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ABSTRACT

A generalization of the Fokker-Schwarzschild-Tetrode-Wheeler-Feynman electromagnetic theory onto the superspace is considered. The classical vector and spinor fields belonging to the Maxwell supermultiplet are built of the world-line coordinates of the charged particles in superspace.

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The action at a distance approach developed by Fokker-Schwarzschild-Tetrode-Wheeler-Feynman (FSTWF) revealed a deep connection between classical electromagnetic field and world-line coordinates of relativistic charged particles [1]. The fundamental character of the connections between fields and world coordinates was realized in a new light in string theory where the original classical string lagrangian was formulated in terms of string world-sheet coordinates [2]. The unification of the FSTWF approach with string theory performed by Ramond and Kalb resulted in the discovery of antisymmetric gauge field [3]. It seems natural to expect that a detailed study of the FSTWF approach in the frame of superstring theory may elucidate the problems of building string field theory.

Here we consider the possibility of supersymmetrization for FSTWF approach and show that the classical fields of the Maxwell supermultiplet may be constructed of the world-line coordinates of particles in superspace.

As is known, in the FSTWF approach classical electromagnetic field a^{μ} is built using the world-line coordinates $x^{\mu}(t)$ and $y^{\mu}(\tau)$ of interacting charged particles

$$a^{\mu}(x) = e \int d\tau \dot{y}^{\mu}(\tau)\delta(s_0^2), \tag{1}$$

where $s_0^{\mu} \equiv x^{\mu} - y^{\mu}(\tau)$ is the relativistic interval and $\delta(s_0^2)$ is the Dirac δ-function. The effective electromagnetic field in (1) satisfies the Maxwell equations and the Lorentz gauge condition

$$\begin{aligned} &\partial^{\mu} f_{\mu\nu}(x) = -4\pi j_{\nu}(x), \ \varepsilon_{\mu\nu\rho\sigma}\partial^{\nu} f^{\rho\sigma}(x) = 0, \\ &\partial^{\mu} a_{\mu}(x) = 0, \end{aligned} \tag{2}$$

while the current $j^{\mu}(x)$ is given by the usual expression

$$j^{\mu}(x) = e \int d\tau \dot{y}^{\mu} \delta^{(4)}(s_0), \tag{3}$$

and e is the electric charge of the interacting particles.

In order to supersymmetrize the electromagnetic potential (1) and the current (3) extend the original Minkowski space by introducing several additional Grassmann spinor coordinates.

The superpartner of a_μ in (1) is photino, an electrically neutral particle. Therefore choose the Weyl spinors $\theta^\alpha(t)$, $\bar\theta_{\dot\alpha}(t)$ and $\xi^\alpha(\tau)$, $\bar\xi_{\dot\alpha}(\tau)$, which describe the world–lines of particles in superspace (together with $x^\mu(t)$ and $y^\mu(\tau)$) as additional Grassmann coordinates. The supersymmetry transformations for the world-line coordinates $z^M = (x^\mu(t), \theta^\alpha(t), \bar\theta_{\dot\alpha}(t))$ and $\zeta^M = (y^\mu(\tau), \xi^\alpha(\tau), \bar\xi_{\dot\alpha}(\tau))$ are [4]

$$\delta x^\mu = i\theta\sigma^\mu\bar\epsilon - i\epsilon\sigma^\mu\bar\theta, \quad \delta\theta^\alpha = \epsilon^\alpha, \; \delta\bar\theta_{\dot\alpha} = \bar\epsilon_{\dot\alpha}$$
$$\delta y^\mu = i\xi\sigma^\mu\bar\epsilon - i\epsilon\sigma^\mu\bar\xi, \quad \delta\xi^\alpha = \epsilon^\alpha, \; \delta\bar\xi_{\dot\alpha} = \bar\epsilon_{\dot\alpha}. \tag{4}$$

The simplest generalizations of the interval s_0^μ and the velocity $\dot y^\mu(\tau)$ invariant under global supersymmetry transformations (4) are the following

$$s^\mu = x^\mu - y^\mu - i(\theta\sigma^\mu\bar\xi - \xi\sigma^\mu\bar\theta),$$
$$\omega_\tau^\mu = \dot y^\mu - i(\dot\xi\sigma^\mu\bar\xi - \xi\sigma^\mu\dot{\bar\xi}). \tag{5}$$

However, for the construction suggested below it is more convenient to use the complex coordinates x_L^μ, $x_R^\mu = (x_L^\mu)^*$ [5, 6]

$$x_L^\mu \equiv x^\mu + i\theta\sigma^\mu\bar\theta, \quad y_L^\mu \equiv y^\mu + i\xi\sigma^\mu\bar\xi,$$
$$x_R^\mu \equiv x^\mu - i\theta\sigma^\mu\bar\theta, \quad y_R^\mu \equiv y^\mu - i\xi\sigma^\mu\bar\xi \tag{6}$$

and the intervals s_L^μ, $s_R^\mu = (s_L^\mu)^*$, Δ^α, $\bar\Delta_{\dot\alpha}$ which are also invariant under the SUSY transformations (4)

$$s_L^\mu = x_L^\mu - y_R^\mu - 2i\theta\sigma^\mu\bar\xi = s^\mu + i\Delta\sigma^\mu\bar\Delta, \; \Delta^\alpha = \theta^\alpha - \xi^\alpha,$$
$$s_R^\mu = x_R^\mu - y_L^\mu + 2i\xi\sigma^\mu\bar\theta = s^\mu - i\Delta\sigma^\mu\bar\Delta, \; \bar\Delta^{\dot\alpha} = \bar\theta^{\dot\alpha} - \bar\xi^{\dot\alpha}, \tag{7}$$

but satisfy the chirality conditions[†]

$$D_\alpha s^\mu_R = \left(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\nu\bar\theta)_\alpha\partial_\nu\right) s^\mu_R = D_\alpha\bar\Delta_{\dot\beta} = 0,$$
$$\bar D_{\dot\alpha} s^\mu_L = \left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i(\theta\sigma^\nu)_{\dot\alpha}\partial_\nu\right) s^\mu_L = \bar D_{\dot\alpha}\Delta^\beta = 0. \tag{8}$$

Due to the evident conditions

$$s^\mu_L\Big|_{\Delta=0} = s^\mu_R\Big|_{\Delta=0} = s^\mu_0, \tag{9}$$

the intervals s^2_L and s^2_R may be naturally used as the arguments of δ-functions ($\delta(s^2_L)$ and $\delta(s^2_R)$) entering into the unknown integral representations for the gauge superfields $A^M = (A^\mu, A^\alpha, \bar A_{\dot\alpha})$. The strength components F_{MN} built of the superfields A_M and are not independent superfields due to the standard constraints [6]

$$F_{\alpha\beta} = F_{\dot\alpha\dot\beta} = F_{\alpha\dot\beta} = 0. \tag{10}$$

These constraints are automatically satisfied if the vector superfield A_μ has the form

$$A_\mu = -\frac{i}{4}\tilde\sigma^{\dot\alpha\alpha}_\mu\left(D_\alpha\bar A_{\dot\alpha} + \bar D_{\dot\alpha}A_\alpha\right) \tag{11}$$

and the spinor superfields A^α and $\bar A_{\dot\alpha}$ are chiral ones

$$D_\alpha A_\beta = 0, \quad \bar D_{\dot\alpha}\bar A_{\dot\beta} = 0. \tag{12}$$

Since $\bar A_{\dot\alpha} = -(A_\alpha)^*$ the considered problem is reduced to the construction of a chiral superfield A^α as a function of the world-line coordinates in the given superspace.

Due to the invariant character of A_α its dependence on the world-line coordinates must be realized through the chiral invariants of supersymmetry, the intervals s^μ_R, $\bar\Delta_{\dot\alpha}$ and velocities $\omega^\mu(\tau)$, $\dot\xi^\alpha(\tau)$, $\dot{\bar\xi}_{\dot\alpha}(\tau)$. In view of the constraints (10) and the condition that A_α must be equal to zero at $\xi, \theta \mapsto 0$, it is convenient to seek the discussed representation for A_α in the form of the power series of the invariant $\bar\Delta_{\dot\alpha}$

$$A_\alpha(x,\theta,\bar\theta) = e\int d\tau \mathcal{K}_{\alpha\dot\alpha}(\omega_\tau, \xi, \dot\xi, \bar\Delta)\bar\Delta^{\dot\alpha}\delta(s_R^2), \tag{13}$$

where $\mathcal{K}_{\alpha\dot\alpha}\Big|_{\bar\Delta=0} \neq 0$[‡]. The chiral supersymmetric invariant kernel $\mathcal{K}_{\alpha\dot\alpha}$ may be determined by substituting (13) into the representation (11) and imposing the condition that the superfield A_μ should coincide with the representation (1) when $\xi,\theta \mapsto 0$, i.e.

$$\mathcal{K}_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha}\left[\omega_{\tau\mu} + 2i(\bar\Delta\tilde\sigma_\mu\dot\xi)\right]. \tag{14}$$

Then the spinor superfields A_α and $\bar A_{\dot\alpha} = -(A_\alpha)^*$ are presented in the required integral form

$$\begin{aligned} A_\alpha &= e\int d\tau(\omega_{\tau\mu}\sigma^\mu_{\alpha\dot\alpha}\bar\Delta^{\dot\alpha} + 2i\dot\xi_\alpha\bar\Delta\bar\Delta)\delta(s_R^2), \\ \bar A_{\dot\alpha} &= -e\int d\tau(\omega_{\tau\mu}\Delta^\alpha\sigma^\mu_{\alpha\dot\alpha} - 2i\dot{\bar\xi}_{\dot\alpha}\Delta\Delta)\delta(s_L^2). \end{aligned} \tag{15}$$

In accordance with the relation (11) the superfield generalization of the FSTWF representation (1) may be written as

$$\begin{aligned} A_\mu = -ie\int d\tau\Big[&\omega_{\tau\mu} - \varepsilon_{\mu\nu\rho\lambda}\omega^\nu_\tau(\Delta\sigma^\rho\bar\Delta)\partial^\lambda + i\big((\Delta\sigma_\mu\dot{\bar\xi}) \\ &-(\dot\xi\sigma_\mu\bar\Delta)\big) + \frac{1}{4}\Delta\Delta\bar\Delta\bar\Delta\omega^\nu_\tau\big(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\big) \\ &+\big(\Delta\Delta(\dot{\bar\xi}\tilde\sigma_{\mu\rho}\bar\Delta) + (\dot\xi\sigma_{\mu\rho}\Delta)\bar\Delta\bar\Delta\big)\,\partial^\rho\Big]\delta(s^2). \end{aligned} \tag{16}$$

Since the representation (16) automatically fixes the Lorentz gauge condition

$$\partial_\mu A^\mu = 0,$$

the residual gauge symmetry

$$A'_\mu = A_\mu + i\partial_\mu\Lambda,\ A'_\alpha = A_\alpha + iD_\alpha\Lambda,\ \bar A'_{\dot\alpha} = \bar A_{\dot\alpha} + i\bar D_{\dot\alpha}\Lambda \tag{17}$$

is defined by the real scalar superfield $\Lambda(x,\theta,\bar\theta)$ restricted by the conditions

$$\Box\Lambda = 0, \quad D^\alpha D_\alpha\Lambda = 0, \quad \bar D_{\dot\alpha}\bar D^{\dot\alpha}\Lambda = 0. \tag{18}$$

The nonzero components of the strength $F_{MN}(x,\theta,\bar\theta)$ corresponding to the superpotentials $A_M(x,\theta,\bar\theta)$ (15), (16) are built of the chiral superfields W_α and $\bar W_{\dot\alpha}=(W_\alpha)^*$ [6]

$$W^\alpha \equiv \frac{i}{4}F_{\mu\dot\alpha}\tilde\sigma^{\mu\dot\alpha\alpha}=\frac{1}{8}\bar D_{\dot\beta}\bar D^{\dot\beta}A^\alpha+\frac{i}{2}\partial^{\dot\alpha\alpha}\bar A_{\dot\alpha},$$
$$\bar W^{\dot\alpha} \equiv \frac{i}{4}\tilde\sigma^{\mu\dot\alpha\alpha}F_{\mu\alpha}=-\frac{1}{8}D^\beta D_\beta\bar A^{\dot\alpha}+\frac{i}{2}\partial^{\dot\alpha\alpha}A_\alpha, \tag{19}$$

where $\partial^{\dot\alpha\alpha}\equiv\tilde\sigma^{\mu\dot\alpha\alpha}\partial/\partial x^\mu$. In view of (15) these fields may be presented as the following functions of the world-line coordinates

$$W^\alpha=-ie\int d\tau\Big[\dot\xi^\alpha+i\dot\xi^\alpha\Delta\sigma^\mu\bar\Delta\partial_\mu+\frac{1}{4}\dot\xi^\alpha\Delta\Delta\bar\Delta\bar\Delta\Box$$
$$+\omega_{\tau\mu}\Big(2(\Delta\sigma^{\mu\nu})^\alpha\partial_\nu-\frac{i}{2}\Delta\Delta(\bar\Delta\tilde\sigma_\nu)^\alpha(\partial^\mu\partial^\nu$$
$$-\eta^{\mu\nu}\Box)\Big)-i\Delta\Delta(\dot{\bar\xi}\tilde\sigma_\mu)^\alpha\partial^\mu\Big]\delta(s^2). \tag{20}$$

The superfields W and $\bar W$ may be used for constructing a superfield generalization of the electromagnetic current (3) if we consider the relation

$$-4\pi\mathcal{J}=D^\alpha W_\alpha+\bar D_{\dot\alpha}\bar W^{\dot\alpha}=i\partial^{\dot\alpha\alpha}\left(\bar D_{\dot\alpha}A_\alpha-D_\alpha\bar A_{\dot\alpha}\right). \tag{21}$$

The superfield current $\mathcal{J}$ is invariant under the residual gauge transformations (17) since $\delta\mathcal{J}=-\frac{i}{16\pi}[DD,\bar D\bar D]\Lambda=0$. The right-hand side of Eq. (21), after the substitution of the superpotentials (15), may be written in the form $\Box\Phi(x,\theta,\bar\theta)$, where Φ is a real scalar superfield

$$\Phi=-2e\int d\tau\Delta^\alpha\left(\mathcal{K}_{\alpha\dot\alpha}\delta(s_R^2)+\mathcal{K}^*_{\alpha\dot\alpha}\delta(s_L^2)\right)\bar\Delta^{\dot\alpha} \tag{22}$$

with the kernel $\mathcal{K}_{\alpha\dot\alpha}$ given by (14). An equivalent form for Φ (22) is

$$\Phi=-4e\int d\tau\Big[\omega_\tau^\mu\left(\Delta\sigma_\mu\bar\Delta\right)$$
$$+i(\dot\xi\Delta)\bar\Delta\bar\Delta-i\Delta\Delta(\dot{\bar\xi}\bar\Delta)\Big]\delta(s^2). \tag{23}$$

The use of the fundamental property [7] of the interval (5)

$$\Box\delta(s^2) = -4\pi\delta^{(4)}(s^\nu), \tag{24}$$

together with the representation (23) shows that Eq. (21) is presented in the form of the superfield wave equation

$$\Box\Phi(x,\theta,\bar\theta) = -4\pi\mathcal{J}(x,\theta,\bar\theta) \tag{25}$$

with the superfield current $\mathcal{J}(x,\theta,\bar\theta)$ given by the integral

$$\begin{aligned}\mathcal{J} = -4e\int d\tau\Big(&\omega_\tau^\mu(\Delta\sigma_\mu\bar\Delta)\\ &+i(\dot\xi\Delta)\bar\Delta\bar\Delta - i\Delta\Delta(\dot{\bar\xi}\bar\Delta)\Big)\delta^{(4)}(s).\end{aligned} \tag{26}$$

Eq. (25) is a superfield generalization of the Maxwell equations in the Lorentz gauge and $\Phi(x,\theta,\bar\theta)$ has the physical interpretation of the prepotential $V(x,\theta,\bar\theta)$ [6] evaluated in the gauge

$$\left\{DD,\bar D\bar D\right\}V = 0 \ \Rightarrow V(x,\theta,\bar\theta) = \frac{1}{4}\Phi(x,\theta,\bar\theta). \tag{27}$$

This interpretation follows from the relations

$$W_\alpha = -\frac{1}{16}\bar D\bar D D_\alpha\Phi, \quad \bar W_{\dot\alpha} = -\frac{1}{16}DD\bar D_{\dot\alpha}\Phi, \tag{28}$$

which connect W_α (19) and Φ (23). With the notations used in [6] the superfield gauge condition (27) is split into the following component gauge conditions

$$\begin{aligned}&\partial^{\dot\alpha\alpha}\chi_\alpha(x) = i\bar\lambda^{\dot\alpha}(x),\ \partial^{\dot\alpha\alpha}\bar\chi_{\dot\alpha}(x) = -i\lambda^\alpha(x),\\ &\Box C(x) = -D(x),\ M(x) = N(x) = 0,\\ &\partial_\mu v^\mu(x) = 0\end{aligned} \tag{29}$$

for the component fields $\chi, \bar\chi, C, M, N, v$ entering into the superfield $V(x,\theta,\bar\theta)$. The use of the conditions (29) permits to present $V(x,\theta,\bar\theta)$ by the following component decomposition

$$V = \frac{1}{4}\Phi = C + i\theta^\alpha \chi_\alpha - i\bar{\theta}_{\dot\alpha}\bar{\chi}^{\dot\alpha} - (\theta\sigma_\rho\bar{\theta})v^\rho$$
$$+\frac{i}{2}\theta\theta\bar{\theta}_{\dot\alpha}\bar{\lambda}^{\dot\alpha} - \frac{i}{2}\bar{\theta}\bar{\theta}\theta^\alpha\lambda_\alpha + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}D, \qquad (30)$$

with the vector $v^\mu(x)$, spinors $\lambda^\alpha(x)$, $\bar{\lambda}_{\dot\alpha}(x)$ and an auxiliary field $D(x)$ forming the Maxwell multiplet. The integral representations for these component fields may be obtained after the expansion of the power series of θ and $\bar{\theta}$ for the prepotential Φ (23). This gives the following expansion for the electromagnetic potential $v^\mu(x)$

$$v_\mu(x) = iA_\mu\Big|_{\theta=0} = e\int d\tau\Big[\dot{y}_\mu - \varepsilon_{\mu\nu\rho\lambda}\dot{y}^\nu(\xi\sigma^\rho\bar{\xi})\partial^\lambda$$
$$+\left(\xi\xi(\dot{\bar{\xi}}\tilde{\sigma}_{\mu\rho}\bar{\xi}) + (\dot{\xi}\sigma_{\mu\rho}\xi)\bar{\xi}\bar{\xi}\right)\partial^\rho$$
$$+\frac{1}{4}\xi\xi\bar{\xi}\bar{\xi}\dot{y}^\nu\left(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\right)\Big]\delta(s_0^2), \qquad (31)$$

which is the zero term in the component expansion of the vector superpotential A_μ (16). The representation (31) is the desired supersymmetric generalization of the FSTWF potential (1). For the spin field λ^α which is the superpartner of the electromagnetic strength $v^{\mu\nu}(x) = \partial^\mu v^\nu - \partial^\nu v^\mu$ the discussed expansion gives

$$\lambda^\alpha(x) = e\int d\tau\Big[\dot{\xi}^\alpha - i\dot{\xi}\xi(\bar{\xi}\tilde{\sigma}_\mu)^\alpha\partial^\mu + \frac{i}{2}\xi\xi(\dot{\bar{\xi}}\tilde{\sigma}_\mu)^\alpha\partial^\mu$$
$$-\frac{1}{2}\dot{\xi}^\alpha\xi\xi\bar{\xi}\bar{\xi}\Box + \dot{y}_\mu\Big(-2(\xi\sigma^{\mu\nu})^\alpha\partial_\nu$$
$$+\frac{i}{2}\xi\xi(\bar{\xi}\tilde{\sigma}_\nu)^\alpha(\partial^\mu\partial^\nu - \eta^{\mu\nu}\Box)\Big)\Big]\delta(s_0^2). \qquad (32)$$

The integral representation for the spinor $\bar{\lambda}^{\dot\alpha}$ follows from (32) after its complex conjugation. Finally, for the auxiliary field $D(x)$ derive the integral representation in the form

$$D(x) = e\int d\tau\Big[\dot{y}_\mu(\xi\sigma^\mu\bar{\xi}) - i\left(\xi\xi(\dot{\bar{\xi}}\bar{\xi}) - (\dot{\xi}\xi)\bar{\xi}\bar{\xi}\right)\Big]\Box\delta(s_0^2). \qquad (33)$$

The obtained fields v^μ (31) and $\lambda^\alpha(x)$, $\bar{\lambda}_{\dot\alpha}(x)$ (32) satisfy the Maxwell and Dirac wave equations with currents

$$\Box v^{\mu}(x) = -4\pi j^{(2)\mu}(x),\ \partial_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}(x) = -4\pi j^{(1)}_{\alpha}(x),$$
$$\partial^{\dot\alpha\alpha}\lambda_{\alpha}(x) = -4\pi\bar j^{(1)\dot\alpha}(x) \tag{34}$$

and $D(x) = -4\pi j^{(0)}$. These equations follow from the decomposition of the superfield equation (25) after the substitution of the representation (23) and the component expansion of the superfield current (26)

$$\mathcal{J} = -4j^{(0)} + 4\theta^{\alpha}j^{(1)}_{\alpha} - 4\bar\theta_{\dot\alpha}\bar j^{\dot\alpha(1)} - 4(\theta\sigma_{\rho}\bar\theta)j^{(2)\rho}$$
$$-2i\theta\theta\bar\theta_{\dot\alpha}\partial^{\dot\alpha\alpha}j^{(1)}_{\alpha} + 2i\bar\theta\bar\theta\theta^{\alpha}\partial_{\alpha\dot\alpha}\bar j^{(1)\dot\alpha} + \theta\theta\bar\theta\bar\theta\Box j^{(0)}. \tag{35}$$

The explicit form of the components (35) of the current multiplet $\mathcal{J}(x,\theta,\bar\theta)$ is obtained from the integral representations (31), (32) after acting by the differential operators of D'Alamber and Dirac. As a result, the following supersymmetric generalization for the FSTWF electromagnetic current (3) is derived

$$j^{(2)}_{\mu} = e\int d\tau\Big[\dot y_{\mu} - \varepsilon_{\mu\nu\rho\lambda}\dot y^{\nu}(\xi\sigma^{\rho}\bar\xi)\partial^{\lambda} + \frac{1}{4}\xi\xi\bar\xi\bar\xi\dot y^{\nu}(\partial_{\mu}\partial_{\nu}$$
$$-\eta_{\mu\nu}\Box) + \left(\xi\xi(\dot{\bar\xi}\tilde\sigma_{\mu\rho}\bar\xi) + (\dot\xi\sigma_{\mu\rho}\xi)\bar\xi\bar\xi\right)\partial^{\rho}\Big]\delta^{(4)}(s_0). \tag{36}$$

The conservation of the generalized electromagnetic current (36), i.e. the equation $\partial^{\mu}j^{(2)}_{\mu} = 0$, is an evident consequence of the representation (36). The spinor component of the supercurrent (35) is given by the expression

$$j^{(1)}_{\alpha} = e\int d\tau\Big[\dot y_{\mu}[(\sigma^{\mu}\bar\xi)_{\alpha} - \frac{i}{2}\xi_{\alpha}\bar\xi\bar\xi\partial^{\mu} - i(\sigma^{\mu\rho}\xi)_{\alpha}\bar\xi\bar\xi\partial_{\rho}]$$
$$+i\dot\xi_{\alpha}\bar\xi\bar\xi - \frac{1}{2}(\sigma^{\rho}\dot{\bar\xi})_{\alpha}\xi\xi\bar\xi\bar\xi\partial_{\rho}\Big]\delta^{(4)}(s_0). \tag{37}$$

The complex conjugation of Eq. (37) leads to the current $\bar j^{(1)}_{\dot\alpha}$. Note that the expressions (31)- (37) may be simplified by using the Dirac identities (24) with the interval s^{μ}_0 substituted instead of s^{μ}.

The physical consequences of the supersymmetrization process considered here may be observed even in the simplest case when $\dot{\mathbf{y}} = \dot{\xi} = \dot{\bar{\xi}} = 0$ and the reparametrization gauge is fixed by the condition $y_0 = \tau$. In this case the scalar v_0 and vector $\mathbf{v}$ components of the 4-potential v_μ (31) take the following form

$$v_0 = \frac{e}{r} + e\pi\xi_0^2\bar{\xi}_0^2\delta^{(3)}(\mathbf{r}), \qquad \mathbf{r} = \mathbf{x} - \mathbf{y},$$
$$\mathbf{v} = e\frac{[\Sigma \times \mathbf{r}]}{r^3}, \quad \Sigma = (\xi_0\boldsymbol{\sigma}\bar{\xi}_0), \quad \xi_0^\alpha = \xi_0^\alpha\Big|_{\tau=x_0}. \tag{38}$$

Then the last Eq. (38) shows that the Grassmann variables describe the contribution to the vector potential $\mathbf{v}$ of the magnetic moment $\mathbf{s}$ of the charged particle-source. This explanation is consistent with the well- known interpretation of the Grassmann variables as those describing the spin degrees of freedom of the particle in the classical limit $\hbar \mapsto 0$. Concerning the first Eq. (38) we see that the scalar potential v_0 satisfies the Laplace equation

$$\Delta v_0 = -4\pi[\frac{e}{2}\delta^{(3)}(\mathbf{r} + \Sigma) + \frac{e}{2}\delta^{(3)}(\mathbf{r} - \Sigma)]. \tag{39}$$

Eq. (39) shows that the correction (38) to the Coulomb law is caused by the "smearing" of the particle's electric charge e over the space region of the order of the Compton wavelength of the particle. This "smearing" of the classical trajectory is related to the effect of pair creation ("Zitterbewegung"- effect) [8], which does not disappear in the limit $\hbar \mapsto 0$, because in this case the spin degrees of freedom described by the Grassmann spinors are conserved.

If we have the explicit form (15), (16) of the superpotentials $A^M = (A^\mu, A^\alpha, \bar{A}_{\dot{\alpha}})$ it is easy to construct a superfield generalization of the FSTWF classical action for interacting charged fermions. To this end note that the integrand of the interacting term of the FSTWF action may be presented in the standard form $e\,dx^\mu A_\mu$, with A_μ given by Eq. (1). The supersymmetrization of this 1-form is realized by means of the replace [?]: $e\,dx^\mu A_\mu \mapsto e\,\omega^M(dz)A_M$ with the superfield $A^M = (A^\mu, A^\alpha, \bar{A}_{\dot\alpha})$ given by the expressions (??), (??) and $\omega^M = (\omega_t^\mu dt, \dot\theta^\alpha dt, \dot{\bar\theta}_{\dot\alpha} dt)$. Therefore the superfield generalization of the FSTWF classical action for two charged interacting particles with masses m_1 and m_2 and spins equal 1/2 has the form

$$
\begin{aligned}
S^{SUSY}_{FSTWF} = & \frac{1}{2}\int dt\Big(\frac{\omega_t^2}{g_t} + g_t m_1^2\Big) + \frac{1}{2}\int d\tau\Big(\frac{\omega_\tau^2}{g_\tau} + g_\tau m_2^2\Big) \\
& + e\int dt\Big(\omega_t^\mu A_\mu + \dot\theta_t^\alpha A_\alpha + \dot{\bar\theta}_{t\dot\alpha} A^{\dot\alpha}\Big).
\end{aligned}
\tag{40}
$$

with the integrals (??), (??) substituted instead of A^μ, A^α, $\bar{A}_{\dot\alpha}$ and g_t, g_τ playing the role of einbeins. It is not so difficult to verify that the action (??) is a symmetric one under permutations of the particles.

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† All differential operators $\partial, D, \Box$ used below act on the coordinates $(x, \theta, \bar{\theta})$ of the constructed superfields.

‡ We omit the dependence of A^{α}, $\bar{A}_{\dot{\alpha}}$ and A^{μ} on $\mathbf{y}$ and ξ_0^{α}, which are defined by the zeroes of δ-function.

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