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# Photoproduction of massive gauge bosons in *pp*, *pPb* and *PbPb* collisions



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#### ABSTRACT

In this letter we present, for the first time, the results for the photoproduction of massive gauge bosons in proton – proton, proton – Lead and Lead – Lead collisions at the Large Hadron Collider (LHC), High – Energy LHC (HE – LHC) and Future Circular Collider (FCC). Predictions for the rapidity distributions and total cross sections are presented. We predict a large number of events in the rapidity range probed by the LHC detectors, which implies that this process can be used to probe the photoproduction of massive gauge bosons as well to perform the search of Beyond Standard Model physics.

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The experimental results for photon - induced processes at Tevatron, RHIC and LHC have motivated a series of studies that propose to test our understanding of different aspects of the Standard Model as well to constrain possible scenarios of the Beyond Standard Model (BSM) physics using the analysis of different final states produced in  $\gamma\gamma$  and  $\gamma h$  interactions at hadronic collisions (For a review see, e.g. Ref. [1]). In this letter we will analyze the possibility of improve our understanding about the couplings between the gauge bosons considering the photoproduction of Z and W in pp, pPb and PbPb collisions for the energies of the next run of the LHC, as well of the High - Energy LHC [2] and Future Circular Collider [3]. In particular, we will estimate, for the first time, the total cross sections and expected number of events considering the rapidity range covered by the LHC detectors. Our goal is to verify if the experimental analysis of this process is feasible, which will allow to use it to test the Standard Model predictions, as well to search effects from new physics.

The photoproduction of massive gauge bosons ( $G = Z^0, W^{\pm}$ ) in hadronic collisions is represented in Fig. 1, where we have con-

$$\sigma(h_{1} + h_{2} \to h \otimes G + X) = \int d\omega \, n_{h_{1}}(\omega) \, \sigma_{\gamma h_{2} \to GX} \left( W_{\gamma h_{2}} \right)$$

$$+ \int d\omega \, n_{h_{2}}(\omega) \, \sigma_{\gamma h_{1} \to GX} \left( W_{\gamma h_{1}} \right) ,$$

$$(1)$$

where  $\otimes$  represents the presence of one rapidity gap in the final state,  $\omega$  is the photon energy in the center-of-mass frame (c.m.s.),  $n_{h_i}(\omega)$  is the equivalent photon flux for the hadron  $h_i$ ,  $W_{\gamma h}$  is the c.m.s. photon-hadron energy given by  $W_{\gamma h}^2 = 2\omega\sqrt{s_{NN}}$ , with  $\sqrt{s_{NN}}$  being the c.m.s energy of the hadron-hadron system. Considering the requirement that photoproduction is not accompanied by hadronic interactions (ultra-peripheral collision) an analytic approximation for the equivalent photon flux of a nucleus can be calculated, which is given by [4,5]

$$n_{A}(\omega) = \frac{2 Z^{2} \alpha_{em}}{\pi \omega} \left[ \bar{\eta} K_{0}(\bar{\eta}) K_{1}(\bar{\eta}) - \frac{\bar{\eta}^{2}}{2} \mathcal{U}(\bar{\eta}) \right]$$
 (2)

where  $K_0(\eta)$  and  $K_1(\eta)$  are the modified Bessel functions,  $\bar{\eta} = \omega (R_{h_1} + R_{h_2})/\gamma_L$  and  $U(\bar{\eta}) = K_1^2(\bar{\eta}) - K_0^2(\bar{\eta})$ . Moreover,  $\gamma_L$  is the Lorentz boost of a single beam and we consider  $R_p = 0.6$  fm and

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sidered that both incident hadrons can be the source of photons. The experimental signature for such events is the presence of one rapidity gap, associated to the photon exchange, and one intact hadron in the final state. The associated total cross section is given by [4]

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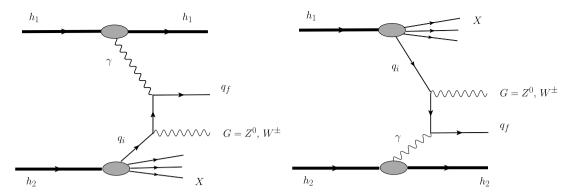


Fig. 1. Photoproduction of massive gauge bosons in hadronic collisions.

 $R_A = 1.2 A^{1/3}$  fm in our calculations for pPb and PbPb collisions. For the photon spectrum associated to the proton we will assume that it is given by [6],

$$n_{p}(\omega) = \frac{\alpha_{\text{em}}}{2\pi \omega} \left[ 1 + \left( 1 - \frac{2\omega}{\sqrt{s_{NN}}} \right)^{2} \right] \times \left( \ln \Omega - \frac{11}{6} + \frac{3}{\Omega} - \frac{3}{2\Omega^{2}} + \frac{1}{3\Omega^{3}} \right), \tag{3}$$

with the notation  $\Omega=1+[\,(0.71\,\text{GeV}^2)/Q_{\text{min}}^2\,]$  and  $Q_{\text{min}}^2=\omega^2/[\gamma_L^2\,(1-2\,\omega/\sqrt{s_{NN}})\,]\approx(\omega/\gamma_L)^2$ . This expression is derived considering the Weizsäcker-Williams method and using an elastic proton form factor (For more details see Refs. [6,7]). The value  $0.71\,\text{GeV}^2$  in  $\Omega$  arises from the electric form factor  $G_E(Q^2)$  which is parametrized by the dipole form  $G_E(Q^2)=(1+Q^2/0.71\,\text{GeV}^2)^{-2}$  [6,7]. The photoproduction cross section for the  $\gamma h \to GX$  process is given by [8,9]

$$\sigma_{\gamma h \to GX}(W_{\gamma h}) = \int_{x_{min}}^{1} dx \sum_{q,\bar{q}} f_{q/h}(x, Q^2) \,\hat{\sigma}_G(\hat{s}), \tag{4}$$

where  $f_{q/h}$  are the parton distribution functions in the hadron target (h=p or Pb), x is the Bjorken variable that describes the momentum fraction of the proton carried by the quark,  $x_{min}=M_G^2/W_{\gamma h}^2$ ,  $\hat{\sigma}_G$  is the cross section for the subprocess  $\gamma q_i \rightarrow Gq_f$  and  $\hat{s}=x\cdot W_{\gamma h}^2$ . For  $G=Z^0$  we have that final and initial quarks are of the same flavour,  $q_f=q_i$  (see Fig. 1), and the subprocess cross section,  $\hat{\sigma}_Z(\hat{s})$ , is given at leading order by [8–10]

$$\hat{\sigma}_{Z} = \frac{\alpha G_{F} M_{Z}^{2}}{\sqrt{2} \,\hat{\mathbf{s}}} \, g_{q}^{2} e_{q}^{2} \left[ \left( 1 - 2\hat{\mathbf{z}} + 2\hat{\mathbf{z}}^{2} \right) \log \left( \frac{\hat{\mathbf{s}} - M_{Z}^{2}}{\Lambda^{2}} \right) + \frac{1}{2} \left( 1 + 2\hat{\mathbf{z}} - 3\hat{\mathbf{z}}^{2} \right) \right], \tag{5}$$

where  $G_F$  is the Fermi coupling constant,  $\hat{z} = M_Z^2/\hat{s}$ ,  $g_q^2 = \frac{1}{2}(1 - 4|e_q|x_W + 8e_q^2x_W^2)$ ,  $e_q$  is the quark charge and  $x_W = \sin^2\theta_W = 0.23$  [12], with  $\theta_W$  being the Weinberg angle. On the other hand, for G = W, we have  $q_f \neq q_i$  and  $\hat{\sigma}_W$  is given by [8–10]

$$\begin{split} \hat{\sigma}_W &= \sigma_0 \, |V_{if}|^2 \left\{ (|e_q|-1)^2 (1-2\hat{z}+2\hat{z}^2) \log(\frac{\hat{s}-M_W^2}{\Lambda^2}) \right. \\ & \left. - \left[ (1-2\hat{z}+2\hat{z}^2) - 2|e_q|(2+2\hat{z}^2) - 1 \right] \log \hat{z} \right. \\ & \left. + \left[ \frac{2}{\hat{z}} + \left( \frac{1}{2} + \frac{3(1+|e_q|^2)}{2} \right) \hat{z} + 2|e_q| + \frac{|e_q|^2}{2} \right] (1-\hat{z}) \right\} \end{split}$$

where  $\sigma_0=\frac{\alpha G_F M_W^2}{\sqrt{2\hat{s}}}$ ,  $\hat{z}=M_W^2/\hat{s}$  and quantities  $V_{if}$  are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The scale  $\Lambda^2$  in the Eqs. (5) and (6) is a cutoff scale that regulates the singularity present at LO when the final state quark  $q_f$  becomes collinear with the initial state photon. Such singularity does not occur at LO for a non-vanishing transverse momentum  $p_T$  of the massive gauge boson, since in this case the  $p_T$  of the boson has to be balanced by the final state quark. As in previous studies [9–11], we will assume that  $\Lambda=0.4$  GeV. Moreover, we will consider that the parton distribution functions for the proton are described by the CTEQ parametrization proposed in Ref. [13], while for photon – nucleus interactions we will assume that the nuclear parton distributions can be expressed as  $f_{q/A}(x,Q^2)=A\cdot R_q(x,Q^2)\cdot f_{q/p}(x,Q^2)$ , where the function  $R_q(x,Q^2)$  parameterizes the nuclear effects in the parton distributions and is described by the EPPS16 parametrization [14].

The rapidity distribution for the photoproduction of massive gauge bosons in pp, pPb and PbPb collisions can be calculated considering that the rapidity Y of the boson in the final state is directly related to the photon energy  $\omega$  by the relation  $Y \propto \ln{(\omega/M_G)}$ . Explicitly, the rapidity distribution is written down as,

$$\frac{d\sigma}{dY}[h_1 + h_2 \to h \otimes G + X] 
= \left[\omega n_{h_1}(\omega) \sigma_{\gamma h_2 \to G X}\right]_{\omega_I} + \left[\omega n_{h_2}(\omega) \sigma_{\gamma h_1 \to G X}\right]_{\omega_R}$$
(7)

where  $\omega_L \, (\propto e^Y)$  and  $\omega_R \, (\propto e^{-Y})$  denote photons from the  $h_1$  and  $h_2$  hadrons, respectively. As the cross section increases with the energy, we have that the first term on the right-hand side of the Eq. (7) peaks at positive rapidities while the second term peaks for negative rapidities. Consequently, given the photon flux, the study of the rapidity distribution can be used to constrain the photoproduction cross section for a given energy. Moreover, the rapidity distributions for pp and PbPb collisions will be symmetric about midrapidity (Y=0). In contrast, for pPb collisions,  $d\sigma/dY$  is given by sum of the  $\gamma \, p \to GX$  and  $\gamma \, Pb \to GX$  contributions, with the photon being generated by the nucleus and by the proton, respectively. The  $\gamma \, p$  contribution is dominant because the equivalent photon spectrum of the nucleus is enhanced by a factor  $Z^2$  in comparison to the proton one. Consequently, the rapidity distribution is asymmetric with respect to Y=0.

In Fig. 2 we present our predictions for the rapidity distributions for the  $Z^0$  (left panels) and  $W^+$  (right panels) photoproduction in pp (upper panels), pPb (central panels) and PbPb (lower panels) collisions. We consider the planned center – of – mass energies for the next run of the LHC, as well for the future High – Energy LHC [2] and Future Circular Collider [3]. We will focus on the  $W^+$  photoproduction, since its predictions are slightly larger than for  $W^-$  (see e.g. Fig. 1 in Ref. [11]). Such difference arises from the fact that the main contribution for the  $W^+$  production

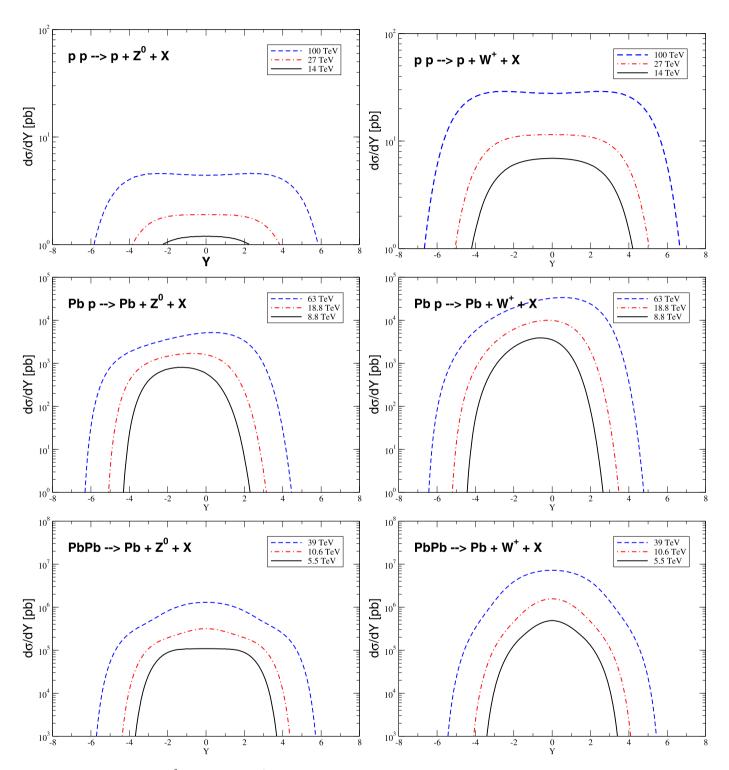


Fig. 2. Rapidity distributions for the  $Z^0$  (left panels) and  $W^+$  (right panels) photoproduction in pp (upper panels), pPb (central panels) and PbPb (lower panels) collisions. The solid, dot-dashed and dashed lines represent the predictions for the LHC, HE – LHC and FCC energies, respectively.

**Table 1**Cross sections and associated number of events in the leptonic decay mode for the photoproduction of massive gauge bosons in pp/pPb/PbPb collisions considering the typical rapidity range covered by a central detector ( $|Y| \le 2.5$ ).

| pp collisions         | $\sigma(Z^0)[pb]$      | # events ( $Z^0  ightarrow \mu^+ \mu^-$ ) | $\sigma(W^+)[pb]$       | # events ( $W^+ 	o \mu^+ \nu_\mu$ ) |
|-----------------------|------------------------|---|-------------------------|-------------------------------------|
| $\sqrt{s}$ = 14 TeV   | 5.67                   | 190.0                                     | 32.35                   | 3438.0                              |
| $\sqrt{s}$ = 27 TeV   | 9.36                   | 315.0                                     | 56.30                   | 5984.0                              |
| $\sqrt{s}$ = 100 TeV  | 22.69                  | 764.0                                     | 142.80                  | 15179.0                             |
| Pbp collisions        | $\sigma(Z^0)[pb]$      | # events ( $Z^0 	o \mu^+ \mu^-$ )         | $\sigma(W^+)[pb]$       | # events ( $W^+ 	o \mu^+ \nu_\mu$ ) |
| $\sqrt{s}$ = 8.8 TeV  | 2.29 ×10 <sup>3</sup>  | 77.0                                      | 11.17 ×10 <sup>3</sup>  | 1187.0                              |
| $\sqrt{s}$ = 18.8 TeV | $5.99 \times 10^{3}$   | 201.0                                     | $34.16 \times 10^3$     | 3631.0                              |
| $\sqrt{s}$ = 63 TeV   | 21.44 ×10 <sup>3</sup> | 20940.0                                   | 135.88 ×10 <sup>3</sup> | 418877.0                            |
| PbPb collisions       | $\sigma(Z^0)[pb]$      | # events ( $Z^0 	o \mu^+ \mu^-$ )         | $\sigma(W^+)[pb]$       | # events ( $W^+ 	o \mu^+ \nu_\mu$ ) |
| $\sqrt{s}$ = 5.5 TeV  | $0.49 \times 10^{6}$   | 165.0                                     | $1.40 \times 10^{6}$    | 1488.0                              |
| $\sqrt{s}$ = 10.6 TeV | $1.24 \times 10^{6}$   | 417.0                                     | $4.74 \times 10^{6}$    | 5038.0                              |
| $\sqrt{s}$ = 39 TeV   | $5.26 \times 10^{6}$   | 19487.0                                   | $27.60 \times 10^{6}$   | 322726.0                            |

**Table 2** Cross sections and associated number of events in the leptonic decay mode for the photoproduction of massive gauge bosons in pp/pPb/PbPb collisions considering the typical rapidity range covered by a forward detector ( $2 \le Y \le 4.5$ ).

| pp collisions               | $\sigma(Z^0)[pb]$     | # events ( $Z^0  ightarrow \mu^+ \mu^-$ ) | $\sigma(W^+)[pb]$     | # events ( $W^+ \rightarrow \mu^+ \nu_\mu$ ) |
|-----------------------------|-----------------------|---|-----------------------|--|
| $\sqrt{s}$ = 14 TeV         | 1.47                  | 49.0                                      | 8.69                  | 923.0  |
| $\sqrt{s} = 27 \text{ TeV}$ | 3.28                  | 110.0                                     | 20.44                 | 2172.0                                       |
| $\sqrt{s}$ = 100 TeV        | 10.95                 | 368.0                                     | 69.35                 | 7371.0                                       |
| Pbp collisions              | $\sigma(Z^0)[pb]$     | # events ( $Z^0 	o \mu^+ \mu^-$ )         | $\sigma(W^+)[pb]$     | # events ( $W^+ \rightarrow \mu^+ \nu_\mu$ ) |
| $\sqrt{s}$ = 8.8 TeV        | $0.0012 \times 10^3$  | 0.04                                      | $0.015 \times 10^{3}$ | 2.0  |
| $\sqrt{s}$ = 18.8 TeV       | $0.070 \times 10^{3}$ | 2.35                                      | $0.67 \times 10^{3}$  | 71.0   |
| $\sqrt{s}$ = 63 TeV         | $2.56 \times 10^{3}$  | 2500.0                                    | $20.18 \times 10^3$   | 62208.0                                      |
| PbPb collisions             | $\sigma(Z^0)[pb]$     | # events ( $Z^0 	o \mu^+ \mu^-$ )         | $\sigma(W^+)[pb]$     | # events $(W^+ \rightarrow \mu^+ \nu_\mu)$   |
| $\sqrt{s}$ = 5.5 TeV        | $0.068 \times 10^{6}$ | 22.0                                      | $0.067 \times 10^{6}$ | 71.0   |
| $\sqrt{s}$ = 10.6 TeV       | $0.21 \times 10^{6}$  | 70.0                                      | $0.32 \times 10^{6}$  | 340.0  |
| $\sqrt{s}$ = 39 TeV         | $1.06 \times 10^{6}$  | 3927.0                                    | $3.43 \times 10^{6}$  | 40106.0                                      |

comes from the interaction of the photon with a quark u, while the photoproduction of a  $W^-$  is determined by the interaction of the photon with a quark d. At small photon – hadron energies, we are probing large value of x, where the number of quarks up is larger than d, which implies that  $\sigma(\gamma h \to W^+ X) > \sigma(\gamma h \to W^- X)$ . The results presented in Fig. 2 indicate that the predictions for the  $W^+$ production are larger than for the  $Z^0$  one, which is expected since  $M_Z > M_W$  as well as from the results for the photoproduction of massive gauge bosons presented in Refs. [9,11]. Moreover, we have that for larger energies the rapidity distribution increases and becomes wider in rapidity. As expected from our previous discussion, the distribution is symmetric for pp and PbPb collisions and asymmetric for pPb one, with the maximum occurring for positive rapidities. Finally, the predictions for midrapidities ( $Y \approx 0$ ) increase with the energy and are a factor  $\approx 10^3 \ (10^6)$  larger in *pPb* (*PbPb*) collisions than in pp collisions. The enhancement in pPb collisions is mainly associated to the  $Z^2$  – factor in the nuclear photon flux. On the other hand, in PbPb collisions, the cross sections receive an additional enhancement proportional to the atomic number A, which is associated to the nuclear dependence of the photon - nucleus cross section, with  $\sigma(\gamma A \to GX) \approx A \cdot \sigma(\gamma p \to GX)$ .

In the Tables 1 and 2 we present our predictions for the total cross sections and number of events considering the rapidity range covered by a typical central detector ( $|Y| \le 2.5$ ), as the ATLAS and CMS detectors, as well as for a forward detector ( $2.0 \le Y \le 4.5$ ) as the LHCb detector. For pp/pPb/PbPb collisions we predict cross sections of the order of  $pb/nb/\mu b$ , with the results for a forward detector being, in general, smaller by a factor  $\ge 3$  than for a central detector. In order to estimate the number the events we will consider the expected integrated luminosities in pp/pPb/PbPb collisions presented in Refs. [2,3], which for the next run of LHC and HE – LHC are  $\mathcal{L}=1$   $fb^{-1}/1$   $pb^{-1}/10$   $nb^{-1}$ , while for the FCC

are  $\mathcal{L}=1\,fb^{-1}/29\,pb^{-1}/110\,nb^{-1}$ . Moreover, we will also take into account the leptonic decay mode, with the associated branching ratios for the processes  $W^+\to\mu\nu_\mu$  and  $Z^0\to\mu^+\mu^-$  being 10.63% and 3.3658%, respectively. The results presented in the Tables 1 and 2 indicate that the number of events is large, especially for  $W^+$  production at midrapidities and FCC energies. In principle, such large number will allow to perform the search of tiny effects, as those associated to the presence of anomalous couplings between the gauge bosons, which are a signal of BSM physics (see e.g. Refs. [10,11,15]). Such aspect will be investigated in a future publication.

Some comments are in order. First, in our analysis the contribution for the massive gauge boson production associated to the hadronic component of the photon, denoted resolved contribution, was not included. Such contribution implies that a G + jet final state can be produced via e.g. the  $q\bar{q} o gG$  subprocess. Previous studies have demonstrated that the resolved contribution slightly increases the magnitude of the cross section and is negligible for the production of a massive gauge boson with a large transverse momentum [9]. Second, our estimates were obtained at leading order. The next - to - leading order (NLO) corrections are predicted to increase the LO direct cross section by  $\approx$  10% [16]. As both corrections increase the cross sections, the results presented in this letter can be considered a lower bound for the magnitude of the number of events expected at the LHC, HE - LHC and FCC. We plan to perform a more detailed analysis, taking into account of these corrections and presenting predictions for the transverse momentum distributions, in a forthcoming study. Finally, it is important to emphasize that the cross sections for the inclusive gauge boson production in hadronic collisions, discussed e.g. in Refs. [17,18], where the massive gauge bosons are produced at leading order via the  $q_i \bar{q}_f \rightarrow G$  subprocess and the incident hadrons break up, are a factor  $\approx 10^3$  larger than for the photoproduction of massive gauge bosons. However, distinctly from the inclusive production, the final state present in the photoproduction is characterized by the presence of an intact hadron in the final state, which can be tagged by forward detectors. In addition, a rapidity gap associated to the photon exchange is also present. Both aspects can be used, in principle, to separated the photon – induced processes [1].

As a summary, in this letter we have performed an exploratory study and estimated, for the first time, the  $Z^0$  and  $W^+$  photoproduction in hadronic collisions at the LHC, HE – LHC and FCC energies. Our study is strongly motivated by the high photon – hadron luminosity present in hadronic collisions at high energies, which become feasible the experimental analysis of different final state that can be used to test some of the more important properties of Standard Model (SM) as well to search by BSM physics. We have estimated the rapidity distributions for pp, pPb and PbPb collisions, cross sections for the rapidity ranges covered by central and forward detectors, as well the corresponding number of events in the leptonic decay mode. Our results indicate that the number of events is large enough to allow a future experimental analysis to probe the Standard Model predictions and possible scenarios of the BSM physics.

## **Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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