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# Measurements of hadron production in $\pi^+$ + C and $\pi^+$ + Be interactions at 60 GeV/c

The NA61/SHINE Collaboration

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2 A. Aduszkiewicz<sup>15</sup>, E.V. Andronov<sup>21</sup>, T. Antičić<sup>3</sup>, V. Babkin<sup>19</sup>, M. Baszczyk<sup>13</sup>, S. Bhosale<sup>10</sup>, A. Blondel<sup>23</sup>,  
3 M. Bogomilov<sup>2</sup>, A. Brandin<sup>20</sup>, A. Bravar<sup>23</sup>, W. Bryliński<sup>17</sup>, J. Brzywczyk<sup>12</sup>, M. Buryakov<sup>19</sup>, O. Busygina<sup>18</sup>,  
4 A. Bzdak<sup>13</sup>, H. Cherif<sup>6</sup>, M. Ćirković<sup>22</sup>, M. Csanad<sup>7</sup>, J. Cybowska<sup>17</sup>, T. Czopowicz<sup>17</sup>, A. Damyanova<sup>23</sup>,  
5 N. Davis<sup>10</sup>, M. Deliyergiyev<sup>9</sup>, M. Deveaux<sup>6</sup>, A. Dmitriev<sup>19</sup>, W. Dominik<sup>15</sup>, P. Dorosz<sup>13</sup>, J. Dumarchez<sup>4</sup>,  
6 R. Engel<sup>5</sup>, G.A. Feofilov<sup>21</sup>, L. Fields<sup>24</sup>, Z. Fodor<sup>7,16</sup>, A. Garibov<sup>1</sup>, M. Gaździcki<sup>6,9</sup>, O. Golosov<sup>20</sup>,  
7 M. Golubeva<sup>18</sup>, K. Grebieszko<sup>17</sup>, F. Guber<sup>18</sup>, A. Haesler<sup>23</sup>, S.N. Igolkin<sup>21</sup>, S. Ilieva<sup>2</sup>, A. Ivashkin<sup>18</sup>,  
8 S.R. Johnson<sup>26</sup>, K. Kadija<sup>3</sup>, E. Kaptur<sup>14</sup>, N. Kargin<sup>20</sup>, E. Kashirin<sup>20</sup>, M. Kielbowicz<sup>10</sup>, V.A. Kireyeu<sup>19</sup>,  
9 V. Klochko<sup>6</sup>, V.I. Kolesnikov<sup>19</sup>, D. Kolev<sup>2</sup>, A. Korzenev<sup>23</sup>, V.N. Kovalenko<sup>21</sup>, K. Kowalik<sup>11</sup>, S. Kowalski<sup>14</sup>,  
10 M. Koziel<sup>6</sup>, A. Krasnoperov<sup>19</sup>, W. Kucewicz<sup>13</sup>, M. Kuich<sup>15</sup>, A. Kurepin<sup>18</sup>, D. Larsen<sup>12</sup>, A. László<sup>7</sup>,  
11 T.V. Lazareva<sup>21</sup>, M. Lewicki<sup>16</sup>, K. Łojek<sup>12</sup>, B. Łysakowski<sup>14</sup>, V.V. Lyubushkin<sup>19</sup>, M. Maćkowiak-  
12 Pawłowska<sup>17</sup>, Z. Majka<sup>12</sup>, B. Maksiak<sup>11</sup>, A.I. Malakhov<sup>19</sup>, A. Marchionni<sup>24</sup>, A. Marcinek<sup>10</sup>, A.D. Marino<sup>26</sup>,  
13 K. Marton<sup>7</sup>, H.-J. Mathes<sup>5</sup>, T. Matulewicz<sup>15</sup>, V. Matveev<sup>19</sup>, G.L. Melkumov<sup>19</sup>, A.O. Merzlaya<sup>12</sup>,  
14 B. Messerly<sup>27</sup>, Ł. Mik<sup>13</sup>, G.B. Mills<sup>25</sup>, S. Morozov<sup>18,20</sup>, S. Mrówczyński<sup>9</sup>, Y. Nagai<sup>26</sup>, M. Naskręt<sup>16</sup>,  
15 V. Ozvenchuk<sup>10</sup>, V. Paolone<sup>27</sup>, M. Pavin<sup>4,3</sup>, O. Petukhov<sup>18</sup>, R. Płaneta<sup>12</sup>, P. Podlaski<sup>15</sup>, B.A. Popov<sup>19,4</sup>,  
16 B. Porfy<sup>7</sup>, M. Posiadała-Zezula<sup>15</sup>, D.S. Prokhorova<sup>21</sup>, D. Pszczel<sup>11</sup>, S. Puławski<sup>14</sup>, J. Puzović<sup>22</sup>,  
17 M. Ravonel<sup>23</sup>, R. Renfordt<sup>6</sup>, E. Richter-Wąs<sup>12</sup>, D. Röhrich<sup>8</sup>, E. Rondio<sup>11</sup>, M. Roth<sup>5</sup>, B.T. Rumberger<sup>26</sup>,  
18 M. Rumyantsev<sup>19</sup>, A. Rustamov<sup>1,6</sup>, M. Rybczynski<sup>9</sup>, A. Rybicki<sup>10</sup>, A. Sadovsky<sup>18</sup>, K. Schmidt<sup>14</sup>,  
19 I. Selyuzhenkov<sup>20</sup>, A. Yu. Seryakov<sup>21</sup>, P. Seyboth<sup>9</sup>, M. Słodkowski<sup>17</sup>, A. Snoch<sup>6</sup>, P. Staszal<sup>12</sup>, G. Stefanek<sup>9</sup>,  
20 J. Stepaniak<sup>11</sup>, M. Strikhanov<sup>20</sup>, H. Ströbele<sup>6</sup>, T. Šušar<sup>3</sup>, A. Taranenko<sup>20</sup>, A. Tefelska<sup>17</sup>, D. Tefelski<sup>17</sup>,  
21 V. Tereshchenko<sup>19</sup>, A. Toia<sup>6</sup>, R. Tsenov<sup>2</sup>, L. Turko<sup>16</sup>, R. Ulrich<sup>5</sup>, M. Unger<sup>5</sup>, F.F. Valiev<sup>21</sup>, D. Ve-  
22 berič<sup>5</sup>, V.V. Vechernin<sup>21</sup>, A. Wickremasinghe<sup>27</sup>, Z. Włodarczyk<sup>9</sup>, A. Wojtaszek-Szwarc<sup>9</sup>, K. Wójcik<sup>14</sup>,  
23 O. Wyszynski<sup>12</sup>, L. Zambelli<sup>4</sup>, E.D. Zimmerman<sup>26</sup>, and R. Zwaska<sup>24</sup>

24 <sup>1</sup> National Nuclear Research Center, Baku, Azerbaijan

25 <sup>2</sup> Faculty of Physics, University of Sofia, Sofia, Bulgaria

26 <sup>3</sup> Ruđer Bošković Institute, Zagreb, Croatia

27 <sup>4</sup> LPNHE, University of Paris VI and VII, Paris, France

28 <sup>5</sup> Karlsruhe Institute of Technology, Karlsruhe, Germany

29 <sup>6</sup> University of Frankfurt, Frankfurt, Germany

30 <sup>7</sup> Wigner Research Centre for Physics of the Hungarian Academy of Sciences, Budapest, Hungary

31 <sup>8</sup> University of Bergen, Bergen, Norway

32 <sup>9</sup> Jan Kochanowski University in Kielce, Poland

33 <sup>10</sup> Institute of Nuclear Physics, Polish Academy of Sciences, Cracow, Poland

34 <sup>11</sup> National Centre for Nuclear Research, Warsaw, Poland

35 <sup>12</sup> Jagiellonian University, Cracow, Poland

36 <sup>13</sup> AGH - University of Science and Technology, Cracow, Poland

37 <sup>14</sup> University of Silesia, Katowice, Poland

38 <sup>15</sup> University of Warsaw, Warsaw, Poland

39 <sup>16</sup> University of Wrocław, Wrocław, Poland

40 <sup>17</sup> Warsaw University of Technology, Warsaw, Poland

41 <sup>18</sup> Institute for Nuclear Research, Moscow, Russia

42 <sup>19</sup> Joint Institute for Nuclear Research, Dubna, Russia

43 <sup>20</sup> National Research Nuclear University (Moscow Engineering Physics Institute), Moscow, Russia

44 <sup>21</sup> St. Petersburg State University, St. Petersburg, Russia

45 <sup>22</sup> University of Belgrade, Belgrade, Serbia

- 46 <sup>23</sup> University of Geneva, Geneva, Switzerland
- 47 <sup>24</sup> Fermilab, Batavia, USA
- 48 <sup>25</sup> Los Alamos National Laboratory, Los Alamos, USA
- 49 <sup>26</sup> University of Colorado, Boulder, USA
- 50 <sup>27</sup> University of Pittsburgh, Pittsburgh, USA
- 51

52 Precise knowledge of hadron production rates in the generation of neutrino beams is necessary for  
53 accelerator-based neutrino experiments to achieve their physics goals. NA61/SHINE, a large-acceptance  
54 hadron spectrometer, has recorded hadron+nucleus interactions relevant to ongoing and future long-baseline  
55 neutrino experiments at Fermi National Accelerator Laboratory. This paper presents three analyses of  
56 interactions of 60 GeV/c  $\pi^+$  with thin, fixed carbon and beryllium targets. Integrated production and  
57 inelastic cross sections were measured for both of these reactions. In an analysis of strange, neutral hadron  
58 production, differential production multiplicities of  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  were measured. Lastly, in an analysis  
59 of charged hadron production, differential production multiplicities of  $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$  and protons  
60 were measured. These measurements will enable long-baseline neutrino experiments to better constrain  
61 predictions of their neutrino flux in order to achieve better precision on their neutrino cross section and  
62 oscillation measurements.

## 63 1 Introduction

64 The NA61 or SPS Heavy Ion and Neutrino Experiment (SHINE) [1] has a broad physics program that  
65 includes heavy ion physics, cosmic ray physics and neutrino physics. Accelerator-generated neutrino beams  
66 rely on beams of high energy protons which are directed towards a fixed target. The interactions of these  
67 protons result in secondary hadrons (especially pion, kaons, protons, neutrons and lambdas), some of which  
68 decay to produce the beam of neutrinos. As most neutrino beam lines use targets that are an interaction  
69 length or longer in length, many of the secondary hadrons can re-interact inside the target and other beam  
70 material (such as the decay pipe walls or material of the focusing horns). Thus, it is important to have  
71 accurate knowledge of not only the primary proton interactions in the target, but also of the re-interactions  
72 of secondary particles.

73 NA61/SHINE has previously measured hadron production in interactions of 31 GeV/c protons with a thin  
74 carbon target for the benefit of the T2K experiment [2, 3, 4, 5]. The NA61/SHINE experiment is also well  
75 suited to making measurements of the beam line interactions that dominate the neutrino production in the  
76 Fermilab long-baseline accelerator neutrino program, including the existing NuMI beam [6], which is  
77 initiated by 120 GeV/c primary protons, and the proposed Long-Baseline Neutrino Facility (LBNF) beam  
78 line [7] that will supply neutrinos for the Deep Underground Neutrino Experiment (DUNE) [8], which will  
79 use 60-120 GeV/c primary protons. The current optimized beam line design for LBNF features a  $\sim 2.2$   
80 m-long graphite target [9], but beryllium and hybrid targets have been considered as well.

81 In DUNE, near the oscillation peak at a neutrino energy of 3 GeV, roughly half of the neutrinos are produced  
82 from the decays of secondary particles generated in the interactions of primary protons ( $p \rightarrow X \rightarrow \nu$ ) [10].  
83 The other half come from the decays of particles generated by the re-interactions of protons or hadrons  
84 (eg.  $p \rightarrow X \rightarrow Y \rightarrow \nu$ ). For the LBNF optimized beam, each neutrino in the near detector results  
85 from an average of 1.8 interactions in the beam line (including the interaction of the primary proton) [11].  
86 After protons, the largest source of these interactions is pions with an average of 0.2 pion interactions  
87 contributing to each neutrino, and these pions typically have momenta in the range from roughly 10 GeV/c  
88 to 70 GeV/c.

89 The current estimates of the flux uncertainties in DUNE [11] near the oscillation maximum are dominated  
90 by uncertainties on existing  $p + C$  measurements such as those described in Ref. [12], proton and neutron  
91 interactions that are not covered by existing data and uncertainties on the re-interactions of pions and  
92 kaons. NA61/SHINE seeks to improve on these uncertainties by making improved measurements of proton  
93 interactions with neutrino target materials (with more phase space coverage and larger statistics) and by  
94 making measurements of meson interactions with target and beam line materials. With the exception of the  
95 HARP measurements [13], there is little existing data on the particle production spectra from interactions  
96 of mesons in the incident momentum range of interest for long-baseline neutrino experiments. This paper  
97 presents new results on the yields of particles resulting from the interactions of 60 GeV/c  $\pi^+$  on carbon  
98 and beryllium targets recorded in 2016.

99 Three types of results are presented in this paper. Section 4 presents measurements of the integrated  
100 production and inelastic cross sections for  $\pi^+ + C$  at 60 GeV/c and  $\pi^+ + Be$  at 60 GeV/c interactions, and  
101 describes the uncertainties on these measurements. Section 5 describes measurements of the differential  
102 multiplicity of neutral hadrons ( $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$ ) produced in these interactions, in bins of the momentum and  
103 angle of the produced hadron. Section 6 describes measurements of the differential multiplicity of the  
104 charged hadrons ( $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$  and  $p$ ) in bins of the momentum and angle of the produced hadron.  
105 Section 7 describes the systematic uncertainties on the results presented in Sections 5 and 6.

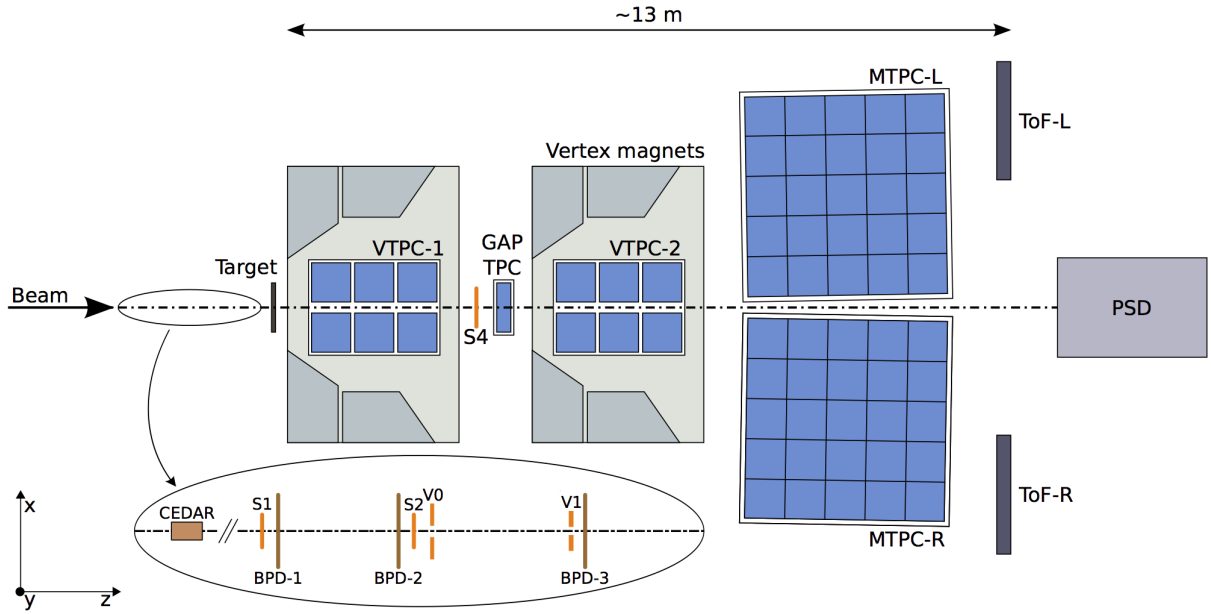


Figure 1: The schematic top-view layout of the NA61/SHINE experiment in the configuration used during the 2016 data taking.

## 2 Detector Setup

Located on a secondary beam line of CERN's Super Proton Synchrotron (SPS), NA61/SHINE probes the interactions of protons, pions, kaons and heavy ions with fixed targets. The 400 GeV/c primary protons from the SPS beam strike a target 535 m upstream of NA61/SHINE, generating the secondary beam. A system of magnets selects the desired beam momentum. Unwanted positrons and electrons are absorbed by a 4 mm lead absorber.

The NA61/SHINE detector [1] is shown in Figure 1. In the 2016 operation configuration, the detector comprises four large Time Projection Chambers (TPCs) and a Time of Flight (ToF) system allowing NA61/SHINE to make spectral measurements of produced hadrons. Two of the TPCs, Vertex TPC 1 (VTPC-1) and Vertex TPC 2 (VTPC-2), are located inside superconducting magnets, capable of generating a combined maximum bending power of 9 T-m. Downstream of the VTPCs are the Main TPC Left (MTPC-L) and Main TPC Right (MTPC-R). Additionally, a smaller TPC, the Gap TPC (GTPC), is positioned along the beam axis between the two VTPCs. Two side time-of-flight walls, ToF-Left and ToF-Right, walls were present. The Projectile Spectator Detector (PSD), a forward hadron calorimeter, sits downstream of the ToF system.

The NA61/SHINE trigger system uses two scintillator counters (S1 and S2) to trigger on beam particles. The S1 counter provides the start time for all counters. Two veto scintillation counters (V0 and V1), each with a hole aligned to the beam, are used to remove divergent beam particles upstream of the target. The S4 scintillator with a 1 cm radius sits downstream of the target and is used to determine whether or not an interaction has occurred. A Cherenkov Differential Counter with Achromatic Ring Focus (CEDAR) [14, 15] identifies beam particles of the desired species. The CEDAR focuses the Cherenkov ring from a beam particle onto a ring of 8 Photomultiplier Tubes (PMTs). The pressure is set to a fixed value so that only

128 particles of the desired species will trigger the PMTs, and typically, a coincidence of at least 6 PMTs is  
129 required to tag a particle for the trigger.

130 The beam particles are selected by defining the beam trigger ( $T_{\text{beam}}$ ) as the coincidence of  $S1 \wedge S2 \wedge$   
131  $\overline{V0} \wedge \overline{V1} \wedge CEDAR$ . The interaction trigger ( $T_{\text{int}}$ ) is defined by the coincidence of  $T_{\text{beam}} \wedge \overline{S4}$  to select  
132 beam particles which have interacted with the target. A correction factor will be discussed in detail in  
133 Section 4.1 to correct for interactions that result in an S4 hit. Three Beam Position Detectors (BPDs),  
134 which are proportional wire chambers, are located 30.39 m, 9.09 m and 0.89 m upstream of the target and  
135 determine the location of the incident beam particle to an accuracy of  $\sim 100 \mu\text{m}$ .

136 Interactions of  $\pi^+$  beams were measured on thin carbon and beryllium targets. The carbon target was  
137 composed of graphite of density  $\rho = 1.80 \text{ g/cm}^3$  with dimensions of 25 mm (W) x 25 mm (H) x 14.8 mm  
138 (L), corresponding to roughly 3.1% of a proton-nuclear interaction length. The beryllium target had a  
139 density of  $\rho = 1.85 \text{ g/cm}^3$  with dimensions of 25 mm (W) x 25 mm (H) x 14.9 mm (L), corresponding to  
140 roughly 3.5% of a proton-nuclear interaction length. The uncertainties in the densities of the targets were  
141 found to be 0.69% for the carbon target and 0.19% for the beryllium target.

### 142 3 Event Selection

143 Several cuts were applied to events to ensure the purity of the samples and to control the systematic effects  
144 caused by beam divergence. The same event cuts are used for the integrated cross section and differential  
145 cross section analyses in order to ensure that the normalization constants obtained from the integrated cross  
146 section analysis are valid for calculating multiplicities in the differential cross section analyses. First, the  
147 so-called WFA (Wave Form Analyzer) cut was used to remove events in which multiple beam particles  
148 pass through the beam line in a small time frame. The WFA determines the timing of beam particles that  
149 pass through the S1 scintillator. If another beam particle passes through the beam line close in time to  
150 the triggered beam particle, it could cause a false trigger in the S4 scintillator and off-time tracks being  
151 reconstructed to the main interaction vertex. To mitigate these effects, a WFA cut of  $\pm 2 \mu\text{s}$  is used.

152 The measurements from the BPDs are important for estimating the effects of beam divergence on the  
153 integrated cross section measurements. To mitigate these effects, tracks are fitted to the reconstructed BPD  
154 clusters, and these tracks are extrapolated to the S4 plane. The so-called "Good BPD" cut requires that  
155 each event includes a cluster in the most downstream BPD and that a track was successfully fit to the BPDs.  
156 Figure 2 shows the resulting BPD extrapolation to the S4 plane for the 60 GeV/c  $\pi^+$  beam. A radial cut  
157 was applied to the BPD tracks extrapolated to the S4, indicated by the red circles on Figure 2, in order to  
158 ensure that non-interacting beam particles strike the S4 counter. This corresponds to a trajectory within 0.7  
159 cm of the S4 center (compared to the S4 radius of 1 cm). It can be seen from these distributions that the  
160 beam, veto counters and the S4 were well-aligned during the data taking.

161 To begin the event selection, only unbiased  $T_{\text{beam}}$  events are considered for the integrated cross section  
162 analysis. For the analysis of spectra, only  $T_{\text{int}}$  events are considered. The numbers of events after the  
163 described selection cuts for the 2 reactions analyzed with the targets inserted and with the targets removed  
164 are shown in Table 1 for the integrated cross section analysis and Table 2 for the spectra analysis.

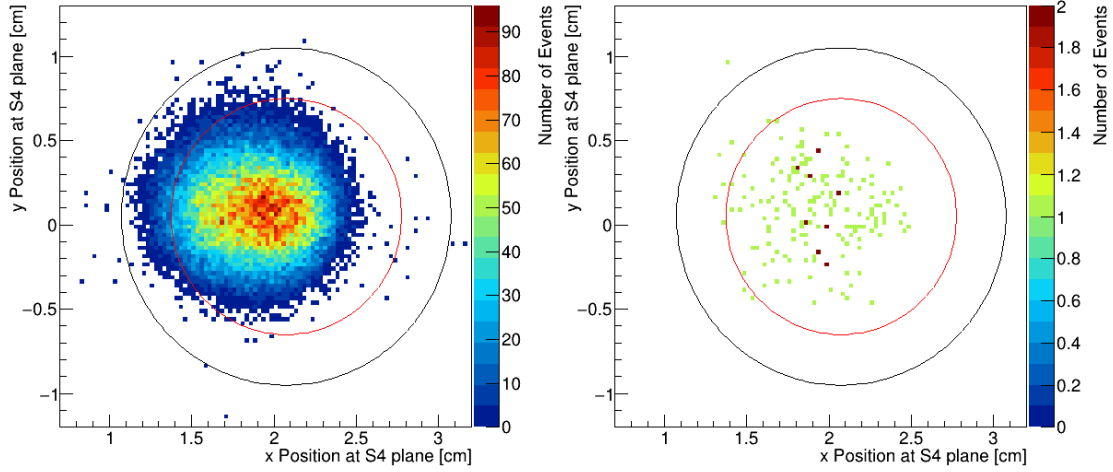


Figure 2: Positions of BPD tracks extrapolated to the S4 plane in target-removed data runs from the  $\pi^+ + \text{C}$  at 60 GeV/c data set. The measured S4 position is shown as a black circle and the BPD radius cut is shown as a red circle in both figures. Left: Events taken by the beam trigger. Right: Events taken by the interaction trigger.

Interaction	60 GeV/c $\pi^+ + \text{C}$		60 GeV/c $\pi^+ + \text{Be}$	
Target	Inserted	Removed	Inserted	Removed
Total	284,684	128,259	222,505	112,583
WFA	263,679	117,876	199,895	99,962
Good BPD	198,169	88,158	122,031	61,010
Radial cut	191,099	86,022	116,944	58,551

Table 1: This table shows the event selection statistics for the  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c integrated cross section analyses with the target inserted and the target removed.

Interaction	60 GeV/c $\pi^+ + \text{C}$		60 GeV/c $\pi^+ + \text{Be}$	
Target	Inserted	Removed	Inserted	Removed
Total	2,324,615	171,074	2,204,152	146,351
WFA	2,155,645	157,380	1,977,486	130,259
Good BPD	1,582,021	101,395	1,176,159	67,860
Radial cut	1,496,524	86,764	1,096,003	57,045

Table 2: This table shows the event selection statistics for the  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c spectra analyses with the target inserted and the target removed.

## 165 4 Integrated Inelastic and Production Cross Section Analysis

166 The total integrated cross section of hadron+nucleus interactions,  $\sigma_{\text{tot}}$ , can be defined as the sum of the  
 167 inelastic cross section,  $\sigma_{\text{inel}}$ , and the coherent elastic cross section,  $\sigma_{\text{el}}$ :

$$\sigma_{\text{tot}} = \sigma_{\text{inel}} + \sigma_{\text{el}}. \quad (1)$$



168 Coherent elastic scattering leaves the nucleus intact. The sum of all other processes due to strong interactions  
 169 makes up the inelastic cross section. The inelastic cross section can be divided into the production cross  
 170 section,  $\sigma_{\text{prod}}$ , and the quasi-elastic cross section,  $\sigma_{\text{qe}}$ :

$$\sigma_{\text{inel}} = \sigma_{\text{prod}} + \sigma_{\text{qe}}. \quad (2)$$

171 In this paper, production interactions are defined as processes in which new hadrons are produced. Quasi-  
 172 elastic interactions include processes other than coherent elastic interactions in which no new hadrons are  
 173 produced, mainly fragmentation of the nucleus. In this paper, measurements of the production cross section,  
 174  $\sigma_{\text{prod}}$ , and inelastic cross section,  $\sigma_{\text{inel}}$ , are presented for  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c  
 175 interactions. These cross section measurements are important for accelerator-based neutrino experiments  
 176 and are needed to normalize the differential cross section yields that will be discussed in Sections 5 and 6.  
 177 This analysis closely follows the method described in Ref. [16], but with some differences, which will be  
 178 discussed below.

#### 179 4.1 Trigger Cross Section

180 For sufficiently thin targets, the probability  $P$  of a beam particle interacting is approximately proportional  
 181 to the thickness,  $L$ , of the target, the number density of the target nuclei,  $n$ , and the interaction cross section,  
 182  $\sigma$ :

$$P = \frac{\text{Number of interactions}}{\text{Number of incident particles}} = n \cdot L \cdot \sigma. \quad (3)$$

183 The density of nuclei can be written in terms of Avogadro's number,  $N_A$ , the material density,  $\rho$ , and the  
 184 atomic mass,  $m_a$ :

$$n = \frac{\rho N_A}{m_a}. \quad (4)$$

185 The counts of beam ( $T_{\text{beam}}$ ) and interaction triggers ( $T_{\text{int}}$ ) that pass the event selection can be used to  
 186 estimate the trigger probability with the target inserted (I) and with the target removed (R):

$$P_{\text{T}}^{\text{I,R}} = \frac{N(T_{\text{beam}} \wedge T_{\text{int}})^{\text{I,R}}}{N(T_{\text{beam}})^{\text{I,R}}}. \quad (5)$$

187 Figure 3 shows an example of the trigger probabilities for each run for the  $\pi^+ + \text{C}$  at 60 GeV/c data  
 188 set. The target-removed runs were interspersed throughout the target-inserted data runs to ensure they  
 189 represented comparable beam conditions. The trigger rates show consistency over the course of the runs,  
 190 which were recorded over a period of about three days. Table 3 gives the trigger probabilities for both  
 191 the target-inserted and target-removed samples of the  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c data  
 192 sets.

Interaction	$p$ (GeV/c)	$P_{\text{Tint}}^{\text{I}}$ (%)	$P_{\text{Tint}}^{\text{R}}$ (%)
$\pi^+ + \text{C}$	60	$2.90 \pm 0.04$	$0.41 \pm 0.02$
$\pi^+ + \text{Be}$	60	$3.28 \pm 0.05$	$0.47 \pm 0.03$

Table 3: This table presents the observed trigger interaction probabilities for both the target-inserted and target-removed samples of the  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c data sets.

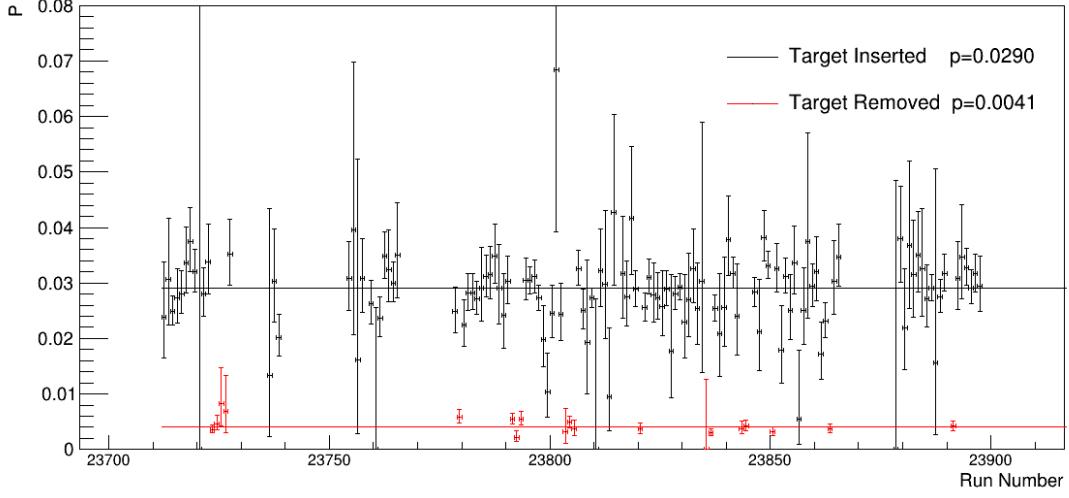


Figure 3: Trigger interaction probabilities for the  $\pi^+ + C$  at 60 GeV/c data set for target-inserted and target-removed runs.

193 Taking into account the trigger probabilities with the target inserted and the target removed,  $P_T^I$  and  $P_T^R$ ,  
 194 the corrected trigger probability,  $P_{\text{trig}}$ , can be obtained:

$$P_{\text{trig}} = \frac{P_T^I - P_T^R}{1 - P_T^R}. \quad (6)$$

195 Analogous to Equation 3, the trigger cross section  $\sigma_{\text{trig}}$  is defined as:

$$\sigma_{\text{trig}} = \frac{m_a}{\rho L_{\text{eff}} N_A} \cdot P_{\text{trig}}, \quad (7)$$

196 where the beam attenuation is taken into account by replacing  $L$  with  $L_{\text{eff}}$ . The effective target length can  
 197 be calculated using the absorption length,  $\lambda_{\text{abs}}$ :

$$L_{\text{eff}} = \lambda_{\text{abs}}(1 - e^{-L/\lambda_{\text{abs}}}), \quad (8)$$

198 where

$$\lambda_{\text{abs}} = m_a / (\rho N_A \sigma_{\text{trig}}). \quad (9)$$

199 By combining Equations 7, 8 and 9,  $\sigma_{\text{trig}}$  can be rewritten as

$$\sigma_{\text{trig}} = \frac{m_a}{\rho L N_A} \ln\left(\frac{1}{1 - P_{\text{trig}}}\right). \quad (10)$$

## 200 4.2 S4 Correction Factors

201 The trigger cross section takes into account the interactions where the resulting particles miss the S4  
 202 scintillator. But even when there has been a production or quasi-elastic interaction in the target, there is a

203 possibility that a forward-going particle will strike the S4 counter. Moreover, not all elastically scattered  
 204 beam particles strike the S4. The trigger cross section must be corrected to account for these effects.  
 205 Combining Equations 1 and 2, the trigger cross section can be related to the production cross section  
 206 through Monte Carlo (MC) correction factors as follows:

$$\sigma_{\text{trig}} = \sigma_{\text{prod}} \cdot f_{\text{prod}} + \sigma_{\text{qe}} \cdot f_{\text{qe}} + \sigma_{\text{el}} \cdot f_{\text{el}}, \quad (11)$$

207 where  $f_{\text{prod}}$ ,  $f_{\text{qe}}$  and  $f_{\text{el}}$  are the fractions of production, quasi-elastic and elastic events that miss the S4  
 208 counter. The cross sections  $\sigma_{\text{qe}}$  and  $\sigma_{\text{el}}$  are also estimated from MC. Equation 11 can be rewritten to obtain  
 209  $\sigma_{\text{prod}}$  and  $\sigma_{\text{inel}}$  as:

$$\sigma_{\text{prod}} = \frac{1}{f_{\text{prod}}} (\sigma_{\text{trig}} - \sigma_{\text{qe}} \cdot f_{\text{qe}} - \sigma_{\text{el}} \cdot f_{\text{el}}) \quad (12)$$

210 and

$$\sigma_{\text{inel}} = \frac{1}{f_{\text{inel}}} (\sigma_{\text{trig}} - \sigma_{\text{el}} \cdot f_{\text{el}}). \quad (13)$$

211 A GEANT4 detector simulation [17, 18, 19] using GEANT4 version 10.4 with physics list FTFP\_BERT  
 212 was used to estimate the MC correction factors discussed above. The MC correction factors obtained for  
 213  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c interactions are presented in Table 4.

Interaction	$p$ (GeV/c)	Monte Carlo Correction Factors					
		$\sigma_{\text{el}}$ (mb)	$f_{\text{el}}$	$\sigma_{\text{qe}}$ (mb)	$f_{\text{qe}}$	$f_{\text{prod}}$	$f_{\text{inel}}$
$\pi^+ + \text{C}$	60	54.1	0.268	15.9	0.813	0.976	0.961
$\pi^+ + \text{Be}$	60	39.6	0.229	13.7	0.813	0.975	0.960

Table 4: Monte Carlo correction factors obtained for analyzing  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c interactions.

### 214 4.3 Beam Composition

215 For the analyses of  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c interactions recorded in 2016, the  
 216 beam composition could be constrained better than in the analysis of interactions recorded in 2015 by  
 217 NA61/SHINE as discussed in [16]. Simulations of the H2 beam line show that the population of muons in  
 218 the 60 GeV/c secondary hadron beam used to record these interactions is at the level of  $1.5 \pm 0.5\%$  [20].  
 219 Nearly all of the muons come from decays of 60 GeV/c pions, so they have a minimum energy of 34 GeV/c.  
 220 GEANT4 simulations were run to estimate the target-inserted and target-removed trigger rates due to  
 221 muons,  $P_{\mu}^I$  and  $P_{\mu}^R$ . These simulations took the momentum distribution of muons into account. Additional  
 222 H2 beam line simulations were run to more precisely estimate the level of positron contamination in the  
 223 beam [21]. A conservative estimate of  $0.5\% \pm 0.5\%$  was attributed to this contamination. The trigger rates  
 224 due to positrons,  $P_e^I$  and  $P_e^R$ , were also estimated with GEANT4 simulations. The effect of muon and  
 225 positron contamination on the trigger cross section was estimated as follows:

$$P_{\text{T}}^{\pi^+} = (P_{\text{T}} - P_e \cdot f_e - P_{\mu} \cdot f_{\mu}) / f_{\pi} \quad (\text{Target I, R}), \quad (14)$$

226 where  $f_e = 0.005$ ,  $f_{\mu} = 0.015$  and  $f_{\pi} = 0.98$ . The resulting corrections applied to  $\sigma_{\text{prod}}$  ( $\sigma_{\text{inel}}$ ) were  
 227  $+0.3\%$  ( $+0.3\%$ ) for  $\pi^+ + \text{C}$  at 60 GeV/c and  $+1.1\%$  ( $1.0\%$ ) for  $\pi^+ + \text{Be}$  at 60 GeV/c.

## 228 4.4 Systematic Uncertainties

229 The integrated cross section results were evaluated for a number of possible systematic effects. The sources  
 230 of uncertainty having a non-negligible effect on the results are the uncertainty in the density of the target, the  
 231 uncertainty in the S4 size, the uncertainty on the beam composition and uncertainties on the S4 correction  
 232 factors. The procedures used to evaluate these sources of systematic uncertainties were discussed in [16],  
 233 so they will not be discussed here.

### 234 4.4.1 Breakdowns of the Integrated Cross Section Uncertainties

235 The target density uncertainties, S4 size uncertainties, beam composition uncertainties and S4 correction  
 236 factor uncertainties associated with the production and inelastic cross sections measurements for  $\pi^+ +$   
 237 C at 60 GeV/ $c$  and  $\pi^+ +$  Be at 60 GeV/ $c$  interactions are presented in Tables 5 and 6.

		Systematic uncertainties for $\sigma_{\text{prod}}$ (mb)					
Interaction	$p$ (GeV/ $c$ )	Density	S4 Size	Beam Purity	MC Stat.	Total Syst. Uncer.	Model Uncer.
$\pi^+ + \text{C}$	60	$\pm 1.3$	$\pm_{1.2}^{1.1}$	$\pm_{1.5}^{1.5}$	$\pm 0.2$	$\pm_{2.4}^{2.3}$	$\pm_{3.8}^{0.2}$
$\pi^+ + \text{Be}$	60	$\pm 0.3$	$\pm_{0.9}^{0.8}$	$\pm_{0.7}^{0.7}$	$\pm 0.1$	$\pm_{1.2}^{1.2}$	$\pm_{3.5}^{0.1}$

Table 5: Breakdown of systematic uncertainties for the production cross section measurements of  $\pi^+ + \text{C}$  at 60 GeV/ $c$  and  $\pi^+ + \text{Be}$  at 60 GeV/ $c$  interactions.

		Systematic uncertainties for $\sigma_{\text{inel}}$ (mb)					
Interaction	$p$ (GeV/ $c$ )	Density	S4 Size	Beam Purity	MC Stat.	Total Syst. Uncer.	Model Uncer.
$\pi^+ + \text{C}$	60	$\pm 1.4$	$\pm_{1.2}^{1.1}$	$\pm_{1.6}^{1.6}$	$\pm 0.2$	$\pm_{2.4}^{2.4}$	$\pm_{2.8}^{0.2}$
$\pi^+ + \text{Be}$	60	$\pm 0.3$	$\pm_{0.9}^{0.9}$	$\pm_{0.7}^{0.7}$	$\pm 0.1$	$\pm_{1.2}^{1.2}$	$\pm_{2.5}^{0.1}$

Table 6: Breakdown of systematic uncertainties for the inelastic cross section measurements of  $\pi^+ + \text{C}$  at 60 GeV/ $c$  and  $\pi^+ + \text{Be}$  at 60 GeV/ $c$  interactions.

## 238 4.5 Integrated Cross Section Results

239 Measurements of production cross sections for  $\pi^+ + \text{C}$  at 60 GeV/ $c$  and  $\pi^+ + \text{Be}$  at 60 GeV/ $c$  are sum-  
 240 marized in Table 7 along with statistical, systematic and physics model uncertainties. The production  
 241 cross section of  $\pi^+ + \text{C}$  at 60 GeV/ $c$  interactions was found to be 166.7 mb, and the production cross  
 242 section of  $\pi^+ + \text{Be}$  at 60 GeV/ $c$  interactions was found to be 140.6 mb. The result obtained for interactions  
 243 of  $\pi^+ + \text{C}$  at 60 GeV/ $c$  with these 2016 data was lower compared to the result obtained with the 2015  
 244 data [16], but it is within the estimated uncertainty. Reasons for this difference could be due to the difference  
 245 in the detector setup, the different target used and statistical fluctuations. These results, the results obtained  
 246 by NA61/SHINE from data recorded in 2015 and the measurements of Carroll et al. [22] are compared in  
 247 Figure 4.

248 The measurements of inelastic cross sections for  $\pi^+ + \text{C}$  at 60 GeV/ $c$  and  $\pi^+ + \text{Be}$  at 60 GeV/ $c$  are  
 249 summarized in Table 8 along with statistical, systematic and physics model uncertainties. The inelastic

250 cross section of  $\pi^+ + \text{C}$  at 60 GeV/c was found to be 182.7 mb, and the inelastic cross section of  $\pi^+ +$   
251  $\text{Be}$  at 60 GeV/c was found to be 154.4 mb. Again, the result obtained for interactions of  $\pi^+ + \text{C}$  at 60 GeV/c  
252 with these 2016 data was lower compared to the result obtained with the 2015 data [16], but it is within the  
253 estimated uncertainty. These results, the results obtained by NA61/SHINE from data recorded in 2015 and  
254 the measurements of Denisov et al. [23] are compared in Figure 5.

Interaction	$p$ (GeV/c)	Production cross section (mb)				
		$\sigma_{\text{prod}}$	$\Delta_{\text{stat}}$	$\Delta_{\text{syst}}$	$\Delta_{\text{model}}$	$\Delta_{\text{total}}$
$\pi^+ + \text{C}$	60	166.7	$\pm 3.5$	$\pm_{2.4}^{2.3}$	$\pm_{3.9}^{0.2}$	$\pm_{5.8}^{4.2}$
$\pi^+ + \text{Be}$	60	140.6	$\pm 3.5$	$\pm_{1.2}^{1.2}$	$\pm_{3.5}^{0.1}$	$\pm_{3.1}^{3.7}$

Table 7: Production cross section measurements of  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c interactions are presented. The central values as well as the statistical ( $\Delta_{\text{stat}}$ ), systematic ( $\Delta_{\text{syst}}$ ) and model ( $\Delta_{\text{model}}$ ) uncertainties are shown. The total uncertainties ( $\Delta_{\text{total}}$ ) are the sum of the statistical, systematic and model uncertainties in quadrature.

Interaction	$p$ (GeV/c)	Inelastic cross section (mb)				
		$\sigma_{\text{inel}}$	$\Delta_{\text{stat}}$	$\Delta_{\text{syst}}$	$\Delta_{\text{model}}$	$\Delta_{\text{total}}$
$\pi^+ + \text{C}$	60	182.7	$\pm 3.6$	$\pm_{2.4}^{2.4}$	$\pm_{2.8}^{0.2}$	$\pm_{5.2}^{4.3}$
$\pi^+ + \text{Be}$	60	154.4	$\pm 3.5$	$\pm_{1.2}^{1.2}$	$\pm_{2.5}^{0.1}$	$\pm_{4.5}^{3.7}$

Table 8: Inelastic cross section measurements of  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c interactions are presented. The central values as well as the statistical ( $\Delta_{\text{stat}}$ ), systematic ( $\Delta_{\text{syst}}$ ) and model ( $\Delta_{\text{model}}$ ) uncertainties are shown. The total uncertainties ( $\Delta_{\text{total}}$ ) are the sum of the statistical, systematic and model uncertainties in quadrature.

## 255 5 Analysis of Neutral Hadron Spectra

256 NA61/SHINE is able to identify a number of species of weakly-decaying neutral hadrons by tracking  
257 their charged decay products. The simplest decay topology NA61/SHINE can identify is the  $V^0$  topology.  
258 This topology refers to track topologies in which an unobserved neutral particle decays into two child  
259 particles, one positively charged and one negatively charged, observed by the tracking system. This paper  
260 presents differential production cross section measurements of produced  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  in interactions of  
261  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c using a  $V^0$  analysis.

### 262 5.1 Selection of $V^0$ Candidates

263 To start with, every pair of one positively charged and one negatively charged track with a distance-of-closest  
264 approach less than 5 cm is considered as a  $V^0$  candidate. Of course, many of these  $V^0$  candidates are not  
265 true  $V^0$ s. For example, a  $V^0$  candidate might consist of two tracks that come from the main interaction  
266 point, the child tracks might come from two different vertices or the child tracks might come from a parent  
267 track, which is not a neutral particle. Additionally, photons converting to  $e^+e^-$  pairs make up part of the  
268  $V^0$  sample.

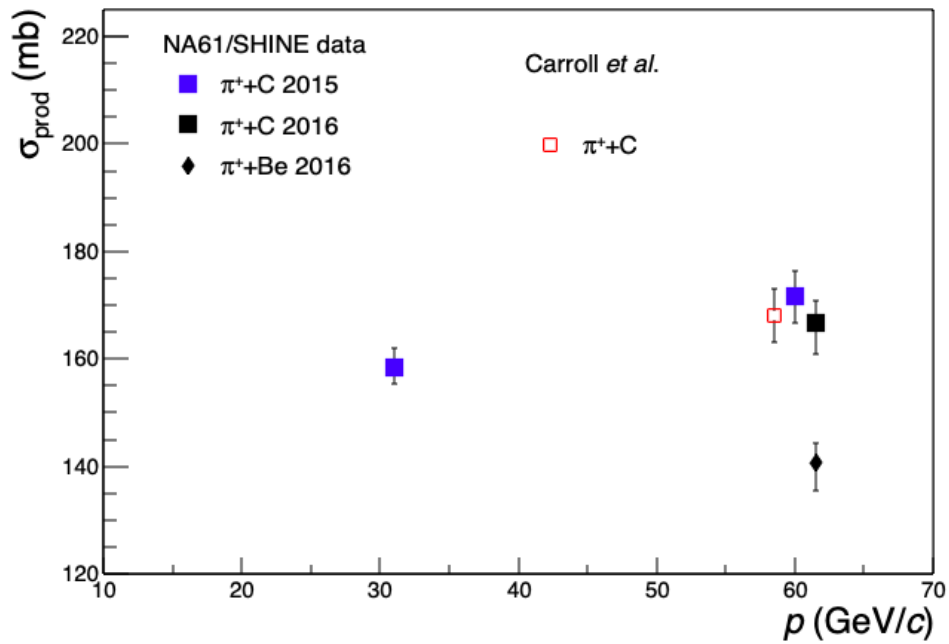


Figure 4: Summary of production cross section measurements. The results are compared to previous results from NA61/SHINE [16] and Carroll et al. [22].

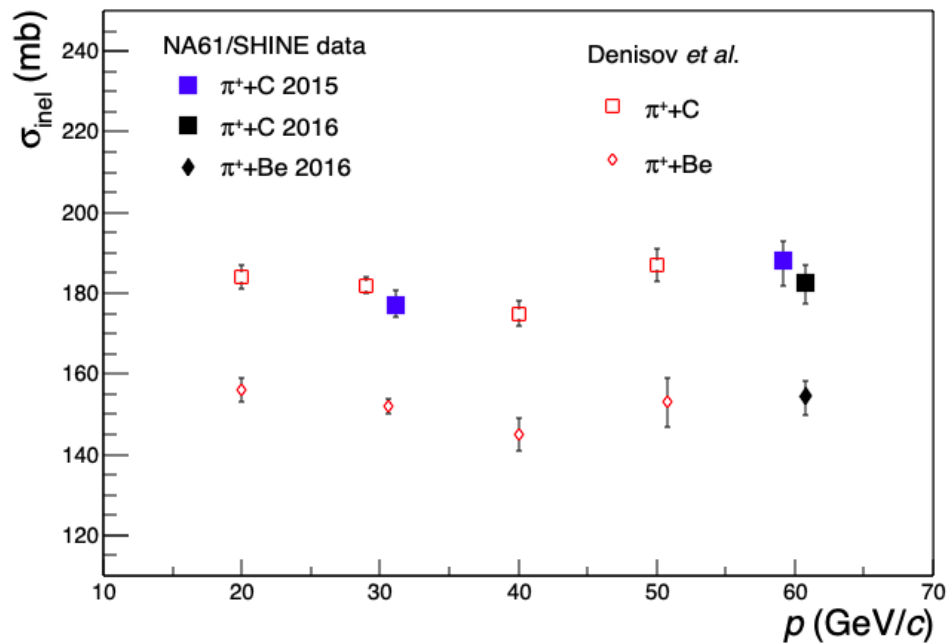


Figure 5: Summary of inelastic cross section measurements. The results are compared to previous results from NA61/SHINE [16] and Denisov et al. [23].

### 269 5.1.1 Topological Cuts

270 The topological cuts are designed to reduce the number of false  $V^0$ s in the collection of  $V^0$  candidates and to  
 271 remove  $V^0$  candidates that have poorly fitted track variables. Only  $V^0$  candidates that have a reconstructed  
 272  $V^0$  vertex downstream of the target are considered.

273 The second topological selection is the requirement that both child tracks have at least 20 reconstructed TPC  
 274 clusters and that at least 10 of those clusters belong to the VTPCs. This cut ensures that the reconstructed  
 275 kinematics of the decay are reliable.

276 The third topological cut is the impact parameter cut, which removes many false  $V^0$  candidates. This  
 277 selection allows an impact parameter from between the extrapolated  $V^0$ s track and the main interaction  
 278 vertex of up to 4 cm in the x dimension and up to 2 cm in the y dimension.

### 279 5.1.2 Purity Cuts

280 The purity cuts are designed to separate the desired neutral hadron species from other neutral species, as  
 281 well as to remove additional false  $V^0$  candidates. The first two purity cuts are applied in the same way to  
 282  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$ . This first selection requires the reconstructed z position of the  $V^0$  vertex to be at least 3.5  
 283 cm downstream of the target center. This cut removes many of the  $V^0$  candidates coming from the main  
 284 interaction vertex and neutral species that decay more quickly than  $K_S^0$ ,  $\Lambda$  or  $\bar{\Lambda}$ .

285 Photons undergoing pair production ( $\gamma \rightarrow e^+e^-$ ) are present in the  $V^0$  sample. Because the photon is  
 286 massless, the transverse momentum of the decay is:

$$p_T = |p_T^+| + |p_T^-| = 0 \text{ GeV}/c. \quad (15)$$

287 In order to remove most of these photons from the sample, the second purity cut requires a  $p_T >$   
 288  $0.03 \text{ GeV}/c$ .

### 289 5.1.3 Purity Cuts for the Selection of $K_S^0$

290 At this point, it is necessary to assume a decay hypothesis. For  $K_S^0$ , the hypothesis is  $K_S^0 \rightarrow \pi^+\pi^-$ .  
 291 Therefore, it is assumed that the  $V^0$  particle has a mass of  $m_{K_S^0} = 0.498 \text{ GeV}/c^2$  and the child particles  
 292 have a mass of  $m_{\pi^\pm} = 0.140 \text{ GeV}/c^2$  [24].

293 To remove  $\Lambda$  and  $\bar{\Lambda}$  from the  $K_S^0$  sample, cuts on the angles that the child particle tracks make with the  $V^0$   
 294 track in the decay frame are applied to the sample. These angles are represented in Figure 6. In order to  
 295 remove  $\Lambda$ ,  $\cos \theta^{+*} < 0.8$  is required and to remove  $\bar{\Lambda}$ ,  $\cos \theta^{-*} < 0.8$  is required.

296 The next selection is an allowed range of the invariant mass. The invariant mass is calculated with the  
 297 reconstructed momenta, assumed masses and energies of the child particles:

$$m_{+-} = \sqrt{m_+^2 + m_-^2 + 2(E_+E_- - \vec{p}_+ \cdot \vec{p}_-)}. \quad (16)$$

298 The invariant mass range cut removes  $V^0$  candidates with unreasonable values of  $M_{\pi^+\pi^-}$ , but is wide  
 299 enough to allow a reliable fit to the background invariant mass distribution. For  $K_S^0$ , this range is chosen to  
 300 be  $[0.4, 0.65] \text{ GeV}/c^2$ .

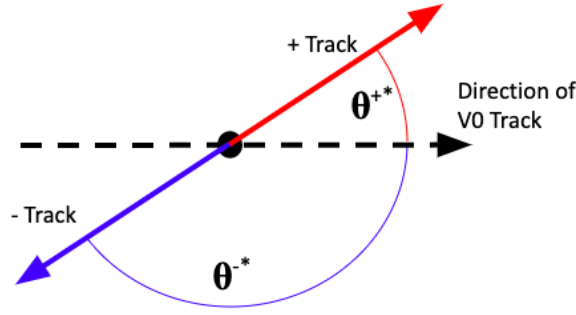


Figure 6: This cartoon shows the relevant angles in  $V^0$  decays in the rest frame of the  $V^0$ . The child particles decay back to back in this frame. The angle at which the positively charged particle is emitted is  $\theta^{+*}$ , and the angle at which the negatively charged particle is emitted is  $\theta^{-*}$ .

301 The final cut applied to the  $K_S^0$  selection is a cut on the proper decay length,  $c\tau$ . The proper decay length  
 302 can be calculated with the estimated momentum of the  $V^0$ ,  $p$ , the assumed mass,  $m$ , and the reconstructed  
 303 length of the  $V^0$  track,  $L$ :

$$c\tau = \frac{pL}{mc}. \quad (17)$$

304 The purpose of this cut is to further reduce the number of false  $V^0$ s and more quickly decaying neutral  
 305 species. The chosen cut is  $c\tau > 0.67$  cm, which is a quarter of the proper decay length provided by the  
 306 PDG [24], 2.68 cm.

#### 307 5.1.4 Purity Cuts for the Selection of $\Lambda$ and $\bar{\Lambda}$

308 An invariant mass range cut and a proper decay length cut are used in the purity selection of  $\Lambda$  and  $\bar{\Lambda}$ . The  
 309 invariant mass hypothesis for the  $\Lambda$  decay is  $\Lambda \rightarrow p\pi^-$  and the hypothesis for the  $\bar{\Lambda}$  is  $\bar{\Lambda} \rightarrow \bar{p}\pi^+$ . An  
 310 invariant mass range of [1.09, 1.215] GeV/ $c^2$  is used in both the  $\Lambda$  and  $\bar{\Lambda}$  analyses.

311 A proper decay length cut is also applied to the  $\Lambda$  and  $\bar{\Lambda}$  selection. The chosen cut is  $c\tau > 1.97$  cm, which  
 312 is a quarter of the proper decay length given by the PDG [24], 7.89 cm.

#### 313 5.1.5 Armenteros-Podolansky Distributions

314 The effect of these selections on the  $V^0$  candidates can be visualized with Armenteros-Podolansky  
 315 distributions, which are distributions of  $\alpha$  vs.  $p_T$ . The parameter  $\alpha$  is the asymmetry in the longitudinal  
 316 momenta of the child tracks with respect to the  $V^0$  track:

$$\alpha = \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}. \quad (18)$$

317 Figure 7 shows the  $V^0$  candidates coming from  $\pi^+ + C$  at 60 GeV/ $c$  interactions before the  $V^0$  selection  
 318 cuts were applied and after the selection cuts were applied for the  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  analyses.



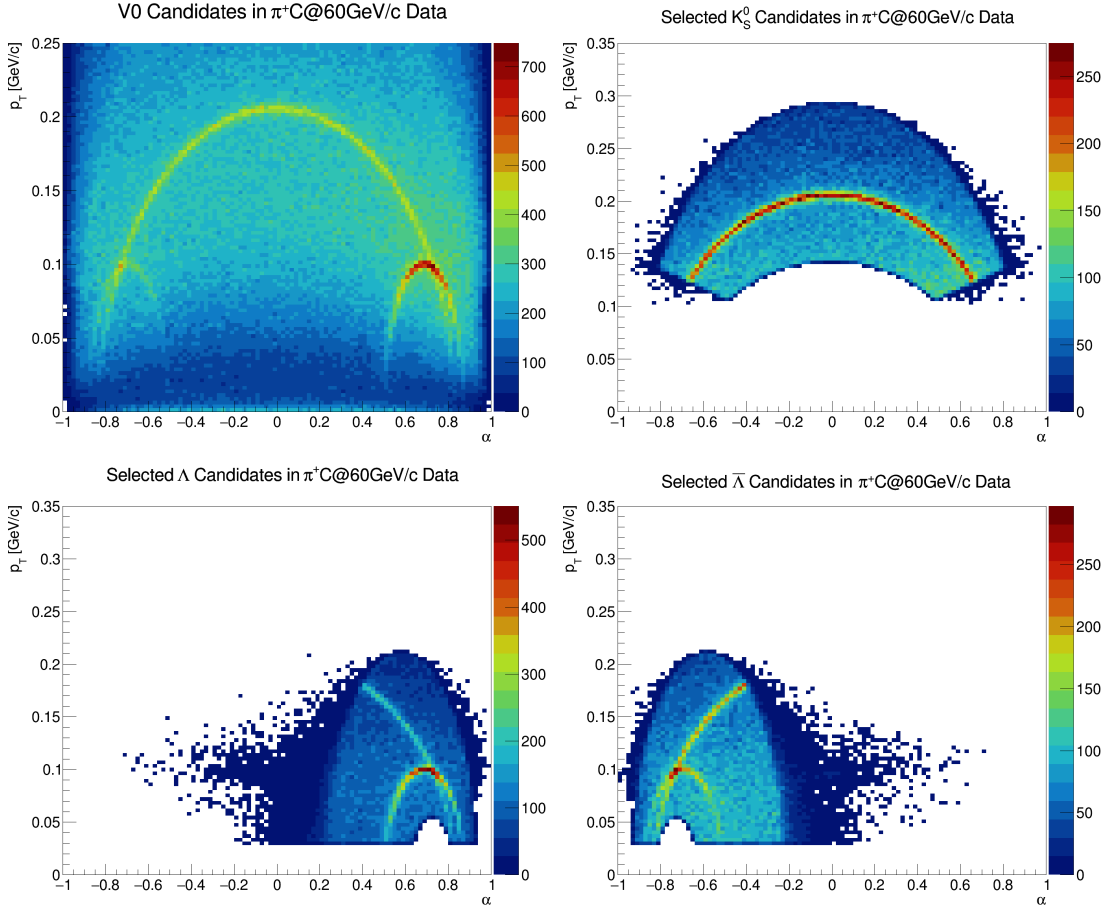


Figure 7: The Armenteros-Podolansky distribution of the  $V^0$  candidates in the  $\pi^+ + C$  at 60 GeV/c analysis before selection cuts were applied is shown in the top left. The distribution is shown after selection cuts are applied for the  $K_S^0$  analysis (top right),  $\Lambda$  analysis (bottom left) and  $\bar{\Lambda}$  analysis (bottom right).

## 319 5.2 Fitting of Invariant Mass Distributions

320 After applying the selection cuts for each particle species, the  $V^0$  candidates are placed into the kinematic  
 321 bins. For each of these kinematic bins, invariant mass distributions consist of both true  $K_S^0$ ,  $\Lambda$  or  $\bar{\Lambda}$  (signal)  
 322 and the remaining background vertices. The objective of the fitting routine is to determine the number  
 323 of true  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  in these invariant mass distributions. These fits are performed the same way on  
 324 target-inserted and target-removed samples.

### 325 5.2.1 Signal Model

326 In order to model the invariant mass distribution of  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  coming from the main interactions,  
 327 template invariant mass distributions were derived from a GEANT4 MC production using the physics list  
 328 FTFP\_BERT.  $V^0$  vertices are reconstructed, selected and binned in the same way as was done with the data.  
 329 For each kinematic bin, MC templates are formed from the distributions of invariant mass from true  $K_S^0$ ,  $\Lambda$   
 330 and  $\bar{\Lambda}$ . These template distributions,  $g_{MC}(m)$ , are generated for both target-inserted and target-removed

331 MC productions. In order to account for shifts in the invariant mass peaks and distortions of the signal shape  
 332 due to misreconstruction of track variables and other possible effects, a mass shift,  $m_0$ , and a smearing are  
 333 applied to  $g_{MC}(m)$ . The smearing is applied by convolving the  $g_{MC}(m)$  with a unit gaussian distribution  
 334 with width  $\sigma_s$ . The full signal distribution can be written as:

$$f_s(m; m_0, \sigma_s) = g_{MC}(m - m_0) \otimes \frac{1}{\sqrt{2\pi}\sigma_s} \exp - \frac{(m - m_0)^2}{2\sigma_s^2}. \quad (19)$$

### 335 5.2.2 Background Model

336 It was observed that the shapes of the backgrounds in the invariant mass distributions vary among the  $K_S^0$ ,  
 337  $\Lambda$  and  $\bar{\Lambda}$  selection as well as among the kinematic bins. The background model was required to be flexible  
 338 enough to account for the variation of background shapes in all of the kinematic bins for  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$ . A  
 339 second order polynomial was chosen to be used to fit the background distributions.

### 340 5.2.3 Fitting Strategy

341 In order to fit for the signal and background contributions to the invariant mass distributions, a continuous  
 342 log-likelihood function is constructed:

$$\log L = \sum_{V^0 \text{ Candidates}} \log F(m; \theta), \quad (20)$$

343 where

$$F(m; \theta) = c_s f_s(m; \theta_s) + (1 - c_s) f_{bg}(m; \theta_{bg}). \quad (21)$$

344 This distribution function incorporates the signal model,  $f_s$ , and the background model,  $f_{bg}$ , with the  
 345 parameter  $c_s$  controlling what fraction of the  $V^0$  candidates are considered to be part of the signal. The  
 346 parameters,  $\theta$ , include  $c_s$  as well as the signal parameters,  $\theta_s$ , discussed in Section 5.2.1 and the background  
 347 parameters,  $\theta_{bg}$ , which are the coefficients of the second degree polynomial. After obtaining  $c_s$  from the  
 348 fits, the raw yield of signal particles is calculated with:  $y^{\text{raw}} = c_s N_{V^0 \text{ Candidates}}$ .

349 Figures 8 and 9 show example fits to  $K_S^0$  and  $\Lambda$  invariant mass distributions from the  $\pi^+ + C$  at 60 GeV/c  
 350 data set. Averaging over the fit results for all kinematic bins, the observed  $K_S^0$  mass was 498.7 MeV/c<sup>2</sup>,  
 351 which is slightly higher than the known value of 497.6 MeV/c<sup>2</sup> [24]. The average of the widths of the  
 352 invariant mass distributions was observed to be 17 MeV/c<sup>2</sup>. The  $\Lambda$  and  $\bar{\Lambda}$  masses were both observed to  
 353 be 1,117 MeV/c<sup>2</sup>, slightly higher than the known value of 1,116 MeV/c<sup>2</sup> [24]. The widths of the  $\Lambda$  and  
 354  $\bar{\Lambda}$  distributions were found to be 6 MeV/c<sup>2</sup> and 7 MeV/c<sup>2</sup>, respectively. These small discrepancies in the  
 355 masses compared to the known values are likely due to small biases in the momentum reconstruction of  
 356 tracks.

### 357 5.3 Corrections

358 The raw yields obtained from the fits discussed in the previous section must be corrected for systematic  
 359 effects. These can roughly be categorized into several effects: branching ratio of the decay, detector  
 360 acceptance, feed-down corrections, reconstruction efficiency and selection efficiency. The combined effect

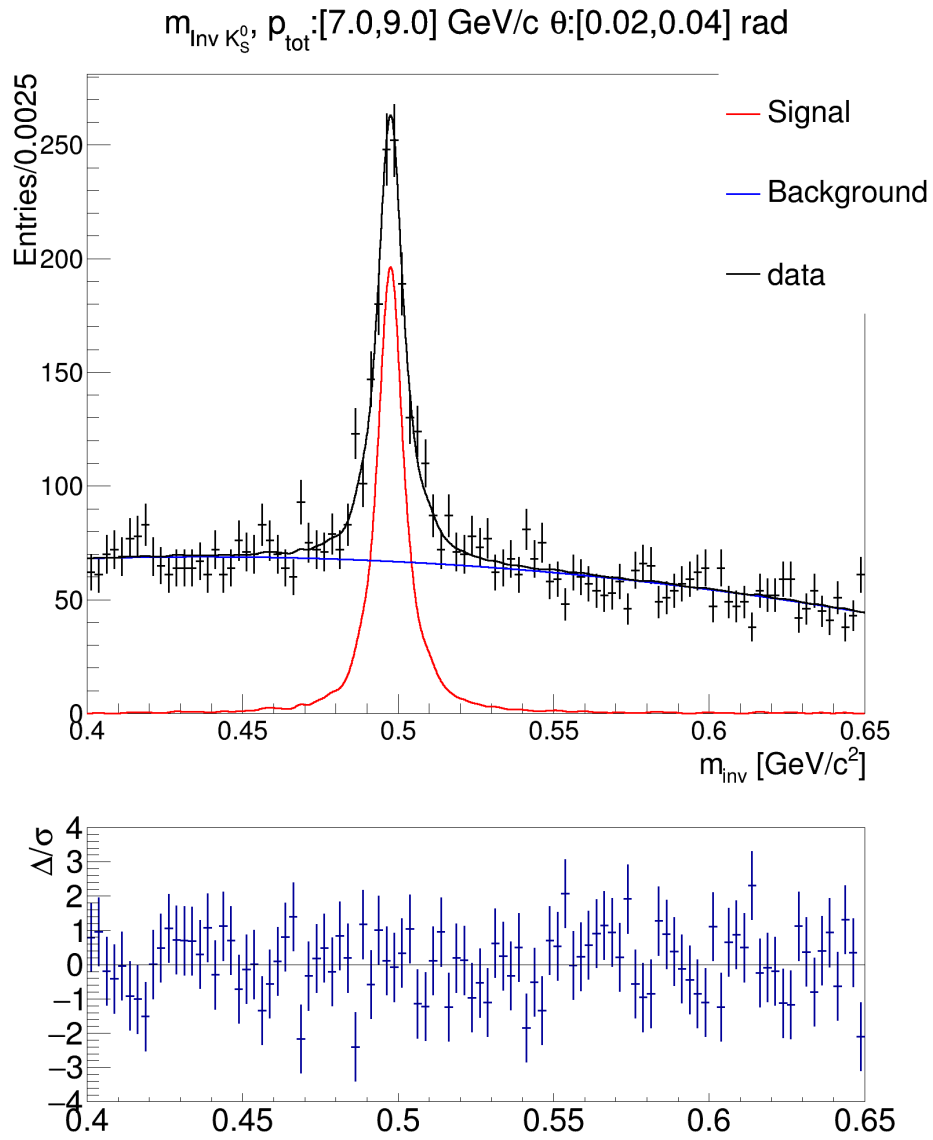


Figure 8: Example fit to the  $K_S^0$  invariant mass distribution in  $\pi^+ + C$  at  $60 \text{ GeV}/c$  data for an example kinematic bin. The  $m_{\text{inv}}$  distribution and the fitted model is shown in the top. The residuals of the fit are shown on the bottom.

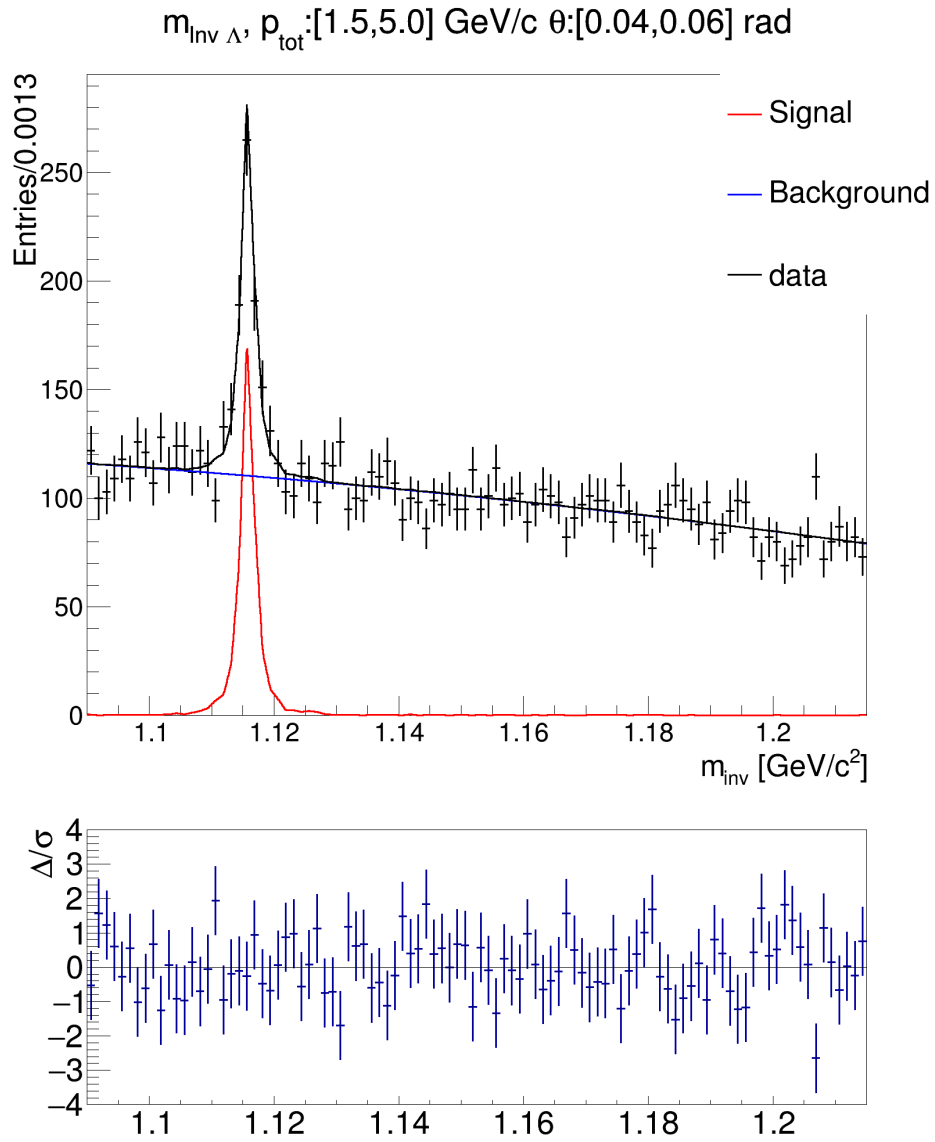


Figure 9: Example fit to the  $\Lambda$  invariant mass distribution in  $\pi^+ + \text{C}$  at  $60 \text{ GeV}/c$  data for an example kinematic bin. The  $m_{\text{inv}}$  distribution and the fitted model is shown on the top. The residuals of the fit are shown on the bottom.

361 of these individual effects can be estimated as a single correction factor from Monte Carlo simulations.  
 362 Using  $K_S^0$  as an example, the correction factor for kinematic bin  $i$  is given by:

$$c_i = \frac{N(\text{simulated } K_S^0)}{N(\text{selected, reconstructed } K_S^0)} = c_{\text{BR}} \times c_{\text{acc.}} \times c_{\text{feed-down}} \times c_{\text{rec. eff.}} \times c_{\text{sel. eff.}} \quad (22)$$

363 The correction factors are calculated in the analogous way for  $\Lambda$  and  $\bar{\Lambda}$ . The correction factors are obtained  
 364 from the MC production using the FTFP\_BERT physics list.

## 365 6 Analysis of Charged Hadron Spectra

366 The analysis of produced charged hadrons is performed with a dE/dx analysis, which uses energy loss  
 367 measured by the TPCs to separate particle species for both positively and negatively charged tracks. In  
 368 particular, it was possible to measure spectra of produced  $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$  and protons with this method.

### 369 6.1 Selection of Tracks

370 The selection criteria are devised to remove off-time tracks and tracks coming from secondary interactions  
 371 mistakenly reconstructed to the main interaction vertex. The selection cuts are also devised to filter out  
 372 tracks with poorly determined track parameters, mainly  $p$ ,  $\theta$  and dE/dx. To start with, all tracks emanating  
 373 from the main interaction vertex are considered for the dE/dx analysis.

#### 374 6.1.1 Track Topologies

375 There are a few ways tracks can be classified into different track topologies, including the initial direction  
 376 of the tracks and which TPC chambers the tracks pass through. The most basic track topology classification  
 377 used in NA61/SHINE analyses is the distinction between so-called right-side tracks (RSTs) and wrong-side  
 378 tracks (WSTs) determined by the charge and direction emitted from the target. RSTs have a reconstructed  
 379  $p_x$  that is in the same direction as the deflection by the vertex magnets. WSTs have a reconstructed  $p_x$   
 380 opposite to the bending direction of the magnetic fields. This can be written more succinctly:

$$\begin{cases} p_x/q > 0 & \text{RST} \\ p_x/q < 0 & \text{WST} \end{cases} \quad (23)$$

381 For the same reconstructed momenta, RSTs and WSTs have very different detector acceptances, numbers  
 382 of clusters and trajectories through different TPC sectors. Therefore, in this analysis, RSTs and WSTs  
 383 undergo different selection criteria, are fit separately and had different corrections applied to them. This  
 384 classification allows for a basic cross check, since these two samples lead to two somewhat independent  
 385 measurements. For the purposes of this analysis, the distinction between RSTs and WSTs is not made for  
 386 the first angular bin ( $[0,10]$  mrad for pions and  $[0,20]$  mrad for kaons and protons), because it is difficult to  
 387 accurately distinguish between RSTs and WSTs near  $\theta = 0$  mrad.

## 388 6.1.2 Phi Cuts

389 The azimuthal acceptance of the NA61/SHINE detector is highly dependent on the track topology and  $\theta$ . In  
390 order to obtain samples of tracks with similar numbers of clusters,  $\phi$  cuts were devised as a function of  $\theta$   
391 bin and track topology and applied to the selection.

## 392 6.1.3 Track Quality Cuts

393 The impact parameter of tracks (distance from the main interaction vertex and the extrapolation of the track  
394 to the plane of the target) is required to be less than 2 cm in order to remove off-time tracks and tracks  
395 produced in secondary interactions.

396 To ensure that the selected tracks have narrow enough  $dE/dx$  distributions to distinguish between particle  
397 species, at least 30 clusters are required in the VTPCs and MTPCs. In order to ensure tracks have  
398 good momentum estimations, there must be at least 4 clusters in the GTPC or 10 clusters in the VTPCs.  
399 Additionally, to remove tracks resulting from secondary interactions that were falsely reconstructed to the  
400 main interaction vertex, a cut is applied to tracks with no reconstructed GTPC and VTPC-1 clusters. This  
401 cut requires there to be fewer than 10 potential clusters in the VTPC-1 and fewer than 7 potential clusters in  
402 the GTPC, where the potential clusters are calculated by extrapolating tracks through the tracking system.

403 Several  $dE/dx$  cuts were applied to remove tracks with nonsensical  $dE/dx$  values (MIP) and rare heavier  
404 mass or doubly-charged particles:

$$\begin{cases} 0 < dE/dx < 2 & p \geq 2.2 \text{ GeV}/c \\ 0 < dE/dx < \langle dE/dx \rangle_{De} + 1 & p < 2.2 \text{ GeV}/c \end{cases} \quad (24)$$

405 These cuts remove much less than 1% of tracks, so no correction is made to account for the  $dE/dx$  cuts.

406 Figure 10 shows the  $dE/dx$ -momentum distribution of the selected positively charged and negatively charged  
407 tracks.

## 408 6.2 Fitting to $dE/dx$ Distributions

409 For each analysis bin, a fit is used to determine the yields of each particle species. Five particles species and  
410 their anti-particles are considered:  $e^+$ ,  $\pi^+$ ,  $K^+$ , protons and deuterons. Positively charged and negatively  
411 charged tracks are simultaneously fit to better constrain the parameters.

### 412 6.2.1 $dE/dx$ Model

413 The mean  $dE/dx$ ,  $\langle \epsilon \rangle$ , of charged particles passing through NA61/SHINE's TPCs depends on the particles'  
414 values of  $\beta$ , which, for particles of the same momentum, depend on their masses. A Bethe-Bloch table  
415 provides initial guesses of  $\langle \epsilon \rangle$  for particle species within each bin.

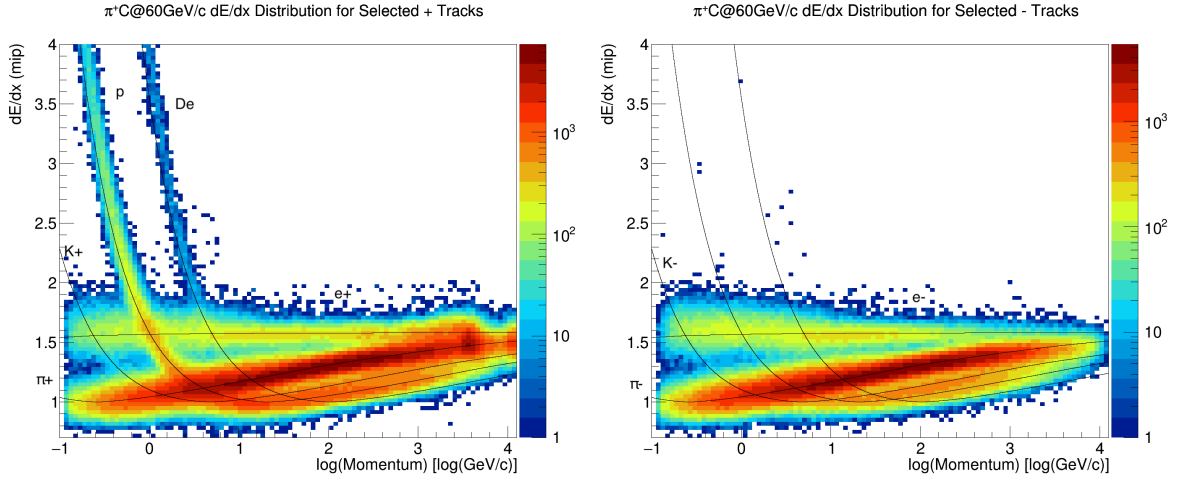


Figure 10: 2-dimensional distributions of  $dE/dx$  and  $p$  are shown for the selected positively (left) and negatively (right) charged tracks in the  $\pi^+ + C$  at 60 GeV/c analysis. The black lines represent the Bethe-Bloch predictions for the  $dE/dx$  mean position of electrons, pions, kaons, protons and deuterons.

416 The  $dE/dx$  distribution function describing the observed  $dE/dx$  of a charged particle passing through the  
 417 TPCs depends on  $\langle \epsilon \rangle$  and the distance traveled through the TPCs. The distribution closely resembles an  
 418 asymmetric gaussian:

$$f(\epsilon, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{1}{2} \left( \frac{\epsilon - \mu}{\delta\sigma} \right)^2 \right], \quad (25)$$

419 where  $\epsilon$  is the measured  $dE/dx$  of a track. The peak  $dE/dx$  of the distribution,  $\mu$ , is related to  $\langle \epsilon \rangle$  through  
 420 the relation:

$$\mu = \langle \epsilon \rangle - \frac{4d\sigma}{\sqrt{2\pi}}, \quad (26)$$

421 where  $d$  is the asymmetry parameter, which controls the asymmetry of the distribution through the relation:

$$\delta = \begin{cases} 1 - d, & \text{if } \epsilon \leq \mu \\ 1 + d, & \text{if } \epsilon > \mu \end{cases}. \quad (27)$$

422 For a detector with uniform readout electronics, the width of the distribution for a single particle depends  
 423 on the number of  $dE/dx$  clusters,  $N_{Cl}$ , and on  $\langle \epsilon \rangle$ :

$$\sigma = \frac{\sigma_0 \langle \epsilon \rangle^\alpha}{\sqrt{N_{Cl}}}, \quad (28)$$

424 where the parameter,  $\alpha$ , controls how the width scales with  $\langle \epsilon \rangle$  and  $\sigma_0$  is the base  $dE/dx$  width of a single  
 425 cluster. However, in NA61/SHINE, nonuniform readout electronics leads to different base widths for  
 426 clusters reconstructed in different areas of the detector. This effect is most apparent in 3 main areas of the  
 427 NA61/SHINE TPC system: the MTPCs, the two most upstream sectors of the VTPCs and the rest of the  
 428 VTPCs. Different base widths characterizes each of these regions:  $\sigma_{0, M}$ ,  $\sigma_{0, Up}$  and  $\sigma_{0, V}$ . The  $dE/dx$  width  
 429 of a single track can be parametrized more precisely by accounting for the numbers of clusters in each TPC

430 region,  $N_{\text{Cl, Up}}$ ,  $N_{\text{Cl, v}}$  and  $N_{\text{Cl, M}}$ :

$$\sigma = \frac{\langle \epsilon \rangle^\alpha}{\sqrt{\frac{N_{\text{Cl, Up}}}{\sigma_{0, \text{Up}}^2} + \frac{N_{\text{Cl, v}}}{\sigma_{0, \text{v}}^2} + \frac{N_{\text{Cl, M}}}{\sigma_{0, \text{M}}^2}}}. \quad (29)$$

431 At this point, some calibration and shape parameters need to be added in to account for imperfect dE/dx  
 432 calibration, variation in pad response, variation in track angle and other effects that can cause  $\langle \epsilon \rangle$  and  $\sigma$   
 433 deviate from the ideal model. Therefore, additional calibration parameters are added to allow the peaks and  
 434 widths of the species distribution functions to vary slightly from the ideal model for each analysis bin.

435 The full form of the single species distribution function is then:

$$f^{i,j}(\epsilon, p, N_{\text{Cl, Up}}, N_{\text{Cl, v}}, N_{\text{Cl, M}}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{cal}}^{i,j}} \exp\left[-\frac{1}{2}\left(\frac{\epsilon - \mu_{\text{cal}}^{i,j}}{\delta\sigma_{\text{cal}}^{i,j}}\right)^2\right], \quad (30)$$

436 where  $\sigma_{\text{cal}}^{i,j}$  and  $\mu_{\text{cal}}^{i,j}$  implicitly depend on the the momentum  $p$ , the number of clusters variables and the  
 437 calibration parameters.

438 With these single-species distribution functions the single-track distribution functions can be built for both  
 439 charges,  $F^+$  and  $F^-$ :

$$F^j(\epsilon, p, N_{\text{Cl, Up}}, N_{\text{Cl, v}}, N_{\text{Cl, M}}) = \sum_i y^{i,j} f^{i,j}(\epsilon, p, N_{\text{Cl, Up}}, N_{\text{Cl, v}}, N_{\text{Cl, M}}) \quad (31)$$

440 where  $y^{i,j}$  is the fractional contribution of species  $i$  to the sample of tracks with charge  $j$ . The yields for  
 441 each charge are constrained such that they sum to 1.

## 442 6.2.2 Fitting Strategy

443 To perform the minimization, a continuous log-likelihood function is constructed:

$$\log L = \sum_{+\text{tracks}} \log F^+(\epsilon, p, N_{\text{Cl, Up}}, N_{\text{Cl, v}}, N_{\text{Cl, M}}; \theta) + \sum_{-\text{tracks}} \log F^-(\epsilon, p, N_{\text{Cl, Up}}, N_{\text{Cl, v}}, N_{\text{Cl, M}}; \theta). \quad (32)$$

444 The log-likelihood function involves a sum over all of the positively and negatively charged tracks for a  
 445 given analysis bin. In addition to the constraint that the yield fractions add up to 1 for each charge, soft  
 446 constraints are applied to avoid the parameters converging to unreasonable values. For example, without  
 447 constraints, it is easy for two species to swap the location of their dE/dx means. For fits to the target-removed  
 448 data, all of the parameters are fixed to the fitted values from the target-inserted fits, except for the particle  
 449 yields. Figure 11 shows a fit to the dE/dx distribution of an example bin. The estimated raw yield of a  
 450 particle species in analysis bin  $k$  is obtained by multiplying the fractional yield obtained from the fit,  $y_k^{i,j}$ ,  
 451 by the number of positively or negatively charged tracks in that bin,  $N_k^i$ :

$$Y_k^{i,j,\text{raw}} = y_k^{i,j} N_k^i. \quad (33)$$

452 For each of the  $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$  and proton analyses, a raw yield is obtained for each bin and for both the  
 453 target-inserted and target-removed samples.



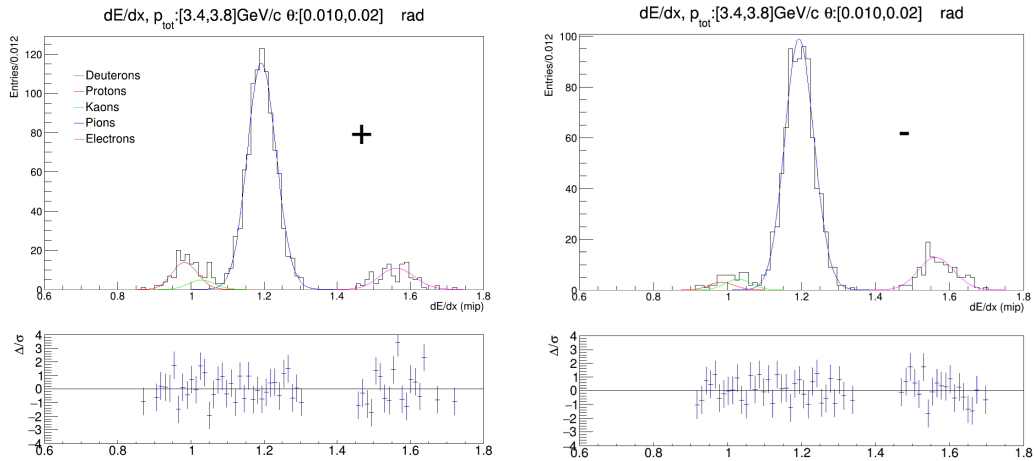


Figure 11: An example fit to a  $dE/dx$  distribution is shown for the analysis of pions. On the top, the  $dE/dx$  distributions are shown for positively charged tracks (left) and negatively charged tracks (right) along with the fitted contributions due to the 5 particle species considered. On the bottom, the residuals of the fit with respect to the  $dE/dx$  distribution are shown.

## 454 6.3 Corrections

### 455 6.3.1 Fit Bias Corrections

456 Simulated  $dE/dx$  distributions were generated in order to estimate the bias and the standard deviation of the  
 457 particle yields obtained from the fitting procedure. 50 simulated  $dE/dx$  distributions for each analysis bin  
 458 were built from the  $dE/dx$  model discussed in the previous section. The kinematic variables of tracks from  
 459 data and the resulting hadron yields were taken as inputs for the  $dE/dx$  simulation. The fit parameters are  
 460 varied according to the spread of fit results observed in data.

461 The biases and standard deviations in the fitted yields are determined from the results of fits to these  
 462 simulated  $dE/dx$  distributions. In general, the biases in the pion yields are small. The biases of the proton  
 463 and kaon yields are larger in the high momentum regions and near the Bethe-Bloch crossing regions,  
 464 where the particle distributions overlap significantly. The biases are used to correct the fit results with  
 465 correction factors,  $c_k^{\text{fit}}$ , and the standard deviations are used to estimate the uncertainties related to the fitting  
 466 procedure.

### 467 6.3.2 Monte Carlo Corrections

468 The raw yields of particles obtained from the  $dE/dx$  fits must be corrected for a number of systematic effects.  
 469 These can roughly be organized into: detector acceptance, feed-down corrections, reconstruction efficiency,  
 470 selection efficiency and in the case of pions, muon contamination. The combined effect of these individual  
 471 effects can be estimated as an overall correction factor from Monte Carlo simulations, as was done in the  
 472  $V^0$  analysis.

473 In the case of corrections for  $\pi^+$  and  $\pi^-$ , because the  $dE/dx$  signal from muons is indistinguishable from  
 474 pions, muon tracks that pass the selection criteria and are fitted to the main interaction vertex must also be

475 accounted for:

$$c_k^{\text{MC}} = \frac{N(\text{produced, simulated } \pi^\pm)_k}{N(\text{selected, reconstructed } \pi^\pm, \mu^\pm)_k} = c_{\text{acc.}} \times c_{\text{feed-down}} \times c_{\text{rec. eff.}} \times c_{\text{sel. eff.}} \times c_{\mu^\pm}. \quad (34)$$

### 476 6.3.3 Feed-down Reweighting

477 The feed-down correction, which can be as large as 20% for protons, is the main component of the MC  
 478 correction factor that depends on the physics model. We cannot assume that the production of  $\Lambda$ ,  $\bar{\Lambda}$  and  $K_S^0$   
 479 is accurately predicted by the physics generators. This incurs an uncertainty on the MC corrections and  
 480 subsequently, on the resulting multiplicity measurements.

481 We can constrain this uncertainty by reweighting our MC productions with the results of the  $V^0$  analyses.  
 482 When counting the number of reconstructed pions and protons passing the selection criteria, a weight is  
 483 applied whenever that reconstructed track comes from a  $K_S^0$ ,  $\Lambda$  or  $\bar{\Lambda}$ :

$$w_\beta = \frac{m_\beta^{\text{data}}}{m_\beta^{\text{MC}}}, \quad (35)$$

484 where  $m_\beta^{\text{data}}$  is the multiplicity measured in bin  $\beta$  of the  $V^0$  analysis and  $m_\beta^{\text{MC}}$  is the multiplicity observed  
 485 in the simulation in that bin.

## 486 7 Systematic Uncertainties on Spectra Measurements

487 A number of possible systematic effects on the multiplicity measurements have also been evaluated.  
 488 These include biases and uncertainties incurred by the fitting procedures, uncertainties associated with the  
 489 MC corrections, uncertainties incurred in the selection procedures and uncertainties associated with the  
 490 reconstruction. On top of the uncertainties described in the following sections, an overall normalization  
 491 uncertainty is attributed to all of the multiplicity measurements. It has been estimated to be  $\pm_1^2\%$  by  
 492 propagating the uncertainties on the normalization constants derived from the integrated cross section  
 493 analysis through the multiplicity calculation, which will be discussed in Section 8.

### 494 7.1 Fit Model Uncertainty

495 In the  $V^0$  analysis, it cannot be assumed that the fits to the invariant mass distributions perfectly separate  
 496 the signal from the background. To check for biases in the fit results, the fitting procedure is performed  
 497 on additional MC productions using GEANT4 physics lists QGSP\_BERT, QBBC and FTF\_BIC. With  
 498 these samples, the numbers of true  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  are known, so the bias and the standard deviation of the fit  
 499 result can be calculated. For  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$ , the fitting bias,  $\mu$ , on the signal fraction,  $c_s$ , was found to be  
 500  $3.3\% \pm 2.7\%$ ,  $4.8\% \pm 4.2\%$  and  $11\% \pm 10\%$ , respectively. The bias is not used as a correction for the fit  
 501 results, but the values of  $\mu \pm \sigma$  are taken as upper and lower uncertainties on the signal fraction, which are  
 502 propagated through the multiplicity calculation.

503 The fit model uncertainties on the charged spectra are obtained from the fits to simulated dE/dx distributions  
 504 discussed in Section 6.3.1. The standard deviations in the particle yields are propagated to the multiplicities  
 505 and taken as the uncertainties associated with the fitting routine.

## 506 7.2 Physics Uncertainties

507 Assuming different underlying physics can lead to different MC correction factors. For example, if the  
508 acceptance changes as a function of  $p$  and  $\theta$ , different MC-predicted  $p$  and  $\theta$  distributions can lead to  
509 different MC correction factors. This uncertainty is evaluated by applying correction factors obtained  
510 with additional MC productions using the physics lists: QGSP\_BERT, QBBC and FTF\_BIC. The upper  
511 and lower bounds on the uncertainties are taken as the maximum and minimum values of the multiplicity  
512 obtained using these additional MC correction factors for each analysis bin.

## 513 7.3 Feed-down Uncertainties

514 The MC corrections account for a background of produced hadrons coming from heavier weakly-decaying  
515 particles. However, it cannot be assumed that the physics generators correctly predict the production rates  
516 of these heavier weakly-decaying hadrons. This uncertainty is evaluated by assuming a 50% uncertainty on  
517 the number of reconstructed feed-down particles when calculating the MC correction factors, unless the  
518 feed-down particle was a reweighted  $K_S^0$ ,  $\Lambda$  or  $\bar{\Lambda}$ . In this case, the upper and lower uncertainties on the  
519 associated neutral hadron spectra are assigned to the weight assigned to the feed-down particles. These  
520 uncertainties are then propagated to the multiplicities. This reweighting treatment results in a significant  
521 reduction of the uncertainties on the  $\pi^+$ ,  $\pi^-$  and proton spectra.

## 522 7.4 Selection Uncertainties

523 Although the MC corrections account for the efficiency of the selection cuts, differences in data and MC  
524 could incur systematic biases in the result. It was found that tracks in data are typically composed of  
525 around 5% fewer clusters than tracks in MC for the same kinematics. To estimate the selection uncertainty,  
526 alternative sets of MC corrections were obtained by artificially decreasing the numbers of clusters in MC  
527 tracks by 5%. Higher multiplicities are obtained when applying these alternative correction factors, which  
528 are taken as the upper bounds of the selection uncertainty.

## 529 7.5 Reconstruction Uncertainties

530 The MC corrections should account for inefficiencies in the reconstruction of tracks and  $V^0$ s if the geometry  
531 and detector response are perfectly modeled by the simulation. Differences between the real detector and  
532 the simulated detector could lead to systematic effects on reconstruction efficiency component of the MC  
533 corrections. To estimate this uncertainty, the detectors were purposefully moved in the detector description  
534 model used by the reconstruction. Specifically, eight alternative productions were made after shifting the  
535 VTPC-1 and VTPC-2 by +.2 mm and -.2 mm in the x direction and +.5 mm and -.5 mm in the y direction.  
536 These shifts are considered to be rather large when compared to the alignment effects seen in the calibration  
537 of the data.

538 The numbers of selected charged tracks and  $V^0$  candidates were calculated from these alternative productions.  
539 The maximum difference in the number of candidate tracks/ $V^0$ s among the productions are calculated  
540 for the x shifts and the y shifts in each analysis bin. The effects of the x and y shifts are then added in  
541 quadrature to estimate the uncertainty for each bin. The resulting uncertainties are generally less than 1%  
542 and do not exceed 4%.

## 543 7.6 Momentum Uncertainties

544 There is an uncertainty on the reconstruction of momentum due to uncertainties in converting the magnet  
 545 currents to magnetic field strength. This uncertainty can be investigated by checking the invariant mass  
 546 distributions fitted in the V0 analysis. The variation in the fitted means of the invariant mass distributions  
 547 of  $K_S^0$  and  $\Lambda$  indicate an uncertainty in the reconstruction of momentum of up to 0.3%. Uncertainties on  
 548 the measured multiplicities due to misreconstructed momenta was determined by varying the momenta  
 549 of tracks by 0.3% and recalculating the numbers of selected tracks and  $V^0$  candidates. This uncertainty  
 550 was determined to be less than 1% for the majority of the analysis bins, but is on the level of the statistical  
 551 uncertainty for some of the analysis bins at the edges of the phase space measured.

## 552 7.7 Breakdowns in Uncertainties

553 The breakdowns in the uncertainties for  $\pi^+$ ,  $K^+$ , proton,  $K_S^0$  and  $\Lambda$  spectra from  $\pi^+ + C$  at 60 GeV/ $c$   
 554 interactions are shown for representative angular bins in Figure 12. These breakdowns include statistical  
 555 uncertainties, fit uncertainties, physics uncertainties, feed-down uncertainties, selection uncertainties  
 556 momentum uncertainties and reconstruction uncertainties. The breakdowns of the uncertainties are largely  
 557 similar for the measured hadron spectra from interactions of  $\pi^+ + Be$  at 60 GeV/ $c$ . Figures in Ref. [25]  
 558 present breakdowns of the uncertainties for the complete set of spectra measurements for interactions of  
 559  $\pi^+ + C$  at 60 GeV/ $c$  and  $\pi^+ + Be$  at 60 GeV/ $c$ .

560 For the neutral spectra, the uncertainties are within 10% in the kinematic regions with good detector  
 561 acceptance and high statistical power. In the low-momentum regions, uncertainties associated with the  
 562 fitting routine tend to dominate the lower uncertainties and selection uncertainties tend to dominate the  
 563 upper uncertainties. The physics model uncertainty is typically the largest component of the uncertainty in  
 564 the high momenta regions.

565 For the charged spectra, the total uncertainties are generally around 5% or less except in the kinematic  
 566 regions with poor acceptance or poor dE/dx separation. In spectra of  $\pi^+$ , the largest uncertainties tend to  
 567 be reconstruction uncertainties at high momenta and dE/dx fit uncertainties at low momenta. In the case of  
 568  $\pi^-$ , dE/dx fit uncertainties, physics model uncertainties and statistical uncertainties contribute the most to  
 569 the total uncertainty. For kaons, dE/dx fit uncertainties are dominant in the majority of the phase space  
 570 measured. For protons, uncertainties related to the physics model and dE/dx fit uncertainties are dominant  
 571 for the majority of the phase space measured.

## 572 8 Differential Production Multiplicity Measurements

573 The differential production multiplicity is the yield of particles produced per production interaction per  
 574 unit momentum per radian in each kinematic bin  $k$ . The production multiplicity for neutral hadrons can be  
 575 written:

$$\frac{d^2 n_k}{dp d\theta} = \frac{\sigma_{\text{trig}} c_k^{MC}}{f_{\text{prod}} \sigma_{\text{prod}} (1 - \epsilon) \Delta p \Delta \theta} \left( \frac{Y_k^I}{N^I} - \frac{\epsilon Y_k^R}{N^R} \right), \quad (36)$$

576 where  $\Delta p \Delta \theta$  is the size of bin  $k$ , and the yields,  $Y_k^{I,R}$ , are the total numbers of particles observed in bin  $k$   
 577 determined by the invariant mass fits for target-inserted and target-removed data. The constants  $\sigma_{\text{trig}}$ ,  $\sigma_{\text{prod}}$ ,  
 578  $f_{\text{prod}}$  and  $\epsilon$  are determined from the integrated cross section analysis and  $N^I$  and  $N^R$  are the numbers of

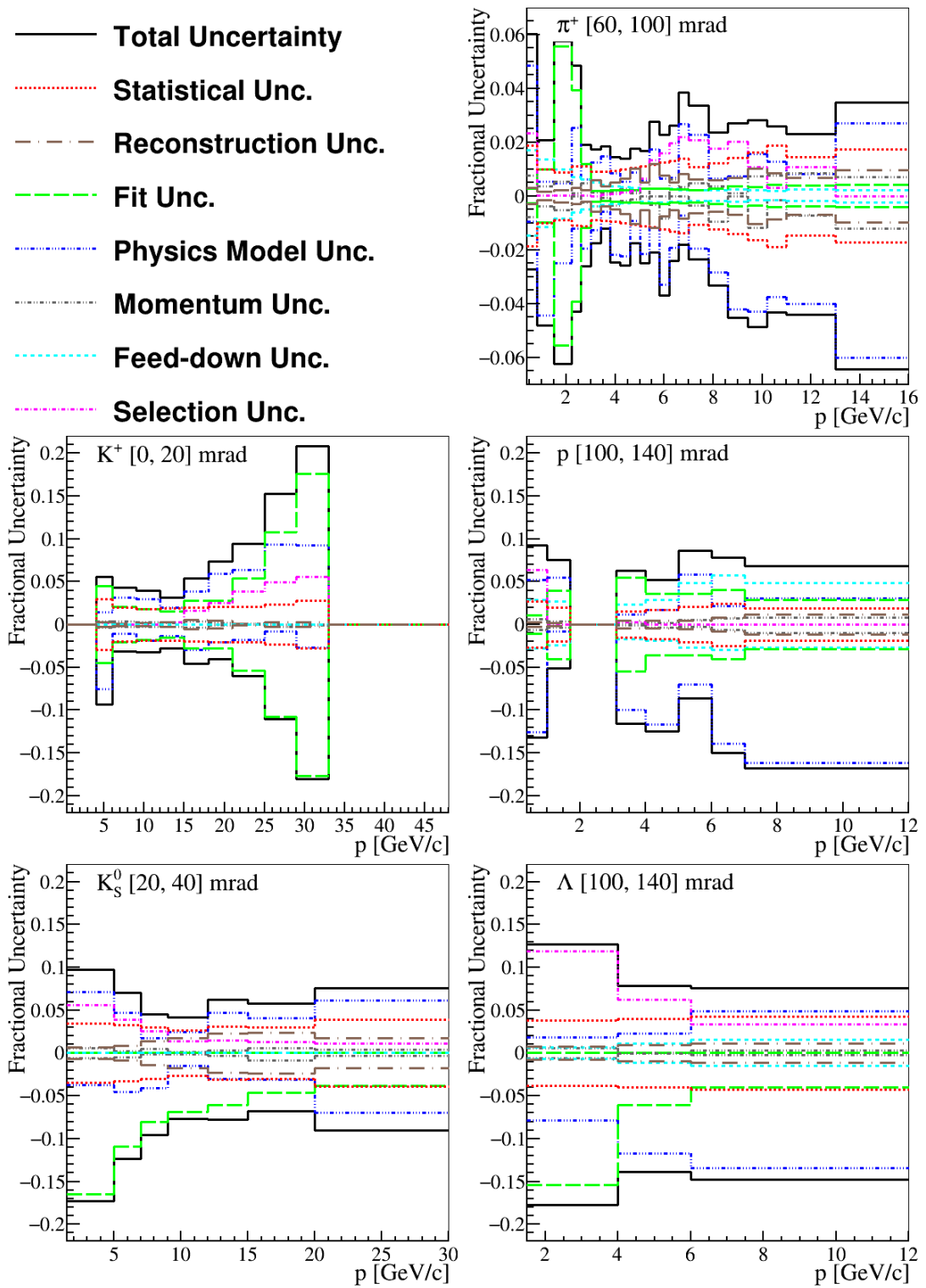


Figure 12: The breakdown of the fractional uncertainties on  $\pi^+$ ,  $K^+$ , proton,  $K_S^0$  and  $\Lambda$  spectra from  $\pi^+$  + C at 60 GeV/c interactions for select representative angular bins. The upper and lower uncertainties are shown on the positive and negative sides of the y axes.

579 selected events with the target inserted and target removed. The differential cross section is related to the  
 580 multiplicity by a factor of  $\sigma_{\text{prod}}$ :

$$\frac{d^2\sigma_k}{dpd\theta} = \sigma_{\text{prod}} \frac{d^2n_k}{dpd\theta}. \quad (37)$$

581 In order to calculate the multiplicity for produced charged hadrons (for each track topology - RST and  
 582 WST), an additional correction factor is required for the fit bias corrections,  $c^{\text{fit}}$ :

$$m_k = \frac{d^2n_k}{dpd\theta} = \frac{\sigma_{\text{trig}} c_k^{\text{MC}} c_k^{\text{fit}}}{f_{\text{prod}} \sigma_{\text{prod}} (1 - \epsilon) \Delta p \Delta \theta} \left( \frac{Y_k^I}{N^I} - \frac{\epsilon Y_k^R}{N^R} \right). \quad (38)$$

583 For kinematic bins for which the detector acceptance and fit reliability is sufficient enough for multiplicity  
 584 measurements in both RST and WST bins, the single-side multiplicities,  $m_{\text{R}}$  and  $m_{\text{W}}$ , are merged by taking  
 585 the weighted average:

$$m_{\text{merged}} = \sigma_{\text{merged}}^2 \left( \frac{m_{\text{R}}}{\sigma_{\text{R}}^2} + \frac{m_{\text{W}}}{\sigma_{\text{W}}^2} \right), \quad (39)$$

586 where the merged uncertainty,  $\sigma_{\text{merged}}$  is calculated with:

$$\frac{1}{\sigma_{\text{merged}}^2} = \frac{1}{\sigma_{\text{R}}^2} + \frac{1}{\sigma_{\text{W}}^2}. \quad (40)$$

587 The uncertainties on the individual RST and WST multiplicities consider both the statistical uncertainties  
 588 and the fit uncertainties:

$$\sigma_{\text{R,W}} = \sqrt{\sigma_{\text{R,W stat}}^2 + \sigma_{\text{R,W fit}}^2}. \quad (41)$$

589 In analysis bins for which the detector acceptance is only sufficient for either RSTs or WSTs, only the  
 590 single-side multiplicity and uncertainty is taken as the result.

591 Multiplicity spectra obtained for  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  in  $\pi^+ + \text{C}$  at 60 GeV/c interactions are presented in  
 592 Figures 13, 14 and 15. The spectra are shown as 1-dimensional momentum spectra for individual bins  
 593 of  $\theta$ . The error bars represent the total uncertainty except for the normalization uncertainty. The results  
 594 are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as  
 595 GiBUU2019 [26] and FLUKA2011.2x.7 [27, 28, 29]. In general,  $K_S^0$  spectra are well-predicted by the  
 596 models, except in the first angular bin. The models exhibit a large variability in their predictions of  $\Lambda$  and  
 597 especially  $\bar{\Lambda}$  spectra. QGSP\_BERT seems to provide the best prediction of  $\Lambda$  spectra, while GiBUU2019  
 598 seems to provide the best prediction of  $\bar{\Lambda}$  spectra. Tables in Ref. [25] present the numerical values of the  
 599 multiplicity measurements of  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$  along with statistical, systematic and total uncertainties for  
 600 each kinematic bin analyzed. The normalization uncertainty of  $\pm 1\%$  is not included in the values of the  
 601 uncertainties shown in these tables but should be attributed to the multiplicity spectra of all hadron species  
 602 analyzed.

603 Multiplicity spectra obtained for charged pions, charged kaons and protons in  $\pi^+ + \text{C}$  at 60 GeV/c  
 604 interactions are shown in Figures 16 through 20. The results are compared to the predictions of the  
 605 GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011. In general,  
 606 charged hadron spectra are well-predicted by the models at the intermediate production angles. At smaller  
 607 production angles, the models deviate from the observed spectra. The best agreement is seen for charged  
 608 pions, which are well-predicted by the models except for the first angular bin. Tables in Ref. [25] present  
 609 the numerical values of the multiplicity measurements of charged pions, charged kaons and protons along  
 610 with statistical, systematic and total uncertainties for each kinematic bin analyzed. The normalization

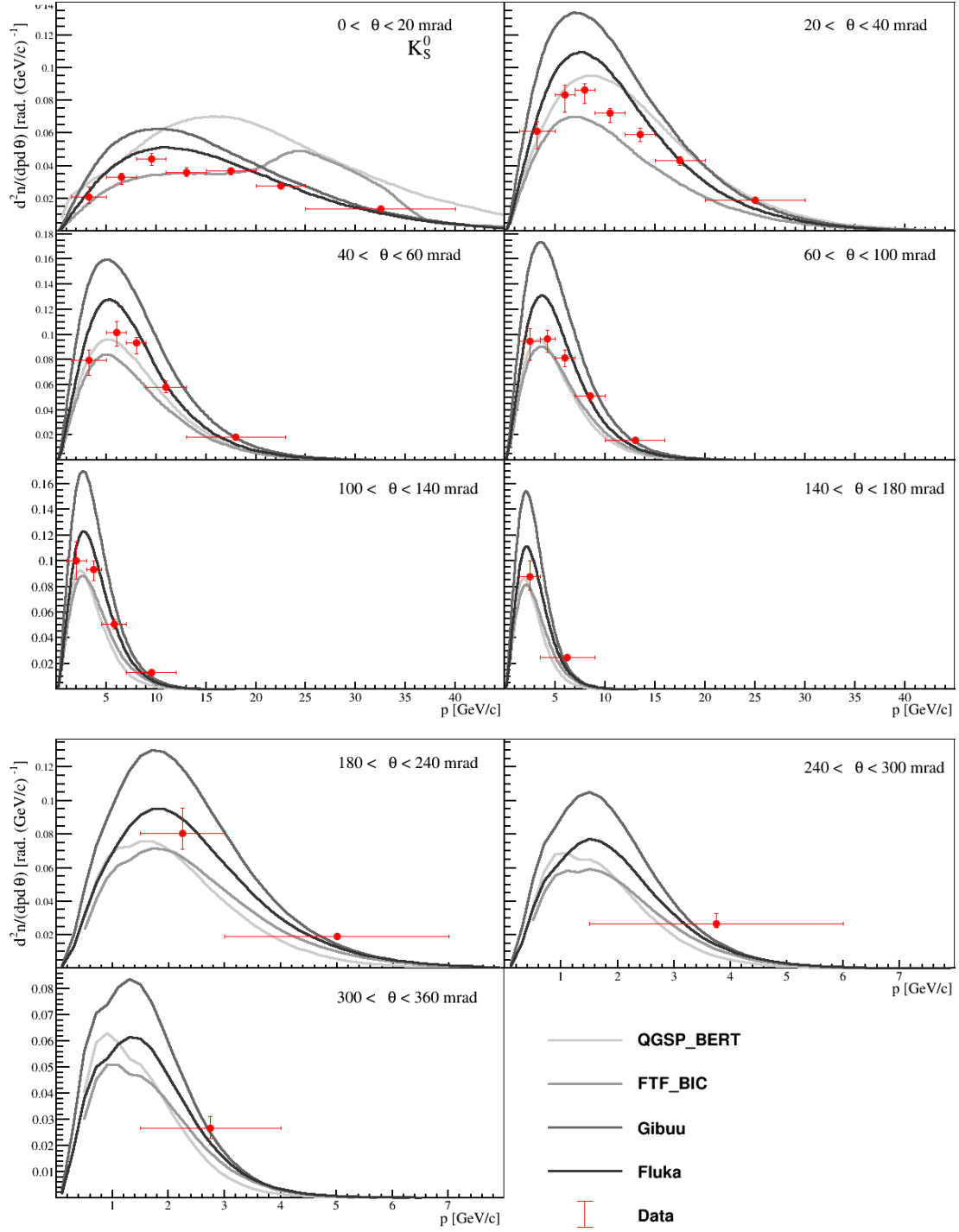


Figure 13:  $K_S^0$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown for different regions of  $\theta$ . The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

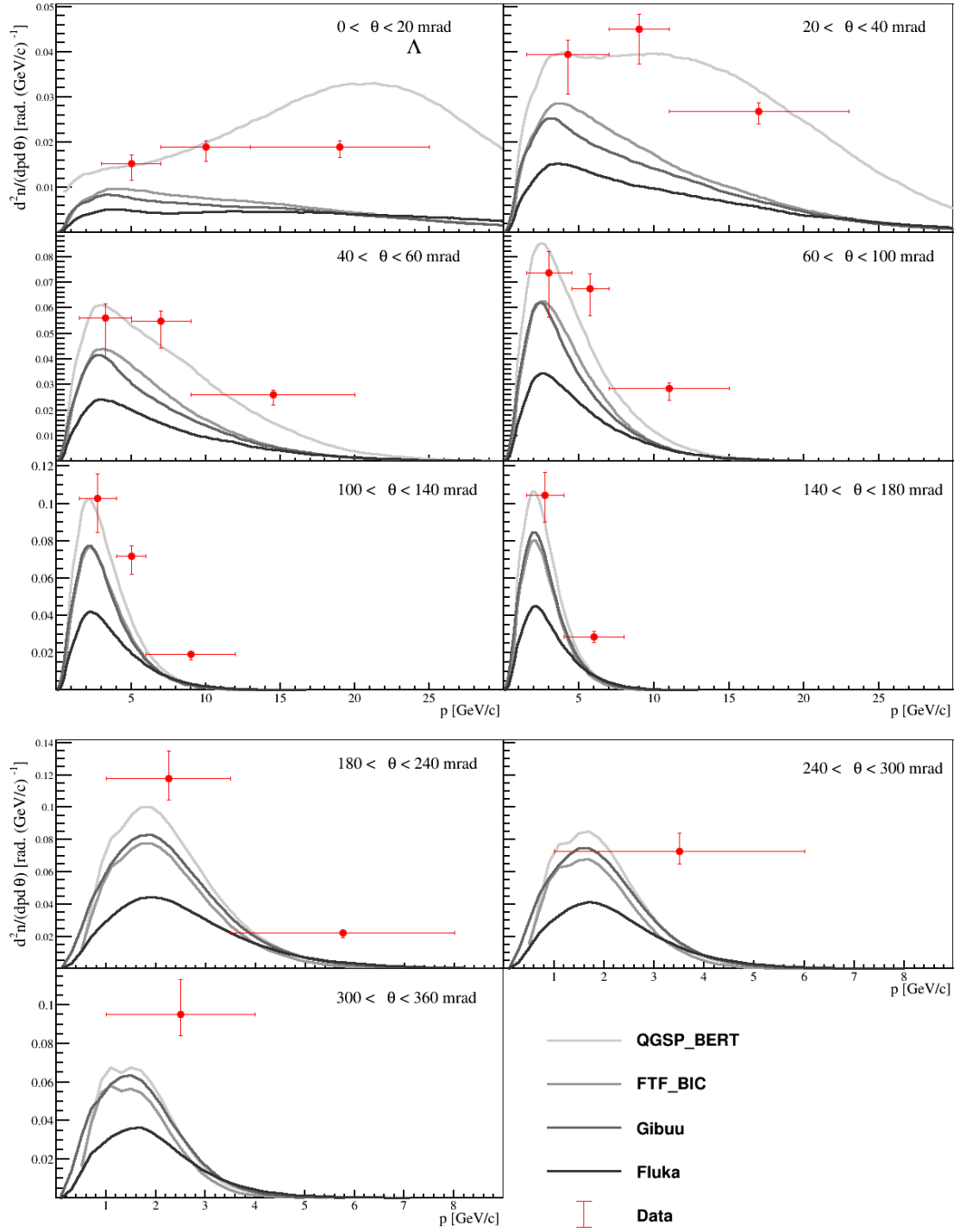


Figure 14:  $\Lambda$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown for different regions of  $\theta$ . The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.



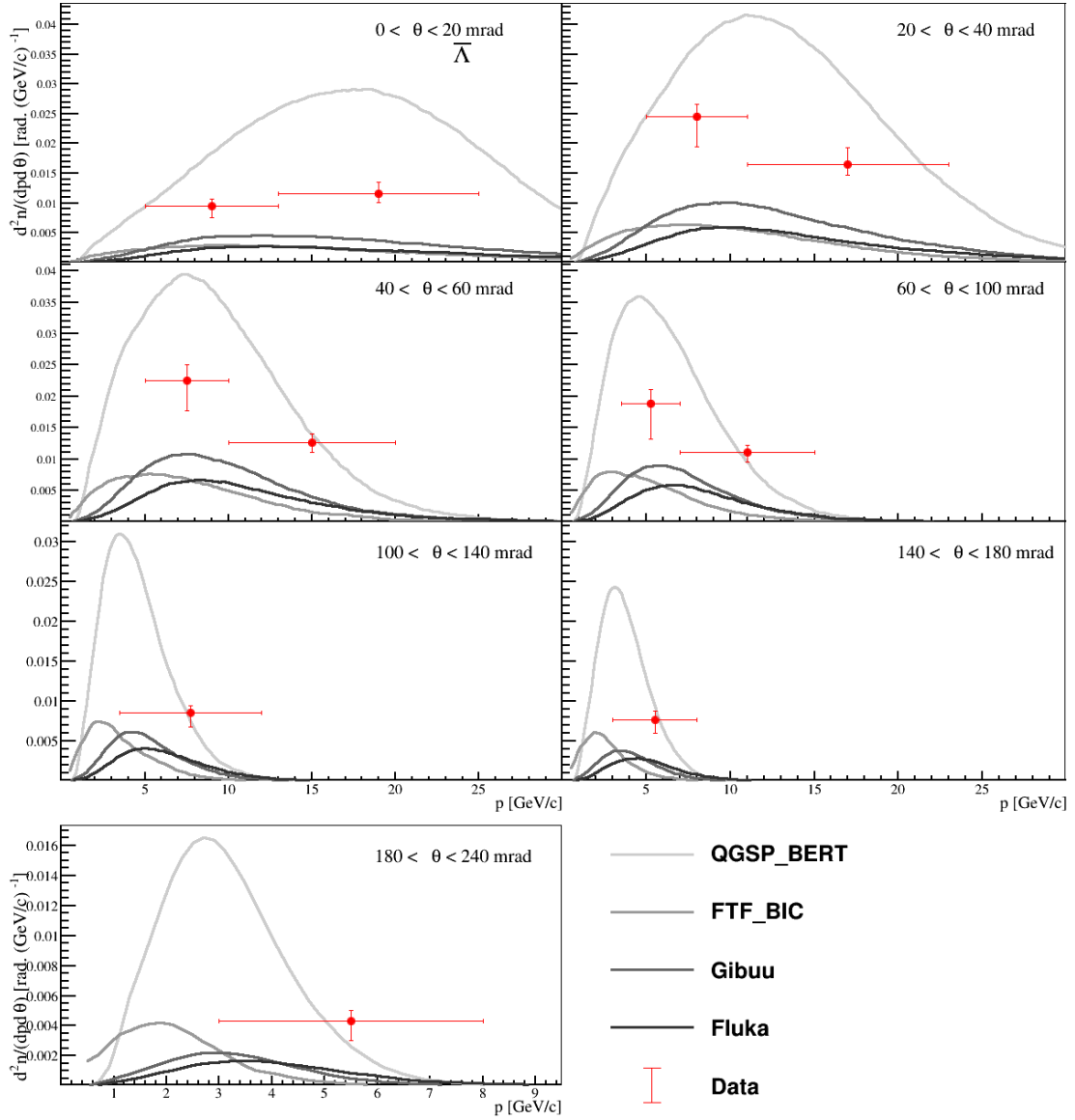


Figure 15:  $\bar{\Lambda}$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown for different regions of  $\theta$ . The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

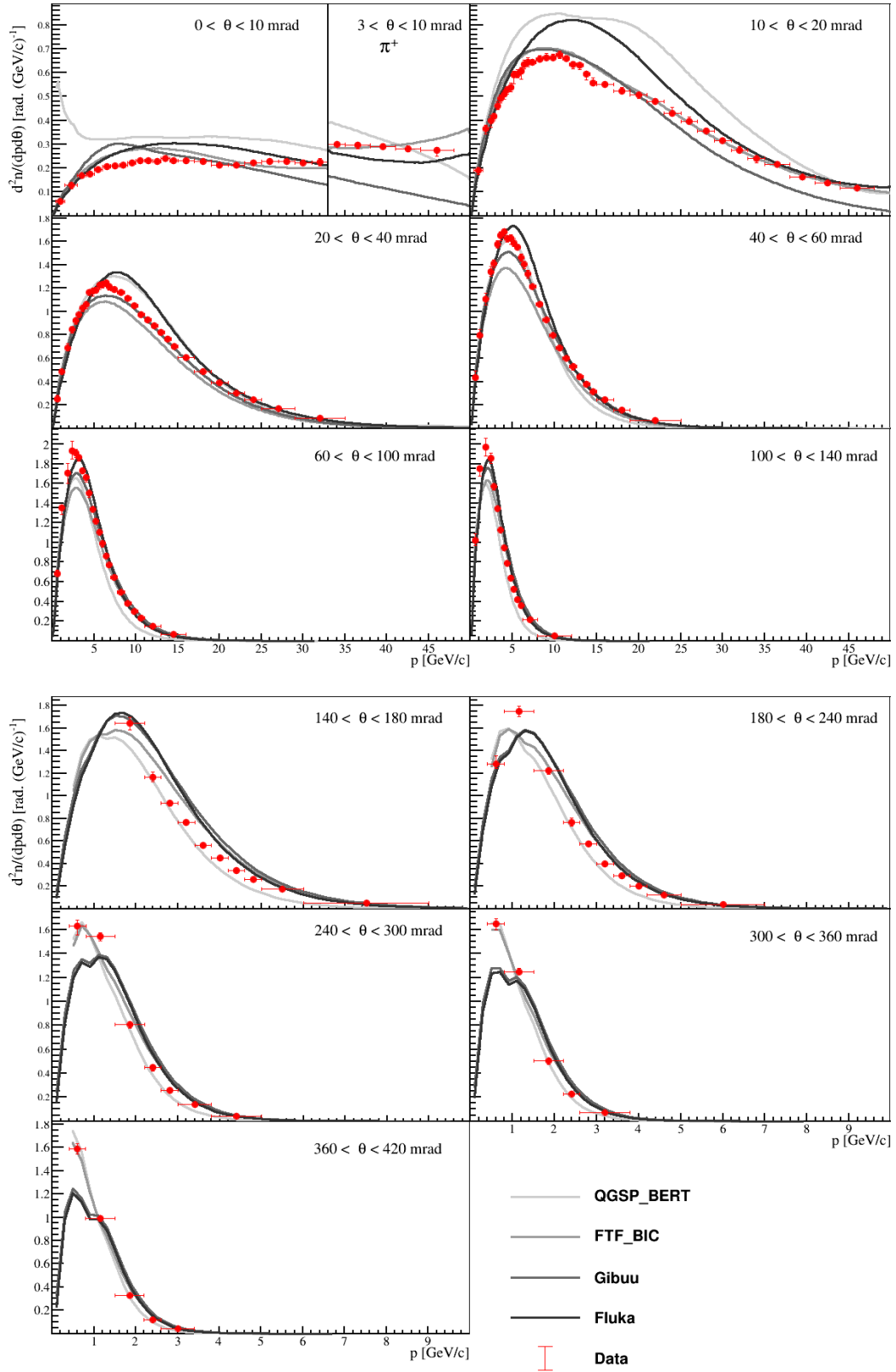


Figure 16:  $\pi^+$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown. The error bars represent total uncertainties except for the normalization uncertainty. Note that the first angular bin ( $[0,10]$  mrad) is divided into two regions. For momenta less than 33 GeV/c, the angular range is  $[0,10]$  mrad and for momenta greater than 33 GeV/c, the angular range is  $[3,10]$  mrad. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

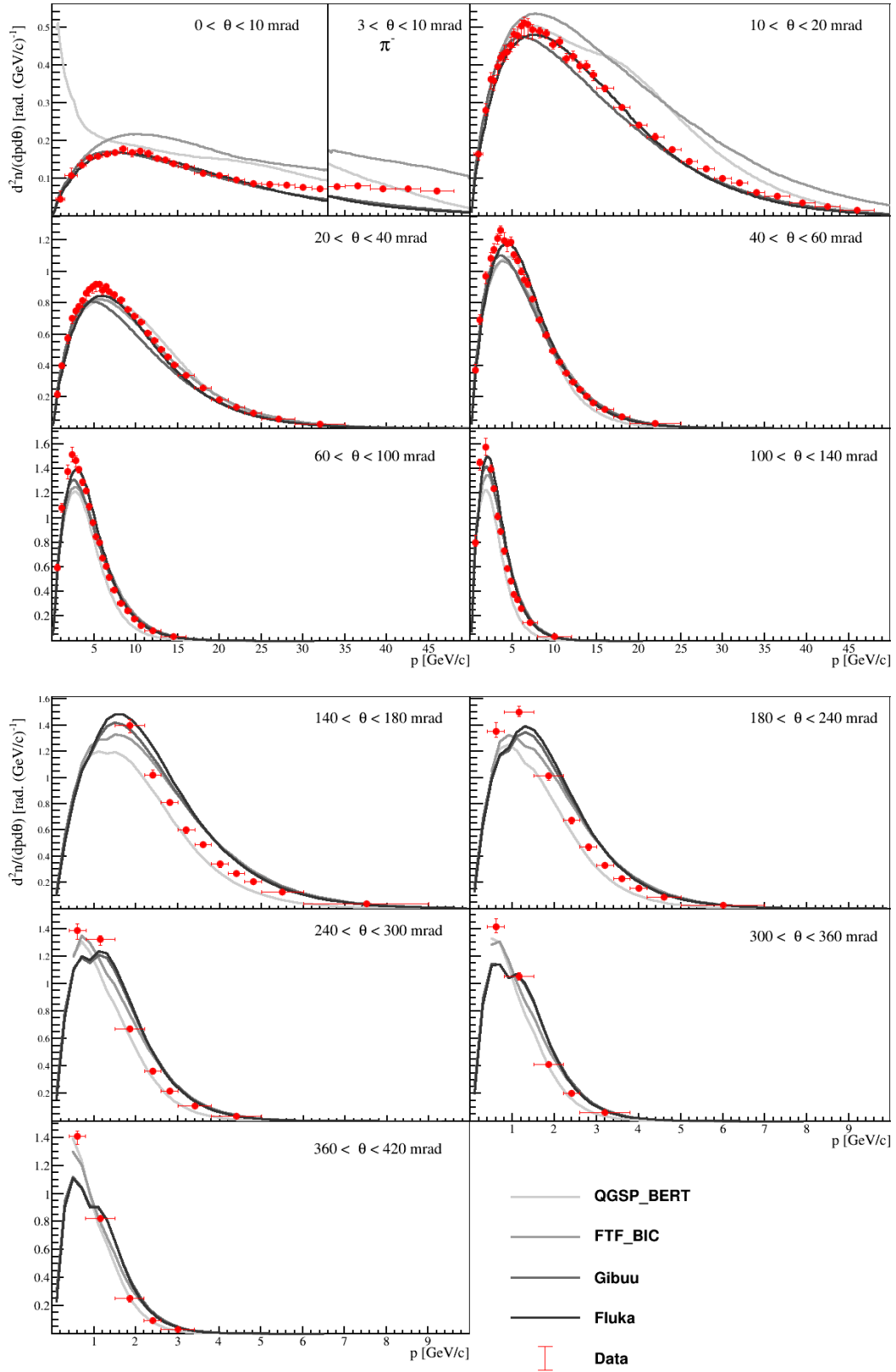


Figure 17:  $\pi^-$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown. The error bars represent total uncertainties except for the normalization uncertainty. Note that the first angular bin ( $[0,10]$  mrad) is divided into two regions. For momenta less than 33 GeV/c, the angular range is  $[0,10]$  mrad and for momenta greater than 33 GeV/c, the angular range is  $[3,10]$  mrad. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

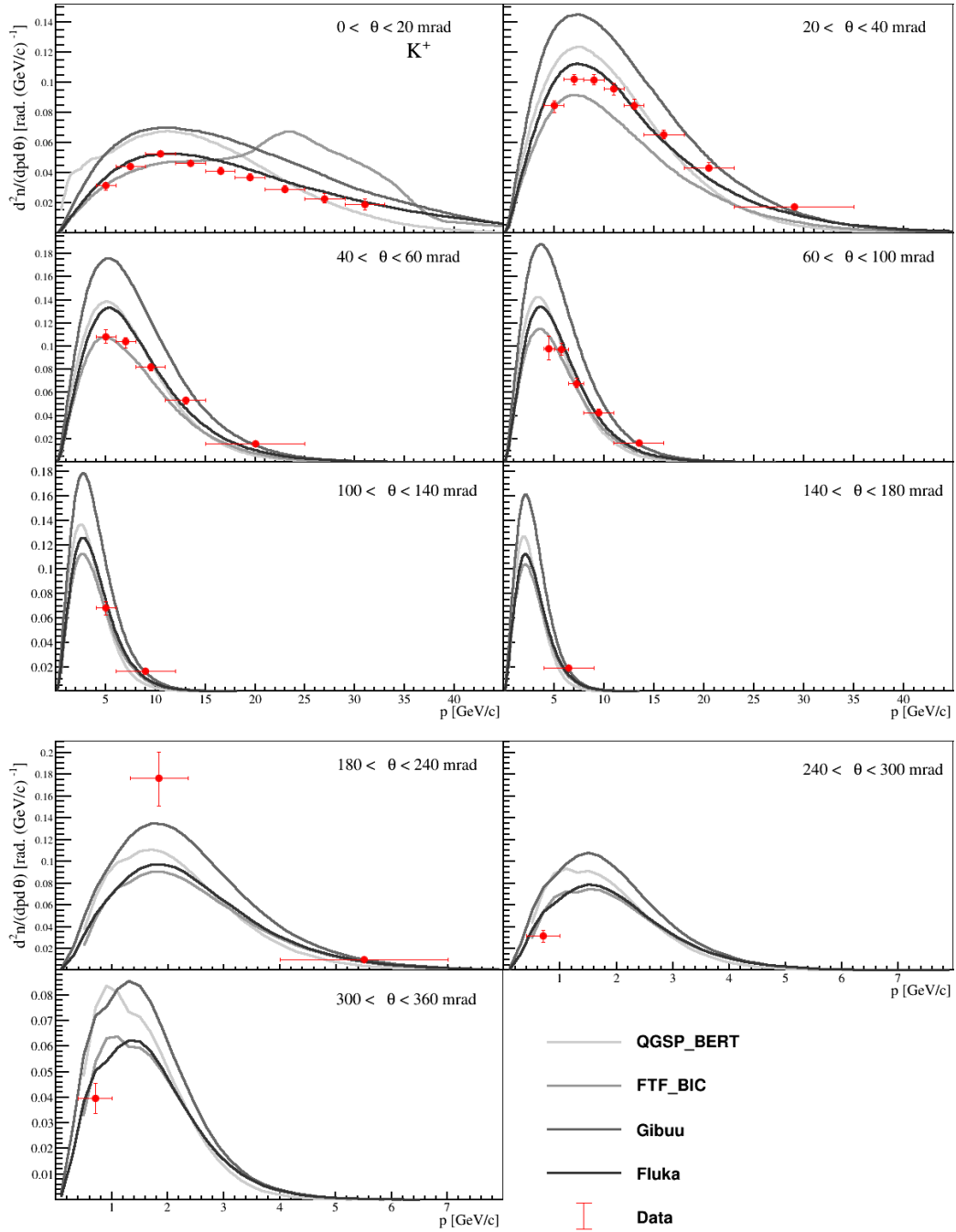


Figure 18:  $K^+$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown. The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

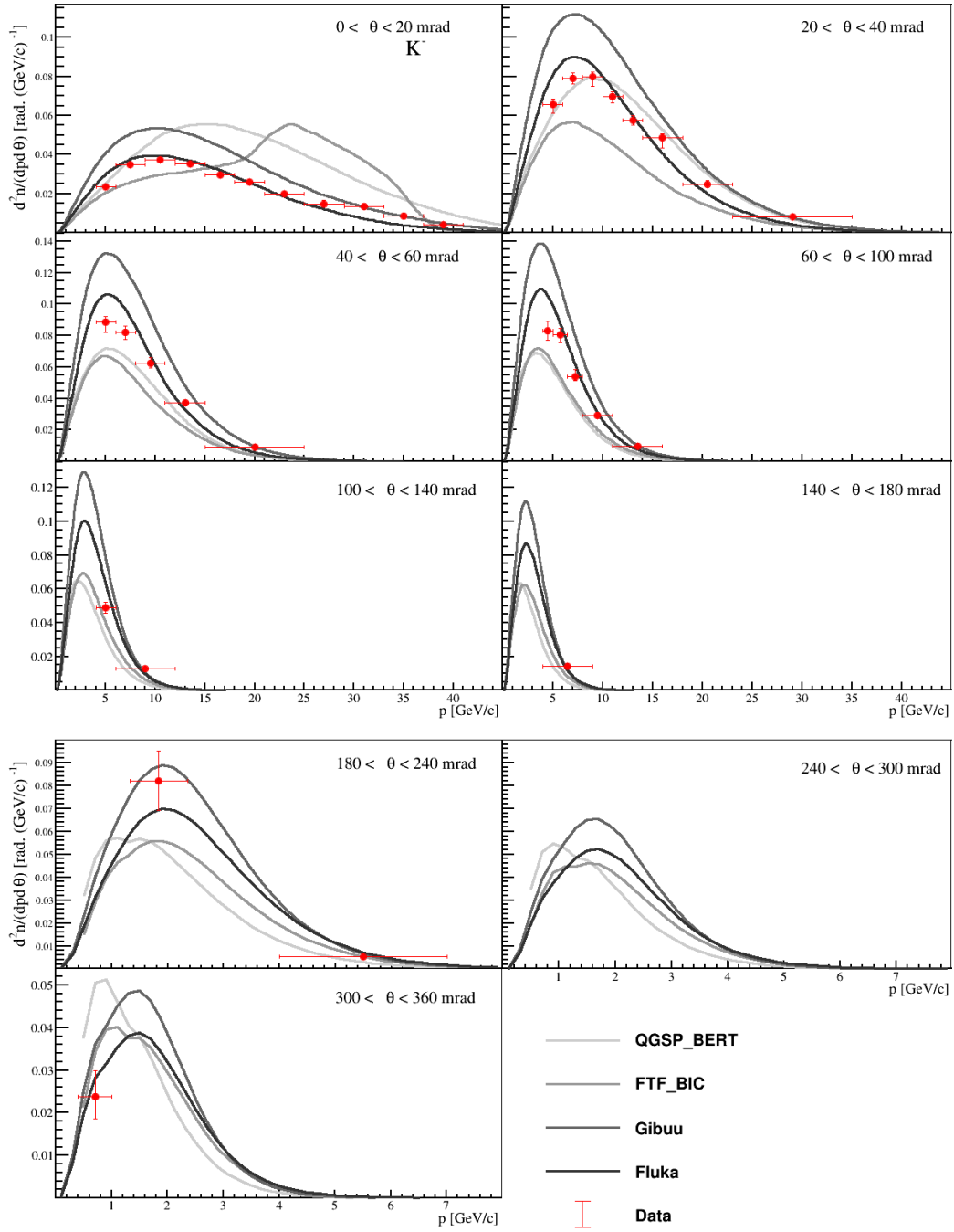


Figure 19:  $K^-$  multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown. The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

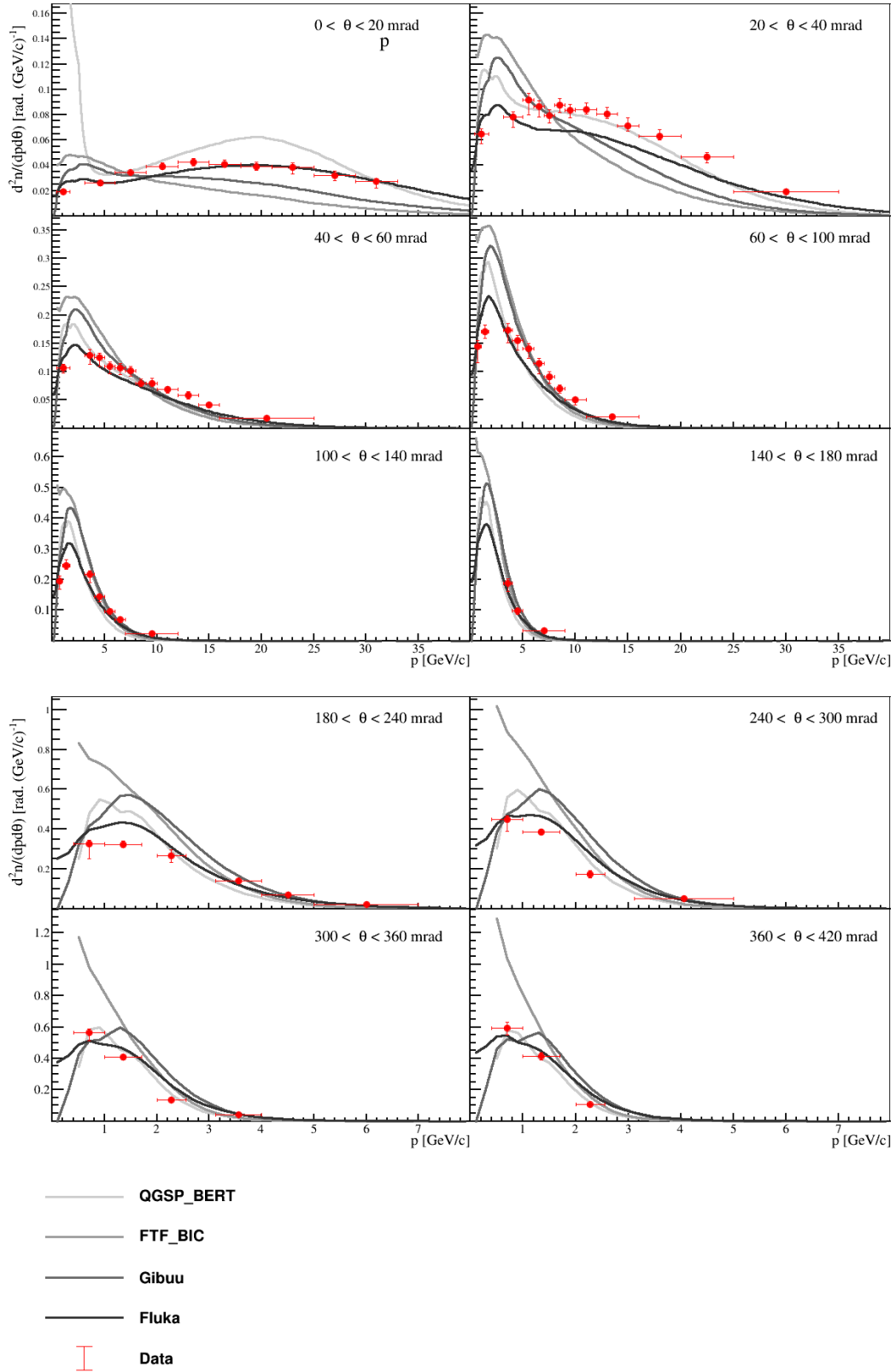


Figure 20: Proton multiplicity spectra from  $\pi^+ + C$  at 60 GeV/c interactions are shown. The error bars represent total uncertainties except for the normalization uncertainty. The results are compared to the predictions of the GEANT4 physics lists: QGSP\_BERT and FTF\_BIC as well as GiBUU2019 and FLUKA2011.

611 uncertainty of  $\pm_1^2\%$  is not included in the values of the uncertainties shown in these tables but should be  
612 attributed to the multiplicity spectra of all hadron species analyzed.

613 Measurements of spectra of produced  $\pi^+$ ,  $K^+$ , proton,  $K_S^0$  and  $\Lambda$  from interactions of  $\pi^+ + \text{Be}$  at 60 GeV/c  
614 are shown in comparison to the results for interactions of  $\pi^+ + \text{C}$  at 60 GeV/c for representative angular  
615 bins in Figure 21. The spectra are largely similar. The most notable difference in the spectra is that the  
616 multiplicities tend to be lower in the regions of low momentum and high production angle in interactions  
617 of  $\pi^+ + \text{Be}$  at 60 GeV/c. The full set of comparisons between the spectra results of  $\pi^+ + \text{Be}$  at 60 GeV/c  
618 and  $\pi^+ + \text{C}$  at 60 GeV/c is presented in Ref. [25].

## 619 9 Summary and Conclusions

620 In summary, hadron production was studied in interactions of  $\pi^+ + \text{C}$  at 60 GeV/c and  $\pi^+ + \text{Be}$  at 60 GeV/c.  
621 For both of these reactions, the integrated production and inelastic cross sections were measured. Further-  
622 more, differential cross sections were measured for produced  $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$ , protons,  $K_S^0$ ,  $\Lambda$  and  $\bar{\Lambda}$ .  
623 The inelastic cross sections measurements are the first to be made at a beam momentum of 60 GeV/c. The  
624 production cross section of interactions of  $\pi^+ + \text{Be}$  at 60 GeV/c was measured for the first time as well.  
625 The differential cross sections were measured for the first time at this beam momentum scale, and compared  
626 to previous measurements at lower beam momenta, a larger kinematic phase space and more particle species  
627 were studied. These results will enable neutrino flux predictions to be constrained in neutrino experiments  
628 using the NuMI beam and future neutrino beam at LBNF. Specifically, these results can be used to reduce  
629 the uncertainties associated with secondary interactions of pions in the carbon targets and the beryllium  
630 elements in these beam lines.

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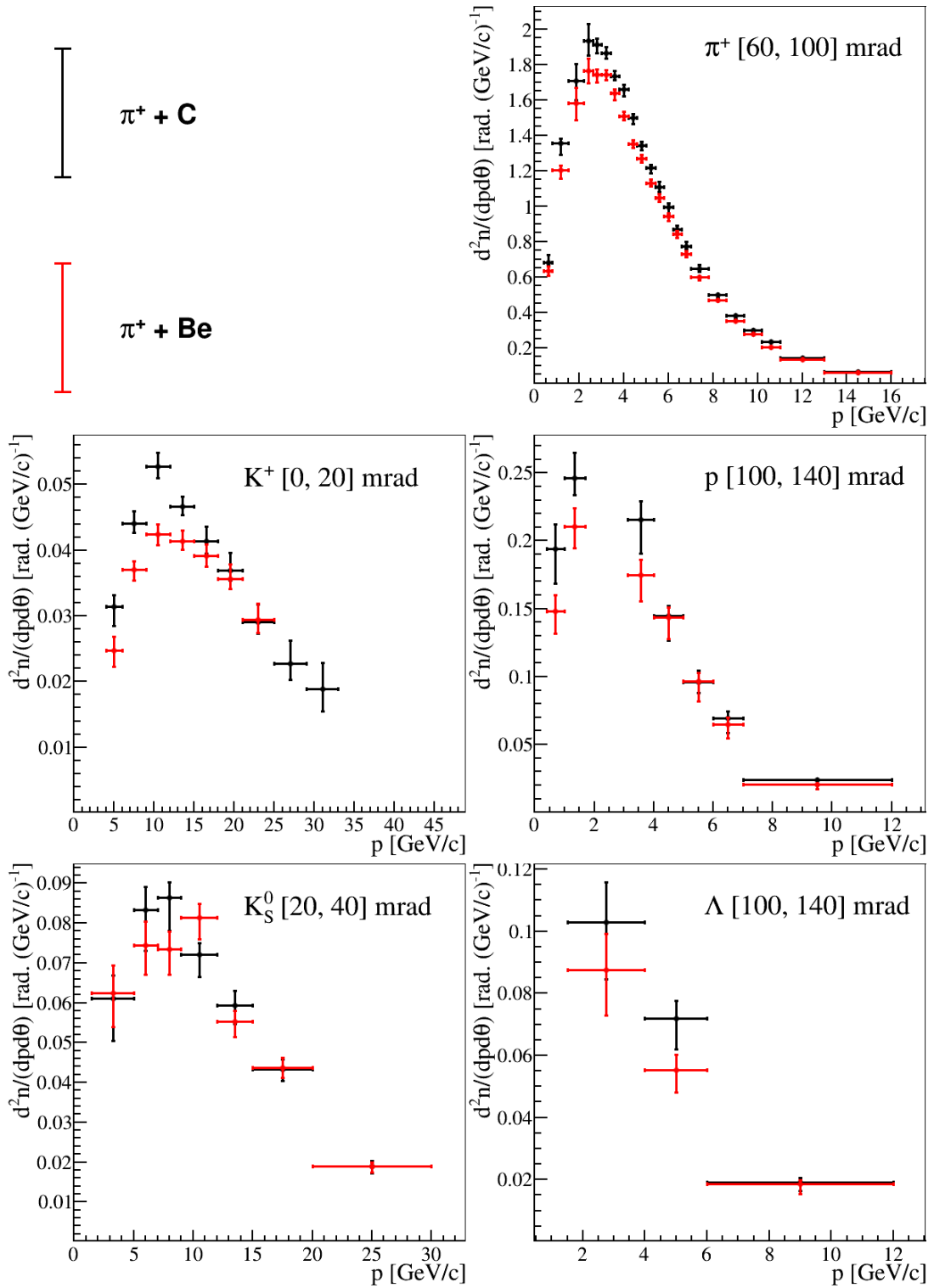


Figure 21: Measurements of spectra from  $\pi^+$  + C at 60 GeV/c and  $\pi^+$  + Be at 60 GeV/c interactions are shown for produced  $\pi^+$ ,  $K^+$ , proton,  $K_S^0$  and  $\Lambda$  for select representative angular bins. The error bars represent total uncertainties except for the normalization uncertainty.



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