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Scaling law
for the $\Upsilon(4S) \rightarrow B\bar{B}$ and $\psi(3770) \rightarrow D\bar{D}$ decay
constants from effective sum rules

Abstract

V.V.Kiselev. Scaling law for the $\Upsilon(4S) \rightarrow B\bar{B}$ and $\psi(3770) \rightarrow D\bar{D}$ decay constants from effective sum rules: ПИЕР Preprint 94-3. — Protvino, 1994. — p.12, refs.: 11.

The sum rules for exclusive production of heavy meson pairs in e^+e^- annihilation are used to evaluate the $\Upsilon(4S) \rightarrow B\bar{B}$ and $\psi(3770) \rightarrow D\bar{D}$ decay widths. Infinitely heavy quark limit is discussed, and the scaling law for the quarkonium-meson coupling constant is derived. The $B\bar{B}$ pair contribution into the leptonic constant $f_{\Upsilon(4S)}$ is estimated.

Аннотация

В.В.Киселев. Масштабный закон для констант распадов $\Upsilon(4S) \rightarrow B\bar{B}$ и $\psi(3770) \rightarrow D\bar{D}$ из эффективных правил сумм: Препринт ИФВЭ 94-3. — Протвино, 1994. — 12с., библиогр.: 11.

Правила сумм для эксклюзивного образования пар тяжелых мезонов в e^+e^- аннигиляции используются для оценки ширины распадов $\Upsilon(4S) \rightarrow B\bar{B}$ и $\psi(3770) \rightarrow D\bar{D}$. Обсуждается предел бесконечно тяжелого кварка, и, таким образом, выводится масштабный закон для констант связи кваркония с мезонами. Оценивается величина вклада пар $B\bar{B}$ в лептонную константу $f_{\Upsilon(4S)}$.

Introduction

Some Υ -states, being heavier than a kinematical threshold of the decay into $B\bar{B}$ pairs, have total widths, 10^3 times larger than hadronic widths of under-threshold states [1]. The fact agrees absolutely with the Okubo-Zweig-Iizuka rule. However, as for approaches of numerical estimates for the decay widths such as $\Gamma(\Upsilon(4S) \rightarrow B\bar{B})$, one finds an essential problem, caused by the non-perturbative character of the calculations, because the strong interaction in the sector of light quarks, coupling with the heavy b -quark into the B meson, can not be described by the perturbative theory of QCD. At present, the width for the decay $\psi(3770) \rightarrow D\bar{D}$ has only be estimated in the framework of the phenomenological nonrelativistic model [2] by calculating the meson wave function overlapping. We use another effective way to calculate nonperturbative quantities, sum rules are applied [3].

So, we preliminarily calculate the total cross sections for the exclusive production of the heavy meson pairs in e^+e^- annihilation using of a simple constituent quark model, which contains a bit of well defined parameters [4]. The application of the sum rules allows one to determine the contribution of the $B\bar{B}$ pairs into the leptonic decay constant of $\Upsilon(4S)$. The contribution is generally determined by the choice of the light quark constituent mass in the B meson. The dispersion relation allows one to calculate the coupling constant of $\Upsilon(4S)$ with B mesons. The constant depends slightly on the choice of the model parameters, such as the constituent mass of light quark, a threshold of a nonresonant contribution into the hadronic spectral density.

We study the infinitely heavy quark limit ($m_{b,c} \rightarrow \infty$), so that for the coupling constant g of the heavy vector quarkonium ($Q\bar{Q}$) with the meson

$(Q\bar{q})$, the scaling law is found

$$\frac{g^2}{M} = \text{const}$$

independently of the heavy quark flavour. This law is considered to be rather reliable for the $\Upsilon(4S) \rightarrow B\bar{B}$ and $\psi(3770) \rightarrow D\bar{D}$ decays.

The model allows one also to consider a possible value of the $m_{\text{light}}/m_{\text{heavy}}$ corrections from a phenomenological point of view.

1. Exclusive Production of Heavy Meson Pair in e^+e^- Annihilation

In the framework of the constituent quark model [4] the exclusive production of the heavy meson pair is determined by the values of the leptonic decay constant f of the meson, the constituent quark masses $m_{Q,q}$ and QCD coupling constant $\alpha_S(4m_q^2)$. One can easily show, if m_Q and m_q are the masses of the heavy and light quarks, respectively, then for the cross sections of the pseudoscalar P and vector V mesons one has the following expressions

$$\begin{aligned} \sigma(e^+e^- \rightarrow (Q\bar{q})_P(\bar{Q}q)_P) &= \frac{\pi^3 \alpha_S^2(4m_q^2) \alpha_{em}^2}{3^7 \cdot 4m_q^6} \frac{m_Q^2}{M^2} f_P^4 (1-v^2)^3 v^3 \times \\ &\quad \left(3e_Q \left(\frac{2m_q}{m_Q} - 1 + v^2 \right) - 3e_q \left(2 - (1-v^2) \frac{m_q}{m_Q} \right) \frac{m_q^3 \alpha_S(4m_Q^2)}{m_Q^3 \alpha_S(4m_q^2)} \right)^2, \\ \sigma(e^+e^- \rightarrow (Q\bar{q})_P(\bar{Q}q)_V) &= \frac{\pi^3 \alpha_S^2(4m_q^2) \alpha_{em}^2}{3^7 \cdot 2m_q^6} f_P^2 f_V^2 (1-v^2)^4 v^3 \times \\ &\quad \left(3e_Q - 3e_q \frac{m_q^3 \alpha_S(4m_Q^2)}{m_Q^3 \alpha_S(4m_q^2)} \right)^2, \\ \sigma(e^+e^- \rightarrow (Q\bar{q})_V(\bar{Q}q)_V) &= \frac{\pi^3 \alpha_S^2(4m_q^2) \alpha_{em}^2}{3^7 \cdot 2m_q^6} f_V^4 (1-v^2)^3 v^3 \times \\ &\quad \left(3e_Q - 3e_q \frac{m_q^3 \alpha_S(4m_Q^2)}{m_Q^3 \alpha_S(4m_q^2)} \right)^2 [3(1-v^2) + (1+v^2)(1-a)^2 + \\ &\quad \frac{a^2}{2}(1-v^2)(1-3v^2)], \end{aligned} \quad (1)$$

where v is the meson velocity in the center of mass system, and

$$a = \frac{m_Q}{M} \frac{1 - \frac{e_q}{e_Q} \frac{m_q^4}{m_Q^4} \frac{\alpha_S(4m_Q^2)}{\alpha_S(4m_q^2)}}{1 - \frac{e_q}{e_Q} \frac{m_q^3}{m_Q^3} \frac{\alpha_S(4m_Q^2)}{\alpha_S(4m_q^2)}}$$

One can see that in the limit of $m_{\text{light}}/m_{\text{heavy}} \ll 1$ the values of the cross sections depend strongly on the choice of the constituent light quark mass m_q .

Note, that near the threshold the cross section ratios for the pseudoscalar and vector mesons are

$$\sigma(PP) : \sigma(PV) : \sigma(VV) = 1 : 4 : 7, \quad (2)$$

where one takes into account $\sigma(PV) = \sigma(B\bar{B}^*) + \sigma(\bar{B}B^*) = 2\sigma(\bar{B}B^*)$.

Eq.(2) agrees with EHQT [5], and it has been obtained in papers of Refs.[2, 6,7].

2. $B\bar{B}$ Contribution into Leptonic Constant $f_{\Upsilon(4S)}$

With the required accuracy in QCD sum rules [3] one found the following expressions

$$\sum_n \frac{f_{V_n}^2 M_{V_n}^2}{M_{V_n}^2 - q^2} \approx \frac{1}{\pi} \int_{s_i}^{s_{th}} \frac{\Im m \Pi_V^{(QCD)pert}(s)}{s - q^2} ds + \Pi_V^{(QCD)nonpert}(q^2), \quad (3)$$

where f_V is a leptonic constant of the vector state $V(Q\bar{Q})$ with the mass M_V ,

$$\begin{aligned} i f_V M_V \epsilon_\mu^{(\lambda)} e^{ipx} &= \langle 0 | J_\mu(x) | V(\lambda) \rangle, \\ J_\mu &= \bar{Q}(x) \gamma_\mu Q(x), \end{aligned} \quad (4)$$

and $\Im m \Pi_V^{(QCD)pert}(s)$ is calculated by means of the QCD perturbation theory, so that

$$\begin{aligned} (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \Pi_V^{(QCD)pert}(q^2) + q_\mu q_\nu \Pi_S^{(QCD)pert}(q^2) &= \\ \int d^4 x e^{iqx} \langle 0 | T J_\mu(x) J_\nu(0) | 0 \rangle, \end{aligned} \quad (5)$$

and

$$\Pi_V^{(QCD)nonpert}(q^2) = \sum C_i(q^2) O^i, \quad (6)$$

where O^i are vacuum expectation values of composit operators like $\langle m \bar{\psi} \psi \rangle$ and $\langle \alpha_s / \pi G_{\mu\nu}^2 \rangle$ and so on. The Wilson coefficients $C_i(q^2)$ are calculated in the perturbative QCD framework. s_i is the kinematical threshold of the QCD perturbative contribution, s_{th} is the threshold energy squared for the nonresonant hadronic contribution, kept equal to the perturbative one at $s > s_{th}$. The summation over the vector resonances in Eq.(3) is performed for the states with the masses $M_{V_n}^2 > s_i$.

It is quite evident that the saturation of the two-point quark current correlator $\langle 0|TJ_\mu(x)J_\nu(0)|0\rangle$ by two-point meson current correlator $\langle 0|TJ_\mu^{B\bar{B}}(x)J_\nu^{B\bar{B}}(0)|0\rangle$ leads to the extraction of the $B\bar{B}$ pair contribution to the $f_{\Upsilon(4S)}$ value for the kinematically admissible lightest state $\Upsilon(4S)$.

One can easily find that

$$\Im m\Pi_V^{B\bar{B}}(q^2) = \frac{q^4}{(4\pi\alpha_{em}e_b)^2} \sigma_{e^+e^- \rightarrow B\bar{B}}(q^2). \quad (7)$$

Determining the B meson velocity by the expression

$$v = \sqrt{1 - \frac{4M_B^2}{s}}, \quad (8)$$

one derives the sum rule for the $B\bar{B}$ pair contribution into $f_{\Upsilon(4S)}$

$$\frac{f_{\Upsilon(4S)}^2(B\bar{B})M_{\Upsilon(4S)}^2}{M_{\Upsilon(4S)}^2 - q^2} = \frac{1}{\pi} \int_0^{v_{th}} \frac{dv^2}{(1-v^2)^3} \left(\frac{M_B^2}{\pi\alpha_{em}e_b}\right)^2 \frac{\sigma_{e^+e^- \rightarrow B\bar{B}}(v)}{1 - q^2/s}, \quad (9)$$

where the hadronic part of the sum rule is presented as the two contributions, the first one is the contribution of the $\Upsilon(4S)$ state, lying above the $B\bar{B}$ production threshold, and the second contribution is nonresonant one, which coincides with the calculated cross section of the $B\bar{B}$ production at $s > s_{th}$.

At $q^2 = 0$ for the n -th derivative of Eq.(9) one gets

$$f_{\Upsilon(4S)}^2(B\bar{B}) = \frac{1}{\pi} \int_0^{v_{th}} \frac{dv^2}{(1-v^2)^{(3-n)}} \left(\frac{M_B^2}{\pi\alpha_{em}e_b}\right)^2 \sigma_{e^+e^- \rightarrow B\bar{B}}(v), \quad (10)$$

where in accordance with Eq.(8) v_{th} is determined by the threshold value of the total energy

$$\sqrt{s_{th}} = 2M_B + \delta. \quad (11)$$

One has supposed $M_{\Upsilon(4S)} \simeq 2M_B$.

Some experience in working with the QCD sum rules makes us believe that s_{th} is determined by the energy, at which the production of both new states and manyparticle systems becomes significant. In the case of interest, this energy is defined by the threshold of the $B\bar{B}^*$ and $B^*\bar{B}^*$ production. The $B^*\bar{B}^*$ production cross section increases greatly, beginning from the threshold, so that $\sigma(B^*\bar{B}^*)/\sigma(B\bar{B}) \simeq 7$, and the $B\bar{B}^*$ production cross section becomes equal to $\sigma(B\bar{B})$ at the same energy of $B^*\bar{B}^*$ threshold. A new channel opens when additional π mesons are produced.

So, one takes δ in Eq.(11) as

$$\delta \approx 2(M_{B^*} - M_B) \simeq 100 \text{ MeV}. \quad (12)$$

Eq.(10) defines the $B\bar{B}$ pair contribution into the leptonic constant of $\Upsilon(4S)$.

From definition (4) it follows that

$$\Gamma(\Upsilon(4S) \rightarrow e^+e^-) = \frac{4\pi}{3} \alpha_{em}^2 e_b^2 \frac{f_{\Upsilon(4S)}^2}{M_{\Upsilon(4S)}}, \quad (13)$$

so that one has experimentally [1]

$$f_{\Upsilon(4S)} \simeq 320 \text{ MeV}. \quad (14)$$

Taking into account the contributions from the charged and neutral B mesons, one gets the following expression

$$r^2 = \frac{4}{\pi f_{\Upsilon(4S)}^2} \int_0^{v_{th}} \frac{dv^2}{(1-v^2)^{3-n}} \left(\frac{M_B^2}{\pi\alpha_{em}e_b}\right)^2 \sigma_{e^+e^- \rightarrow B\bar{B}}(v), \quad (15)$$

for the ratio

$$r = \frac{f_{\Upsilon(4S)}(B\bar{B})}{f_{\Upsilon(4S)}}.$$

From Eq.(1) for the $\sigma(e^+e^- \rightarrow B\bar{B})$ cross section one can see that the r value depends strongly on the choice of the constituent light quark mass, so that σ is proportional to $1/m_q^6$.

3. Coupling Constant of $\Upsilon(4S)$ with B Mesons

Let's consider $f_{\Upsilon(4S)}(B\bar{B})$ as a value of a complex function at the physical point with the condition $\Re ef \gg \Im mf$ near the threshold $q^2 \simeq 4M_B^2$, where the imaginary part of the function tends to zero.

Then, the imaginary part of the transversal correlator, shown on Fig.1, is related with $\Im mf$ as the following

$$\Im m\Pi^{tr}(q^2) = -M_{\Upsilon(4S)} \frac{1}{2} \Im mf_{\Upsilon(4S)}(B\bar{B}), \quad (16)$$

so that at $q^2 = 4M_B^2/(1-v^2)$ one has

$$\Im mf_{\Upsilon(4S)}(B\bar{B}) = g_{\Upsilon B\bar{B}} \frac{\alpha_S}{8 \cdot 27} \frac{f_B^2 m_b}{M_B^5} \left(\frac{M_B}{m_q}\right)^3 |\vec{k}_B|^3 (1-v^2)^2 (1-v^2 - \frac{2m_q}{m_b}), \quad (17)$$

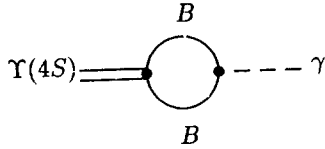


Figure 1. The contribution by the B meson pair into the leptonic decay constant.

where one has used the formula for the photon-meson vertex, derived in the framework of the exclusive cross section calculations

$$L_{AB\bar{B}} = e \cdot e_b F(q^2) A_\mu \cdot k^\mu, \quad (18)$$

with

$$F(q^2) = \frac{2\pi\alpha_S(4m_q^2)}{9} \frac{f_B^2 m_b}{M_B^3} \left(\frac{M_B}{m_q}\right)^3 (1-v^2)\left(1-v^2 - \frac{2m_q}{m_b}\right), \quad (19)$$

and the $\Upsilon B\bar{B}$ vertex has been written down as

$$L_{\Upsilon B\bar{B}} = g_{\Upsilon B\bar{B}} \epsilon_\mu k^\mu, \quad (20)$$

where ϵ is the polarization vector of $\Upsilon(4S)$, $k = (p_B - p_{\bar{B}})/2$.

One can easily find that the consistent consideration of the vector dominance means that near the threshold

$$(q^2 - M_{\Upsilon(4S)}^2) \Im m \Pi_V^{B\bar{B}}(q^2) = 2 f_{\Upsilon(4S)}(B\bar{B}) M_{\Upsilon(4S)} \Im m \Pi^{tr}(q^2), \quad (21)$$

and the sum rules for $(q^2 - M_{\Upsilon(4S)}^2) \Pi_V^{B\bar{B}}(q^2)$ may be rewritten as

$$-f_{\Upsilon(4S)}^2(B\bar{B}) M_{\Upsilon(4S)}^2 = f_{\Upsilon(4S)}(B\bar{B}) M_{\Upsilon(4S)} \frac{2}{\pi} \int_{4M_B^2}^{q_{th}^2} \frac{dq^2}{q^2 - s} \Im m \Pi^{tr}(q^2), \quad (22)$$

so that at physical point $s = 4M_B^2$ one has Eq.(16).

Using the dispersion relation such as Eqs.(9)-(10), one can easily write down at $q^2 = 4M_B^2$

$$\Re f_{\Upsilon(4S)}(B\bar{B}) = \frac{1}{\pi} \int_0^{v_{th}} \frac{dv^2}{(1-v^2)v^2} \Im m f_{\Upsilon(4S)}(B\bar{B}). \quad (23)$$

Taking into account the condition $|\vec{k}| = v\sqrt{s}/2$, one obtains

$$\Re f_{\Upsilon(4S)}(B\bar{B}) = \frac{1}{\pi} \int_0^{v_{th}} \frac{v^2 dv}{\sqrt{1-v^2}} g_{\Upsilon B\bar{B}} \frac{\alpha_S}{4 \cdot 27} \frac{f_B^2 m_b}{M_B^2} \left(\frac{M_B}{m_q}\right)^3 \left(1-v^2 - \frac{2m_q}{m_b}\right), \quad (24)$$

Note, the left hand side of Eq.(24) is found from analogous sum rules (9)-(10) and the $\Re f$ value is proportional to $1/m_q^3$, so that the constant $g_{\Upsilon B\bar{B}}$ does not crucially depend on the value of the constituent light quark mass.

In accordance with Eq.(20), $g_{\Upsilon B\bar{B}}$ is related with the $\Upsilon(4S) \rightarrow B^+ B^-$ decay width through the expression

$$\Gamma(\Upsilon(4S) \rightarrow B^+ B^-) = \frac{1}{24\pi} g_{\Upsilon B\bar{B}}^2 \frac{|\vec{k}|^3}{M_{\Upsilon(4S)}^2}. \quad (25)$$

Thus, one can calculate the $\Upsilon(4S) \rightarrow B^+ B^-$ decay width using the sum rules. Those calculations may be easily performed explicitly in the limit of infinitely heavy quark and at $v_{th} \ll 1$.

4. Infinitely Heavy Quark Limit

In the limit $m_q/m_b \ll 1$, $m_b \approx M_B$, one can easily find

$$\sigma_{e^+e^- \rightarrow B^+B^-}(v) = \frac{\pi^3}{4 \cdot 3^5} e_b^2 \alpha_S^2 (4m_q^2) \alpha_{em}^2 \frac{f_B^4}{m_q^6} (1-v^2)^5 v^3. \quad (26)$$

Then sum rule (10) may be rewritten as

$$f_{\Upsilon(4S)}^2(B^+B^-) = \frac{1}{\pi} \int_0^{v_{th}} v^4 dv \frac{1}{2 \cdot 3^5} \alpha_S^2 (4m_q^2) \frac{f_B^4 M_B^4}{m_q^6} (1-v^2)^{2+n}. \quad (27)$$

At $v_{th} \ll 1$ one gets, that the $B\bar{B}$ pair contribution into $f_{\Upsilon(4S)}$ has the form

$$f_{\Upsilon(4S)}(B^+B^-) \approx \frac{\alpha_S(4m_q^2) f_B^2 M_B^2}{m_q^3} \left(\frac{v_{th}^5}{10 \cdot 3^5}\right)^{1/2}. \quad (28)$$

Furthermore, from sum rule (24), it follows that

$$f_{\Upsilon(4S)}(B^+B^-) \approx g_{\Upsilon B\bar{B}} \frac{\alpha_S(4m_q^2)}{4 \cdot 81\pi} \frac{f_B^2 M_B^2}{m_q^3} v_{th}^3. \quad (29)$$

From Eqs.(28)-(29) one finds

$$g_{\Upsilon B\bar{B}} \simeq 12\pi \sqrt{\frac{3}{10v_{th}}}. \quad (30)$$

Thus, one can see that in the infinitely heavy quark limit the sum rules for the exclusive production of the $B\bar{B}$ pairs in the $\Upsilon(4S)$ resonance lead to the

expression for $g_{\Upsilon B\bar{B}}$, that depends weakly on the threshold of the nonresonant contribution, and the coupling constant is independent of the parameters such as the constituent light quark mass. One finds conditions such as the conditions in the effective heavy quark theory [5], where, for example, the consideration for the form factors of semileptonic decays $B \rightarrow D^{(*)}e\nu$ leads to the statement, that the form factor normalization $\xi(q^2 = q_{max}^2) = 1$ is determined by purely kinematical relations and does not depend on nonperturbative parameters, such as m_q .

5. Numerical Estimates

In accordance with the experimental data [1], Eq.(25) allows one to determine the empirical values of $g_{\Upsilon B\bar{B}}$ and $g_{\psi D\bar{D}}$ for the $\Upsilon(4S)$ and $\psi(3770)$ states, respectively.

5.1. Analysis for $\Upsilon(4S)$

First of all, note that the calculations of the initial five moments of the spectral function in Eq.(10) show that $f_{\Upsilon(4S)}(B\bar{B})$ depends weakly on the momentum number n , so that $\Delta f/f \approx 5\%$, i.e. the sum rules give the stable value of $f_{\Upsilon(4S)}(B\bar{B})$ under the variation of the scheme parameter (n). There is a more essential dependence on the f_B and m_q choices.

In accordance with the QCD sum rules and mass spectrum of the mesons with b -quarks, one has supposed [3,8]

$$\begin{aligned} f_B &= 100 \text{ MeV} , \\ m_q &= 330 \text{ MeV} , \\ m_b &= 4.95 \text{ GeV} , \\ \delta &= 100 \div 140 \text{ MeV} , \\ \Lambda_{QCD} &= 200 \text{ MeV} . \end{aligned} \quad (31)$$

As for the m_q/m_b correction to the exclusive cross section for the production of $B\bar{B}$ pairs, it reduces both the value of $f_{\Upsilon(4S)}(B\bar{B})$ by 15% and $g_{\Upsilon B\bar{B}}$ by 10%, approximately! Thus, one can state that the infinitely heavy quark limit is valid for the $\Upsilon(4S)$ state with the accuracy of about 15%.

¹Note, the value of the first order $1/m_Q$ correction to the exclusive production of the heavy meson pairs is intentionally overestimated, since in accordance with the results of Refs.[9] the first order correction over $1/m_Q$ must vanish at the threshold, so that the kinematical correction, following from Eq.(1), must be compensated by the variation of the leptonic constants due to the $1/m_Q$ correction, for instance, $\Delta f_B/f_B \approx 10\%$. So, at v_{th} close to zero, the corrections to the obtained results are really much less than the presented estimates, and the results are rather reliable.

Numerically one has

$$\begin{aligned} g_{\Upsilon B\bar{B}}^{exp} &= 51 \pm 3 , \\ g_{\Upsilon B\bar{B}}^{theor} &= 53 \pm 8 , \\ \frac{f_{\Upsilon(4S)}(B\bar{B})}{f_{\Upsilon(4S)}} &= (0.12 \div 0.26)\% . \end{aligned} \quad (32)$$

Note, that the main result, concerning the g value, very slightly depends only on the threshold energy choice (δ), that is reliably fixed by the physical reason, which is the production of some particles different from the $B\bar{B}$ pair, so that the lightest final states are $B\bar{B}^*$, \bar{B}^*B , \bar{B}^*B^* .

5.2. Analysis for $\psi(3770)$

For the $\psi(3770) \rightarrow D\bar{D}$ decay, the approach accuracy falls significantly.

First, one observes a strong dependence on the number of the spectral function moments. For the initial five moments one has $\Delta f/f \approx 30\%$.

Second, the difference between the m_q/m_b corrected and uncorrected values of $f_{\psi(3770)}(D\bar{D})$ or $g_{\psi D\bar{D}}$ is of the order of 50% (see the footnote to the previous section).

The variation of the nonresonant contribution threshold is found to be inessential.

In accordance with the QCD sum rules and mass spectrum of the mesons with c -quarks, one has supposed [3,8]

$$\begin{aligned} f_D &= 170 \text{ MeV} , \\ m_q &= 330 \text{ MeV} , \\ m_c &= 1.54 \text{ GeV} , \\ \delta &= 200 \div 280 \text{ MeV} , \\ \Lambda_{QCD} &= 200 \text{ MeV} . \end{aligned} \quad (33)$$

Then

$$\begin{aligned} g_{\psi D\bar{D}}^{exp} &= 31 \pm 2 , \\ g_{\psi D\bar{D}}^{theor} &= 30 \pm 15 , \\ \frac{f_{\psi(3770)}(D\bar{D})}{f_{\psi(3770)}} &= (0.2 \div 0.3)\% . \end{aligned} \quad (34)$$

Note, in the limit $m_q/m_{b,c} \rightarrow 0$, Eq.(30) gives the g values, close to the empirical, both for the Υ and ψ particles.

Discussion and Conclusion

In the present paper we have applied the sum rule approach to analyze some nonperturbative values such as the $B\bar{B}$ pair contribution into the leptonic decay constant of $\Upsilon(4S)$ and the coupling constant of $\Upsilon(4S)$ with the B mesons.

The consideration of the infinitely heavy quark limit allows one to derive a simple explicit expression for $g_{\Upsilon B\bar{B}}$ independent of the nonperturbative parameters such as the constituent light quark mass, so that the $g_{\Upsilon B\bar{B}}$ value is determined by the threshold of the nonresonant contribution

$$g_{\Upsilon B\bar{B}} \simeq 12\pi \sqrt{\frac{3}{10v_{th}}}.$$

Moreover, taking into account that in the EHQT framework the mass difference for the vector and pseudoscalar states satisfies the relation

$$\Delta M \cdot M = const \quad (35)$$

independently of the heavy quark flavour, and

$$v_{th} \simeq \sqrt{2 \frac{\Delta M}{M}}, \quad (36)$$

one gets

$$v_{th} \cdot M = const, \quad (37)$$

and

$$\frac{g^2}{M} = const \quad (38)$$

independently of the heavy quark flavour.

The numerical estimates made in accordance with law (38), are close to the experimental ratio of the g values for $\Upsilon(4S)$ and $\psi(3770)$.

Thus, in the leading order of EHQT the sum rules allow one to state scaling law (38) for the coupling constants of heavy quarkonia with heavy mesons, including the single heavy quark². This law is found to be powerful for the $\Upsilon(4S)$ and $\psi(3770)$ states. However, it has been shown that the approach accuracy is not high, when the sum rules are applied to the c -quark systems. For the b -quarks, the accuracy is rather high and may be considered as admissible.

It has been also shown that the meson pair contribution into the leptonic decay constants are of the order of 0.2%.

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²Note, that the infinitely heavy quark limit was applied to derive the scaling law for the leptonic decay constants of heavy quarkonia (ϕ , ψ , Υ , B_c) in the framework of the QCD sum rules [10].

Naive scaling assumptions for the heavy quarkonium were discussed in Refs. [11].

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