

8 Perspectives of heavy quarkonium production at the FCC-ee

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Owing to its non-relativistic nature, heavy quarkonium, constituting heavy quark–antiquark pairs ($Q\bar{Q} = b\bar{b}$ or $c\bar{c}$) is an ideal object to investigate both perturbative and non-perturbative aspects of QCD. The non-relativistic QCD factorisation formalism [1], built on rigorous effective field theory [2], provides a powerful tool to calculate heavy quarkonium production and decay systematically. In this formalism, the production of heavy quarkonium is factorised into the process-dependent short-distance coefficients (SDCs) multiplied by supposedly universal long-distance matrix elements (LDMEs). The SDC describing the production of a $Q\bar{Q}$ pair in Fock state $n = {}^{2S+1}L_J^{[a]}$ with total spin S , orbital angular momentum L , and total angular momentum J can be calculated perturbatively as an expansion in α_s . The LDMEs related to the probability that Fock state n will evolve into the heavy meson are organised by the velocity scaling rules [3] of non-relativistic QCD (NRQCD), and their values can be determined by fitting to experimental data. Here, the velocity, v_Q , refers to the motion of a heavy quark, Q , in the rest frame of the heavy meson. Although NRQCD has greatly improved our understanding of the heavy quarkonium production mechanism, the long-standing ‘ J/ψ polarisation puzzle’ has not yet been resolved. The SDCs for the relevant colour singlet (CS) channel (${}^3S_1^{[1]}$) and the three colour octet (CO) (${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$, ${}^3P_J^{[8]}$) channels have been obtained by three groups independently, while the corresponding LDMEs were fitted to different sets of experimental data, based on different considerations [4–6]. However, none of these predictions can explain both the J/ψ yield and polarisation data at hadron colliders simultaneously. Recently, the universality of the NRQCD LDMEs was challenged by η_c hadroproduction data [7].

Compared with hadron colliders, in e^+e^- colliders, the production mechanism is simpler, the uncertainties in the theoretical calculations are smaller, and the convergence of perturbative calculations is faster. Moreover, on the experimental side, the much cleaner background makes it possible to study the production of other heavy quarkonia besides the J/ψ and Υ mesons, such as $\eta_{c,b}$ and $\chi_{c,b}$, and to study more production processes, such as the associated production of heavy quarkonium with a photon or a heavy quark pair, in detail. Therefore, heavy quarkonium production in e^+e^- colliders plays an important role in testing NRQCD factorisation, so as to help resolving the ‘ J/ψ polarisation puzzle’. There are two ways to produce heavy quarkonium directly.* One is in e^+e^- annihilation and the other is in $\gamma\gamma$ collisions. We review heavy quarkonium production, concentrating on the J/ψ case, by these two processes in Sections 8.1 and 8.2, respectively, and discuss the prospects of heavy quarkonium production at the FCC-ee beyond the current measurements made at B factories and CERN LEP-II. Section 8.3 contains a summary and an outlook.

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*Here, we mean production other than through the decay of other heavy particles, like the Z boson, Higgs boson, or top quark.

8.1 Heavy quarkonium production through e^+e^- annihilation

The total cross-section for inclusive J/ψ production in e^+e^- annihilation was measured by the Babar [8], Belle [9], and CLEO [10] collaborations at $\sqrt{s} = 10.6$ GeV, yielding

$$\sigma(e^+e^- \rightarrow J/\psi + X) = \begin{cases} 2.5 \pm 0.21 \pm 0.21 \text{ pb} & \text{Babar} \\ 1.47 \pm 0.10 \pm 0.13 \text{ pb} & \text{Belle} \\ 1.9 \pm 0.2 \text{ pb} & \text{CLEO} \end{cases}$$

The NRQCD prediction at leading order (LO) is in the wide range of 0.8-1.7 pb [11–14], including 0.3 pb from the CS mechanism. The Belle collaboration further managed to discriminate the contributions due to the final states $J/\psi + c\bar{c} + X$ and $J/\psi + X_{\text{non-}c\bar{c}}$, and found that $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.74 \pm 0.08_{-0.08}^{+0.09}$ pb and $\sigma(e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}) = 0.43 \pm 0.09 \pm 0.09$ pb [15]. Neither of these results is compatible with LO NRQCD predictions.

The LO NRQCD prediction for $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$ is about 0.15 pb, in which the CO contribution is negligible [16]. To solve the problem, both the next-to-leading-order (NLO) QCD [17] and relativistic corrections [18] were calculated. The relativistic correction was found to be less than one percent of the LO contribution. The effect of the NLO QCD correction is large. Its K factor is about 1.8 for $m_c = 1.5$ GeV and $\alpha_s = 0.26$. After including the feed-down contribution from $\psi(2S)$, the NRQCD prediction at NLO becomes $0.53_{-0.23}^{+0.59}$ pb and largely removes the discrepancy [17]. However, the theoretical prediction depends strongly on the chosen values of m_c and α_s . According to the design [19], the FCC-ee will run at several beam energies. Measuring $J/\psi + c\bar{c}$ production at different energies will definitely help to improve our understanding of the parameter setting in the theoretical calculation.

At high energies, the predominant contribution to $J/\psi + c\bar{c}$ production comes from the fragmentation process. For heavy quarkonium production, it is found that there are two types of fragmentation [20], (1) single-parton fragmentation (SPF) and (2) double-parton fragmentation (DPF). At hadron colliders, experimentally, the $J/\psi + c\bar{c}$ final state is hard to detect and, theoretically, both SPF and DPF contribute, so that it is very difficult to study their properties separately.

In the e^+e^- annihilation process, only SPF contributes. Thus, the differential cross-section in the fragmentation limit can be expressed as

$$d\sigma(e^+e^- \rightarrow J/\psi + c\bar{c}) = 2 \int d\sigma(e^+e^- \rightarrow c\bar{c}) D_{c \rightarrow J/\psi}(z) dz, \quad (8.1)$$

where

$$D_{c \rightarrow J/\psi}(z) = \frac{8\alpha_s^2}{27\pi} \frac{z(1-z)^2(5z^4 - 32z^3 + 72z^2 - 32z + 16)}{(2-z)^6} \frac{|R(0)|^2}{m_c^3}, \quad (8.2)$$

with $z = E_{J/\psi}/\sqrt{s}$, where $|R(0)|$ is the wave function of J/ψ at the origin [21].

At $\sqrt{s} = 10.6$ GeV, the fragmentation contribution can only account for 58% of the complete calculation [16]. The comparison between the complete calculation and the fragmentation approximation is shown in Fig. B.8.1. We observe that, only in the energy range of the FCC-ee or even beyond, the fragmentation contribution provides a good approximation. Conversely, the differential cross-section of $e^+e^- \rightarrow Q\bar{Q}$ is known at $\mathcal{O}(\alpha_s^2)$ [22, 23]. By comparing experimental measurements with higher-order theoretical calculations, the fragmentation function at higher orders can also be extracted.

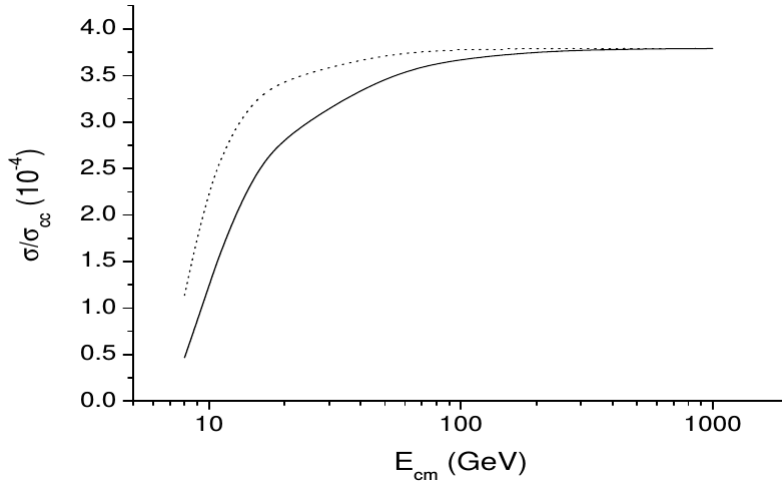


Fig. B.8.1: Cross-section of $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$ normalised to $\sigma(e^+e^- \rightarrow c\bar{c} + X)$ at LO in NRQCD as a function of the centre-of-mass energy. The dotted line denotes the complete result, and the solid line denotes the fragmentation calculation. Figure courtesy ref. [16].

For $J/\psi + X_{\text{non-}c\bar{c}}$ production, in the CS contribution, the NLO QCD corrections [24] and relativistic corrections [25] are equally important. Their K factors are both around 1.2 [24, 25], and the cross-section through NLO in QCD and v^2 becomes $\sigma(e^+e^- \rightarrow J/\psi + gg) \simeq 437$ fb for $\mu = \sqrt{s}/2$ and $m_c = 1.5$ GeV, which almost saturates the Belle measurement and leaves little room for the CO contribution [25]. The NLO QCD corrections to the CO channels $^1S_0^{[8]}$ and $^3P_J^{[8]}$ were also computed [26]. A lower bound on the CO contribution is obtained by using the LDMEs from Ref. [4], yielding 0.3 pb. Therefore, the total NRQCD prediction is larger than the Belle measurements, but does not conflict with the Babar and CLEO measurements if we assume that $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)$ is similar in these three experiments. To understand the CO mechanism in e^+e^- annihilation, further analysis of $J/\psi + X_{\text{non-}c\bar{c}}$ production at 10.6 GeV and in the future at the FCC-ee is necessary.

Besides charmonium, the production of bottomonium in e^+e^- annihilation is also of great interest. However, the collision energy at B factories is so close to the Υ production threshold that perturbative calculations are no longer reliable. Moreover, such a low energy is not sufficient to enable $\Upsilon + b\bar{b}$ production. At the FCC-ee, the collision energy is of the order of 10^2 GeV and, therefore, provides a unique opportunity to study $\Upsilon + X_{\text{non-}b\bar{b}}$ and $\Upsilon + b\bar{b}$ production in e^+e^- annihilation. Theoretically, the NRQCD prediction through NLO can easily be obtained from the known J/ψ calculation by changing the value of \sqrt{s} and replacing m_c with m_b and the LDMEs of J/ψ by those of Υ .

8.2 Heavy quarkonium production in $\gamma\gamma$ collisions

J/ψ photoproduction in $\gamma\gamma$ collisions ($e^+e^- \rightarrow e^+e^- J/\psi + X$) was measured by the DELPHI collaboration at LEP-II [27, 28]. The total cross-section was found to be $\sigma(e^+e^- \rightarrow e^+e^- J/\psi + X) = (45 \pm 9 \pm 17)$ pb [28]. The DELPHI collaboration also measured the transverse momentum (p_T) distribution of the cross-section. Since the higher excited states χ_{cJ} and ψ' can decay into J/ψ via radiative decays or hadronic transitions, their feed-down contributions should also be considered. In such processes, the $c\bar{c}$ pair can either be produced by photons directly (direct photoproduction) or via the light quark and gluon content of the photons (resolved

photoproduction), so that there are three channels: direct, single resolved, and double resolved, all of which contribute formally at the same order in the perturbative expansion and should be included.

Working in the Weizsäcker–Williams approximation to describe the bremsstrahlung photons radiated off the e^\pm beams and using the factorisation theorems of the QCD parton model and NRQCD, the general formula for the differential cross-section for the production of the heavy quarkonium state H can be written as

$$\frac{d\sigma(e^+e^- \rightarrow e^+e^-H + X)}{dx_1 dx_2 dx_a dx_b} = \sum_{a,b,n} f_\gamma(x_1) f_\gamma(x_2) f_{a/\gamma}(x_a) f_{b/\gamma}(x_b) \times d\hat{\sigma}(a + b \rightarrow Q\bar{Q}(n) + X) \langle \mathcal{O}^H(n) \rangle, \quad (8.3)$$

where $f_\gamma(x)$ is the flux function of the photon in the e^\pm beam, $f_{j/\gamma}(x)$ is $\delta(1-x)$ if $j = \gamma$ and otherwise the parton distribution function of parton j in the resolved photon, $d\hat{\sigma}(a + b \rightarrow Q\bar{Q}(n) + X)$ is the partonic cross-section, and $\langle \mathcal{O}^H(n) \rangle$ is the NRQCD LDME.

In the LO calculation, both direct J/ψ production and the feed-down from χ_{cJ} for $J = 0, 1, 2$ and ψ' are included [29]. For J/ψ (ψ') production through relative order $\mathcal{O}(v^4)$, the Fock states include $n = {}^3S_1^{[1,8]}, {}^1S_0^{[8]}, {}^3P_J^{[8]}$, and for χ_{cJ} production at LO in v^2 one needs $n = {}^3P_J^{[1]}, {}^3S_1^{[8]}$. As shown in Fig. B.8.2, the LO NRQCD prediction of $d\sigma/dp_T^2$, evaluated with the LDMEs from the LO fit to Tevatron data [30], agree very well with the DELPHI data, while the CS contribution itself lies far below the data, as the central values are about 16 times smaller. The total cross-section in the range $1 \leq p_T^2 \leq 10 \text{ GeV}^2$ measured by DELPHI is $6.4 \pm 2.0 \text{ pb}$ [27]. The NRQCD prediction is $4.7_{-1.2}^{+1.9} \text{ pb}$ [29], which is also consistent with the DELPHI result, within errors. However, the CS contribution is only $0.39_{-0.09}^{+0.16} \text{ pb}$ [29]. The nice agreement between the NRQCD calculation and the experimental measurement for J/ψ photoproduction is one of the earliest pieces of evidence for the CO mechanism predicted by NRQCD.

In 2011, two groups independently obtained complete NLO QCD corrections to J/ψ direct hadroproduction for the first time [31, 32]. However, their LDMEs are different because they fitted to data in different p_T ranges. To eliminate such problems and further check the universality of the NRQCD LDMEs at NLO, a global analysis to worldwide data including $\gamma\gamma$ collisions was conducted. The resulting three CO LDMEs, $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle = (4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$, $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle = (2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$, and $\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle = (-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$, which obey the velocity scaling rules, were found to explain all the J/ψ yield data fairly well, except for the case of $\gamma\gamma$ collisions [4]. In contrast to the situation at LO, the DELPHI data systematically overshoot the NLO NRQCD prediction, as may be seen in Fig. B.8.3. However, Figs. B.8.2 and B.8.3 indicate that the uncertainties in the experimental measurements are very large. There are only 36 ± 7 $J/\psi \rightarrow \mu^+\mu^-$ events in total (and 16 thereof in the region $p_T > 1 \text{ GeV}$), collected with an integrated luminosity of 617 pb^{-1} . The integrated luminosity at the FCC-ee will reach the ab^{-1} level, which is more than three orders of magnitude larger than that of LEP-II. Measuring J/ψ production in $\gamma\gamma$ collisions at the FCC-ee would not only serve as a cross-check of the LEP-II results, but also provide results with high accuracy. Such a study could surely clarify the current conflict and deepen our understanding of the heavy quarkonium production mechanism in $\gamma\gamma$ collisions.

Unlike the case of e^+e^- annihilation, $J/\psi + c\bar{c} + X$ production in $\gamma\gamma$ collisions is predicted to have a smaller cross-section than $J/\psi + X_{\text{non-}c\bar{c}}$ production. While $\gamma\gamma \rightarrow J/\psi + X_{\text{non-}c\bar{c}}$ proceeds dominantly via single resolved photoproduction, $\gamma\gamma \rightarrow J/\psi + c\bar{c} + X$ proceeds dominantly via

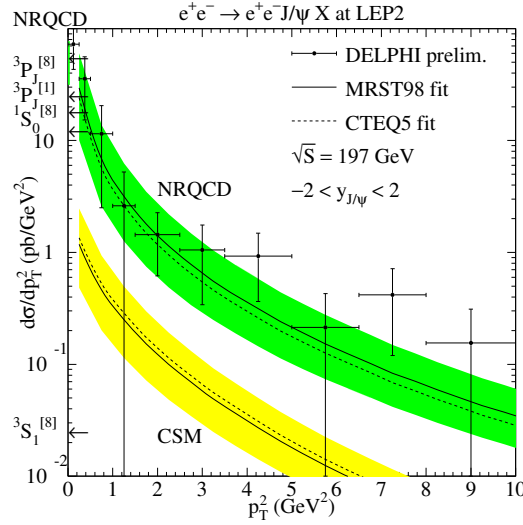


Fig. B.8.2: Comparison between NRQCD and CS model predictions of $d\sigma/dp_T^2$ as functions of p_T^2 for $\gamma\gamma \rightarrow J/\psi$ at LO and DELPHI measurement at LEP-II. The solid and dashed lines are calculated with the MRST98 LO and CTEQ5 parton distribution functions, respectively. The bands indicate the theoretical uncertainties. Figure courtesy ref. [29].

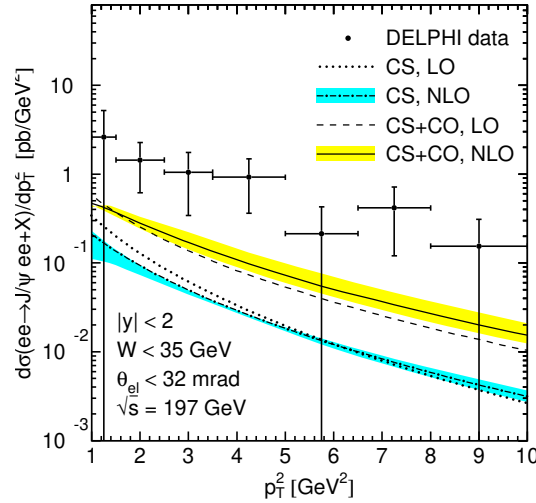


Fig. B.8.3: Comparison of LEP-II data on $\gamma\gamma \rightarrow J/\psi$ with NLO NRQCD predictions evaluated with LDMEs obtained via a global data analysis. Figure courtesy ref. [4].

direct photoproduction [33]. The total cross-section in the region $p_T^{J/\psi} > 1$ GeV is predicted to be about 0.16–0.20 pb, depending on the chosen values of α_s and the CS LDME [33, 34]. Its NLO NRQCD correction has also been calculated, and the K factor is found to be 1.46, enhancing the total cross-section in the region $p_T^{J/\psi} > 1$ GeV to become around 0.23–0.29 pb, which is too small to be analysed at LEP-II [34]. The cross-section becomes larger as the e^+e^- collision energy increases. Based on the results given in Ref. [34], we estimate the numbers of $J/\psi + c\bar{c}$ events accumulated with the FCC-ee at the ZZ and ZH thresholds to be around 2×10^6 each, assuming the kinematic-cut conditions for the FCC-ee to be the same as for LEP-II. Such large data samples should be enough to usefully study $J/\psi + c\bar{c} + X$ production in $\gamma\gamma$ collisions.

8.3 Summary and outlook

The production mechanisms of heavy quarkonium, especially of the J/ψ meson, have not yet been fully understood within the framework of NRQCD factorisation. We have discussed here two modes of J/ψ production at e^+e^- colliders, through e^+e^- annihilation and $\gamma\gamma$ collisions. In the e^+e^- annihilation case, for $J/\psi + c\bar{c} + X$ production, the NRQCD prediction and the Belle measurement agree within errors; however, for $J/\psi + X_{\text{non-}c\bar{c}}$ production, the Belle result favours the CS model prediction and is overshoot by NRQCD predictions evaluated using any of the available LDME sets, although the latter are mutually inconsistent. We note that the NRQCD predictions seem to be compatible with the Babar and CLEO results. As for J/ψ production in $\gamma\gamma$ collisions, the NRQCD prediction can explain the LEP-II data, whose uncertainties are large, at LO, but fails once the NLO correction is included.

The FCC-ee will run at different energy points with considerable integrated luminosity, of $\mathcal{O}(\text{ab}^{-1})$ or even $\mathcal{O}(10^2 \text{ ab}^{-1})$ at the Z boson peak [19], which will provide a perfect environment to judge the disagreements independently. Moreover, it can significantly enrich our knowledge of heavy quarkonium production in e^+e^- collisions, especially by studying bottomonium production, the fragmentation function of $c \rightarrow J/\psi$, and $J/\psi + c\bar{c}$ production in $\gamma\gamma$ collisions.

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