

de Sitter Conjectures in $\mathcal{N} = 1$ SupergravitySergio Ferrara¹, Magnus Taroni² and Antoine Van Proeyen²,¹ Theoretical Physics Department, CERN CH-1211 Geneva 23, Switzerland

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² KU Leuven, Institute for Theoretical Physics, Celestijnenlaan 200D, B-3001 Leuven, Belgium**Abstract**

Supergravity theories at $D = 4$ allow to formulate the Swampland de Sitter conjectures in the complex field space of scalar components of chiral multiplets. We formulate the refined de Sitter conjecture by using the Kähler invariant \mathcal{G} -function and explore a class of models in the Landscape/Swampland scenario which obey and/or violate such conjectures. Furthermore we give a new construction of exponential potentials in supergravity. These depend on a chiral superfield with a Kähler potential parametrizing an $SU(1,1)/U(1)$ geometry. We show that the construction allows for modifications to supergravity theories causing them to obey the de Sitter conjectures.

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1 Introduction

It is well known that while AdS geometries may provide both broken and unbroken supersymmetry in the vacuum of scalar field configurations, de Sitter geometry always implies broken supersymmetry in any phase of the theory. The refined de Sitter conjecture may imply that also metastable vacua may not exist. In the present note we consider classes of simple theories with broken supersymmetry and see whether the Swampland criteria are verified. Our conclusion is that Swampland string conjectures are generically not satisfied in supergravity so that many supergravity theories are not the Effective Field Theories (EFT) of some ultraviolet complete theory. On the other end these constraints may be avoided if string theory with broken supersymmetry is studied beyond its weak coupling perturbative regime.

Based on numerous constructions in string theory conjectures have been put forward that pushes (meta-)stable de Sitter vacua into the Swampland, see [1–5] or the review [6]. We cite from [4]:

Refined de Sitter Conjecture. *A potential $V(\phi)$ for scalar fields in a low energy effective theory of any consistent quantum gravity must satisfy either,*

$$|\nabla V| \geq \frac{c}{M_p} \cdot V, \quad (1.1)$$

or

$$\min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_p^2} \cdot V, \quad (1.2)$$

for some universal constants $c, c' > 0$ of order 1, where the left-hand side of (1.2) is the minimum eigenvalue of the Hessian $\nabla_i \nabla_j V$ in an orthonormal frame.

The refined conjecture in (1.1), (1.2) was first formulated in [3] based on tachyonic tree level de Sitter constructions in IIA/IIB string theory (an earlier suggestion for a modification to [1] was made in [2] but was not formulated as a covariant bound on the scalar potential). More general arguments for the refined conjecture that don't rely on specific string theory compactifications were later on given in [4].

In this work we reformulate the de Sitter Swampland conjectures into $\mathcal{N} = 1$, $D = 4$ supergravity, where the scalars in the conjectures are complex scalars z^α . The first conjecture implies that extrema of the potential do not appear for positive cosmological constant:

$$\nabla_\alpha V = 0 \rightarrow V \leq 0. \quad (1.3)$$

There is no precise value for the constant c , and thus the above implication is the main part of the conjecture that we can check. For positive potential the full conjecture is that

$$V > 0 \rightarrow \kappa^{-2} \frac{\partial_\alpha V g^{\alpha\bar{\beta}} \partial_{\bar{\beta}} V}{V^2} \geq \frac{1}{2} c^2. \quad (1.4)$$

If the first conjecture is not satisfied, the second conjecture may offer an escape for the model. It has thus mainly to be considered when there are vacua with *positive cosmological*

constant. They then still pass the conjectures if the lowest value for second derivatives of the potential is enough negative (depending on the value of c'). Again, since we do not know c' , we will check whether there is a negative value.

The conjectures can further be written as constraints on the Kähler-invariant functional \mathcal{G} , as we will show explicitly below.

The paper contains several short sections made as follows:

In sections 2 and 3 we recall the supergravity mass formulae for non-supersymmetric configurations. In section 4 the refined de Sitter conjectures are formulated in terms of the Kähler-invariant function \mathcal{G} , the fermionic shift and fermion mass matrices. In section 5 and 6 we study the conjectures in the case of the Polónyi model and in the case of R-symmetric potentials for one complex scalar, and obtain bounds on parameters in the theories. Finally in section 7 we consider the case of exponential scalar potentials. Models of this kind can describe present stage quintessence. We show how they can be simply constructed by the inclusion of a chiral multiplet with a Kähler potential typical in α -attractor models to the theory. In particular, we consider the potential induced by nilpotent scalars as in the KKLT construction. These classes of models are more close to stringy potentials and indeed the Swampland conjectures are often satisfied in this case. We further discuss how the construction of single exponential potentials can be used to modify supergravity theories such that they obey the de Sitter conjectures.

2 Mass units

It is easiest to work first with the engineering dimensions as in [7, Sec.18.3.1]. This uses dimensionless scalars. Explicit $\kappa = M_p^{-1}$ are then introduced in

$$\mathcal{K}: \kappa^{-2}, \quad g_{\alpha\bar{\beta}}: \kappa^{-2}, \quad W: \kappa^{-3}. \quad (2.1)$$

E.g. for the simple flat Kähler case we write

$$\mathcal{K} = \kappa^{-2} z^\alpha \delta_{\alpha\bar{\beta}} \bar{z}^{\bar{\beta}}. \quad (2.2)$$

One can redefine at the end $z' = \kappa^{-1}z$ to avoid factors κ in the kinetic term, and z' has then the physical dimension 1.

The potential (here for only chiral multiplets)

$$V = -3\kappa^2 e^{\kappa^2 \mathcal{K}} W \bar{W} + e^{\kappa^2 \mathcal{K}} \nabla_\alpha W g^{\alpha\bar{\beta}} \bar{\nabla}_{\bar{\beta}} \bar{W} \quad (2.3)$$

has then mass dimension 4 as it should be.

The invariant function

$$\mathcal{G} = \kappa^2 \mathcal{K} + \log(\kappa^6 W \bar{W}), \quad (2.4)$$

is dimensionless, and in terms of this:

$$V_F = \kappa^{-4} e^{\mathcal{G}} \left(\mathcal{G}_\alpha \mathcal{G}^{\alpha\bar{\beta}} \mathcal{G}_{\bar{\beta}} - 3 \right), \quad (2.5)$$

where $\mathcal{G}_\alpha = \partial_\alpha \mathcal{G}$, \dots and $\mathcal{G}^{\alpha\bar{\beta}}$ is the inverse of $\mathcal{G}_{\alpha\bar{\beta}} = \kappa^2 g_{\alpha\bar{\beta}}$.

Example: in this way $\partial_\alpha V g^{\alpha\bar{\beta}} \partial_{\bar{\beta}} V$ has, due to the inverse metric, a mass dimension 2 lower than V^2 . This is thus consistent with (1.4). When we go to the z' coordinates this factor κ^2 comes from the two derivatives to the scalars.

3 Mass relations

Several quantities that were introduced to study mass relations in [8] are useful in this context. E.g. one defines

$$X \equiv \kappa^{-2} \frac{\nabla_\alpha W g^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W}}{W \bar{W}}. \quad (3.1)$$

In the \mathcal{G} -formulation, this is

$$X = \mathcal{G}_\alpha \mathcal{G}^\alpha = \kappa^{-2} \mathcal{G}_\alpha g^{\alpha\bar{\beta}} \mathcal{G}_{\bar{\beta}} = \mathcal{G}_\alpha \mathcal{G}^{\alpha\bar{\beta}} \mathcal{G}_{\bar{\beta}}, \quad (3.2)$$

where $\mathcal{G}^{\alpha\bar{\beta}}$ is the inverse of $\mathcal{G}_{\alpha\bar{\beta}} = \kappa^2 g_{\alpha\bar{\beta}}$ and is thus dimensionless. Similarly, we define here

$$\mathcal{G}^\alpha \equiv \mathcal{G}^{\alpha\bar{\beta}} \mathcal{G}_{\bar{\beta}} = \kappa^{-2} g^{\alpha\bar{\beta}} \mathcal{G}_{\bar{\beta}}. \quad (3.3)$$

This allows to write the potential as

$$V = \kappa^{-4} e^{\mathcal{G}} (X - 3). \quad (3.4)$$

The full holomorphic mass matrix for the fermions is

$$M_{\alpha\beta} = \sqrt{\frac{\bar{W}}{W}} e^{\mathcal{G}/2} \left[\mathcal{G}_{\alpha\beta} + \frac{X-2}{X} \mathcal{G}_\alpha \mathcal{G}_\beta \right], \quad \mathcal{G}_{\alpha\beta} = \nabla_\alpha \partial_\beta \mathcal{G}. \quad (3.5)$$

Since the overall factors are going to cancel at the end, we define

$$M_{\alpha\beta} = \sqrt{\frac{\bar{W}}{W}} = \sqrt{\frac{\bar{W}}{W}} e^{\mathcal{G}/2} \mathcal{M}_{\alpha\beta}, \quad \mathcal{M}_{\alpha\beta} = \mathcal{G}_{\alpha\beta} + \frac{X-2}{X} \mathcal{G}_\alpha \mathcal{G}_\beta. \quad (3.6)$$

It appears in the derivative of the potential

$$V_\alpha \equiv \partial_\alpha V = \kappa^{-4} e^{\mathcal{G}} \left[\mathcal{G}_{\alpha\beta} \mathcal{G}^\beta + (X-2) \mathcal{G}_\alpha \right] = \kappa^{-4} e^{\mathcal{G}} \mathcal{M}_{\alpha\beta} \mathcal{G}^\beta. \quad (3.7)$$

For analyzing the condition (1.2), we provide the second derivatives

$$\begin{aligned} \kappa^4 V_{\alpha\bar{\alpha}} &= e^{\mathcal{G}} \left[g_{\alpha\bar{\alpha}} (X-2) - \mathcal{G}_\alpha \mathcal{G}_{\bar{\alpha}} + (\mathcal{G}_{\alpha\beta} + \mathcal{G}_\alpha \mathcal{G}_\beta) g^{\beta\bar{\beta}} (\mathcal{G}_{\bar{\alpha}\bar{\beta}} + \mathcal{G}_{\bar{\alpha}} \mathcal{G}_{\bar{\beta}}) + R_{\alpha\bar{\alpha}}{}^{\beta\bar{\beta}} \mathcal{G}_\beta \mathcal{G}_{\bar{\beta}} \right], \\ \kappa^4 V_{\alpha\beta} &= e^{\mathcal{G}} \left[-(\mathcal{G}_{\alpha\beta} + \mathcal{G}_\alpha \mathcal{G}_\beta) + (\mathcal{G}_{\alpha\beta\gamma} + \mathcal{G}_\alpha \mathcal{G}_\beta \mathcal{G}_\gamma + 3\mathcal{G}_{(\alpha\beta} \mathcal{G}_{\gamma)}) \mathcal{G}^\gamma \right]. \end{aligned} \quad (3.8)$$

4 Conjectures in $\mathcal{N} = 1$ supergravity

Using (3.4) and (3.7), the left-hand side of (1.4) can be written as

$$\kappa^{-2} \frac{\partial_\alpha V g^{\alpha\bar{\beta}} \partial_{\bar{\beta}} V}{V^2} = \frac{\mathcal{M}_{\alpha\beta} \mathcal{G}^\beta \mathcal{G}^{\alpha\bar{\alpha}} \mathcal{M}_{\bar{\alpha}\bar{\beta}} \mathcal{G}^{\bar{\beta}}}{(X-3)^2}, \quad (4.1)$$

and thus the first of the de Sitter Swampland conjectures can be formulated as

$$\sqrt{2\mathcal{M}_{\alpha\beta} \mathcal{G}^\beta \mathcal{G}^{\alpha\bar{\alpha}} \mathcal{M}_{\bar{\alpha}\bar{\beta}} \mathcal{G}^{\bar{\beta}}} \geq c(X-3). \quad (4.2)$$

An alternative expression using

$$\mathcal{H} \equiv \log(\kappa^4 V) = \mathcal{G} + \log(X-3), \quad \mathcal{H}_\alpha = \partial_\alpha \log(\kappa^4 V) = \mathcal{G}_\alpha + \frac{\partial_\alpha X}{X-3}, \quad (4.3)$$

is

$$\kappa^{-2} \frac{\partial_\alpha V g^{\alpha\bar{\beta}} \partial_{\bar{\beta}} V}{V^2} = \mathcal{H}_\alpha \mathcal{G}^{\alpha\bar{\beta}} \mathcal{H}_{\bar{\beta}} = X + \frac{1}{X-3} (\mathcal{G}^\alpha \partial_\alpha X + \text{h.c.}) + \frac{1}{(X-3)^2} \partial_\alpha X \mathcal{G}^{\alpha\bar{\alpha}} \partial_{\bar{\alpha}} X. \quad (4.4)$$

Thus for $V > 0$, according to the first conjecture we should have $\mathcal{H}_\alpha \mathcal{H}^\alpha \geq \frac{1}{2} c^2$.

Note that, using $\nabla_\alpha \mathcal{G}^\beta = \delta_\alpha^\beta$:

$$\partial_\alpha X = \mathcal{G}_{\alpha\beta} \mathcal{G}^\beta + \mathcal{G}_\alpha = \mathcal{M}_{\alpha\beta} \mathcal{G}^\beta + (3-X) \mathcal{G}_\alpha, \quad (4.5)$$

and therefore the connection with the formulation in (4.1) can be readily made by

$$\mathcal{H}_\alpha = \frac{\mathcal{M}_{\alpha\bar{\beta}} \mathcal{G}^{\bar{\beta}}}{X-3}. \quad (4.6)$$

For the second conjecture we introduce latin indices a, b to denote the holomorphic as well as anti-holomorphic coordinates. Since $\partial_a V = \mathcal{H}_a V$, we find for the second derivatives of the potential

$$\nabla_a \partial_b V = \mathcal{H}_{ab} V + \mathcal{H}_a \mathcal{H}_b V, \quad (4.7)$$

where $\mathcal{H}_{ab} = \nabla_a \partial_b \mathcal{H}$. The second de Sitter conjecture for the canonically normalized scalars becomes

$$\min \left(\frac{\partial z^a}{\partial \phi^i} \frac{\partial z^b}{\partial \phi^j} [\mathcal{H}_{ab} + \mathcal{H}_a \mathcal{H}_b] \right) \leq -\kappa^2 c'. \quad (4.8)$$

In terms of \mathcal{G} :

$$\begin{aligned} \nabla_a \partial_b V &= V \left(\mathcal{G}_{ab} + \mathcal{G}_a \mathcal{G}_b + \frac{2}{X-3} \mathcal{G}_{(a} \partial_{b)} X + \frac{1}{X-3} \nabla_a \partial_b X \right), \\ \partial_b X &= \mathcal{G}^a \mathcal{G}_{ba} = \mathcal{G}^{\bar{\alpha}} \mathcal{G}_{b\bar{\alpha}} + \mathcal{G}^\alpha \mathcal{G}_{b\alpha}, \\ \nabla_a \partial_b X &= \mathcal{G}_a{}^c \mathcal{G}_{bc} + \mathcal{G}^c \mathcal{G}_{abc} = \mathcal{G}_a{}^\alpha \mathcal{G}_{b\alpha} + \mathcal{G}_a{}^{\bar{\alpha}} \mathcal{G}_{b\bar{\alpha}} + \mathcal{G}^\alpha \mathcal{G}_{ab\alpha} + \mathcal{G}^{\bar{\alpha}} \mathcal{G}_{ab\bar{\alpha}}. \end{aligned} \quad (4.9)$$

5 Polónyi model

The Polónyi model [9] is of importance for the description of supersymmetry breaking within the supergravity framework. It contains a single chiral multiplet on a flat Kähler manifold guided by the superpotential $W = \kappa^{-3}\mu(z + \beta)$. Therefore

$$\mathcal{G} = z\bar{z} + \log(\mu^2|z + \beta|^2), \quad \mathcal{G}_z = \bar{z} + \frac{1}{z + \beta}, \quad \mathcal{G}_{z\bar{z}} = 1, \quad \mathcal{G}_{zz} = -\frac{1}{(z + \beta)^2}. \quad (5.1)$$

The parameters μ, β are taken to be real. Since β is of order 1, we will restrict to $0 \leq \beta < 2$.

We write z in terms of two real canonically normalized fields $z = \frac{1}{\sqrt{2}}(x + iy)$. In the direction y the minimum appears for $y = 0$. The potential V is then solely a function of x . We find

$$\begin{aligned} X &= |\mathcal{G}_z|^2 = \left(\frac{x}{\sqrt{2}} + \frac{1}{\frac{x}{\sqrt{2}} + \beta} \right)^2, \\ V(x) &= \kappa^{-4}\mu^2 e^{x^2/2} \left(\frac{x}{\sqrt{2}} + \beta \right)^2 \left(\left(\frac{x}{\sqrt{2}} + \frac{1}{\frac{x}{\sqrt{2}} + \beta} \right)^2 - 3 \right), \\ \mathcal{M}_{zz} &= -\frac{1}{\left(\frac{x}{\sqrt{2}} + \beta\right)^2} + (X - 2), \\ \mathcal{M}_{zz}\mathcal{G}^z &= \frac{(x^2 + \sqrt{2}\beta x + 2)(x^3 + \sqrt{2}\beta x^2 - 4\sqrt{2}\beta)}{2\sqrt{2}(x + \sqrt{2}\beta)^2}. \end{aligned} \quad (5.2)$$

Since the potential is proportional to μ^2 , this parameter is irrelevant in (1.4).

The formula in (4.1) then becomes

$$\kappa^{-1} \frac{|\nabla V|}{V} = \frac{|x^5 + 2\sqrt{2}\beta x^4 + 2(\beta^2 + 1)x^3 - 2\sqrt{2}\beta x^2 - 8\beta^2 x - 8\sqrt{2}\beta|}{2\sqrt{2}(x^4 + 2\sqrt{2}\beta x^3 + 2(\beta^2 - 1)x^2 - 8\sqrt{2}\beta x + 4 - 12\beta^2)}. \quad (5.3)$$

The real root of (5.3) for $0 \leq \beta < 2$ is

$$x = \frac{\sqrt{2}}{3} \left[-\beta + 3\beta^{1/3} \left(1 - \frac{\beta^2}{27} + \sqrt{1 - 2\frac{\beta^2}{27}} \right)^{1/3} + \frac{\beta^{5/3}}{3 \left(1 - \frac{\beta^2}{27} + \sqrt{1 - 2\frac{\beta^2}{27}} \right)^{1/3}} \right]. \quad (5.4)$$

One can check that the single extremum for $0 \leq \beta < 2$ is a minimum. By looking at the zeros of the potential for $0 \leq \beta < 2$

$$V(x_0) = 0 : x_0 = \frac{1}{\sqrt{2}} \left(\sqrt{3} - \beta \pm \sqrt{-1 + 2\sqrt{3}\beta + \beta^2} \right) \quad (5.5)$$

we see the lowest value of β for which real zero value for the potential exists is $\beta = 2 - \sqrt{3}$. In that case $x_0 = \sqrt{2}(\sqrt{3} - 1)$ and, as one can show, it is then also the minimum of the

potential. In fact, it is the value chosen by Polónyi, who was looking for a Minkowski vacuum. This model has also been studied in [10], where Figure 1 is also drawn. One can see in that Figure (where $\kappa = \mu = 1$) that the potential is negative in the interval between

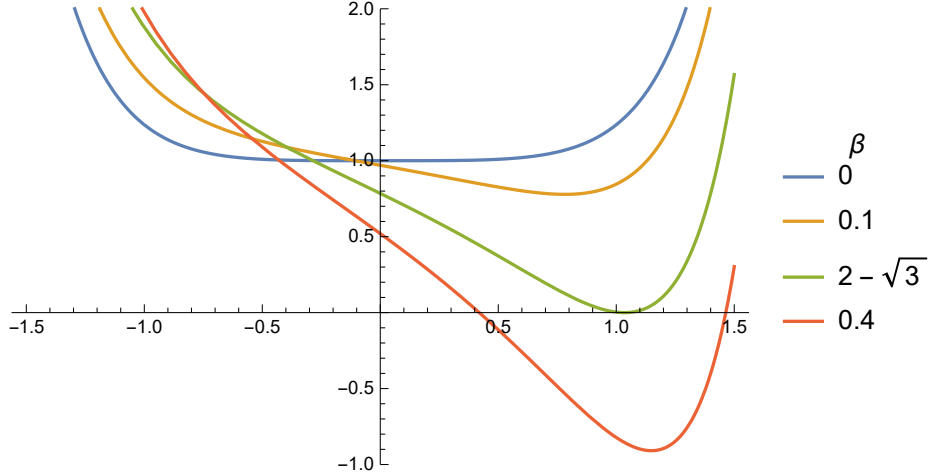


Figure 1: The scalar potential of the Polónyi model for $\beta = \{0, 0.1, 2 - \sqrt{3}, 0.4\}$. For $0 \leq \beta \leq 2 - \sqrt{3}$ the minimum is positioned at $V \geq 0$. When $\beta > 2 - \sqrt{3}$ there is a minimum at $V < 0$.

the zero modes in (5.5), where it will reach its minimum value. Thus we find

$$\begin{aligned} 0 \leq \beta < 2 - \sqrt{3} = 0.27 & : \text{minimum at } V > 0 : (1.3) \text{ violated,} \\ \beta \leq 2 - \sqrt{3} & : \text{minimum at } V \leq 0 : (1.3) \text{ satisfied,} \end{aligned} \quad (5.6)$$

The value of the ratio $|\nabla V|/V$ is shown in Figure 2.

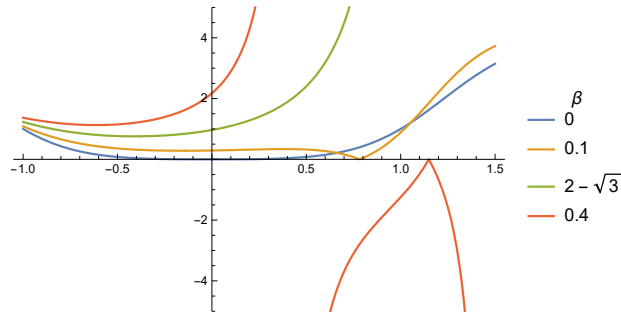


Figure 2: The function $|\nabla V|/V$ for the Polónyi model

There is only one extremum which is a minimum and the second derivative is everywhere positive. Thus for $0 \leq \beta < 2 - \sqrt{3}$ both the de Sitter conjectures (1.1),(1.2) are violated.

6 R-symmetric one scalar model

We consider a flat Kähler model with one scalar and R -symmetry. Then the superpotential should be homogeneous, i.e. $W = \kappa^{-3}z^\alpha$. The case $\alpha = 1$ thus overlaps with $\beta = 0$ in the previous section. We thus have

$$\mathcal{G} = z\bar{z} + \alpha \log(z\bar{z}), \quad \mathcal{G}_z = \frac{1}{z}(z\bar{z} + \alpha), \quad \mathcal{G}_{z\bar{z}} = 1, \quad \mathcal{G}_{zz} = -\frac{\alpha}{z^2}. \quad (6.1)$$

In this case, nearly everything depends only on

$$\rho = z\bar{z}. \quad (6.2)$$

This is in fact the statement of R -symmetry. We find

$$\begin{aligned} X &= |\mathcal{G}_z|^2 = \rho^{-1}(\alpha^2 + 2\alpha\rho + \rho^2), \\ V &= \kappa^{-4}e^\rho \rho^{\alpha-1}(\alpha^2 + (2\alpha - 3)\rho + \rho^2), \\ \mathcal{M}_{zz} &= -\frac{\alpha}{z^2} + (X - 2)\frac{\mathcal{G}_z}{\mathcal{G}_{z\bar{z}}}. \end{aligned} \quad (6.3)$$

We thus find that V is everywhere positive if $\alpha > 3/4$.

Since $\mathcal{G}_z/\mathcal{G}_{z\bar{z}} = \bar{z}/z = z^{-2}\rho$, we find

$$\begin{aligned} \mathcal{M}_{zz} &= \frac{1}{z^2}[\alpha(\alpha - 1) + 2(\alpha - 1)\rho + \rho^2], \\ \mathcal{M}_{zz}\bar{\mathcal{G}}^z &= \frac{1}{\rho z}[\alpha^2(\alpha - 1) + 3\alpha(\alpha - 1)\rho + (3\alpha - 2)\rho^2 + \rho^3]. \end{aligned} \quad (6.4)$$

Since the metric is trivial and the right-hand side of (4.1) is then a modulus squared, we can write

$$\kappa^{-1}\frac{|\nabla_z V|}{V} = |\rho^{-1/2}| \frac{|\alpha^2(\alpha - 1) + 3\alpha(\alpha - 1)\rho + (3\alpha - 2)\rho^2 + \rho^3|}{\alpha^2 + (2\alpha - 3)\rho + \rho^2}. \quad (6.5)$$

The overall factor $\rho^{1/2}$ appears from the change of variables from z to ρ .

$$\kappa^{-1}|\nabla_z V| = \kappa^{-1}|\rho^{1/2}\nabla_\rho V| = \kappa^{-4}|e^\rho \rho^{\alpha-3/2}|\alpha^2(\alpha - 1) + 3\alpha(\alpha - 1)\rho + (3\alpha - 2)\rho^2 + \rho^3|. \quad (6.6)$$

The first derivative of the potential has 4 roots

$$|\nabla V| = 0 \Leftrightarrow \rho \rightarrow \{0, 1 - \alpha - \sqrt{1 - \alpha}, 1 - \alpha + \sqrt{1 - \alpha}, -\alpha\}. \quad (6.7)$$

Whenever the first derivative of the potential has a root the first de Sitter conjecture can be trivially violated. It is therefore interesting to study these specific points. The overall factor $\rho^{\alpha-3/2}$ in (6.6) causes $\rho = 0$ to be a root when $\alpha > 3/2$. But in the de Sitter conjecture the overall factor is reduced to $\rho^{-1/2}$, so there is no effect of this root on the violation of the first de Sitter conjecture. The second and third root are only real for $\alpha \leq 1$. There is thus a distinct behaviour around the value $\alpha = 1$, which can also be seen from Figure 3.

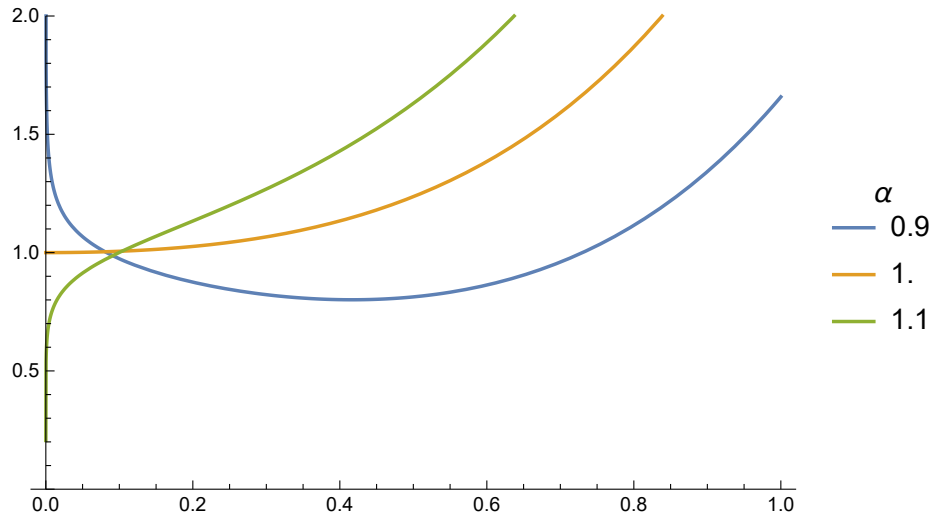


Figure 3: The scalar potential $V(\rho)$ of the one scalar model for values $\alpha \sim 1$

6.1 Evaluation of first conjecture

For $\alpha > 1$ there is no vacuum, and thus the *first condition can be satisfied*. To consider the condition once a value of c is imposed, see Figure 4. The minimum of $|\nabla V|/V \sim \mathcal{O}(1)$.

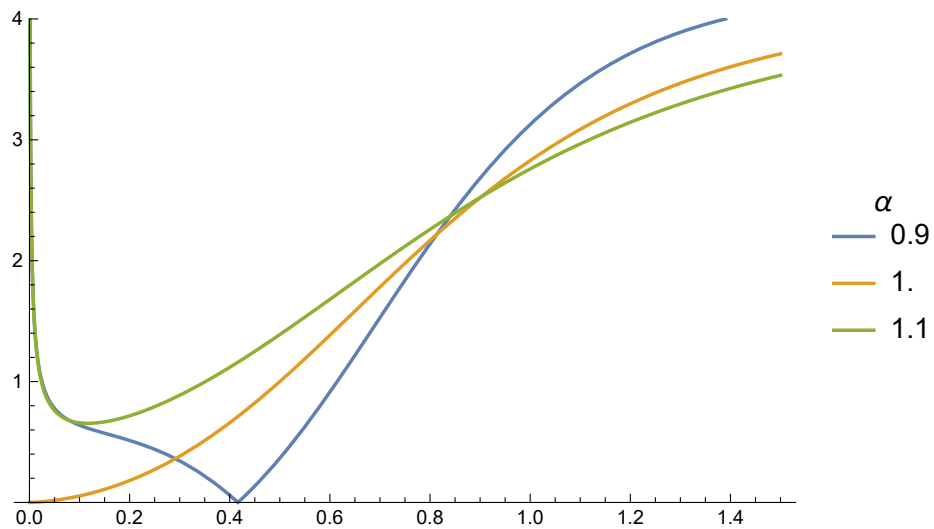


Figure 4: The function $|\nabla V|/V$ of the one scalar model for values $\alpha \sim 1$

For $\alpha \leq 1$, the third root in (6.7) occurs for a positive ρ (or $\rho = 0$ for $\alpha = 1$) and thus defines an extremum. As mentioned above the potential is positive for $\alpha > 3/4$, and thus we can identify different classes of potentials.

For $\frac{3}{4} < \alpha \leq 1$, the potential is positive for $\alpha > 3/4$ and thus we have an extremum with a positive V , hence a *violation* of (1.3).

For $\alpha \leq 3/4$ the potential becomes negative in the following range of values for ρ

$$V(\rho) \leq 0 \quad \text{for} \quad \frac{1}{2} \left(3 - 2\alpha - \sqrt{3}\sqrt{3-4\alpha} \right) \leq \rho \leq \frac{1}{2} \left(3 - 2\alpha + \sqrt{3}\sqrt{3-4\alpha} \right). \quad (6.8)$$

Different cases are shown in Figure 5.

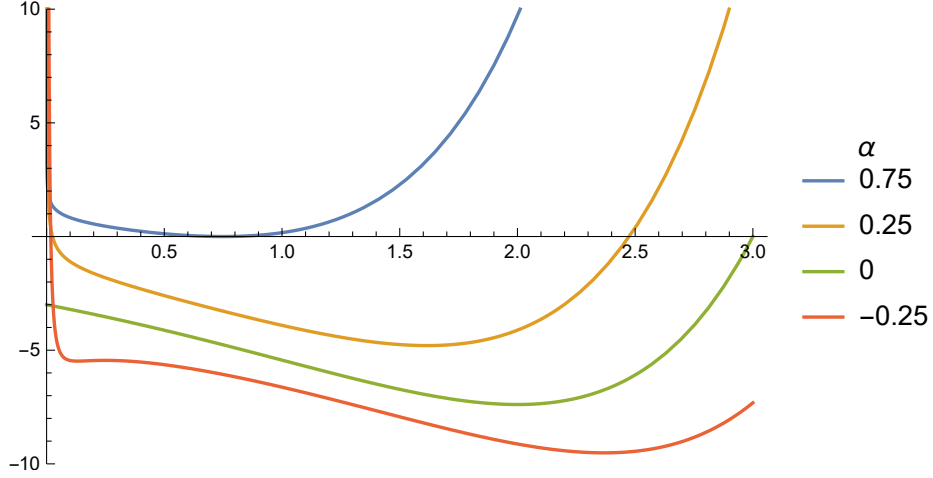


Figure 5: The scalar potential $V(\rho)$ of the one scalar model for values $\alpha \leq 3/4$

For $0 \leq \alpha \leq \frac{3}{4}$ the potential has only one extremum positioned at $\rho = 1 - \alpha + \sqrt{1 - \alpha}$ which falls in the interval of negative potential (6.8). We conclude that the first de Sitter conjecture *is not violated by the extrema of these potentials*. The ratio is shown in Figure 6.

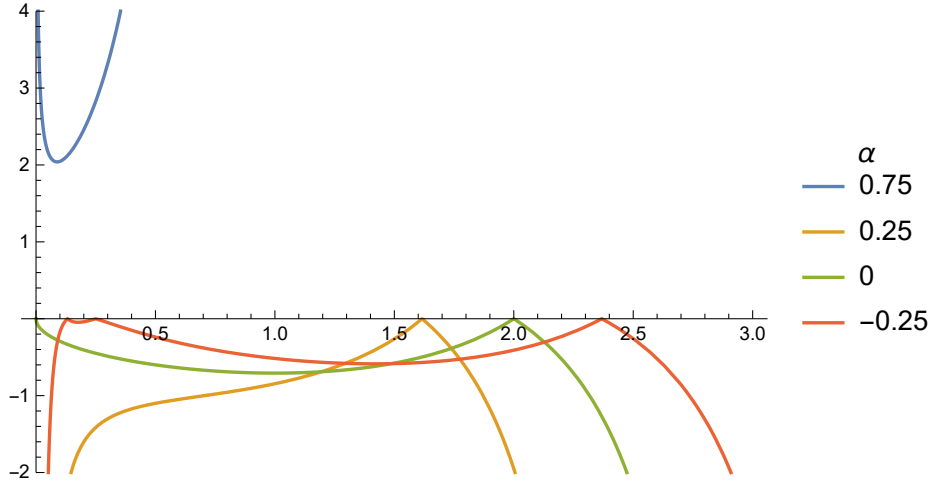


Figure 6: The function $|\nabla V|/V$ of the one scalar model for values $\alpha \leq 3/4$

For $\alpha < 0$ the potential has three extrema positioned at

$$\rho = \left\{ -\alpha, 1 - \alpha - \sqrt{1 - \alpha}, 1 - \alpha + \sqrt{1 - \alpha} \right\}. \quad (6.9)$$

One can check that for $\alpha \leq 0$

$$\begin{aligned}
\kappa^4 V(-\alpha) &= -3e^{-\alpha}(-\alpha)^\alpha \leq 0, \\
\kappa^4 V(1 - \alpha - \sqrt{1 - \alpha}) &= e^{-\alpha - \sqrt{1 - \alpha} + 1} (-\alpha - \sqrt{1 - \alpha} + 1)^{\alpha - 1} (2\alpha + \sqrt{1 - \alpha} - 1) \leq 0, \\
\kappa^4 V(1 - \alpha + \sqrt{1 - \alpha}) &= e^{-\alpha + \sqrt{1 - \alpha} + 1} (-\alpha + \sqrt{1 - \alpha} + 1)^{\alpha - 1} (2\alpha - \sqrt{1 - \alpha} - 1) \leq 0.
\end{aligned} \tag{6.10}$$

So again we see that around the extrema the potentials are negative which implies that the *first de Sitter conjecture can be satisfied* when $\alpha \leq 0$.

Thus, we found only a problem with the conjecture for the range $\frac{3}{4} < \alpha \leq 1$.

6.2 Evaluation of second conjecture

To evaluate the second conjecture (1.2) we look for the lowest eigenvalue of the matrix $\partial_i \partial_j V$ where the indices run over the canonically normalized fields $x = \sqrt{2} \operatorname{Re}(z)$, $y = \sqrt{2} \operatorname{Im}(z)$. It is easiest to first consider the second derivative of the potential to the fields $\sqrt{\rho}$ and φ : $z = \sqrt{\rho} e^{i\varphi}$. Since the potential does not depend on φ there is only one of the second derivatives that is non-zero

$$\begin{aligned}
V'' \equiv \left(\frac{\partial}{\partial \sqrt{\rho}} \right)^2 V &= 2 e^\rho \rho^{-2+\alpha} \left(\alpha^2 (3 - 5\alpha + 2\alpha^2) + \alpha \rho (3 - 11\alpha + 8\alpha^2) \right. \\
&\quad \left. + \rho^2 (-2 - 7\alpha + 12\alpha^2) + \rho^3 (-1 + 8\alpha) + 2\rho^4 \right).
\end{aligned} \tag{6.11}$$

Considering polar coordinates $(\sqrt{\rho}, \varphi)$, there is also a second covariant derivative $V_{\varphi\varphi}$ proportional to V' . The eigenvalues of the matrix of covariant derivatives with respect to x and y are then

$$\lambda = \frac{1}{2} \{ \rho^{-1/2} V', V'' \}. \tag{6.12}$$

where the factor 1/2 originates from the factors $\sqrt{2}$ in the definition of x and y (in order that they have canonical kinetic energy).

At the extremum, the non-zero eigenvalue is thus proportional to V'' . In Figure 7, we draw the eigenvalues for different values of α . To satisfy the second conjecture (1.2), the smallest one should be negative, which is not satisfied e.g. in the extremum where $\lambda_1 = 0$, and $\lambda_2 > 0$.

We conclude that the second conjecture does not lead to an escape for the troublesome range $\frac{3}{4} < \alpha \leq 1$.

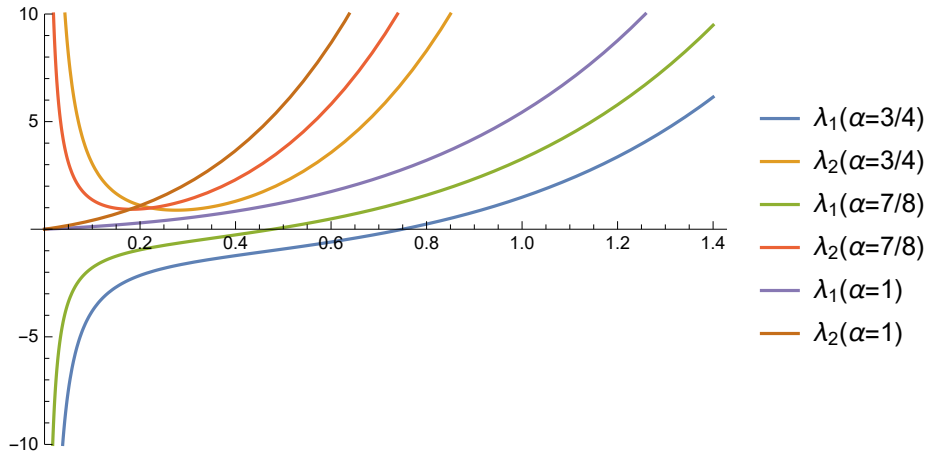


Figure 7: The eigenvalues $\{\lambda_1, \lambda_2\} = \frac{1}{2} \{\rho^{-1/2}V', V''\}$ of the one scalar model for $\alpha = 7/8$. At the extremum none of the eigenvalues are negative $\{0, \lambda_2\} \geq 0$.

7 Models with exponential scalar potentials

One class of models that can trivially satisfy the conjecture in (1.1) are the single exponential scalar potentials

$$V(\phi) = V_0 e^{\xi\phi}, \quad V_0 > 0. \quad (7.1)$$

These models can describe the effects of a slowly changing vacuum energy caused by a rolling field. They are therefore the most simplest of the present stage quintessence models and are in agreement with observations when $\xi \lesssim 0.6$ [11]. In this section we show how such models and extensions thereof can be constructed in a supergravity setup. We hereby use and expand on the work done in [12–14].

We start with Kähler and superpotential, K and W , describing the dynamics of a set of chiral superfields z^α by a scalar potential V . The next step is to add a chiral superfield Φ to the Kähler potential of the previous theory

$$\widehat{K} = K - a\kappa^{-2} \log(\Phi + \bar{\Phi}), \quad a > 0. \quad (7.2)$$

The Kähler potential of the field Φ in (7.2) parametrizes an $SU(1, 1)/U(1)$ symmetric geometry and is well-known in the literature where it is used in inflationary α -attractor models [15] with $a = 3\alpha$. The chiral superfield Φ appears in the superpotential with a power law

$$\widehat{W} = \Phi^{-b/2} W. \quad (7.3)$$

The scalar potential of the extended theory is

$$\widehat{V} = (\Phi + \bar{\Phi})^{-a} |\Phi|^{-b} \left(V + \frac{e^{\kappa^2 K}}{a\kappa^4 |\Phi|^2} \left[\frac{b}{2} \left(\frac{b}{2} + a \right) (\Phi + \bar{\Phi})^2 + a^2 |\Phi|^2 \right] \right), \quad (7.4)$$

where we used V to describe the scalar potential of the original theory. The potential in (7.4) is minimal in the Φ plane for $\text{Im } \Phi = 0$. Considering therefore only the real part of the

scalar field, one finds that after canonically normalizing

$$\Phi + \bar{\Phi} = e^{-\sqrt{2/a}\phi}, \quad (7.5)$$

the scalar potential takes the form¹

$$\widehat{V} = e^{\sqrt{2\gamma}\phi} V + \gamma \kappa^{-2} |m_{3/2}|^2, \quad (7.6)$$

with $\gamma = (a+b)^2/a$ and the gravitino mass given by

$$|m_{3/2}|^2 = \kappa^{-2} e^{\widehat{G}} = \kappa^4 e^{\sqrt{2\gamma}\phi + \kappa^2 K} |W|^2. \quad (7.7)$$

By demanding that there exists a stable minimum for the fields $z^\alpha, \bar{z}^{\bar{\alpha}}$, we find the condition

$$\partial_\alpha \widehat{V} = 0 \quad \Leftrightarrow \quad \partial_\alpha V + \gamma \kappa^2 \partial_\alpha (e^{\kappa^2 K} |W|^2) = 0, \quad (7.8)$$

and its holomorphic counterpart. Using the expression of the scalar potential and $F^\alpha = e^{\kappa^2 K/2} g^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W}$, the condition in (7.8) can be rewritten as

$$\gamma F^\beta \nabla_\alpha \bar{F}_\beta + (2 - \gamma) V_\alpha = 0. \quad (7.9)$$

Whenever the condition in (7.9) can be satisfied, at that specific point the only dynamical field will be the rolling scalar following the exponential potential

$$\widehat{V} = e^{\sqrt{2\gamma}\phi} \left(V_0 + \gamma \kappa^2 e^{\kappa^2 K} |W|^2 \Big|_{z, \bar{z} = z_0, \bar{z}_0} \right). \quad (7.10)$$

Notice from (7.9) that when the vacuum of the scalar potential V is supersymmetric ($F^\alpha = 0$) the condition (7.8) is immediately satisfied. Therefore in the case of supersymmetric vacua the addition of the chiral multiplet Φ leaves the position of the vacua unperturbed. Furthermore notice that for supersymmetric vacua, the uplift to a de Sitter vacuum can only happen when $\gamma > 3$. However in this case the phenomenological constraint $\xi \lesssim 0.6$ is not satisfied. This implies that in order for the models just described to be phenomenologically viable the breaking of supersymmetry, needed for the de Sitter uplift, cannot be solely caused by the quintessence field Φ .

The inclusion of a quintessence scalar, in the way we presented in (7.2), (7.3), provides a simple way to escape the violation of the de Sitter conjectures. The effects on the original model are minimal as can be seen from (7.6) which can also be written as

$$\widehat{V} = e^{\sqrt{2\gamma}\phi + \kappa^2 K} \left(\nabla_\alpha W g^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W} - (3 - \gamma) \kappa^2 |W|^2 \right). \quad (7.11)$$

¹Since the kinetic term

$$\frac{a\kappa^{-2}}{(\Phi + \bar{\Phi})^2} \partial\Phi\partial\bar{\Phi}$$

is scale invariant. There is a freedom in redefining the field which allows us to fix the scale of the potential.

Therefore when γ is sufficiently small the main features of the theory will be preserved. From (1.1) one can see that the lower bound on γ in order for the first de Sitter conjecture to be everywhere satisfied is

$$\gamma \geq \left(\frac{c^2}{2} - \frac{\partial_\alpha \widehat{V} g^{\alpha\bar{\beta}} \partial_{\bar{\beta}} \widehat{V}}{\kappa^2 \widehat{V}^2} \right) \quad \text{for } \widehat{V} > 0. \quad (7.12)$$

Example 1. The simplest example is the case where the uplift is caused by a nilpotent superfield $X^2 = 0$. Since this multiplet does not contain any scalars, condition (7.9) is trivially satisfied. The Kähler and superpotential are given by

$$\widehat{K} = X\bar{X} - a\kappa^{-2} \log(\Phi + \bar{\Phi}), \quad W = \Phi^{-b/2}(m\kappa^{-2} + fX). \quad (7.13)$$

The scalar potential of this model is

$$\widehat{V} = e^{\sqrt{2\gamma}\phi} (f^2 - \kappa^{-2}m^2(3 - \gamma)). \quad (7.14)$$

The scalar potential is similar to the one found in [16–18]. One important difference is that for the model in (7.14) the difference between the supersymmetry breaking scale and the gravitino mass is decoupled from the vacuum energy by the presence of an extra parameter γ associated to the quintessence.

Example 2. Looking back at the Polónyi model in Figure 5, we can see how the addition of a quintessence scalar changes the picture. The Kähler-invariant functional \mathcal{G} is

$$\mathcal{G} = z\bar{z} - a \log(\Phi + \bar{\Phi}) + \log(\mu^2|z + \beta|^2) + \frac{b^2}{4} \log(\Phi\bar{\Phi}). \quad (7.15)$$

After going to canonically normalized coordinates $z = \frac{1}{\sqrt{2}}(x + iy)$, $\Phi + \bar{\Phi} = e^{-\sqrt{2/a}\phi}$ one can check that $y = 0$ forms a minimum, and the scalar potential becomes

$$\widehat{V}(x, \phi) = \kappa^{-4} \mu^2 e^{\sqrt{2\gamma}\phi} e^{x^2/2} \left(\frac{x}{\sqrt{2}} + \beta \right)^2 \left(\left(\frac{x}{\sqrt{2}} + \frac{1}{\frac{x}{\sqrt{2}} + \beta} \right)^2 + \gamma - 3 \right). \quad (7.16)$$

There is a stable Minkowski vacuum for $\beta = 2 - \sqrt{3 - \gamma}$. We saw that without the inclusion of a quintessence scalar the values $0 \leq \beta < 2 - \sqrt{3}$ were disallowed by the de Sitter conjecture. For the model in (7.16) this corresponds to the range $0 \leq \beta < 2 - \sqrt{3 - \gamma}$. However notice that now the de Sitter conjecture is satisfied for this range of β as long as $\sqrt{2\gamma} \geq c$.

Until now we only considered the case of single exponential potentials, but the analysis above can be extended to multiple exponential functions and fields by enlarging the Kähler and superpotential to

$$\widehat{K} = K - \kappa^{-2} \sum_{i=1}^N a_i \log(\Phi_i + \bar{\Phi}_i), \quad \widehat{W} = W \prod_{i=1}^N \left(\sum_{j=1}^{M_i} \Phi_i^{-b_{ij}/2} \right). \quad (7.17)$$

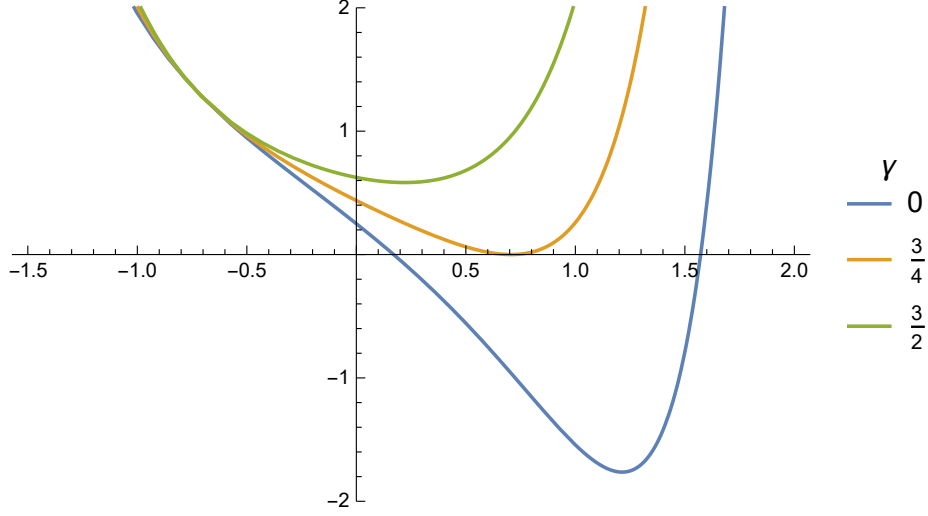


Figure 8: The potential $\widehat{V}(x, 0)$ of the Polonyi model with a quintessence scalar that is positioned at $\phi = 0$ for $\beta = 1/2$ with $\gamma = \{0, 3/4, 3/2\}$. The addition of the quintessence scalar causes an uplift of the vacuum energy corresponding to the steepness of the potential in the direction of ϕ .

The requirement for stable minima in the z^α , $\bar{z}^{\bar{\alpha}}$ directions will again be an equation of the form in (7.9), but with $\gamma(\phi_i)$ generically being a function of the quintessence fields. Therefore in order for these minima to remain fixed during the running of the quintessence scalars, both terms in (7.9) should vanish independently at the same position. This is a highly restrictive condition, but can be trivially fulfilled when the uplift is caused by a nilpotent superfield(s).

Example 3. As a last example one can consider the case where an extra power law is added to (7.13):

$$\widehat{K} = X\bar{X} - a\kappa^{-2} \log(\Phi + \bar{\Phi}), \quad W = (\Phi^{-b_1/2} + \Phi^{-b_2/2}) (m\kappa^{-2} + fX). \quad (7.18)$$

The scalar potential in this case is given by

$$\widehat{V} = \Lambda_1 e^{\sqrt{2\gamma_1}\phi} + \Lambda_2 e^{\sqrt{2\gamma_2}\phi} + \Lambda_3 e^{(\sqrt{\gamma_1/2} + \sqrt{\gamma_2/2})\phi}, \quad (7.19)$$

where $\gamma_1 = (a + b_1)^2/a$, $\gamma_2 = (a + b_2)^2/a$ and

$$\begin{aligned} \Lambda_1 &= f^2 - \kappa^{-2}m^2 (3 - \gamma_1), \\ \Lambda_2 &= f^2 - \kappa^{-2}m^2 (3 - \gamma_2), \\ \Lambda_3 &= f^2 - \kappa^{-2}m^2 (3 - \sqrt{\gamma_1\gamma_2}). \end{aligned} \quad (7.20)$$

8 Conclusion

The de Sitter conjectures (1.1), (1.2), assumed to be imposed by quantum gravity, constrain the class of EFTs belonging to the landscape. Whenever supersymmetry is broken, supergravity theories will not automatically satisfy the conjectures. We derived the constraints on the Kähler-invariant \mathcal{G} -function in these cases and explicitly discussed their impact on several models. We studied three types: the Polónyi model, R -symmetric one scalar models and models with exponential potentials.

The Polónyi model violates both conjectures in the range $0 \leq \beta < 2 - \sqrt{3}$. The troublesome range for the R -symmetric one scalar model is $3/4 < \alpha \leq 1$ where both the conjectures are also violated. For all other values at least one of the conjectures is satisfied. These parameter ranges might be expanded when the value of constant c is taken into account.

Models with exponential potentials are important for quintessence phenomenology. In this paper we provided a method to obtain exponential potentials in supergravity such that the phenomenological constraint $\xi \lesssim 0.6$ can be satisfied while also having a positive vacuum energy $V_0 > 0$. The method can furthermore be used to rescue models from disobeying the de Sitter conjectures by including a quintessence scalar into the theory.

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