Determination of the branching ratios $\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0)$ and $\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi e\nu)$

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ABSTRACT

Improved branching ratios were measured for the $K_L \to 3\pi^0$ decay in a neutral beam at the CERN SPS with the NA31 detector:
$\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0) = 1.611 \pm 0.037$ and
$\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi e\nu) = 0.545 \pm 0.010$.
From the first number an upper limit for $\Delta I = 5/2$ and $\Delta I = 7/2$ transitions in neutral kaon decay is derived. Using older results for the Ke3/Kμ3 fraction, the $3\pi^0$ branching ratio is found to be $\Gamma(K_L \to 3\pi^0)/\Gamma_{tot} = (0.211 \pm 0.003)$, about a factor three more precise than from previous experiments.

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1. Introduction

The hadronic decays of particles involving heavy quarks are not yet well understood. Perhaps the best known example of an empirical law is the $\Delta I = 1/2$ rule, which states that in strange particle decays the $\Delta I = 1/2$ amplitude dominates over the $\Delta I = 3/2$ amplitude. These are the only amplitudes available in $K \rightarrow 2\pi$ decays. While transitions with $\Delta I = 5/2$ and $\Delta I = 7/2$ are possible in $K \rightarrow 3\pi$ decays, they are not allowed in the conventional first order weak interaction. The I=3 state of pions is only accessible through additional mechanisms like the electromagnetic interaction that do not conserve isotopic spin. The best limits for $\Delta I = 5/2$ and $\Delta I = 7/2$ transitions in $K \rightarrow 3\pi$ decays were obtained from charged kaon decays. A similar precision could not be reached in neutral kaon decays because of the difficulty of measuring the $K_L \rightarrow 3\pi^0$ decay. Only recently a small, but expected, deviation from a pure phase-space density of final states in this decay was observed 1). Additional information can be obtained from a comparison of the branching ratios of $K_L \rightarrow 3\pi^0$ and $K_L \rightarrow \pi^+\pi^-\pi^0$ decays.

In this paper new measurements are presented of the branching ratios of $K_L \rightarrow 3\pi^0/K_L \rightarrow \pi^+\pi^-\pi^0$ and $K_L \rightarrow 3\pi^0/K_L \rightarrow \pi e\nu$. As a byproduct, the absolute branching ratio of the $K_L \rightarrow 3\pi^0$ decay is determined with better precision. This decay is convenient for a determination of the kaon flux in experiments on neutral decays of the K_L at high energies because it is essentially free of backgrounds from other decays. The branching ratio of $K_L \rightarrow 3\pi^0/K_L \rightarrow$ *all charged* has been measured once 2), with a relative uncertainty of 4 %.

After a brief description of the apparatus in section 2, the event selection and the analysis of these three decays are discussed in section 3. The results are presented in section 4.

2. Apparatus and data taking

The detector used for this experiment was designed for a measurement of direct CP-violation in a high energy neutral kaon beam (experiment NA31) at the CERN SPS. It is described in detail in previous publications 3,4). For the benefit of the reader a sketch of the experimental layout is shown in fig. 1; it does not include the transition radiation detector. A special run with trigger conditions different from those in the NA31 experiment was made for the purpose of the present experiment.

The detector consists of two multiwire proportional chambers, each with four planes of different wire orientation. The chambers are separated by 23 m and are followed by a lead/liquid-argon electromagnetic calorimeter, an iron/scintillator hadron calorimeter and a muon veto counter system. A transition radiation detector, located between the second chamber and the calorimeters, is used for additional electron identification. The lateral division in 1.25 cm wide horizontal and vertical strips in the electromagnetic calorimeter allows a measurement of shower position and width. Due to the calorimeter length (25 radiation lengths), electromagnetic showers rarely leak into the hadron calorimeter. A scintillator hodoscope in front of the electromagnetic calorimeter provides trigger and timing information for charged particles; a scintillator plane in the liquid argon serves the same purpose for neutral decays. The decay region is surrounded by four rings of veto counters which are used in the trigger to suppress decays in which a photon or a charged particle escapes the detector. An evacuated beam pipe with an average radius of 10 cm traverses the whole detector and allows particles in the primary beam (primarily neutrons and photons) to pass directly to the beam dump and not produce backgrounds in the detector. The momentum range of kaons accepted in the analysis is from 70 GeV to 150 GeV.

The trigger for neutral decays requires a coincidence of scintillators in opposite halves of the electromagnetic calorimeter along with a total electromagnetic energy of more than 35 GeV, as well as less than 12 GeV in the hadron calorimeter. With the pulse heights recorded in the horizontal and vertical strips of the electromagnetic calorimeter, the number of peaks in each projection is determined on-line by fast analog electronics. At least five peaks in one projection are required for $3\pi^0$ decays. The distance between the decay point and the calorimeter, calculated from the first and second moments of the photon distributions under the assumption that all the photons come from the decay of a neutral kaon, has to exceed 55 m.

Charged decays are selected by a coincidence of hodoscope counters in opposite quadrants. For these events an energy deposition of more than 30 GeV in the calorimeters is required in the trigger. Further cuts then are imposed on the fraction of events for which on-line reconstruction is possible. All events with a photon shower more than 15 cm from any charged particle in either the vertical or horizontal projection are accepted. In order to suppress Ke3 decays, charged events without a photon candidate and with two space points in each wire chamber are "downscaled" and only one of five of these events is recorded.

A total of 1.2×10^6 triggers were recorded during one day at the end of the last running-period of NA31. The K_L beam was run at half the normal intensity. Before and after the run the detector was calibrated *in situ* with $K_S \to 2\pi$ decays from a K_S beam. Details of the data analysis that follows are given in Ref.5.

3. Data analysis

The data analysis closely follows the procedure established previously [6,7,8]. For convenience the same fiducial volume for the decay points (between 2 m and 48m from the end of the K_L collimator) was chosen as in the NA31 experiment. This choice is originally motivated by the extent over which the K_S target can be moved. For each of the decay modes the kaon flux is determined - up to a constant factor - as a function of the kaon energy from the observed data and the calculated acceptance. The relative branching ratios are then measured in bins of kaon energy. The final result is averaged over energy. As a convenient way to describe the kaon energy spectrum, a function of the form $f(E) \propto p(E) \cdot E^{\alpha} \cdot e^{-\beta E}$ with a second order polynomial $p(E)$ is used in the Monte Carlo programs with parameters adjusted to describe the $K_L \to 3\pi^0$ data (see fig. 2). This spectrum describes correctly all three decay rates; it is also consistent with the spectrum observed in the NA31 experiment from $K_L \to 2\pi^0$ decays in the same beam. A precise description of the spectrum is not crucial; what matters in this experiment is the ratio of acceptances. The pertinent points of the analysis are given in the following sections.

3.1 $K_L \to 3\pi^0$ decays

Events with at least 6 reconstructed photons whose energies are greater than 5 GeV and which are separated from each other by a minimum distance of 5 cm (3 cm in both x- and y-projections if they fall in the same quadrant) are retained for further analysis. Each photon is required to be at least 16 cm from the center of the beam line to avoid energy leakage into the beam pipe. The longitudinal position of the decay point is reconstructed using the kaon invariant mass and is required to be between 2 m and 48 m downstream of the final K_L collimator. The best pairing of the photons to $3\pi^0$s is chosen to determine the vertex and momentum of the kaon, but no cuts on the 2-photon invariant masses are made. No cut is made on the center of gravity of the photon showers, since only 0.08% of events have a center of gravity in excess of 10 cm. This is well outside the edge of the beam profile at 5 cm from the beam centerline. The energy scale is calibrated with 0.1% relative precision in K_S runs using the known position of an anticounter at the exit of the K_S collimator. The systematic difference between calibrations before and after the K_L run is below 0.2%. The energy scale is consistent within $(0.2 \pm 0.1)\%$ with the scale deduced from $K_L \to \pi^+\pi^-\pi^0$ decays, in which the calculated decay point of the π^0 can be compared with the measured intersection of π^+ and π^- tracks. The total kaon energy is confined to the interval from 70 GeV to 150 GeV.

The acceptance is calculated for a uniform density in the Dalitz plot. The quadratic term does not contribute a statistically significant effect to the acceptance. The simulations of the photon energy and impact point fluctuations are made with parameters determined in the NA31 experiment [3]. The measured distributions of π^0 masses, photon energies and impact points agree well with the simulation (see, e.g., the photon energy distribution shown in fig. 3). The acceptance depends on the kaon energy (fig. 4). It is $(6.55 \pm 0.07)\%$ on average for the decay $K_L \to 3\pi^0$. The uncertainty reflects the variation in the data with changing cuts. Most sensitive are the cuts in the minimum photon energy (varied from 3 GeV to 7 GeV) and the projected distance between photons (varied from 2 cm to 4 cm). The relative change of the acceptance is 0.7% and 0.8%, respectively. The spectrum obtained from $K_L \to 3\pi^0$ decays is compared in fig. 5 with the Monte Carlo simulation. The agreement is satisfactory. This is not surprising, since the parameters of the Monte Carlo are adjusted to describe this spectrum. The position of the decay point along the the beam (fig. 6) has an almost uniform distribution, well simulated by Monte Carlo.

Corrections are necessary for photon conversion, π^0 Dalitz decays, for accidentals and for reconstruction losses. They are summarized in table 1. Events with space points reconstructed in the first (upstream) wire chamber are not accepted. The material in front of the first chamber is equivalent to 0.0048 radiation lengths, essentially given by the thickness of the large vacuum window on the downstream end of the evacuated decay region. Accidental tracks in the first wire chamber are distinguished from conversions on the basis of their projected distance from showers in the calorimeter. Accidental photons are determined from the observed rate of 7γ events. In 0.38% of 6γ events an additional photon with more than 2 GeV is found; in 50% of these cases the photon has an energy exceeding 5 GeV. From overlays of observed 5γ events and 6γ events with these accidental photons we estimate that 0.11% of 6γ events are lost and 0.08% are gained by accidental activity. Reconstruction losses are estimated from a sample of 6768 Monte Carlo events which pass all cuts and in which full shower profiles as measured in the detector are simulated.

3.2 $K_L \to \pi e \nu$ decays

Events with exactly two reconstructed space points in the first chamber and at least two space points in the second chamber are candidates for Ke3 decays. Cuts in the distance of a track from the center of the beam pipe in the second chamber (18 cm) and in the position of the decay point (2m $\leq$ z $\leq$ 48 m) are imposed. Electrons are identified by their pulse height in the transition radiation detector (TRD) and by the small amount of energy deposited in the hadron calorimeter. The mean of the three lowest pulseheights from the four proportional chambers of the TRD has to exceed 720 counts. (For comparison: the mean pulseheights are 460 counts for pions and 1270 counts for electrons.) This limit is slightly energy dependent to assure a constant electron identification ($(95.3 \pm 0.2)\%$) for all particle momenta. In addition, the energy deposited in the hadron calorimeter has to be less than 10 % of the total particle energy. This cut is $(98.8 \pm 0.2)\%$ efficient for electrons. A combined cut is therefore $(94.2 \pm 0.3)\%$ efficient for electrons; it reduces the chance for a pion to be accepted as an electron to about 0.3%. Since the branching ratios of decays without an electron are smaller than the Ke3 branching ratio, the contamination from other decays in the Ke3 sample is less than 0.1%. The cuts do not make use of the normally narrow shower width of the electron; radiative $Ke3$ events are therefore accepted. Only events with more than one visible photon are rejected. The losses due to accidental photons are negligible.

Events are further restricted by kinematical cuts. The measured energies of pions and electrons have to exceed 15 GeV, the pion to electron energy ratio is required to be between 0.4 and 2.5, and their sum must exceed 40 GeV. The calibration of pion energies followes the established NA31 procedure, which relies on calibrations with pion beams and regular updates with muon data. This calibration is further refined in the present experiment with data from the K_S runs by the requirement that the invariant mass of the charged pions is centered on the K^0 mass. The reconstruction of the kaon energy in Ke3 decays is not possible event by event because of the well-known ambiguity in the longitudinal neutrino momentum. The kaon spectrum is determined by an iterative procedure with appropriate weights for both solutions. The procedure was tested with simulated events; it reproduces the known kaon energy spectrum correctly over the kaon energy interval from 70 GeV to 150 GeV. The algorithm is described in the Appendix. The agreement between the kaon spectrum from Ke3 decays with the Monte Carlo is reasonable (see fig. 7). The direct comparison of the $K_L \to \pi e \nu$ and $K_L \to 3\pi^0$ spectra shows even better agreement; the ratio of the decay rates is consistent with being energy independent (see fig. 12). The decay point distribution for data and Monte Carlo is shown in fig. 8.

Corrections are necessary for wire chamber inefficiencies, for accidental tracks and δ-rays in the first wire chamber, for pion punch-through and for trigger losses (see table 1). Inefficiencies in the wire chambers, caused by missing signals or by failure of the reconstruction program in case of additional activity, are determined from Ke3 events defined by the patterns of energy deposition in the calorimeters and a clean signature (only 2 spacepoints) in one of the drift chambers. The event losses are $(0.06 \pm 0.03)\%$ in the first chamber and $(0.16 \pm 0.04)\%$ in the second chamber yielding a combined chamber loss of $(0.22 \pm 0.05)\%$. Accidentals and δ-rays can give rise to extra hits in the chambers. In the first chamber, however, events with more than two hits are rejected. Losses by δ-rays and accidentals are determined from a sample of Ke3 events which show two tracks with a common vertex in the fiducial volume and in which no restriction on the number of space points in the first chamber is applied. Pion punch-through, that is muons originating from a pion shower in the calorimeter and hitting the muon veto counter, is determined from events taken without a muon veto. Trigger losses, essentially in the on-line processors, are determined with a sample of events in which the on-line trigger is avoided but the trigger decision is recorded.

The acceptance is determined by a Monte Carlo simulation of the K_L decay and of the detector performance. In the simulation a standard Dalitz-plot density$^{9)}$ is assumed; pion decay-in-flight is included. The energy resolution of the detector is parametrized by $\sigma = 0.65 * \sqrt{E}$ for pions and $\sigma = 0.075 * \sqrt{E}$ for electrons. A change of the hadronic resolution function to $0.72 * \sqrt{E}$ causes a relative change of the acceptance by 0.2%. The estimated uncertainty in the pion energy scale of 0.3 % also gives a 0.2% uncertainty in the acceptance. The main error in the acceptance comes from the cut in the sum of electron and pion energies (varied from 40 GeV to 50 GeV) and in the ratio of these energies (varied from 2.5 to 2.2). Both variations causes a relative change of 0.6% in the acceptance. A variation in the vertex cut from 2 m to 10 m changes the acceptance by 0.5% of its value. The average acceptance is $(31.93 \pm 0.35)\%$.

3.3 $K_L \rightarrow \pi^+\pi^-\pi^0$ decays

Events are selected which have two tracks satisfying the same geometric conditions as described above for Ke3 decays and with two photons of more than 5 GeV in the calorimeter. The tracks are required to have no electron signature in the combination of TRD and calorimeters with the same definition of electrons as described in section 3.2. About 6 % of electrons are not recognized in this way. Contamination from Ke3 decays, however, is reduced to below the 0.1% level because there are two photons in the $\pi^+\pi^-\pi^0$ final state. The photons must be separated from the charged particles by more than 20 cm (5 cm in x- and y-projections) in order to reduce the overlap of hadron and photon showers in the electromagnetic calorimeter. One additional photon of less than 5 GeV is allowed. Losses and gains of events due to accidental photons are estimated by overlays with accidental photons found in the $3\pi^0$ sample. The energies of the charged pions have to exceed 15 GeV and are further restricted by cuts in the energy ratio ($0.4 \leq E_1/E_2 \leq 2.5$) and the energy sum ($E_1 + E_2 > 40$ GeV). The energies are originally measured by calorimetry. The accurracy of this measurement is improved by imposing transverse momentum balance in the decay. The location of the center of gravity of the pions on the line joining their impact points is better determined when the impact points of the π^0 and kaon are used. The improvement in accuracy of the pion energies is a factor 2 on average. The kaon momentum spectrum and the decay point distribution are shown in fig. 9 and 10. The selected sample is free of background (see fig. 11); the invariant mass distribution is slightly asymmetric because of residual overlap of tails from photon and pion showers.

Corrections for accidentals, δ-rays, Dalitz decays, pion punch-through, trigger losses and photon conversions (see table 1) are made as discussed in sections 3.1 and 3.2. The acceptance is calculated with a Monte Carlo program using the known Dalitz-plot density [9)]. Systematic uncertainties in the acceptance are estimated as before from the variations with changes in the cuts. Most sensitive are the cuts in the minimal radial distance of the pion trajectories to the beam line in the second drift chamber (varied from 18 cm to 22 cm) and in the minimal photon energy (varied from 3 GeV to 7 GeV). They cause a relative change of the acceptance of 0.8%

and 1.0%, respectively. The average acceptance is $(8.64 \pm 0.16)\%$.

4. Results

Among 1.2×10^6 triggers, 38403 events with $3\pi^0$, 28035 $\pi^+\pi^-\pi^0$ events and 85520 $\pi e \nu$ events satisfy the selection criteria. To calculate the number of kaon decays, the downscaling procedure has to be taken into account. Since only a fraction of the charged decays is downscaled, an effective downscaling factor has to be determined. It is 3.899 ± 0.008 for Ke3 events and 1.104 ± 0.0005 for $\pi^+\pi^-\pi^0$ events. Branching ratios are determined in bins of 10 GeV for the kaon energy range 70 GeV $< E_K <$ 150 GeV. No systematic variations with energy are found (see fig. 12, 13 and 14 and table 2). The branching ratios are therefore averaged with the results:

$$\begin{aligned}\frac{\Gamma(K_L \to 3\pi^0)}{\Gamma(K_L \to \pi e \nu)} &= 0.545 \pm 0.004(stat.) \pm 0.009(syst.) \qquad (1)\\ &= 0.545 \pm 0.010\end{aligned}$$

$$\begin{aligned}\frac{\Gamma(K_L \to 3\pi^0)}{\Gamma(K_L \to \pi^+\pi^-\pi^0)} &= 1.611 \pm 0.014(stat.) \pm 0.034(syst.) \qquad (2)\\ &= 1.611 \pm 0.037\end{aligned}$$

$$\begin{aligned}\frac{\Gamma(K_L \to \pi^+\pi^-\pi^0)}{\Gamma(K_L \to \pi e \nu)} &= 0.336 \pm 0.003(stat.) \pm 0.007(syst.) \qquad (3)\\ &= 0.336 \pm 0.008\end{aligned}$$

The dominant errors are systematic uncertainties mainly in the acceptance determination; correlations due to common cuts are taken into account. For the final result statistical and systematic errors are added in quadrature.

Using the known branching ratios [9)]

$\Gamma(K_L \to \pi\mu\nu)/(\Gamma(K_L \to \pi e\nu) = 0.697 \pm 0.010$,
$\Gamma(K_L \to 2\pi)/\Gamma_{tot} = (2.94 \pm 0.05) \times 10^{-3}$,
$\Gamma(K_L \to 2\gamma)/\Gamma_{tot} = (0.57 \pm 0.027) \times 10^{-3}$ we obtain

$$\frac{\Gamma(K_L \to 3\pi^0)}{\Gamma_{tot}} = \frac{\Gamma(3\pi^0)}{\Gamma(3\pi^0) + \Gamma(\pi^+\pi^-\pi^0) + \Gamma(Ke3)(1 + \frac{\Gamma(K\mu3)}{\Gamma(Ke3)}) + \Gamma(others)} \tag{4}$$

$$= 0.2105 \pm 0.0028, \tag{5}$$

where $\Gamma(others) = \Gamma(K_L \to 2\pi) + \Gamma(K_L \to 2\gamma)$.

With only constant terms in the Dalitz-plot density and no contribution from $\Delta I = 5/2$ and $\Delta I = 7/2$ transitions the ratio (2) is expected to be 1.5. Phase space, Coulomb corrections [10)], linear [11)] and quadratic [1,11)] terms in the Dalitz-plot density change this number to (1.586 ± 0.004). If the measured ratio deviates from this value, $\Delta I = 5/2$ and $\Delta I = 7/2$ amplitudes to the 3π final state with I=3 are not negligible. We obtain, in the notation of Ref.10,

$$\frac{Re\left\{e^{i\delta_{31}}(m_{35} - \frac{4}{3}m_{37})\right\}}{m_{11} - 2m_{13}} < 0.024 \tag{6}$$

with 90 % confidence level. Here m_{11} and m_{13} are the amplitudes of the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions to the I=1 state , and δ_{31} is the difference of the s-wave scattering phase shifts between the I=3 and the symmetric I=1 states of three pions. A similar limit, on a different combination of $\Delta I = 5/2$ and $\Delta I = 7/2$ amplitudes, was obtained [10)] previously from 3π-decays of charged kaons.

Table 1: *Gains and losses and systematic uncertainties*

	losses in %	uncertainty in %
$K_L \to 3\pi^0$		
Dalitz-decays $\pi^0 \to e^+e^-\gamma$	3.68	0.10
γ conversion	2.43	0.60
accidental tracks in first drift chamber	0.68	0.20
accidental photons in e.m. calorimeter	0.03	0.10
trigger inefficiency	0.03	0.03
reconstruction inefficiency	0.19	0.05
correction factor for $K_L \to 3\pi^0$	1.072±0.007	
$K_L \to \pi e \nu$		
δ-electrons and accidental tracks in first drift chamber	4.3	0.2
electron-pion identification	6.2	0.3
pion punch-through	0.4	0.2
trigger inefficiency	0.39	0.24
wire chamber inefficiency	0.22	0.05
correction factor for $K_L \to \pi e \nu$	1.119±0.005	
$K_L \to \pi^+\pi^-\pi^0$		
δ-electrons and accidental tracks in first drift chamber	4.3	0.2
electron-pion identification	0.2	0.1
Dalitz-decays $\pi^0 \to e^+e^-\gamma$	1.20	0.03
γ conversion	0.81	0.2
accidental photons in e.m. calorimeter	0.09	0.10
pion punch-through	0.8	0.4
trigger inefficiency	0.38	0.24
wire chamber inefficiency	0.22	0.05
correction factor for $K_L \to \pi^+\pi^-\pi^0$	1.082±0.006	

Table 2: *Uncorrected branching ratios with statistical errors*

Energy in GeV	$\frac{\Gamma(K_L \to 3\pi^0)}{\Gamma(K_L \to \pi e \nu)}$	$\frac{\Gamma(K_L \to 3\pi^0)}{\Gamma(K_L \to \pi^+\pi^-\pi^0)}$	$\frac{\Gamma(K_L \to \pi^+\pi^-\pi^0)}{\Gamma(K_L \to \pi e \nu)}$
70 - 80	0.558 ± 0.009	1.613 ± 0.030	0.346 ± 0.005
80 - 90	0.579 ± 0.009	1.659 ± 0.030	0.349 ± 0.005
90 - 100	0.564 ± 0.009	1.686 ± 0.033	0.334 ± 0.006
100 - 110	0.579 ± 0.010	1.642 ± 0.036	0.353 ± 0.007
110 - 120	0.561 ± 0.011	1.491 ± 0.040	0.377 ± 0.010
120 - 130	0.567 ± 0.013	1.607 ± 0.055	0.353 ± 0.012
130 - 140	0.575 ± 0.015	1.703 ± 0.078	0.338 ± 0.015
140 - 150	0.574 ± 0.018	1.539 ± 0.094	0.373 ± 0.022
70 - 150	0.569 ± 0.004	1.626 ± 0.014	0.348 ± 0.003

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Appendix

The determination of the K^0 spectrum in $K_L \rightarrow \pi e \nu$ decays is complicated by the absence of kinematic information on the neutrino. Therefore, an iterative procedure to find the true spectrum is applied[12)]. If $\vec{p}_{e\pi}$ and $E_{e\pi}$ are the measured momentum and energy of the $(e\pi)$-system in the laboratory,

$$\begin{aligned} \vec{p}_{e\pi} &= \vec{p}_e + \vec{p}_\pi \\ E_{e\pi} &= E_e + E_\pi \end{aligned}$$

with invariant mass

$$m_{e\pi}^2 = E_{e\pi}^2 - \vec{p}_{e\pi}^{\,2},$$

the center of mass momentum of this system is

$$\begin{aligned} p_{e\pi}^{*^2} &= E_{e\pi}^{*^2} - m_{e\pi}^2 \\ &= \left(\frac{m_K^2 + m_{e\pi}^2}{2m_K}\right)^2 - m_{e\pi}^2 \\ &= \left(\frac{m_K^2 - m_{e\pi}^2}{2m_K}\right)^2 . \end{aligned}$$

Since $p_{e\pi}^{*^2} = p_\nu^{*^2}$, the total neutrino momentum is known, and by momentum conservation also the transverse momentum $p_\perp$ relative to the K_L direction, but for the longitudinal momentum there is an ambiguity:

$$p_l^\nu = \pm\sqrt{p_\nu^{*^2} - p_\perp^2}.$$

The two solutions give two possibilities for the parent kaon energy, E^+ and E^-. Let E be the true kaon energy, N(E) the probability density of the spectrum, and $n(E^+, E^-)$ the observed density of events. All decays with true energy in the interval (E,E+dE) will end up in two bands in the (E^+, E^-) space (see fig. 15), with the band limited by the lines

$$(E, E^-), (E + dE, E^-), \qquad E^- < E,$$

if the neutrino goes forward, and in the band

$$(E^+, E), (E^+, E + dE), \qquad E < E^+,$$

if the neutrino goes backward in the center of mass system. Let w^+ be the probability for the former case , and w^- the probability for the latter case, with

$$\int_0^E w^+(E, E^-)dE^- + \int_E^\infty w^-(E^+, E)dE^+ = 1.$$

The observed density $n(E^+, E^-)$ is obviously due to decays with $E = E^+$ and $E = E^-$,

$$n(E^+, E^-) = N(E^+)w^+(E^+, E^-) + N(E^-)w^-(E^+, E^-),$$

or, divided by $n(E^+, E^-)$:

$$1 = g^+(E^+, E^-) + g^-(E^+, E^-),$$

where $g^+(g^-)$ denote the probabilities that $E^+(E^-)$ is the correct solution for the kaon energy. The kaon spectrum is obtained from the integral equation

$$N(E) = \int_0^E n(E, E^-)g^+(E, E^-)dE^- + \int_E^\infty n(E^+, E)g^-(E^+, E)dE^+. \tag{7}$$

The probabilities $w^+(E, E^-)$ and $w^-(E^+, E)$ are determined from a simulation of the decays and the detector response. Because of measurement errors it is possible to obtain transverse neutrino momenta beyond the kinematical limit. In these cases the event is treated as being right at the kinematical limit, with the consequence $E^+ = E^-$. The solution of (7) is then obtained by insertion of the
n-th iteration of the spectrum on the right hand side to obtain the (n+1)st iteration. Convergence is reached after about 10 steps. Data are finally corrected for a 2% scale change in the simulation between input and output spectra due to the finite energy resolution of the detector.

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Figure captions

1. The experimental layout; the transition radiation detector is not included.
2. Spectrum of kaons at the target as determined from $K_L \to 3\pi^0$ decays. Events with $70 GeV \leq E_K \leq 150 GeV$ are accepted.
3. Distribution of minimum photon energy, E_γ, in $K_L \to 3\pi^0$ events. Events with $E_\gamma > 5 GeV$ are accepted.
4. Detector acceptance for three decay channels.
5. Measured kaon energy distribution of $K_L \to 3\pi^0$ events, not corrected for acceptance and decay probability.
6. Vertex distribution of $K_L \to 3\pi^0$ events.
7. Reconstructed kaon energy distribution of $K_L \to \pi e \nu$ events, not corrected for acceptance and decay probability.
8. Vertex distribution of $K_L \to \pi e \nu$ events.
9. Measured kaon energy distribution of $K_L \to \pi^+\pi^-\pi^0$ events, not corrected for acceptance and decay probability.
10. Vertex distribution of $K_L \to \pi^+\pi^-\pi^0$ events.
11. Invariant mass distribution of accepted $K_L \to \pi^+\pi^-\pi^0$ events.
12. Ratio of decay rates $\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi e \nu)$ as a function of kaon energy. The shaded area gives the final result including the systematic uncertainty.
13. Ratio of decay rates $\Gamma(K_L \to 3\pi^0)/\Gamma(K_L \to \pi^+\pi^-\pi^0)$ as a function of kaon energy. The shaded area gives the final result including the systematic uncertainty.
14. Ratio of decay rates $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi e \nu)$ as a function of kaon energy. The shaded area gives the final result including the systematic uncertainty.
15. The two bands in which an event with true energy in (E, E+dE) will appear (see text).

Table captions

1. (no caption)
2. To obtain the final result, these branching ratios have to be multiplied by the correction factors in table 1.