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THE STABILITY OF NON-ABELIAN FLUX TUBES AND THEIR LENGTH[1]

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Abstract

We investigated the effect of external field boundary conditions of a background chromomagnetic field on the instability (discovered by Nielsen and Olesen) for the vacuum of a non-Abelian theory. We find that the vacuum is neither stabilized by a one-dimensional nor a two-timensional cut-off of this magnetic field. However, the vacuum in the presence of flux tubes whose length is restricted to be under a critical value, L_0, is stable. There is a tendency for flux tubes of length greater than L_0 to spontaneously fragment into segments each of which is smaller than L_0. This corresponds to a dual picture which allows stable electric fluxes

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ABSTRACT

We investigated the effect of external field boundary conditions of a background chromomagnetic field on the instability (discovered by Nielsen and Olesen) for the vacuum of a non-Abelian field theory. We find that the vacuum is neither stabilized by a one-dimensional nor a two dimensional cutoff of this magnetic field. However, the vacuum in the presence of flux tubes whose length is restricted to be under a critical value, L_0, is stable. There is a tendency for flux tubes of lengths greater than L_0 to spontaneously fragment into segments each of which is smaller than L_0. This corresponds to a dual picture which allows stable electric flux tubes.

1. Introduction

It had been found in 1977 by Savvidy[1] that the infrared singularities lead to an instability of the "trivial" vacuum (i.e. the vacuum in which all fields have zero expectation value) towards the formation of a nontrivial magnetic condensate. On the other hand, Nielsen and Olesen[2] found that even this vacuum cannot be the true one since it is intrinsically unstable as well. They demonstrated this by examining the field equations in the context of an SU(2) Yang-Mills theory and showing that the fluctuations about the magnetic condensate contain exponentially growing modes.

In the discussion of Nielsen and Olesen,[2] in which they found the vacuum instability, they assumed a constant magnetic field. This, of course, assumes that the magnetic field extends over all space and is not confined to any localized region. In the later part of the paper of Ambjørn, et. al.[3] it was conjectured that the instability disappears when the background field develops a domain structure (spaghetti vacuum). However, the proof that such a structure for the vacuum would be stable, was lacking. The results of this present work, makes such a vacuum structure less plausible since we have shown that infinitely long, background filaments, taken individually, are intrinsically unstable. We examine the stability associated with a variety of external chromo-magnetic field configurations which we discuss below. More details may be found in our recent publication.[4]

2. Instability as a Function of Background Field Configuration

We examine the effect on the vacuum stability when boundary conditions are imposed on the background magnetic field. What follows will be in the context of *pure* SU(2) non-Abelian gauge theory. The Lagrangian is therefore given by

$$\mathcal{L} = -\frac{1}{4}\,[G^a_{\mu\nu}]^2 \tag{1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + g\varepsilon^{abc}A^{(b)}_\mu A^{(c)}_\nu \tag{2}$$

We define a W-boson field by $W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}[A^{(1)}_\mu \pm (-iA^{(2)}_\mu)]$ and the "photon" by $A_\mu \equiv A^{(3)}_\mu$.

In terms of $W^\pm$ and A_μ, $\mathcal{L}$ becomes

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}|D_\mu W^+_\nu - D_\nu W^+_\mu|^2 + \frac{1}{2}g\,F_{\mu\nu}W^-_\rho S^{\mu\nu\rho\sigma}W^+_\sigma$$
$$-\frac{1}{4}g^2\,W^-_\rho S^{\mu\nu\rho\sigma}W^+_\sigma W^{-\,\alpha} S_{\mu\nu\alpha\beta}W^{+\beta} \tag{3}$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu; \qquad D_\mu=\partial_\mu+igA_\mu \quad \text{and} \quad S_{\mu\nu\rho\sigma}=-i(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \tag{4}$$

The equations of motion for the W-field in the presence of an external field A_μ resulting from (3) are (neglecting the terms cubic in $W_\mu \equiv W^-_\mu$ and W^+_μ)

$$[D_\lambda D^\lambda g^{\sigma\nu} - D_\lambda D^\sigma g^{\lambda\nu} + igF^{\sigma\nu}]W_\nu = 0 \tag{5}$$

If we assume the background Landau gauge condition:[2] $(\partial^\nu+igA^\nu)W_\nu=0$, Eq. (5) becomes

$$[(\partial_\lambda+igA_\lambda)(\partial^\lambda+igA^\lambda)g^{\sigma\nu} + 2igF^{\sigma\nu}]W_\nu = 0 \tag{6}$$

If in addition we consider the external field to be a constant "color magnetic field"[2], we find the result found by Nielsen and Olesen that the square of the energy eigenvalue, E^2 can take on negative eigenvalues. This leads exponentially growing modes which is results in the vacuum instability. Physically, the anomalous magnetic moment of the "non-Abelian" W-field (gluons for QCD) lowers the E^2 of the W-modes in the constant magnetic field in such a way as to overcome the zero point kinetic energy so that the total zero point energy is imaginary.

2.1 One-Dimensional Cutoff

We considered a constant magnetic field along the z-axis such that in a perpendicular direction, say $x \equiv x^1$, the field goes to zero for $x<-L$ and for $x>L$ for some given $L>0$ where L is a constant. Eq. (6) reduces to a Schrödinger-like equation[4]

$$\left(-\frac{d^2}{dx^2} + V(x)\right)u_\mu(x) = E^2 u_\mu(x) \tag{7}$$

No matter how small we take the magnetic field to be, it can be shown that Eq. (7) has at least one negative eigenvalue (i.e. bound state) and thus for that state, $E^2<0$ and the vacuum instability remains.

2.2 Cylindrically Symmetric Configuration

(a) Quantized Magnetic Flux

We considered a constant magnetic flux along the z-axis within a cylinder bounded by the surface $\rho_0=\sqrt{x^2+y^2}$. The solution of Eq. (6) for this case the standard confluent hypergeometric function for $\rho<\rho_0$ and a Bessel function for $\rho>\rho_0$. Here, too, independent of how small the external magnetic field is taken to be, E^2 is negative for the lowest energy state and hence, the instability remains.

(b) Shielded Magnetic Flux Cylinder

In this case, we assume that the vector potential is confined in the cylinder and so it vanishes for $\rho>\rho_0$. Here, too, we find from analyzing Eq. (6) that independent of the field strength or the value of ρ_0, there is an instability associated with the lowest state.

3. Instability and the Geometry of Flux Tubes

If we now consider a flux tube of finite length, and the flux quantization condition[4], Eq. (6) becomes

$$\frac{d^2U_\mu^{(E)}}{d\rho^2} - \left(\frac{3}{4\rho^2} + k_z^2\right)U_\mu^{(E)} = -E^2U_\mu^{(E)}; \qquad (\rho>\rho_0) \qquad (8)$$

$$\frac{d^2U_\mu^{(E)}}{d\rho^2} + \left(\frac{1}{4\rho^2} - \frac{g^2\rho^2H^2}{4} - k_z^2 \pm 2gH\right)U_\mu^{(E)} = -E^2U_\mu^{(E)}; \qquad (\rho<\rho_0) \qquad (9)$$

where H is the magnitude of the magnetic field and $\hbar k_z$ the momentum of the W-field which has a minimum value which is $\frac{\hbar}{L}$,(L is the length of the flux tube) given by the uncertainty principle. If the flux tube is short enough, i.e. for $k_z^2>2gH$ or $(\frac{1}{L})^2>2gH$ the above equations (8) and (9) have no bound state solutions and the flux tubes are stable. However, if they are longer than $L_0=\sqrt{\frac{1}{2gH}}$ then the above equations have a bound state solution and the flux tubes are unstable. The boundaries of this configuration correspond to magnetic monopoles. To make contact with strings connecting quarks and antiquarks, the above picture has be regarded as representing a dual model for QCD.

4. Confinement and Duality in QCD

It is useful to relate our results to the duality considerations of both 't Hooft[5] and Mandelstam[6] which studied the connection between the stability of electric and magnetic flux tubes. Our result that magnetic flux tubes greater than a critical length are inherently unstable is consistent with only one of the possibilities considered by 't Hooft, namely the existence of stable flux tubes in the electric domain. Our result can also be important in the context of QCD vacuum building using the abelian chromomagnetic monopoles.[7] An isolated abelian monopole has an infinitely long Dirac string attached to it and is therefore unstable. This also means that the flux tubes connecting monopole and antimonopole can not be too long, which should lead to a peak in the distribution of the flux tube length in the monopole-antimonopole configurations.

1. G. Savvidy, *Phys. Lett. B71*, (1977) 133.
2. N. Nielsen and P. Olesen, *Nucl. Phys.* **B144** (1978) 376.
3. J. Ambjørn, N.K. Nielsen and P. Olesen, *Nucl. Phys.* **B152** (1979) 75.
4. E. I. Guendelman, D. A. Owen and A. Leonidov, *Int. J. Mod. Phys.* **A8** (1993) 4745.
5. G. 't Hooft, *Nucl. Phys.* **B153** (1979) 141.
6. S. Mandelstam, *Phys. Rev.* **D19** (1979) 2391.
7. K. Yee. Monopoles and Quark Confinement: Introduction and Overview. Preprint LSUHEP-022094, hep-ph/9404363 (1994).