in the modulus of the pair fields, and show that the dynamics is then that of two coupled TDNLSL's. The thread that unites all these approaches is ultimately Galilean invariance. Since the microscopic starting point is always Galilean invariant, one expects any theory based on an effective action to preserve this symmetry, a point emphasized in Ref. 15, and the Schrödinger Lagrangian is the simplest such available.

We begin with the BCS Lagrangian for s-wave pairing and in the absence of external fields:

$$L = \sum_\sigma \psi_\sigma^*(x) \left(i\partial_t + \frac{\nabla^2}{2m} + \mu \right) \psi_\sigma(x) + g\psi_\uparrow^*(x)\psi_\downarrow^*(x)\psi_\downarrow(x)\psi_\uparrow(x), \quad (1)$$

where ψ_σ describes electrons with spin $\sigma = (\uparrow, \downarrow)$, $\mu = k_F^2/2m$ is the Fermi energy in the normal state, and $x = (\mathbf{x}, t)$. Introducing the auxiliary (pair) fields $\Delta(x)$ and $\Delta^*(x)$, and integrating out the electron fields, one obtains the effective action

$$S[\Delta, \Delta^*] = -i \,\mathrm{Tr} \ln G^{-1} - \frac{1}{g} \int d^4x |\Delta(x)|^2, \quad (2)$$

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where the Nambu Green function satisfies

$$\begin{pmatrix} O_1 & \Delta(x) \\ \Delta^*(x) & O_2 \end{pmatrix} G(x,x') = \delta(x-x') \tag{3}$$

with $O_1 = i\partial_t + \nabla^2/2m + \mu$, $O_2 = i\partial_t - \nabla^2/2m - \mu$, and Tr includes internal and space-time indices. To obtain from (2) an effective Lagrangian in terms of the degrees of freedom represented by Δ, a possible procedure[7] is to set $\Delta(x) = \Delta_0 + \Delta'(x)$ where Δ_0 is the position of the minimum of S for space-time independent Δ, and where Δ' is assumed to be slowly varying in both space and time. One then expands Tr $\ln G^{-1}$ in powers of derivatives of Δ'. There are, however, two (related) objections to this. First, we are dealing with the spontaneous breaking of a local U(1) phase invariance, which implies that at a temperature far from the transition temperature, the most important degree of freedom is the phase of Δ, which is the relevant Goldstone field. It is this field, rather than the real and/or imaginary parts of Δ, which should carry the low-frequency and long-wavelength dynamics. Second, the ansatz $\Delta(x) = \Delta_0 + \Delta'(x)$ violates the Galilean invariance possessed by (1), which implies[15] that

$$\Delta(\mathbf{r} - \mathbf{v}t, t)\exp(2im\mathbf{v}\cdot\mathbf{r} - imv^2 t) \tag{4}$$

should satisfy the same equation of motion as $\Delta(\mathbf{r},t)$. We shall return to the question of Galilean invariance below.

We therefore set

$$\Delta(x) = e^{i\theta(x)}|\Delta(x)| \tag{5}$$

and $|\Delta(x)| = |\Delta_0| + \delta|\Delta(x)|$, where we are interested in the low-frequency and long-wavelength fluctuations of $\theta(x)$ and $\delta|\Delta(x)|$. However, although $\delta|\Delta(x)|/|\Delta_0|$ is expected to be small, and a simple expansion of the sort mentioned above for Tr $\ln G^{-1}$ could easily be set up in terms of derivatives of $\delta|\Delta(x)|$ if $\theta(x)$ were zero, it is crucial to recognize that $\theta(x)$ is not small in general, so that the phase factor in (5) cannot be expanded, but must be treated as a whole. This prevents a straightforward expansion of Tr $\ln G^{-1}$ when (5) is substituted into (2). Fortunately, this difficulty can be easily circumvented.[9,16] Defining $U(x) = \exp[i\theta(x)\tau_3/2]$, we can write

$$\begin{aligned}\mathrm{Tr}\ln G^{-1} &= \mathrm{Tr}\ln G^{-1}UU^{-1} = \mathrm{Tr}\ln U\tilde{G}^{-1}U^{-1} \\ &= \mathrm{Tr}\ln \tilde{G}^{-1},\end{aligned} \tag{6}$$

where

$$\tilde{G}^{-1} = G_0^{-1}(1 - G_0\Sigma),$$

$$G_0^{-1} = \begin{pmatrix} O_1 & |\Delta_0| \\ |\Delta_0| & O_2 \end{pmatrix}, \tag{7}$$

and

$$\begin{aligned}\Sigma = &-i\nabla^2\theta/4m - i\nabla\theta\cdot\nabla/2m \\ &+[\dot\theta/2 + (\nabla\theta)^2/8m]\tau_3 - \delta|\Delta|\,\tau_1.\end{aligned} \tag{8}$$

Minimizing (2) with $\theta = \delta|\Delta| = 0$ yields the usual gap equation for $|\Delta_0|$. The dynamics of θ and $|\Delta|$ is contained in

$$S_{\text{eff}}[\theta, \delta|\Delta|] = i\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}(G_0\Sigma)^n - \frac{1}{g}\int|\Delta|^2 d^4x\ , \tag{9}$$

where we note that Σ contains just $\delta|\Delta(x)|$ and derivatives of $\theta(x)$, in terms of which (assumed small) quantities a useful expansion can be conducted, following standard techniques.[17]

We now concentrate on $\theta(x)$, and set $\delta|\Delta| = 0$ for the time being. Carrying out the calculation to the indicated order in derivatives we obtain

$$L_{\text{eff}}(\theta) = -\rho_0[\dot\theta/2 + (\nabla\theta)^2/8m] + N(0)[\dot\theta/2 + (\nabla\theta)^2/8m]^2, \tag{10}$$

where $\rho_0 = k_F^3/3\pi^2$ is the electron density at $T = 0$, $N(0)$ is the density of states (for one spin projection) at the Fermi surface and we have adopted a convenient normalization; note that $N(0) = \rho_0/2mv_a^2$, where $v_a = v_F/\sqrt{3}$ is the velocity of propagation of the Bogoliubov-Anderson mode. Equation (10) is the same as Eq. (4) in Ref. 9 if θ is replaced by 2ϕ [see also Eq. (2.6) of Ref. 14]. The equation of motion for θ which follows from (10) is

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0, \tag{11}$$

where

$$\rho = \rho_0 - N(0)[\dot\theta + (\nabla\theta)^2/4m] \tag{12}$$

and

$$\mathbf{j} = \rho\nabla\theta/2m. \tag{13}$$

Equations (11)–(13) are, in fact, precisely those obtained (to this order in derivatives) by putting $\delta|\Delta| = 0$ in Eqs. (21)–(23) of Ref. 15. We now show how the dynamics contained in (11)–(13) can be reinterpreted in terms of a TDNLSL.

We first remark that the forms of (11) and (13) strongly suggest that the quantities ρ and $\mathbf{j}$ have the physical significance of a number density and of a number current density, respectively. In fact, simple linear response theory (assuming, as always, a derivative expansion) gives[18]

$$\delta\rho \approx -N(0)\dot\theta,\ \ \mathbf{j} \approx \rho_0\nabla\theta/2m, \tag{14}$$

where $\delta\rho$ is the departure of the density from the equilibrium value ρ_0, and $\mathbf{j}$ is the number density current. We now consider the implications of Galilean invariance. For densities ρ and $\mathbf{j}$ obeying (11), Galilean symmetry requires

$$\rho'(\mathbf{r}',t') = \rho(\mathbf{r},t),\ \ \mathbf{j}'(\mathbf{r}',t') = \mathbf{j}(\mathbf{r},t) - \mathbf{v}\rho(\mathbf{r},t), \tag{15}$$

where $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$, $t' = t$; that is, ρ and $\mathbf{j}$ transform as t and $\mathbf{r}$, respectively. From the discussion around Eq. (4) above, the behavior of θ under Galilean transformations is given by

$$\theta'(\mathbf{r}',t') = \theta(\mathbf{r},t) + mv^2 t - 2m\mathbf{v}\cdot\mathbf{r}, \tag{16}$$

from which it follows that $\delta\rho$ as given by (14) is not invariant, and that $\mathbf{j}$ transforms incorrectly. However, although $\dot\theta$ is not a Galilean invariant, the combination

$\dot{\theta} + (\nabla\theta)^2/4m$ is. [This is actually the justification for singling out the terms given in (10) from the total $n = 2$ contribution in (9).] Thus the requirement that $\delta\rho$ be Galilean invariant leads precisely to the expression (12) for $\rho - \rho_0$, which can now be identified as $\delta\rho$. Similarly, if we replace ρ_0 in the expression (14) for $\mathbf{j}$ by ρ, we find that $\mathbf{j}$ transforms correctly and the expression for $\mathbf{j}$ is that in (13).

We are therefore led to seek a theory involving two fields ρ and θ, such that the equation of motion for θ is (11) and that for ρ is (12), with ρ, θ and $\mathbf{j}$ related by (13). Consider the TDNLSL

$$L_\psi = i\psi^*\dot{\psi} - \frac{1}{4m}\nabla\psi^* \cdot \nabla\psi - V, \tag{17}$$

where the mass has been chosen to be $2m$, and V will be assumed to be a function of $|\psi|$ only. Let us set

$$\psi = \sqrt{\rho}\, \exp(i\theta). \tag{18}$$

Then inserting (18) into (17) and discarding a total derivative, L_ψ becomes

$$L_\psi = -\rho\dot{\theta} - \rho(\nabla\theta)^2/4m - (\nabla\rho)^2/16m\rho - V(\rho). \tag{19}$$

The equation of motion for θ is then (11), with $\mathbf{j}$ given by (13), while that for ρ is

$$\frac{dV}{d\rho} = -[\dot{\theta}+(\nabla\theta)^2/4m]-(\nabla\rho)^2/16m\rho^2+\nabla^2\rho/8m\rho^2. \tag{20}$$

If we now choose

$$V = (\rho - \rho_0)^2/2N(0) \tag{21}$$

and solve (20) by expanding in derivatives, we recover precisely (12) at the relevant order.

Thus the introduction of the auxiliary variable ρ, which can be expressed in terms of θ via its equation of motion, has allowed us to rewrite the dynamics of the Goldstone field θ, as given by $L_{\text{eff}}(\theta)$, in terms of the TDNLSL (17). The variable ρ is interpreted physically as the number density. It must be stressed, however, that the wave function ψ introduced in this way [see (18)] is quite distinct from the pair field Δ, despite the fact that they have the same phase θ. In our development thus far, the modulus $|\Delta|$ has been held fixed, whereas ρ varies; there is no simple relation between $|\Delta|$ and $\sqrt{\rho}$.

The dynamical theory described by (17) is a special case of the general time-dependent nonlinear Schrödinger theory for the condensate wave function derived in Ref. 13 in which the form of the potential V was not explicitly calculated. Here, it has been necessary to fix V in order to carry out the elimination of the variable ρ, to the required order in derivatives. (We remark that in Ref. 13 ψ is normalized to the density of Cooper pairs, rather than, as here, to the electron density.)

Before discussing the modifications caused by the inclusion of the field $\delta|\Delta(x)|$, we make one further comment on the Lagrangian (10). A simple alternative route to (10) is to start from a Lagrangian which describes just the Bogoliubov-Anderson mode, viz.

$$L_a = \frac{1}{2}\dot{\theta}^2 - \frac{1}{2}v_a^2(\nabla\theta)^2. \tag{22}$$

Now (22) is clearly not invariant under (16). But, as we have seen, the combination $\dot{\theta} + (\nabla\theta)^2/4m$ is invariant. Hence if in (22) we replace $\dot{\theta}^2$ by $[\dot{\theta} + (\nabla\theta)^2/4m]^2$ and $(\nabla\theta)^2$ by $4m\dot{\theta} + (\nabla\theta)^2$ we will have a Galilean invariant Lagrangian; and the result of these replacements is just proportional to $L_{\text{eff}}(\theta)$. Actually, the $(\nabla\theta)^2$ term in (22) is of course invariant by itself, up to constants and a total derivative. Indeed, the term $\dot{\theta}$ introduced above, and present in $L_{\text{eff}}(\theta)$, is also a total derivative and does not affect the equations of motion. Nevertheless it is important physically, as it ensures that the density ρ has the equilibrium value ρ_0.[14]

In view of its relative unfamiliarity, it may be worth noting that $L_{\text{eff}}(\theta)$ (or equivalently L_ψ) embodies the usual phenomenology of superfluid dynamics at $T = 0$ (see, for example, Refs. 19 and 20). We identify $\nabla\theta/2m$ with the superfluid velocity $\mathbf{v}_s$, and multiply ρ and $\mathbf{j}$ of (12) and (13) by m to convert them to mass density and flux, ρ_m and $\mathbf{j}_m$. Equation (11) is then the law of mass conservation, following from the fact that L_{eff} does not depend explicitly on θ. Equation (12) is equivalent[15] to Bernoulli's equation, if we make use of $\delta\rho \approx 2N(0)\delta\mu$ and $\delta p \approx \rho_0\delta\mu$. Since L_{eff} does not depend explicitly on t, we have the energy conservation relation

$$\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{Q} = 0, \tag{23}$$

where using the canonical definitions (with suitable normalization), one finds

$$E \approx \frac{1}{2}\rho_m v_s^2, \quad \mathbf{Q} = \mathbf{j}_m \left(\frac{1}{2}\mathbf{v}_s^2 + \delta\mu\right), \tag{24}$$

and we have dropped a quantity of order $\delta\rho\delta\mu$ in E. Finally, since L_{eff} is translation invariant we have the momentum conservation relation

$$\frac{\partial \mathbf{j}_m}{\partial t} + \nabla \cdot \Pi = 0, \tag{25}$$

where the momentum flux density tensor is

$$\Pi_{ij} = \rho_m v_{si} v_{sj} + \delta p\, \delta_{ij}. \tag{26}$$

Equation (25) is equivalent to Euler's equation. In Ref. 14, the proportionality between the momentum density and the number current $\mathbf{j}$ [defined by $\partial L/\partial(\nabla\theta)$], which is included in (25), was taken as a constraint on possible Lagrangians L. If L is a function solely of the Galilean scalar $g = \dot{\theta} + (\nabla\theta)^2/4m$, then

$$\frac{\partial L}{\partial\dot{\theta}} = \frac{\partial L}{\partial g}, \quad \frac{\partial L}{\partial(\nabla\theta)} = \frac{\partial L}{\partial g}\frac{\nabla\theta}{2m}. \tag{27}$$

Since θ is a phase variable, we can interpret $\partial L/\partial\dot{\theta}$ and $\partial L/\partial(\nabla\theta)$ as being proportional to a conserved number density ρ and number current density $\mathbf{j}$, respectively, so that (27) becomes just (13). The momentum density is then automatically proportional to $\mathbf{j}$. Once again, Galilean invariance is the essential principle.

We now turn to the inclusion of the field $\delta|\Delta(x)|$.

$L_{\rm eff}(\theta,\delta|\Delta|)$ can be extracted from (9), up to a given order in derivatives, but calculations rapidly become laborious. For our present purpose, we will simply use the result of Ref. 15 which, using the normalization of (10), gives

$$\begin{aligned} L_{\rm eff}(\theta,\epsilon) = &-\rho_0\left[\dot\theta/2+\frac{(\nabla\theta)^2}{8m}+\frac{(\nabla\epsilon)^2}{8m}\right] \\ &+N(0)\left[\dot\theta/2+\frac{(\nabla\theta)^2}{8m}+\frac{(\nabla\epsilon)^2}{8m}\right]^2 \\ &+N(0)\left[\dot\epsilon/2+\frac{\nabla\epsilon\nabla\theta}{4m}\right]^2-3N(0)|\Delta_0|^2\epsilon^2, \end{aligned} \tag{28}$$

where $\epsilon(x)\equiv\delta|\Delta(x)|/(\sqrt{3}|\Delta_0|)$ and we have retained corresponding terms in ϵ and θ. The quadratic terms in ϵ yield the amplitude collective mode with a finite gap found in Ref. 18 (and are also in agreement with the result of Ref. 7); we have omitted higher powers of ϵ. The term in $\dot\epsilon/2$ is made Galilean invariant by the addition of $\nabla\epsilon\cdot\nabla\theta/4m$, since $\epsilon(x)$ is a scalar. We now find that in order to rewrite (28) in "Schrödinger" form we have to introduce a second auxiliary density feld, which we call ρ_ϵ, in addition to the earlier density ρ, which now becomes ρ_θ. Thus four fields ($\theta,\epsilon,\rho_\theta$, and ρ_ϵ) are required, and (28) is actually equivalent to two coupled Schrödinger equations. That is, the equations of motion for θ and ϵ which follow from (28) are identical to those arising from

$$\begin{aligned} L_{\psi_1,\psi_2} = &\,i\psi_1^*\dot\psi_1-\frac{1}{4m}\nabla\psi_1^*\nabla\psi_1-(|\psi_1|^2-\rho_0/2)/N(0) \\ &+i\psi_2^*\dot\psi_2-\frac{1}{4m}\nabla\psi_2^*\nabla\psi_2-(|\psi_2|^2-\rho_0/2)/N(0) \\ &+\frac{3}{2}N(0)|\Delta_0|^2[\mathrm{Im}\ln(\psi_1/\psi_2)]^2, \end{aligned} \tag{29}$$

where

$$\begin{aligned} \psi_1 &= \sqrt{(\rho_\theta+\rho_\epsilon)/2}\;\exp[i(\theta+\epsilon)], \\ \psi_2 &= \sqrt{(\rho_\theta-\rho_\epsilon)/2}\;\exp[i(\theta-\epsilon)]. \end{aligned} \tag{30}$$

For example, corresponding to (12), we have

$$\rho_\theta=\rho_0-N(0)\left(\dot\theta+\frac{(\nabla\theta)^2}{4m}+\frac{(\nabla\epsilon)^2}{4m}\right)=\frac{\partial L_{\rm eff}(\theta,\epsilon)}{\partial\dot\theta} \tag{31}$$

and

$$\rho_\epsilon=-N(0)\left(\dot\epsilon+\frac{\nabla\epsilon\nabla\theta}{2m}\right)=\frac{\partial L_{\rm eff}(\theta,\epsilon)}{\partial\dot\epsilon}. \tag{32}$$

Equation (29) represents a system of two TDNLSL's coupled via the "mass" term in (28). Expressions for all the conserved quantities can be found as before, and will include quantum corrections to the semiclassical results of (23)–(26).

The inclusion of electromagnetism in the above formalism is straightforward. Consider the formulation in terms of $L_{\rm eff}(\theta,\epsilon)$. Since θ is the phase of a field with charge $-2e$, gauge invariance implies that $\dot\theta$ and $\nabla\theta$ must appear in the combinations $\dot\theta-2eA_0$ and $\nabla\theta+2e\mathbf{A}$ ($e>0$), where A_0 and $\mathbf{A}$ are the electromagnetic potentials. The field ϵ, on the other hand, is electromagnetically neutral. The leading order electromagnetic charge and current densities are obtained by multiplying $\delta\rho$ and $\mathbf{j}$ in (22) by $-e$ and making the above replacements for $\dot\theta$ and $\nabla\theta$. One then obtains the usual results.[7] In terms of the Schrödinger formulation, one simply makes the expected minimal coupling substitutions: $i\partial_t\to i\partial_t+2eA_0$ and $-i\nabla\to-i\nabla+2e\mathbf{A}$ in (14) or (29) [note from (30) that both ψ_1 and ψ_2 have charge $-2e$].

When the above analysis is extended to higher-order derivative terms, it is clear on dimensional grounds that some characteristic scale must enter. In fact, such higher terms enter in the form $\partial_t/|\Delta_0|$ and $v_F\nabla/|\Delta_0|\sim\xi\nabla$ [see, for example, Eq. (35) of Ref. 9], where ξ is the coherence length. The basis of the expansion is therefore the usual assumption[15] that the characteristic frequency ω, and wave number k, of variations of $\Delta(x)$ satisfy $\omega\ll|\Delta_0|$, $k\ll\xi^{-1}$. Indeed, (28) already yields a static solution for ϵ which decays exponentially over a characteristic distance $\xi/6$. Such a solution is of the type expected far from a vortex core. Inclusion of appropriate higher derivative terms should make possible some predictions about the vortex core structure.

The TDNLSL formulation provides, we believe, a simple and unifying framework for the discussion of dynamical effects in BCS superconductors at $T=0$. Results which have been known for many years,[2,3,6,15] as well as those obtained by quite different methods only recently,[9,14] are all seen to be in agreement with each other, and with the TDNLSL formulation.

Note added: After completion of this work we received an unpublished paper by Stone,[21] in which a similar conclusion is reached concerning the effective TDNLSL when $|\Delta|$ is constant.

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