Proton Decay: Flipped vs Unflipped SU(5)

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ABSTRACT

We analyze nucleon decay modes in a no-scale supersymmetric flipped SU(5) GUT model, and contrast them with the predictions for proton decays via dimension-6 operators in a standard unflipped supersymmetric SU(5) GUT model. We find that these GUT models make very different predictions for the ratios $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$, $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to \pi^0 e^+)$, $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(p \to K^0 \mu^+)/\Gamma(p \to \pi^0 \mu^+)$, and that predictions for the ratios $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to \pi^0 e^+)$ also differ in variants of the flipped SU(5) model with normal- or inverse-ordered light neutrino masses. Upcoming large neutrino experiments may have interesting opportunities to explore both GUT and flavour physics in proton and neutron decays.

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1

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1 Introduction

The advent of a new generation of high-mass underground neutrino detectors—JUNO [1], DUNE [2] and Hyper-Kamiokande [3]—will also open up new prospects for searches for proton (and neutron) decays into an array of channels with sensitivities an order of magnitude beyond current experiments. This motivates a re-evaluation of possible nucleon decay modes in different grand unified theories (GUTs), and analyses of specific signatures that may discriminate between the different models. A well-known example is the distinction that can be drawn between the minimal nonsupersymmetric SU(5) GUT [4]—in which the most characteristic proton decay mode is expected to be $p \to \pi^0 e^+$ induced by dimension-6 operators—and the minimal supersymmetric SU(5) GUT [5]—in which the dominant decay mode is expected to be $p \to K^+ \bar{\nu}$ [6] induced by dimension-5 operators [7]. The prospective sensitivities of the new generation of neutrino detectors to these decay modes has been documented [1–3], and the rate for $p \to K^+ \bar{\nu}$ in the minimal supersymmetric SU(5) GUT has recently been re-evaluated, including an assessment of the uncertainties in the lifetime estimate [8].

As is well known, the difference between the dominant nucleon decays in the minimal supersymmetric and non-supersymmetric versions of SU(5) is linked to the difference between their respective decay mechanisms. Proton decay in minimal non-supersymmetric SU(5) is mediated by dimension-6 operators [9], whereas in minimal supersymmetric SU(5) $p \to K^+ \bar{\nu}$ is mediated by dimension-5 operators [7]. The rate for dimension-5 proton decay is high enough to put pressure on minimal supersymmetric SU(5) [10, 11], though this problem is mitigated by the higher sparticle masses [8, 12–19] now required by fruitless LHC searches [20, 21]. Nevertheless, this issue has added to the motivations for considering the supersymmetric flipped SU(5) GUT [22–25], in which an economical missing-partner mechanism [24, 26, 27] suppresses dimension-6 proton decay. This model is also of interest because it can easily be accommodated within string theory [25, 28], and a unified cosmological scenario for inflation, dark matter, neutrino masses and baryogenesis has been constructed [29–32] in the combined framework of flipped SU(5) and string-motivated [33] no-scale supergravity [34–36].

The dominant final states for proton decay in supersymmetric flipped SU(5) are not expected to contain strange particles, with many of the favoured decay modes expected to be similar to those in minimal supersymmetric SU(5), including $p \to \pi^0 e^+$ and $\pi^+ \bar{\nu}$ [37]. It is therefore important to assemble a kit of diagnostic tools that the upcoming experiments can use to discriminate between the flipped and unflipped SU(5) GUT models. [1] This issue has been discussed previously [39–43], and the purpose of this paper is to update the available diagnostic kit in the framework of the unified cosmological framework that we have proposed previously [29–32], stressing the connection between the flavour structure of nucleon decay operators and the pattern of mixing between neutrinos and their mass ordering.

We identify two primary proton decay signatures of the no-scale flipped SU(5) model [29–32] that may also cast light on the mass-ordering of light neutrinos. One signature is the ratio $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$, and the other is $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to \pi^0 e^+)$. [2] In minimal

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[1]See [38] for proposed diagnostic tools for other GUT models.
[2]Here and subsequently, the sum over the three light neutrino species is to be understood.

SU(5) one expects $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+) \sim 0.008$, whereas this ratio is ~ 0.1 in flipped SU(5) with normally-ordered (NO) light neutrinos and ~ 23 with inversely-ordered (IO) neutrinos. In the case of $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to \pi^0 e^+)$, the IO flipped SU(5) model predicts a ratio ~ 95 and the NO model predicts a ratio ~ 3.2, whereas the minimal SU(5) model allows values as low as 0.4. In addition to these headline signatures, we also find that the ratio $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ would be larger in flipped SU(5) than in minimal SU(5), ~ 0.02 vs ~ 0.003, whereas the ratio $\Gamma(p \to K^0 \mu^+)/\Gamma(p \to \pi^0 \mu^+) \sim 0.02$ in the flipped SU(5) model, as opposed to ~ 17 in minimal SU(5). It is clear therefore, that measurements of proton decay in more than one final state could discriminate between underlying GUT models, and we show that searches for neutron decays may also play an important role.

The outline of this paper is the following. In Section 2 we review relevant features of the no-scale flipped SU(5) GUT model, and in Section 3 we study proton (and some neutron) decay modes in this model, giving expressions in terms of the relevant hadronic matrix elements and discussing their uncertainties. The corresponding expressions in unflipped SU(5) are discussed in Section 4. In Section 5 we present predictions for ratios of proton decay rates in the flipped and unflipped SU(5) GUTs, and we review our conclusions and discuss future prospects in Section 6.

2   The No-Scale Flipped SU(5) Model

In the no-scale flipped SU(5) $\times$ U(1) GUT model [22–25, 29–32], the three generations of the minimal supersymmetric extension of the Standard Model (MSSM) matter fields are embedded, together with three right-handed singlet neutrino chiral superfields, into three sets of $\mathbf{10}$, $\mathbf{\bar{5}}$, and $\mathbf{1}$ representations of SU(5), which we denote by F_i, $\bar{f}_i$ and ℓ_i^c, respectively, where $i = 1, 2, 3$ is the generation index. In units of $1/\sqrt{40}$, the U(1) charges of the F_i, $\bar{f}_i$ and ℓ_i^c are $+1$, -3, and $+5$, respectively. The assignments of the quantum numbers for the right-handed leptons, up- and down-type quarks are "flipped" with respect to the standard SU(5) assignments, giving the model its flippant name.

In addition to these matter fields, the minimal flipped SU(5) model contains a pair of $\mathbf{10}$ and $\mathbf{\overline{10}}$ Higgs fields, H and $\bar{H}$, respectively, a pair of $\mathbf{5}$ and $\mathbf{\bar{5}}$ Higgs fields, h and $\bar{h}$, respectively, and four singlet fields, ϕ_a ($a = 0, \ldots, 3$). The vacuum expectation values (VEVs) of the H and $\bar{H}$ fields break the SU(5) $\times$ U(1) gauge group down to the SM gauge group, and subsequently the VEVs of the doublet Higgs fields H_d and H_u, which reside in h and $\bar{h}$, respectively, break the SU(2)$_L$ $\times$ U(1)$_Y$ gauge symmetry down to the U(1) of electromagnetism.

The renormalizable superpotential in this model is given by

$$W = \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i \ell_j^c h + \lambda_4 HHh + \lambda_5 \bar{H}\bar{H}\bar{h}$$
$$+ \lambda_6^{ia} F_i \bar{H} \phi_a + \lambda_7^a h \bar{h} \phi_a + \lambda_8^{abc} \phi_a \phi_b \phi_c + \mu^{ab} \phi_a \phi_b \,. \tag{1}$$

We assume here that the model possesses an approximate $\mathbb{Z}_2$ symmetry, under which only the H field is odd while the rest of the fields are even. This symmetry is supposed to be violated by some Planck-scale suppressed operators, which prevent the formation of domain

3

walls when the field H acquires a VEV. This $\mathbb{Z}_2$ symmetry forbids some unwanted terms, such as $F_i H h$ and $\bar{f}_i H \bar{h}$, which would cause baryon/lepton-number violation as well as R-parity violation. The $\mathbb{Z}_2$ symmetry also forbids a vector-like mass term for H and $\bar{H}$, which is advantageous for suppressing rapid proton decay induced by colour-triplet Higgs exchange.

We embed the flipped SU(5) model in minimal $N = 1$ supergravity, which we assume to have a Kähler potential of no-scale form [36], as is motivated by the low-energy structure of string theory [33]. In this case the potential V has an F- and D-flat direction along a linear combination of the singlet components in H and $\bar{H}$. These fields develop VEVs in this direction, as discussed in detail in Ref. [30]. After H and $\bar{H}$ acquire VEVs in this 'flaton' direction, the coloured components in these fields form vector-like multiplets with those in h and $\bar{h}$ via the couplings λ_4 and λ_5 in (1). On the other hand, the electroweak doublets H_d and H_u in h and $\bar{h}$ do not acquire masses from the flaton VEV—this is an economical realization of the missing-partner mechanism [24] that solves naturally the doublet-triplet splitting problem.

As discussed in detail in Ref. [29], this model offers the possibility of successful Starobinsky-like [44] inflation, with one of the singlet fields, ϕ_0, playing the role of the inflaton [45]. For $\mu^{00} = m_s/2$ and $\lambda_8^{000} = -m_s/(3\sqrt{3}M_P)$ in (1) with the inflaton mass $m_s \simeq 3 \times 10^{13}$ GeV and $M_P \equiv (8\pi G_N)^{-1/2}$ the reduced Planck mass, the measured amplitude of the primordial power spectrum is successfully reproduced and the tensor-to-scalar ratio $r \simeq 3 \times 10^{-3}$, well within the range allowed by the Planck results and other data [46]. This prediction can be tested in future CMB experiments such as CMB-S4 [47] and LiteBIRD [48]. The predicted value of the tilt in the scalar perturbation spectrum, n_s, is also within the range favoured by Planck and other data at the 68% CL [46].

As seen in Eq. (1), the inflaton ϕ_0 can couple to the matter sector via the couplings λ_6 and λ_7. In Ref. [29], two distinct cases, $\lambda_6^{i0} = 0$ (Scenario A) or $\lambda_6^{i0} \neq 0$ (Scenario B), were studied. We focus on Scenario B in this work. In this scenario, one of the three singlet fields other than ϕ_0, which we denote by ϕ_3, does not have the λ_6 coupling; i.e., $\lambda_6^{i3} = 0$, whereas $\lambda_6^{ia} \neq 0$ for $i = 1, 2, 3$ and $a = 0, 1, 2$. We also assume $\lambda_7^a = 0$ for $a = 0, 1, 2$. To realize this scenario, we introduce a modified R-parity, under which the fields in this model transform as

$$F_i, \bar{f}_i, \ell_i^c, \phi_0, \phi_1, \phi_2 \to -F_i, -\bar{f}_i, -\ell_i^c, -\phi_0, -\phi_1, -\phi_2 \ ,$$
$$H, \bar{H}, h, \bar{h}, \phi_3 \to H, \bar{H}, h, \bar{h}, \phi_3 \ . \tag{2}$$

We note that this modified R-parity is slightly violated by the coupling λ_8^{000}. Nevertheless, since this R-parity-violating effect is only very weakly transmitted to the matter sector, the lifetime of the lightest supersymmetric particle (LSP) is still much longer than the age of the Universe [30, 49], so the LSP can be a good dark matter candidate. We also note that the singlet ϕ_3 can acquire a VEV without spontaneously breaking the modified R-parity. In this case, the coupling λ_7^3, which is allowed by the modified R-parity, generates an effective μ term for h and $\bar{h}$, $\mu = \lambda_7^3 \langle \phi_3 \rangle$, just as in the next-to-minimal supersymmetric extension of the SM.

As discussed in detail in Refs. [29–32], the λ_6 coupling in this model controls i) inflaton decays and reheating; ii) the gravitino production rate and therefore the non-thermal

4

abundance of the LSP; iii) neutrino masses; and iv) the baryon asymmetry of the Universe via leptogenesis [50]. In particular, we showed in Refs. [31, 32] by scanning over possible values of λ_6 that the observed values of neutrino masses, the dark matter abundance, and baryon asymmetry can be explained simultaneously, together with a soft supersymmetry-breaking scale in the multi-TeV range. In this paper, we study nucleon decays in the scenario developed in Refs. [29–32].

Without loss of generality, we adopt the basis where λ_2^{ij} and μ^{ab} are real and diagonal. In this case, the MSSM matter fields and right-handed neutrinos are embedded into the SU(5) representations as in [39]: [3]

$$F_i \ni \left\{ Q_i,\ V_{ij}e^{-i\varphi_j}d_j^c,\ (U_{\nu^c})_{ij}\,\nu_j^c \right\}\ ,$$
$$\bar{f}_i \ni \left\{ u_i^c\ , L_j\,(U_l)_{ji} \right\}\ ,$$
$$\ell_i^c = (U_{l^c})_{ij}\,e_j^c\ , \tag{3}$$

where the V_{ij} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, U_{ν^c}, U_l, and U_{l^c} are unitary matrices, and the phase factors φ_i satisfy the condition $\sum_i \varphi_i = 0$ [39]. The components of the doublet fields Q_i and L_i are written as

$$Q_i = \begin{pmatrix} u_i \\ V_{ij}d_j \end{pmatrix}\ , \qquad L_i = \begin{pmatrix} (U_{\mathrm{PMNS}})_{ij}\nu_j \\ e_i \end{pmatrix}\ , \tag{4}$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. [4]

The diagonal components of λ_2^{ij} and μ^{ab} $(a, b = 0, 1, 2)$ are given by

$$\lambda_2 \simeq \frac{1}{\langle \bar{h}_0 \rangle}\mathrm{diag}(m_u, m_c, m_t)\ , \qquad \mu = \frac{1}{2}\mathrm{diag}(m_s, \mu^1, \mu^2)\ , \tag{5}$$

where we take $m_s = 3 \times 10^{13}$ GeV (see above). In what follows we express these matrices as $\lambda_2^{ij} = \lambda_2^i \delta^{ij}$ and $\mu^{ab} = \mu^a \delta^{ab}$. The first equation in Eq. (5) is only an approximate expression, since in general renormalization-group effects and threshold corrections cause λ_2 to deviate from the up-type Yukawa couplings at low energies. However, since these effects are at most $\mathcal{O}(10)\%$ and depend on the mass spectrum of the theory, we neglect them in the following analysis.

The neutrino/singlet-fermion mass matrix can be written as

$$\mathcal{L}_{\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix} \nu_i & \nu_j^c & \tilde{\phi}_a \end{pmatrix}\begin{pmatrix} 0 & \lambda_2^{ij}\langle\bar{h}_0\rangle & 0 \\ \lambda_2^{ij}\langle\bar{h}_0\rangle & 0 & \lambda_6^{ja}\langle\tilde{\nu}_{\bar{H}}^c\rangle \\ 0 & \lambda_6^{ja}\langle\tilde{\nu}_{\bar{H}}^c\rangle & \mu^a \end{pmatrix}\begin{pmatrix} \nu_i \\ \nu_j^c \\ \tilde{\phi}_a \end{pmatrix} + \mathrm{h.c.}\ , \tag{6}$$

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[3]We use the basis in which $U_u = U_{u^c} = U_\phi = 1$, where these matrices are as defined in Ref. [39]. Moreover, we have removed the overall phase factor U_6 using the field redefinition of F_i and $\bar{f}_i$ and expressed the diagonal phase matrix U_7 as $(U_7)_{ij} = e^{i\varphi_i}\delta_{ij}$.

[4]We define the PMNS matrix as in the Review of Particle Physics (RPP) [51], and that $U_{\mathrm{PMNS}} = U_{\mathrm{MNS}}^*$ in the notation of Ref. [39].

where $i, j = 1, 2, 3$ and $a = 0, 1, 2$, and $\tilde{\phi}_0$ corresponds to the fermionic superpartner of the inflaton field ϕ_0. The mass matrix of the right-handed neutrinos is then obtained from a first seesaw mechanism:

$$(m_{\nu^c})_{ij} = \sum_{a=0,1,2} \frac{\lambda_6^{ia} \lambda_6^{ja}}{\mu^a} \langle \tilde{\nu}_{\bar{H}}^c \rangle^2 \ , \tag{7}$$

where $\langle \tilde{\nu}_{\bar{H}}^c \rangle$ denotes the VEV of the F- and D-flat direction of the singlet components of H and $\bar{H}$: we take $\langle \tilde{\nu}_{\bar{H}}^c \rangle = 10^{16}$ GeV in the following analysis. We diagonalize the mass matrix in Eq. (7) using a unitary matrix U_{ν^c}:

$$m_{\nu^c}^D = U_{\nu^c}^T m_{\nu^c} U_{\nu^c} \ . \tag{8}$$

The light neutrino mass matrix is then obtained through a second seesaw mechanism [52, 53]:

$$(m_\nu)_{ij} = \sum_k \frac{\lambda_2^i \lambda_2^j (U_{\nu^c})_{ik} (U_{\nu^c})_{jk} \langle \bar{h}_0 \rangle^2}{(m_{\nu^c}^D)_k} \ . \tag{9}$$

This mass matrix is diagonalised by a unitary matrix U_ν, so that

$$m_\nu^D = U_\nu^* m_\nu U_\nu^\dagger \ . \tag{10}$$

We note that, given a matrix λ_6^{ia}, the eigenvalues of the m_ν and m_{ν^c} matrices, as well as the mixing matrices U_{ν^c} and U_ν, are uniquely determined as functions of μ^1 and μ^2 via Eqs. (7–9). The PMNS matrix is given by U_l in Eq. (3) and U_ν in Eq. (10):

$$U_{\text{PMNS}} = U_l^* U_\nu^T \ . \tag{11}$$

Using the measured values of the PMNS matrix elements, we can use this relation to obtain U_l from U_ν. The matrix U_l plays an important role in determining the partial decay widths of proton decay modes, as we will see in the subsequent Section.

3   Nucleon Decay in Flipped SU(5)

We are now ready to discuss nucleon decay in our model. In view of the suppression of the dimension-5 contribution mediated by coloured Higgs fields thanks to the missing-partner mechanism in the flipped SU(5) GUT [24], the main contribution to nucleon decay is due to exchanges of SU(5) gauge bosons. The relevant gauge interaction terms are

$$K_{\text{gauge}} = \sqrt{2} g_5 \big(-\epsilon_{\alpha\beta} (u_a^c)^\dagger X_a^\alpha U_l^T L^\beta + \epsilon^{abc} (Q^{a\alpha})^\dagger X_b^\alpha V P^\dagger d_c^c + \epsilon_{\alpha\beta} (\nu^c)^\dagger U_{\nu^c}^\dagger X_a^\alpha Q^{a\beta} + \text{h.c.} \big) \ , \tag{12}$$

where g_5 is the SU(5) gauge coupling constant, the X_a^α are the SU(5) gauge vector superfields, $P_{ij} \equiv e^{i\varphi_i} \delta_{ij}$, α, β are SU(2)$_L$ indices, and a, b, c are SU(3)$_C$ indices.

Below the GUT scale, the effects of SU(5) gauge boson exchanges are in general described by the dimension-six effective operators

$$\mathcal{L}_6^{\text{eff}} = C_{6(1)}^{ijkl} \mathcal{O}_{ijkl}^{6(1)} + C_{6(2)}^{ijkl} \mathcal{O}_{ijkl}^{6(2)} \ , \tag{13}$$

where

$$\mathcal{O}_{ijkl}^{6(1)} = \int d^2\theta d^2\bar{\theta} \; \epsilon_{abc}\epsilon_{\alpha\beta} \left(u_i^{c\dagger}\right)^a \left(d_j^{c\dagger}\right)^b e^{-\frac{2}{3}g'B} \left(e^{2g_3G}Q_k^{\alpha}\right)^c L_l^{\beta} \; , \tag{14}$$

$$\mathcal{O}_{ijkl}^{6(2)} = \int d^2\theta d^2\bar{\theta}\epsilon_{abc}\epsilon_{\alpha\beta} \; Q_i^{a\alpha}Q_j^{b\beta} e^{\frac{2}{3}g'B} \left(e^{-2g_3G}u_k^{c\dagger}\right)^c e_l^{c\dagger} \; , \tag{15}$$

with G and B the $SU(3)_C$ and $U(1)_Y$ gauge vector superfields, respectively, and g_3 and g' the corresponding gauge couplings. In the unflipped SU(5) GUT both of the Wilson coefficients $C_{6(1,2)}^{ijkl}$ are non-zero, but in flipped SU(5) only $C_{6(1)}^{ijkl}$ is non-zero, and is given by [5]

$$C_{6(1)}^{ijkl} = \frac{g_5^2}{M_X^2}(U_l)_{li}V_{kj}^* e^{i\varphi_j} \; , \tag{16}$$

where M_X is the SU(5) gauge boson mass. The Wilson coefficients are run down to low energy scales using the renormalisation group equations. The renormalisation factors for $C_{6(n)}^{ijkl}$ ($n = 1, 2$) between the GUT scale and the electroweak scale, A_{S_n}, are evaluated at the one-loop level [6] as [55, 56]:

$$A_{S_1} = \left[\frac{\alpha_3(\mu_{\text{SUSY}})}{\alpha_3(\mu_{\text{GUT}})}\right]^{\frac{4}{9}} \left[\frac{\alpha_2(\mu_{\text{SUSY}})}{\alpha_2(\mu_{\text{GUT}})}\right]^{-\frac{3}{2}} \left[\frac{\alpha_1(\mu_{\text{SUSY}})}{\alpha_1(\mu_{\text{GUT}})}\right]^{-\frac{1}{18}}$$
$$\times \left[\frac{\alpha_3(m_Z)}{\alpha_3(\mu_{\text{SUSY}})}\right]^{\frac{2}{7}} \left[\frac{\alpha_2(m_Z)}{\alpha_2(\mu_{\text{SUSY}})}\right]^{\frac{27}{38}} \left[\frac{\alpha_1(m_Z)}{\alpha_1(\mu_{\text{SUSY}})}\right]^{-\frac{11}{82}} ,$$
$$A_{S_2} = \left[\frac{\alpha_3(\mu_{\text{SUSY}})}{\alpha_3(\mu_{\text{GUT}})}\right]^{\frac{4}{9}} \left[\frac{\alpha_2(\mu_{\text{SUSY}})}{\alpha_2(\mu_{\text{GUT}})}\right]^{-\frac{3}{2}} \left[\frac{\alpha_1(\mu_{\text{SUSY}})}{\alpha_1(\mu_{\text{GUT}})}\right]^{-\frac{23}{198}}$$
$$\times \left[\frac{\alpha_3(m_Z)}{\alpha_3(\mu_{\text{SUSY}})}\right]^{\frac{2}{7}} \left[\frac{\alpha_2(m_Z)}{\alpha_2(\mu_{\text{SUSY}})}\right]^{\frac{27}{38}} \left[\frac{\alpha_1(m_Z)}{\alpha_1(\mu_{\text{SUSY}})}\right]^{-\frac{23}{82}} , \tag{17}$$

where m_Z, μ_{SUSY}, and μ_{GUT} denote the Z-boson mass, the SUSY scale and the GUT scale, respectively, and $\alpha_A \equiv g_A^2/(4\pi)$ with g_A ($A = 1, 2, 3$) the gauge coupling constants of the SM gauge groups. We give the electroweak-scale matching conditions for each decay mode in what follows. Below the electroweak scale, we take into account the perturbative QCD renormalization factor, which is computed in Ref. [57] at the two-loop level: $A_L = 1.247$. We then calculate the partial decay widths of various proton decay modes by using the corresponding hadronic matrix elements, for which we use the results obtained from the QCD lattice simulation performed in Ref. [58]. The relevant hadronic matrix elements are listed in Table 1.

In the following we summarise the partial decay widths for the proton decay modes that we discuss in this paper, as well as two relevant neutron decay modes. [7]

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[5]However, although $C_{6(2)}^{ijkl}$ vanishes in flipped SU(5), we retain it in the following formulae so that it can also be used for the unflipped case.

[6]The two-loop RGEs for these coefficients above the SUSY-breaking scale are given in Ref. [54].

[7]We note that these partial decay widths do not depend on the phases φ_i.

Table 1: *hadronic matrix elements used in our analysis, which are taken from Ref. [58]. The statistical and systematic uncertainties are indicated by (...)(...). The subscripts e and μ indicate that the matrix elements are evaluated at the corresponding lepton kinematic points.*

Matrix element	Value [GeV^2]
$\langle\pi^0\|(ud)_R u_L\|p\rangle_e$	$-0.131(4)(13)$
$\langle\pi^0\|(ud)_R u_L\|p\rangle_\mu$	$-0.118(3)(12)$
$\langle\pi^+\|(ud)_R d_L\|p\rangle$	$-0.186(6)(18)$
$\langle K^0\|(us)_R u_L\|p\rangle_e$	$0.103(3)(11)$
$\langle K^0\|(us)_R u_L\|p\rangle_\mu$	$0.099(2)(10)$
$\langle K^+\|(us)_R d_L\|p\rangle$	$-0.049(2)(5)$
$\langle K^+\|(ud)_R s_L\|p\rangle$	$-0.134(4)(14)$

$p \to \pi^0 e^+$

The relevant effective operators below the electroweak scale are

$$\mathcal{L}(p \to \pi^0 l_i^+) = C_{RL}(udul_i)\big[\epsilon_{abc}(u_R^a d_R^b)(u_L^c l_{Li})\big] + C_{LR}(udul_i)\big[\epsilon_{abc}(u_L^a d_L^b)(u_R^c l_{Ri})\big] \ , \qquad (18)$$

where

$$C_{RL}(udul_i) = C_{6(1)}^{111i}(m_Z) \ ,$$
$$C_{LR}(udul_i) = V_{j1}\big[C_{6(2)}^{1j1i}(m_Z) + C_{6(2)}^{j11i}(m_Z)\big] \ . \qquad (19)$$

Note that, since $C_{6(2)}^{ijkl} = 0$ in flipped SU(5), the second term in Eq. (18) vanishes for this model. The partial decay width can be expressed as follows in terms of these coefficients at the hadronic scale:

$$\Gamma(p \to \pi^0 l_i^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \big[|\mathcal{A}_L(p \to \pi^0 l_i^+)|^2 + |\mathcal{A}_R(p \to \pi^0 l_i^+)|^2\big] \ , \qquad (20)$$

where

$$\mathcal{A}_L(p \to \pi^0 l_i^+) = C_{RL}(udul_i)\langle\pi^0|(ud)_R u_L|p\rangle \ ,$$
$$\mathcal{A}_R(p \to \pi^0 l_i^+) = C_{LR}(udul_i)\langle\pi^0|(ud)_R u_L|p\rangle \ . \qquad (21)$$

Setting $i = 1$ in Eq. (20), we obtain

$$\Gamma(p \to \pi^0 e^+)_{\text{flipped}} = \frac{g_5^4 m_p |V_{ud}|^2 |(U_l)_{11}|^2}{32\pi M_X^4}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \left(\langle\pi^0|(ud)_R u_L|p\rangle_e\right)^2 \ , \qquad (22)$$

where m_p and m_π denote the masses of the proton and pion, respectively, and here and subsequently the subscript on the hadronic matrix element indicates that it is evaluated at the corresponding lepton kinematic point.

From Eq. (22), we can readily compute the partial lifetime of the $p \to \pi^0 e^+$ mode as

$$\tau(p \to \pi^0 e^+)_{\text{flipped}} \simeq 7.9 \times 10^{35} \times |(U_l)_{11}|^{-2} \left(\frac{M_X}{10^{16} \text{ GeV}}\right)^4 \left(\frac{0.0378}{\alpha_5}\right)^2 \text{ yrs .} \qquad (23)$$

We note that this tends to be longer than the lifetime predicted in unflipped SU(5) by a factor (see also Eq. (45))

$$\frac{\tau(p \to \pi^0 e^+)_{\text{flipped}}}{\tau(p \to \pi^0 e^+)_{\text{unflipped}}} \simeq \frac{A_{S_1}^2 + (1 + |V_{ud}|^2)^2 A_{S_2}^2}{A_{S_1}^2 |(U_l)_{11}|^2} \simeq \frac{4.8}{|(U_l)_{11}|^2} \ , \qquad (24)$$

as found in Refs. [11, 37, 39, 41]. [8]

$p \to \pi^0 \mu^+$

By using the effective Lagrangian in Eq. (18) and the rate in Eq. (20) for $i = 2$, we have

$$\Gamma(p \to \pi^0 \mu^+)_{\text{flipped}} = \frac{g_5^4 m_p |V_{ud}|^2 |(U_l)_{21}|^2}{32\pi M_X^4} \left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \left(\langle \pi^0 | (ud)_R u_L | p\rangle_\mu\right)^2 \ , \qquad (25)$$

and the partial lifetime of the $p \to \pi^0 \mu^+$ mode is

$$\tau(p \to \pi^0 \mu^+)_{\text{flipped}} \simeq 9.7 \times 10^{35} \times |(U_l)_{21}|^{-2} \left(\frac{M_X}{10^{16} \text{ GeV}}\right)^4 \left(\frac{0.0378}{\alpha_5}\right)^2 \text{ yrs .} \qquad (26)$$

$n \to \pi^- l^+$

We note in passing that the rates of neutron decay modes that include a charged lepton can be obtained from $\Gamma(p \to \pi^0 l_i^+)$ through SU(2) isospin relations:

$$\Gamma(n \to \pi^- l_i^+) = 2\Gamma(p \to \pi^0 l_i^+) \ , \qquad (27)$$

which applies to both the flipped and unflipped SU(5) models.

$p \to \pi^+ \bar{\nu}_i$

The relevant effective Lagrangian term in this case is

$$\mathcal{L}(p \to \pi^+ \bar{\nu}_i) = C_{RL}(uddv_i)\left[\epsilon_{abc}(u_R^a d_R^b)(d_L^c \nu_{Li})\right] \ , \qquad (28)$$

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[8]Values of $(U_l)_{11}$ in specific flipped SU(5) GUT scenarios are discussed later: see Eqn. (60).

with the following matching condition at the electroweak scale

$$C_{RL}(uddv_i) = -V_{j1}C_{6(1)}^{11ji}(m_Z) \ . \tag{29}$$

The partial decay width is then computed as

$$\Gamma(p \to \pi^+ \bar{\nu}_i) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 |\mathcal{A}(p \to \pi^+\bar{\nu}_i)|^2 \ , \tag{30}$$

with

$$\mathcal{A}(p \to \pi^+\bar{\nu}_i) = C_{RL}(uddv_i)\langle\pi^+|(ud)_R d_L|p\rangle \ . \tag{31}$$

We then have

$$\Gamma(p \to \pi^+\bar{\nu}_i)_{\text{flipped}} = \frac{g_5^4 m_p |(U_l)_{i1}|^2}{32\pi M_X^4}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \left(\langle\pi^+|(ud)_R d_L|p\rangle\right)^2 \ . \tag{32}$$

$n \to \pi^0 \bar{\nu}_i$

There is a relation between the partial decay widths for $n \to \pi^0\bar{\nu}_i$ and those of $p \to \pi^+\bar{\nu}_i$ given by isospin:

$$\Gamma(n \to \pi^0\bar{\nu}_i) = \frac{1}{2}\Gamma(p \to \pi^+\bar{\nu}_i) \ , \tag{33}$$

which applies to both the flipped and unflipped SU(5) models.

$p \to K^0 e^+$

The effective interactions in this case are given by

$$\mathcal{L}(p \to K^0 l_i^+) = C_{RL}(usul_i)\big[\epsilon_{abc}(u_R^a s_R^b)(u_L^c l_{Li})\big] + C_{LR}(usul_i)\big[\epsilon_{abc}(u_L^a s_L^b)(u_R^c l_{Ri})\big] \ , \tag{34}$$

with

$$\begin{aligned} C_{RL}(usul_i) &= C_{6(1)}^{121i}(m_Z) \ , \\ C_{LR}(usul_i) &= V_{j2}\big[C_{6(2)}^{1j1i}(m_Z) + C_{6(2)}^{j11i}(m_Z)\big] \ . \end{aligned} \tag{35}$$

We then obtain the partial decay width

$$\Gamma(p \to K^0 l_i^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 \big[|\mathcal{A}_L(p \to K^0 l_i^+)|^2 + |\mathcal{A}_R(p \to K^0 l_i^+)|^2\big] \ , \tag{36}$$

where m_K is the kaon mass and

$$\begin{aligned} \mathcal{A}_L(p \to K^0 l_i^+) &= C_{RL}(usul_i)\langle K^0|(us)_R u_L|p\rangle \ , \\ \mathcal{A}_R(p \to K^0 l_i^+) &= C_{LR}(usul_i)\langle K^0|(us)_R u_L|p\rangle \ . \end{aligned} \tag{37}$$

In particular, for $i = 1$, we have

$$\Gamma(p \to K^0 e^+)_{\text{flipped}} = \frac{g_5^4 m_p |V_{us}|^2 |(U_l)_{11}|^2}{32\pi M_X^4}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \left(\langle K^0|(us)_R u_L|p\rangle_e\right)^2 \ . \tag{38}$$

$\underline{p \to K^0\mu^+}$

With $i = 2$ in Eq. (36), we have

$$\Gamma(p \to K^0\mu^+)_{\text{flipped}} = \frac{g_5^4 m_p |V_{us}|^2 |(U_l)_{21}|^2}{32\pi M_X^4} \left(1 - \frac{m_K^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \left(\langle K^0|(us)_R u_L|p\rangle_\mu\right)^2 \; . \quad (39)$$

$\underline{p \to K^+\bar{\nu}_i}$

The low-energy effective interactions for this decay mode is given by

$$\mathcal{L}(p \to K^+\bar{\nu}_i) = C_{RL}(usd\nu_i)\left[\epsilon_{abc}(u_R^a s_R^b)(d_L^c \nu_i)\right] + C_{RL}(uds\nu_i)\left[\epsilon_{abc}(u_R^a d_R^b)(s_L^c \nu_i)\right] \; , \quad (40)$$

with

$$C_{RL}(usd\nu_i) = -V_{j1}C_{6(1)}^{12ji}(m_Z) \; ,$$
$$C_{RL}(uds\nu_i) = -V_{j2}C_{6(1)}^{11ji}(m_Z) \; . \quad (41)$$

We note that the unitarity of the CKM matrix leads to

$$V_{j1}C_{6(1)}^{12ji} = V_{j2}C_{6(1)}^{11ji} = 0 \; , \quad (42)$$

in the case of flipped SU(5). As a result, we have

$$\Gamma(p \to K^+\bar{\nu}_i) = 0 \; , \quad (43)$$

as found in Ref. [39].

4   Dimension-Six Proton Decay in Unflipped SU(5)

In this Section we review briefly the proton decay calculation in unflipped SU(5), assuming that proton decay is dominantly induced by dimension-6 SU(5) gauge boson exchange, i.e., that the dimension-5 contribution of colour-triplet Higgs exchange is negligible. This assumption is valid, e.g., when the sfermion masses are sufficiently large, i.e., $\gtrsim 100$ TeV [8, 12–18] or if a suitable missing-partner mechanism is invoked [26, 27]. For more detailed discussions of the calculation of proton decay induced by SU(5) gauge boson exchange in unflipped SU(5), see Refs. [8, 14, 18, 59, 60].

In unflipped SU(5), the Wilson coefficients of the effective operators in Eq. (13) are given by

$$C_{6(1)}^{ijkl} = -\frac{g_5^2}{M_X^2}e^{i\varphi_i}\delta^{ik}\delta^{jl} \; ,$$
$$C_{6(2)}^{ijkl} = -\frac{g_5^2}{M_X^2}e^{i\varphi_i}\delta^{ik}(V^*)^{jl} \; . \quad (44)$$

The rest of the calculation is exactly the same as before, so we just summarize the resultant expression for each partial decay width.

11

$\underline{p \to \pi^0 e^+}$

$$\Gamma(p \to \pi^0 e^+) = \frac{g_5^4 m_p}{32\pi M_X^4}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 \left(\langle\pi^0|(ud)_R u_L|p\rangle_e\right)^2 \left[A_{S_1}^2 + (1 + |V_{ud}|^2)^2 A_{S_2}^2\right] . \quad (45)$$

$\underline{p \to \pi^0 \mu^+}$

$$\Gamma(p \to \pi^0 \mu^+) = \frac{g_5^4 m_p}{32\pi M_X^4}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 A_{S_2}^2 \left(\langle\pi^0|(ud)_R u_L|p\rangle_\mu\right)^2 \left[|V_{ud}V_{us}^*|^2\right] . \quad (46)$$

$\underline{p \to \pi^+ \bar{\nu}}$

$$\Gamma(p \to \pi^+ \bar{\nu}_i) = \frac{g_5^4 m_p |V_{ud}|^2}{32\pi M_X^4}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2 \delta^{1i} \left(\langle\pi^+|(ud)_R d_L|p\rangle\right)^2 . \quad (47)$$

$\underline{p \to K^0 e^+}$

$$\Gamma(p \to K^0 e^+) = \frac{g_5^4 m_p}{32\pi M_X^4}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 A_L^2 A_{S_2}^2 \left(\langle K^0|(us)_R u_L|p\rangle_e\right)^2 \left[|V_{ud}V_{us}^*|^2\right] . \quad (48)$$

$\underline{p \to K^0 \mu^+}$

$$\Gamma(p \to K^0 \mu^+) = \frac{g_5^4 m_p}{32\pi M_X^4}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 A_L^2 \left(\langle K^0|(us)_R u_L|p\rangle_\mu\right)^2 \left[A_{S_1}^2 + (1 + |V_{us}|^2)^2 A_{S_2}^2\right] .$$
$$(49)$$

$\underline{p \to K^+ \bar{\nu}}$

$$\Gamma(p \to K^+ \bar{\nu}_i) = \frac{g_5^4 m_p}{32\pi M_X^4}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 A_L^2 A_{S_1}^2$$
$$\times \left[\delta^{1i}|V_{us}|^2 \left(\langle K^+|(ud)_R s_L|p\rangle\right)^2 + \delta^{2i}|V_{ud}|^2 \left(\langle K^+|(us)_R d_L|p\rangle\right)^2\right] . \quad (50)$$

5 Comparison of Proton Decay Rates in Flipped and Unflipped SU(5)

As we now discuss, the predictions for proton decay branching fractions in the flipped SU(5) GUT model are different from those generated by dimension-6 operators in the standard unflipped SU(5) GUT, [9] which may enable future experiments to distinguish these two GUT scenarios. To this end, we focus on the following five quantities and compare the predictions for them in flipped and unflipped SU(5) GUTs:

  i) $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$,

  ii) $\sum_i \Gamma(p \to \pi^+ \bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$,

  iii) $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$,

  iv) $\Gamma(p \to K^0 \mu^+)/\Gamma(p \to \pi^0 \mu^+)$,

  v) $p \to K^+ \bar{\nu}$.

5.1 $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$

From Eqs. (22) and (25), we find that this ratio in the flipped SU(5) is given by

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} = \frac{\left(\langle \pi^0|(ud)_R u_L|p\rangle_\mu\right)^2 |(U_l)_{21}|^2}{\left(\langle \pi^0|(ud)_R u_L|p\rangle_e\right)^2 |(U_l)_{11}|^2} \ . \tag{51}$$

We see that this ratio depends on the unitary matrix U_l, which is determined from U_ν and the PMNS matrix U_{PMNS} via Eq. (11). We also note that by taking the ratio between the two partial decay widths $\Gamma(p \to \pi^0 \mu^+)$ and $\Gamma(p \to \pi^0 e^+)$, many of the factors in these quantities such as the SU(5) gauge boson mass, M_X, the SU(5) gauge coupling constant, g_5, and the renormalization factors, A_L and A_{S_1}, are cancelled, which makes the prediction for this ratio rather robust.

In unflipped SU(5), on the other hand, we obtain (see Eqs. (45) and (46)):

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{unflipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{unflipped}}} = \frac{\left(\langle \pi^0|(ud)_R u_L|p\rangle_\mu\right)^2}{\left(\langle \pi^0|(ud)_R u_L|p\rangle_e\right)^2} \frac{|V_{ud} V_{us}^*|^2}{\left[R_A^2 + (1 + |V_{ud}|^2)^2\right]} \ , \tag{52}$$

where

$$R_A \equiv \frac{A_{S_1}}{A_{S_2}} = \left[\frac{\alpha_1(\mu_{\text{SUSY}})}{\alpha_1(\mu_{\text{GUT}})}\right]^{\frac{2}{33}} \left[\frac{\alpha_1(m_Z)}{\alpha_1(\mu_{\text{SUSY}})}\right]^{\frac{6}{41}} \ . \tag{53}$$

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[9] We assume here that the contributions of dimension-5 operators are suppressed, either by large sparticle and/or triplet Higgs masses, or by some missing-partner mechanism.

We find $R_A \simeq 1$ in a typical supersymmetric mass spectrum, and for $R_A = 1$ we have: [10]

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{unflipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{unflipped}}} \simeq 0.008 \ . \tag{55}$$

Hence, the branching fraction of the muon mode is predicted to be smaller than that of the electron mode by approximately two orders of magnitude in the unflipped SU(5) GUT. This prediction is again rather robust: the uncertainty is $\mathcal{O}(10)\%$, which mainly comes from the errors in the hadronic matrix elements. We note also that the contribution of the color-triplet Higgs exchange to these decay modes in supersymmetric SU(5) is suppressed by small Yukawa couplings, and thus is negligible unless there is flavor violation in the sfermion mass matrices [13].

To determine the predicted value of the ratio in flipped SU(5) given by Eq. (51), we perform a parameter scan similar to that in Refs. [31, 32]. We first write the Yukawa matrix λ_6 in the form

$$\lambda_6 = r_6 M_6 \ , \tag{56}$$

where r_6 is a real constant, which plays a role of a scale factor, and M_6 is a generic complex 3×3 matrix. We then scan r_6 with a logarithmic distribution over the range $(10^{-4}, 1)$ choosing a total of 1000 values. For each value of r_6, we generate 1000 random complex 3×3 matrices M_6 with each component taking a value of $\mathcal{O}(1)$.

As discussed in Refs. [31, 32], for each 3×3 matrix λ_6, the eigenvalues of the m_ν and m_{ν^c} matrices and the mixing matrices U_{ν^c} and U_ν are obtained as functions of μ^1 and μ^2 in Eq. (5). We then determine these two μ parameters by requiring that the observed values of the squared mass differences, $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{3\ell}^2 \equiv m_3^2 - m_\ell^2$, are reproduced within the experimental uncertainties, where $\ell = 1$ for the NO case and $\ell = 2$ for the IO case. For the experimental input, we use the results from ν-fit 4.0 given in Ref. [62]. By using U_ν determined in this manner, we then compute the matrix U_l using the relation (11). We parametrise the PMNS matrix elements following the RPP convention [51]:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix} , \tag{57}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ with the mixing angles $\theta_{ij} = [0, \pi/2]$, the Dirac CP phase $\delta \in [0, 2\pi]$, and the order $m_1 < m_2$ is chosen without loss of generality. Again we use the values obtained in Ref. [62] for θ_{12}, θ_{23}, θ_{13}, and δ. As for the Majorana phases α_2 and α_3, we set $\alpha_2 = \alpha_3 = 0$ in this analysis since, as we shall see below, the result scarcely depends

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[10]This result is consistent with the formula given in Ref. [61]

$$\left(\frac{\Gamma(p \to \mu^+ + X)}{\Gamma(p \to e^+ + X)} \right)_{X \text{ nonstrange}} = \frac{\sin^2\theta_c \cos^2\theta_c}{(1 + \cos^2\theta_c)^2 + 1} \simeq 0.01 \ , \tag{54}$$

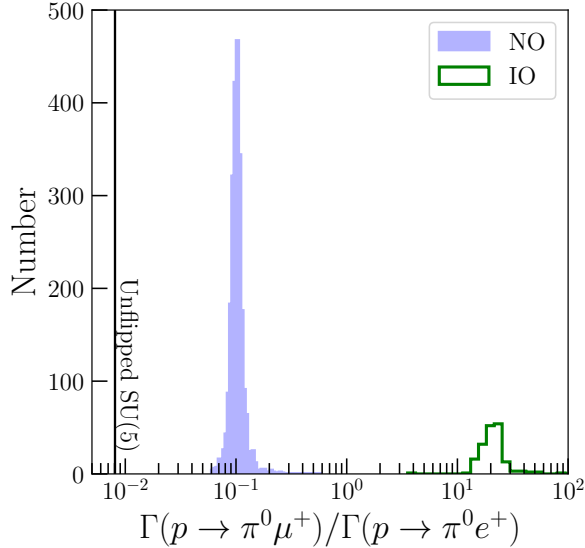where θ_c is the Cabibbo angle: $\sin\theta_c \simeq 0.2245$.

Figure 1: *Histograms of $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ in the flipped SU(5) GUT model for the NO and IO cases in blue and green, respectively. The vertical line corresponds to the unflipped SU(5) prediction.*

on these phases. We generate the same number of λ_6 matrices for each mass ordering, and find solutions for 2399 and 180 matrix choices for the NO and IO cases, respectively, out of a total of 10^6 parameter sets sampled. This difference indicates some preference for the NO case in our model.

In Fig. 1 we display histograms of the ratio $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ in the NO and IO scenarios in blue and green, respectively. The vertical black solid line represents the predicted value in unflipped SU(5). As we see, the flipped SU(5) Model predicts this ratio to be ~ 0.10 and ~ 23 for the NO and IO cases, respectively. To understand the origin of these values, we first note that, due to the hierarchical structure of m_ν in Eq. (9), U_ν has a simple form:

$$U_\nu \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \tag{58}$$

for NO, where $\sin\theta$ is found to be ~ 0.38, and

$$U_\nu \simeq \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{59}$$

for IO, where the first matrix in the right-hand side arranges the order of the neutrino mass eigenvalues in accordance with the RPP convention. The relevant matrix elements of

15

$U_l = U^*_{\text{PMNS}} U_\nu$ are then given by

$$(U_l)_{11} \simeq \begin{cases} (U^*_{\text{PMNS}})_{11} = c_{12}c_{13} & \text{NO} \\ (U^*_{\text{PMNS}})_{13} = s_{13}e^{i\delta - i\frac{\alpha_3}{2}} & \text{IO} \end{cases}, \tag{60}$$

$$(U_l)_{21} \simeq \begin{cases} (U^*_{\text{PMNS}})_{21} = -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & \text{NO} \\ (U^*_{\text{PMNS}})_{23} = s_{23}c_{12}e^{-i\frac{\alpha_3}{2}} & \text{IO} \end{cases}, \tag{61}$$

which leads to

$$\frac{\Gamma(p \to \pi^0\mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} \simeq \frac{(\langle\pi^0|(ud)_R u_L|p\rangle_\mu)^2}{(\langle\pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{|s_{12}c_{23} + c_{12}s_{23}s_{13}e^{-i\delta}|^2}{(c_{12}c_{13})^2} \simeq 0.10 , \tag{62}$$

for NO, and

$$\frac{\Gamma(p \to \pi^0\mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} \simeq \frac{(\langle\pi^0|(ud)_R u_L|p\rangle_\mu)^2}{(\langle\pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{(s_{23}c_{12})^2}{s_{13}^2} \simeq 22.9 , \tag{63}$$

for IO. These approximate estimates are in good agreement with the results given in Fig. 1. We also note that these two expressions do not depend on the unknown Majorana phases, α_2 and α_3. As a consequence, although we have fixed these phases to be zero in our analysis, we expect that the results in Fig. 1 will not be changed even if we take different values for these phases.

The values of $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ predicted in the NO and IO flipped SU(5) scenarios are rather insensitive to the mass of the lightest neutrino, as seen in Fig. 2. On the other hand, we also see there that the spread in predicted values increases with the lightest neutrino mass. It may be challenging for the envisaged next-generation neutrino experiments to measure any deviation from the central values of the model predictions, but the NO and IO predictions remain well separated and hence distinguishable.

The predicted values of $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ in flipped SU(5) are much larger than the standard unflipped SU(5) prediction, which is $\simeq 0.008$. We may therefore be able to distinguish these two models in future proton decay experiments by measuring the partial lifetimes of these two decay modes. We can also determine the neutrino mass ordering in the case of flipped SU(5). Proton decay experiments are relatively sensitive to both of these decay modes, leading to the strongest available constraints on proton partial lifetimes: the current limit on $\tau(p \to \pi^0 e^+)$ from Super-Kamiokande is 2.4×10^{34} yrs and that on $\tau(p \to \pi^0\mu^+)$ is 1.6×10^{34} yrs [63, 64] which can be compared to the predicted partial lifetimes given in Eq. (23) and (26), respectively. This makes the ratio $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ given in Eq. (51) interesting for testing the prediction of flipped SU(5) in future proton decay experiments such as Hyper-Kamiokande [3].
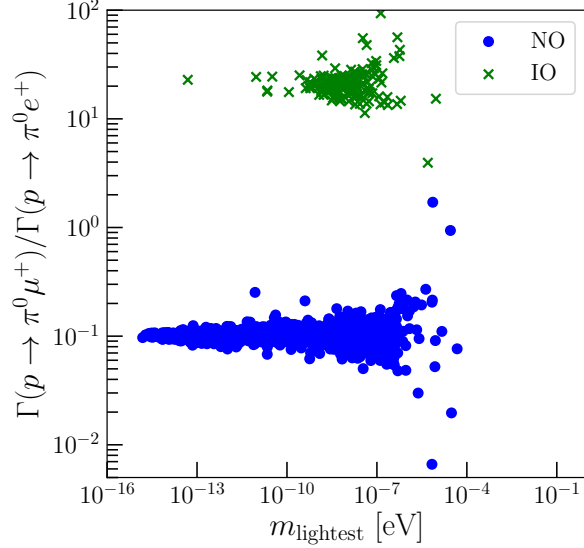
Figure 2: *Scatter plots of values of $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ in the flipped SU(5) GUT model for the NO and IO cases (blue and green, respectively), as functions of the lightest neutrino mass.*

5.2 $\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$

Next we consider the ratio $\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$. Eqs. (32) and (22) imply that for the flipped SU(5) we have

$$\frac{\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} = \frac{(\langle\pi^+|(ud)_R d_L|p\rangle)^2}{(\langle\pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{1}{|V_{ud}|^2 |(U_l)_{11}|^2} \, , \tag{64}$$

whereas for unflipped SU(5) we can use Eqs. (47) and (45) to obtain

$$\frac{\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)_{\text{unflipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{unflipped}}} = \frac{(\langle\pi^+|(ud)_R d_L|p\rangle)^2}{(\langle\pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{R_A^2 |V_{ud}|^2}{\left[R_A^2 + (1 + |V_{ud}|^2)^2\right]} \, . \tag{65}$$

Setting $R_A = 1$ again, we find

$$\frac{\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)_{\text{unflipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{unflipped}}} \simeq 0.4 \, . \tag{66}$$

We note, however, that in the supersymmetric standard SU(5) GUT colour-triplet Higgs exchange also induces $p \to \pi^+\bar{\nu}$ (see, for instance, Refs. [8, 13, 14, 17]), which can be much larger than the contribution in Eq. (47). Therefore, the value in Eq. (66) should be regarded as a lower limit on $\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$ in standard unflipped SU(5).
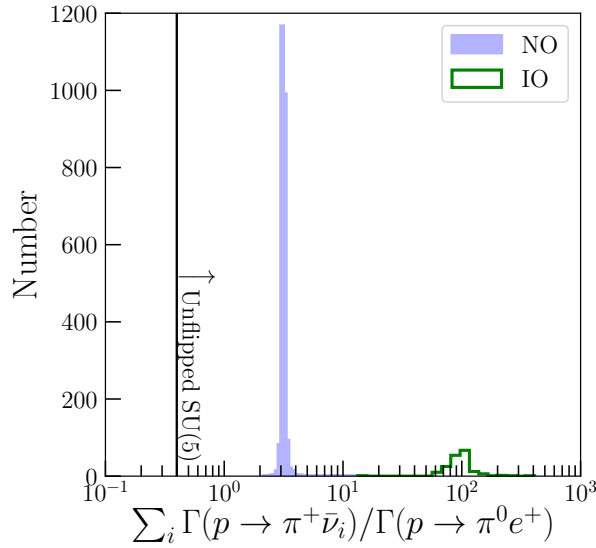
We show in Fig. 3 histograms of $\sum_i \Gamma(p \to \pi^+\bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$ in the flipped SU(5) model for the NO and IO cases in blue and green, respectively. Unflipped SU(5) has the

lower limit indicated by the vertical solid line. As in the previous subsection, we can again estimate this ratio using the approximation given in Eq. (60):

$$\frac{\sum_i \Gamma(p \to \pi^+ \bar{\nu}_i)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} = \frac{(\langle \pi^+|(ud)_R d_L|p\rangle)^2}{(\langle \pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{1}{|V_{ud}|^2 (c_{12} c_{13})^2} \simeq 3.15 \;, \qquad (67)$$

for NO, and

$$\frac{\sum_i \Gamma(p \to \pi^+ \bar{\nu}_i)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} = \frac{(\langle \pi^+|(ud)_R d_L|p\rangle)^2}{(\langle \pi^0|(ud)_R u_L|p\rangle_e)^2} \frac{1}{|V_{ud}|^2 s_{13}^2} \simeq 94.8 \;, \qquad (68)$$

for IO, which agree with the results shown in Fig. 3.

Figure 3: *Histograms of $\sum_i \Gamma(p \to \pi^+ \bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$ in flipped SU(5) for the NO and IO cases in blue and green, respectively. The unflipped SU(5) prediction has a lower limit shown as the vertical solid line.*

This ratio is, however, less powerful for distinguishing the flipped and unflipped SU(5) GUTs than $\Gamma(p \to \pi^0 \mu^+)/\Gamma(p \to \pi^0 e^+)$. First, due to the potential contribution of the colour-triplet Higgs exchange, we have only a lower limit on the unflipped SU(5) prediction. Since the predicted values in the flipped SU(5) are larger than this lower limit, the unflipped SU(5) prediction can in principle mimic the flipped SU(5) predictions. Secondly, the sensitivities of experiments to $p \to \pi^+ \bar{\nu}$ and $n \to \pi^0 \bar{\nu}$ tend to be much worse than that to $p \to \pi^0 \mu^+$; the present bound on $p \to \pi^+ \bar{\nu}$ from Super-Kamiokande is $\tau(p \to \pi^+ \bar{\nu}) > 3.9 \times 10^{32}$ yrs and that on $\tau(n \to \pi^0 \bar{\nu}) > 1.1 \times 10^{33}$ yrs [65], which are much lower than the limit on $p \to \pi^0 \mu^+$. On the other hand, the value of $\sum_i \Gamma(p \to \pi^+ \bar{\nu}_i)/\Gamma(p \to \pi^0 e^+)$ predicted in the flipped SU(5) model in the IO case is so large that this might be detectable.

5.3 $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$

The ratio $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ in flipped SU(5) is computed from Eqs. (38) and (22) to be

$$\frac{\Gamma(p \to K^0 e^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} = \frac{(m_p^2 - m_K^2)^2}{(m_p^2 - m_\pi^2)^2} \frac{(\langle K^0 | (us)_R u_L | p \rangle_e)^2}{(\langle \pi^0 | (ud)_R u_L | p \rangle_e)^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \simeq 1.8 \times 10^{-2} \ . \tag{69}$$

As we see, this ratio does not depend on the matrix U_l. In unflipped SU(5), we use Eqs. (48) and (45) to find

$$\frac{\Gamma(p \to K^0 e^+)_{\text{unflipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{unflipped}}} = \frac{(m_p^2 - m_K^2)^2}{(m_p^2 - m_\pi^2)^2} \frac{(\langle K^0 | (us)_R u_L | p \rangle_e)^2}{(\langle \pi^0 | (ud)_R u_L | p \rangle_e)^2} \frac{|V_{ud} V_{us}^*|^2}{\left[R_A^2 + (1 + |V_{ud}|^2)^2 \right]} \simeq 3.3 \times 10^{-3} \ , \tag{70}$$

for $R_A = 1$. The contribution of the colour-triplet Higgs exchange to $p \to K^0 e^+$ is negligible unless flavour violation occurs in sfermion mass matrices [13, 14], so this value can be regarded as a prediction of unflipped SU(5). As we see, this unflipped SU(5) prediction is much lower than the flipped SU(5) prediction (69), and thus we can in principle also use the ratio $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ to distinguish between these two GUT models.

5.4 $\Gamma(p \to K^0 \mu^+)/\Gamma(p \to \pi^0 \mu^+)$

From Eqs. (39) and (25), we have

$$\frac{\Gamma(p \to K^0 \mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 \mu^+)_{\text{flipped}}} = \frac{(m_p^2 - m_K^2)^2}{(m_p^2 - m_\pi^2)^2} \frac{(\langle K^0 | (us)_R u_L | p \rangle_\mu)^2}{(\langle \pi^0 | (ud)_R u_L | p \rangle_\mu)^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \simeq 0.02 \ . \tag{71}$$

Again, this ratio does not depend on the matrix U_l. In unflipped SU(5), Eqs. (49) and (46) lead to

$$\frac{\Gamma(p \to K^0 \mu^+)_{\text{unflipped}}}{\Gamma(p \to \pi^0 \mu^+)_{\text{unflipped}}} = \frac{(m_p^2 - m_K^2)^2}{(m_p^2 - m_\pi^2)^2} \frac{(\langle K^0 | (us)_R u_L | p \rangle_\mu)^2}{(\langle \pi^0 | (ud)_R u_L | p \rangle_\mu)^2} \frac{\left[R_A^2 + (1 + |V_{us}|^2)^2 \right]}{|V_{ud} V_{us}^*|^2} \simeq 16.7 \ , \tag{72}$$

for $R_A = 1$. The contribution of colour-triplet Higgs exchange to $p \to K^0 \mu^+$ is small unless flavour violation occurs in sfermion mass matrices [13, 14]. Therefore, this ratio can again be used to distinguish between the flipped and unflipped SU(5) GUTs.

5.5 $p \to K^+ \bar{\nu}$

This process tends to be the dominant decay mode in the supersymmetric standard unflipped SU(5) GUT model [7]. In flipped SU(5), on the other hand, as seen in Eq. (43), we have [39]

$$\Gamma(p \to K^+ \bar{\nu}_i) = 0 \ . \tag{73}$$

This is a distinctive prediction in flipped SU(5)—if this decay mode is discovered in future proton decay experiments, flipped SU(5) is excluded.

6 Discussion and Prospects

We have explored in this paper various nucleon decay modes in the flipped SU(5) GUT model developed in [29–32], which builds upon earlier studies [22–25, 39]. We have presented flipped SU(5) predictions in scenarios with both normal-ordered neutrino masses (NO) and inverse ordering (IO), and compared them with the predictions of the standard unflipped SU(5) GUT. Our results for the ratios of decay rates $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$, $\Gamma(p \to \pi^+\bar{\nu})/\Gamma(p \to \pi^0 e^+)$, $\Gamma(p \to K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(p \to K^0\mu^+)/\Gamma(p \to \pi^0\mu^+)$ are compiled in Fig. 4. [11] In all cases we see clear differences between the predictions of flipped SU(5) and standard SU(5), and in the cases of $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(p \to \pi^+\bar{\nu})/\Gamma(p \to \pi^0 e^+)$ we also see clear distinctions between the NO and IO predictions.
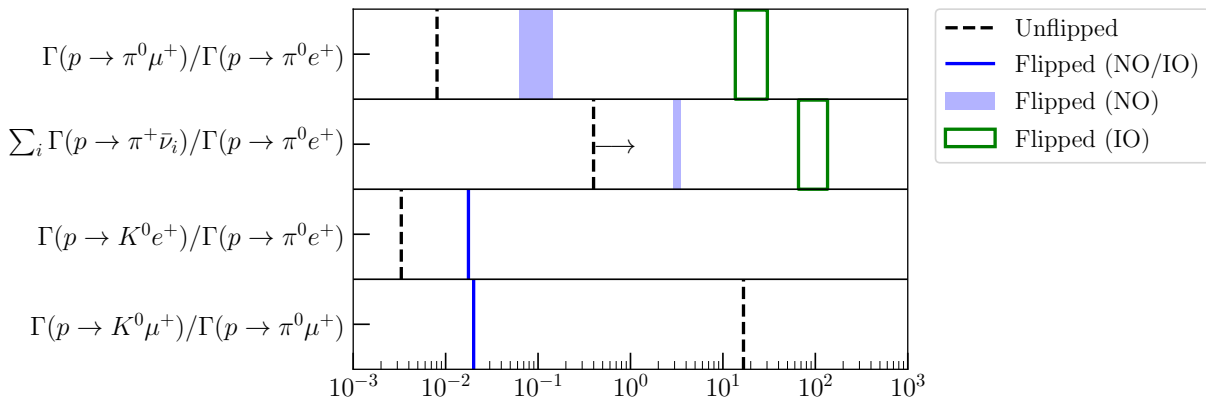

Figure 4: *Compilation of the ratios of proton decay rates predicted in standard unflipped SU(5) (dashed black lines), no-scale flipped SU(5) with neutrino masses that are normal-ordered (NO, blue shading) or inverse-ordered (IO, green boxes). Cases where the flipped SU(5) predictions are independent of the neutrino mass ordering are indicated by solid blue lines.*

The 'Golden Ratio' from the point of view of our analysis is $\Gamma(p \to \pi^0\mu^+)/\Gamma(p \to \pi^0 e^+)$. We recall that Super-Kamiokande has similar sensitivities to these two decay modes, and has established limits on their partial lifetimes of 1.6 and 2.4×10^{34} yrs, respectively [63, 64]. [12] We expect that future proton decay experiments such as Hyper-Kamiokande [3] should have an order of magnitude greater sensitivity to both these decay modes, and hence have a window of opportunity to probe both the NO and IO predictions. Indeed, in the IO case a search for $p \to \pi^0\mu^+$ with a sensitivity to a partial lifetime of 10^{35} yrs would constrain the model as much as a sensitivity to $p \to \pi^0 e^+$ of about 2×10^{36} yrs.

Our results highlight the importance of targeting proton decay modes involving final-state particles from different generations, since our 'Golden Ratio' and two others, $\Gamma(p \to$

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[11]We note also the flipped SU(5) prediction that $\Gamma(p \to K^+\bar{\nu})$ vanishes.

[12]The corresponding searches for $n \to \pi^- l^+$ are less constraining; the present limits on the lifetimes of these decay modes, $\tau(n \to \pi^- e^+) > 5.3 \times 10^{33}$ yrs and $\tau(n \to \pi^- \mu^+) > 3.5 \times 10^{33}$ yrs [66], are weaker than those on $p \to \pi^0 l^+$, while the predicted partial decay widths are larger, as shown in Eq.(27).

$K^0 e^+)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(p \to K^0 \mu^+)/\Gamma(p \to \pi^0 \mu^+)$, involve mixtures of first- and second-generation leptons and quarks.

The fourth ratio, $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to \pi^0 e^+)$, does not involve identifiable second-generation fermions, but second- and third-generation neutrinos contribute to the enhanced values of the ratio predicted in the two flipped SU(5) scenarios we have studied. The current limit on $\tau(p \to \pi^+ \bar{\nu})$ is only 3.9×10^{32} yrs [65]. However, in the IO model this lifetime would be two orders of magnitude shorter than $\tau(p \to \pi^0 e^+)$, so the current limit corresponds to $\tau(p \to \pi^0 e^+) > 3.7 \times 10^{34}$ yrs. Hence the search for $p \to \pi^+ \bar{\nu}$ currently sets a tighter constraint on the IO model than that set by the $p \to \pi^0 e^+$ search. We are unaware of estimates of the improved sensitivity to $p \to \pi^+ \bar{\nu}$ of the upcoming large neutrino experiments, but increasing the sensitivity to $p \to \pi^+ \bar{\nu}$ by the same factor as anticipated for $p \to \pi^0 e^+$ [3] would constrain the IO model as much as a sensitivity to the latter mode of $> 3 \times 10^{35}$ yrs. The current limit $\tau(n \to \pi^0 \bar{\nu}) > 1.1 \times 10^{33}$ yrs [65] constrains the IO model even more, since it corresponds to $\tau(p \to \pi^0 e^+) > 5 \times 10^{34}$ yrs. Again, we are unaware of any estimate of the sensitivity in a future experiment, but an order-of-magnitude improvement would correspond to $\tau(p \to \pi^0 e^+) > 5 \times 10^{35}$ yrs.

These examples show that if the upcoming large neutrino experiments do discover nucleon decay, they will have interesting opportunities to explore both GUT and flavour physics.

Acknowledgements

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