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STRONGLY INTERACTING LIGHT DARK MATTER

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Abstract

We discuss a class of Dark Matter (DM) models that, although inherently strongly coupled, appear weakly coupled at small-energy and fulfill the WIMP miracle, generating a sizable relic abundance through the standard freeze-out mechanism. Such models are based on approximate global symmetries that forbid relevant interactions; fundamental principles, like unitarity, restrict these symmetries to a small class, in such a way that the leading interactions between DM and the Standard Model are captured by effective operators up to dimension-8. The underlying strong coupling implies that these interactions become much larger at high-energy and represent an interesting novel target for LHC missing-energy searches.

1 Introduction and motivations

Studies of processes with missing energy at the LHC constitute an important part of the Dark Matter (DM) research program, that aims at unravelling possible non-gravitational interactions between the Standard Model (SM) and the dark sector. Information from the LHC would be particularly useful for light DM, $m_{\rm DM} \lesssim 10$ GeV, below the threshold for direct detection experiments. In this case, the *WIMP miracle* seems to provide a convincing hint that light DM originates from *weakly coupled* dynamics. Indeed, parameterizing the thermally-averaged annihilation cross section as

$$\langle \sigma v_{\rm rel} \rangle \sim \frac{\alpha_{\rm DM}^2}{m_{\rm DM}^2} \tag{1}$$

with $m_{\rm DM}, \alpha_{\rm DM}$ the DM mass and coupling to the Standard Model (SM) fields, we find for the relic density

$$\Omega_{\rm DM} h^2 \approx \frac{10^{-26}\,{\rm cm}^3/{\rm s}}{\langle \sigma v_{\rm rel} \rangle} \approx 0.1 \left(\frac{0.1}{\alpha_{\rm DM}} \right)^2 \left(\frac{m_{\rm DM}}{10\,{\rm GeV}} \right)^2 \ . \tag{2}$$

A weak coupling $\alpha_{\rm DM} \ll 1$ reproduces the observed value $\Omega_{\rm DM}h^2 \approx 0.1$. Note that, for simplicity, we limit the present discussion to s-wave annihilation. Annihilation in p-wave would imply in eq. (1) the presence of the suppression factor $v_{\rm rel}^2$ due to the relative velocity of the two annihilating particles, roughly $v_{\rm rel} \sim 1/3$ at freeze-out temperature. In this contribution we want to explore how solid this indication is and study the viability of light DM associated with a new *strong*, yet perturbative, coupling which we call $g_* \lesssim 4\pi$. The core aspect of our analysis is *approximate symmetries*, which forbid relevant (renormalizable) SM-DM interactions, but allow irrelevant (non-renormalizable) interactions of dimension D. Referring to M as the *physical* scale suppressing the latter, the amplitude for $2 \to 2$ annihilation, would scale as

$$\alpha_{\rm DM} \sim \frac{g_*^2}{4\pi}\left(\frac{E}{M}\right)^{D-4}, \tag{3}$$

where E denotes the collision energy. At low energies $E \ll M$, such as those relevant at freeze-out, the interaction of eq. (3) appears weak, despite their strongly coupled nature at high-energy: this reconciles an underlying strong coupling with the WIMP miracle. For instance, for D=6, considering that in the relevant non-relativistic limit $E \sim m_{\rm DM}$,

$$\Omega_{\rm DM}h^2 \approx 0.1\left(\frac{4\pi}{g_*}\right)^4\left(\frac{5\,{\rm GeV}}{m_{\rm DM}}\right)^2\left(\frac{M}{3\,{\rm TeV}}\right)^4, \tag{4}$$

showing that even an extremely strongly coupled system $g_* \approx 4\pi$, can reproduce the observed relic abundance, as long as the mediator scale M is in the multi-TeV region. At high-energy $E \lesssim M$, DM interacts strongly with itself and with the SM, eq. (3). This is in fact very appealing for the LHC which, operating at high-energy, has direct access to the strongly coupled regime. Moreover, in this regime, the signal from the strongly coupled sector is expected to be strong, and dominate over the LHC irreducible backgrounds (such as $jZ \to j\nu\nu$). For this reason, because large effects can be obtained even for $E \lesssim M$, DM from a strongly coupled sector provides one of the few examples where the use of a DM Effective Field Theory (EFT) is well motivated even to parametrize LHC DM searches - a topic that has received enormous attention in recent years (see refs. [1, 2, 3] and the literature that followed).

In this contribution we will use symmetry arguments to discuss all structured scenarios where DM is strongly coupled, but fulfills the WIMP miracle. This work in based on [4, 5]. After identifying the relevant symmetries, we use simple power counting rules to build the EFT describing the physics of these scenarios at collider energies, both in the case where DM is a scalar or a fermion. We will see that, in some cases, the EFT for strongly coupled DM differs substantially from the original DM EFT of refs. [1, 2, 3].

2 Analysis and results

So, what symmetries are compatible with irrelevant operators only? For scalars a well-known example is the shift symmetry associated with Nambu–Goldstone bosons (NGBs) from strong dynamics, like QCD pions. In this case the leading interactions appear at D=6 or D=8. For Dirac fermions, on the other hand, chiral symmetry and the absence of gauge interactions are enough to guarantee $D \geq 6$. Alternatively, for Majorana fermions (in analogy with NGBs), non-linearly realized supersymmetry (SUSY) ensures that $D \geq 8$. Indeed the leading interactions of Goldstini from spontaneously broken SUSY only exhibit higher-derivative interactions in the limit where all other SUSY particles are heavy [6]. We will discuss these examples in detail below, but first we want to answer the question of whether, beyond these examples, we can find an infinite set of symmetries such that the low-energy amplitude is suppressed by higher and higher powers of energy, i.e. where $D \geq 10$ constitute the only interactions allowed in the limit of

Table 1: *Building blocks for the effective Lagrangian with different SSB patterns. Dots denote higher order terms in $1/f$.*

G/H	ϕ	d^a_μ	ε^a_μ
$\frac{U(1)}{\mathcal{Z}_2}$	$\phi \in \mathbb{R}$	$\frac{\partial_\mu \phi}{f}$	0
$\frac{SU(2)}{U(1)}$	$\phi \in \mathbb{C}$	$(1 + \frac{\lvert\phi\rvert^2}{f^2} + ...)\frac{\partial_\mu \phi}{f}$	$\frac{\phi^\dagger \overleftrightarrow{\partial}_\mu \phi}{f^2} + ...$
$\frac{SO(6)}{SO(5)}$	$H^i, \phi \in \mathbb{R}$	$\left(1 + \frac{\lvert\phi\rvert^2}{f^2} + \frac{\lvert H\rvert^2}{f^2} + ...\right)\frac{\partial_\mu \phi}{f}$	$\frac{H^\dagger \overleftrightarrow{\partial}_\mu H}{f^2} + ...$

exact symmetry. As a matter of fact the answer is negative. Fundamental principles based on analyticity, unitarity and crossing symmetry of the $2 \to 2$ amplitude provide strict positivity constraints for some of the coefficients of D=8 operators [7]. This implies that generally there is no limit in which a symmetry that protects operators with four fields and $D \geq 10$, forbidding $D \leq 8$, can be considered exact. So the complete set of scenarios with a naturally light strongly coupled DM, that however appears weakly coupled at small E (and therefore fulfills the WIMP miracle) is given by the above examples and is captured by operators of $D \leq 8$.

In the following, we shall focus on the case of scalar DM. Naturally light scalars originate as pseudo-NGBs of the spontaneously symmetry breaking (SSB) pattern G/H. If the sector responsible for SSB is strong, NGB interactions become strong at high-E. These scenarios are particularly interesting in association with the hierarchy problem [8, 9, 10, 11, 12, 13, 14], but also independently from it [15, 16]. Qualitatively different cases of interest can be identified, depending on the particular group structure being considered and the interplay with Higgs physics. First, a light scalar DM can be associated with an abelian $U(1) \to \mathcal{Z}_2$ breaking pattern, while a light composite Higgs originates from e.g. $G/H = SO(5)/SO(4)$ [17]. Alternatively, the DM originates from a non-abelian, e.g. $SU(2) \to U(1)$ or larger, symmetry breaking patterns [8, 13, 15, 14]. Finally, both the Higgs and DM can arise together from a non-factorizable group G, such as $SO(6)/SO(5)$ [18, 9, 10, 12]. The very power of EFTs is that, at low-E, large groups of theories fall in the same universality classes: in our case the generic EFTs that we will now build to describe the above-mentioned scenarios can be matched to any model with approximate symmetries. In all these cases, the NGB interactions are described by the CCWZ construction: the light degrees of freedom ϕ^a are contained in the coset representative $U = \exp(i\phi^a t^a/f) \in G/H$ and appear in the Lagrangian only through the building blocks d^a_μ and ε^A_μ in $U^{-1}\partial_\mu U = i d^a_\mu t^a + i\varepsilon^A_\mu T^A$, where $t^a(T^A)$ are the broken (unbroken) generators in G, f is the analog of the pion decay constant and is related to the mass and couplings of resonances from the (strong) sector that induces SSB through the naive dimensional analysis estimate $f = M/g_*$. Table 1 shows some specific examples. Under a transformation $g \in G$, $U \to gUh(\phi, g)^{-1}$, where $h(\phi, g) \in H$. Then $d_\mu \equiv d^a_\mu t^a$ and $\varepsilon \equiv \varepsilon^A_\mu T^A$ transform under G respectively in the fundamental representation of H and shift as a connection, so that $D^\varepsilon_\mu \equiv \partial_\mu + i\varepsilon_\mu$ is the covariant derivative. With these ingredients, the low energy Lagrangian describing the canonically normalized light scalars only, is simply $\mathcal{L}^{eff} = M^2 f^2 \mathcal{L}\left(d^a_\mu/fM, D^\varepsilon_\mu/M\right)$, with the additional requirement of invariance under the unbroken group H: this automatically guarantees also G invariance.

Clearly DM cannot be an exact massless NGB: the global symmetry must be broken explicitly. We keep track of this breaking by weighting interactions that violate the CCWZ construction with m^2_ϕ/M^2; an assumption that reflects to good extent the expectations in explicit models (see for instance [9]). We further assume the most favorable case in which, to the extent possible, the SM itself is part of the strong dynamics, as discussed in ref. [19], so that DM-SM interactions do not introduce further symmetry breaking effects (we discuss below cases where only some species take part in the new dynamics). This implies in particular that we assume the new dynamics respects the SM (approximate) symmetries: custodial symmetry, CP, flavor symmetry (broken only by the SM Yukawas) and baryon and lepton numbers. Finally we assume the new dynamics can be faithfully described by a single new scale M and coupling g_*. Compatibly with these assumptions, the most general Lagrangian at the leading $D=6$ order in the $1/M$ expansion is,

$$
{}_6\mathcal{L}_{\rm eff}^{DM_\phi} = c_\psi^V \frac{g_*^2}{M^2}\phi^\dagger \overleftrightarrow{\partial}_\mu \phi\, \psi^\dagger \bar\sigma^\mu \psi + c_B^{dip}\frac{g_*}{M^2}\partial_\mu\phi^\dagger \partial_\nu \phi\, B^{\mu\nu} + c_H^S \frac{g_*^2}{M^2}|\partial_\mu \phi|^2\,|H|^2 + c_H^{\not{s}}\frac{g_*^2 m_{\phi,H}^2}{M^2}|\phi|^2\,|H|^2 + c_\psi^{\not{s}}\frac{g_*^2 y_\psi}{M^2}|\phi|^2\psi\psi H \tag{5}
$$

where each operator is weighed by the maximum coefficient that we can expect following the power-counting rules associated with the above mentioned-symmetries. The scaling in powers of the coupling g_* can be unambiguously determined from a bottom-up perspective by restoring $\hbar \neq 1$ in the Lagrangian: the coefficient c_i of an operator $\mathcal{O}_i$ with n fields scales as $c_i \sim (\text{coupling})^{n-2}$. Similarly, at D=8, focussing on operators that contribute to $2 \to 2$ scattering,

$$
{}_8\mathcal{L}_{\rm eff}^{DM} = C_V^{\not{s}}\frac{g_*^2 m_\phi^2}{M^4}|\phi|^2 V^a_{\mu\nu}V^{a\,\mu\nu} + C_\psi^S \frac{g_*^2 y_\psi}{M^4}|\partial^\mu\phi|^2\psi\psi H + C_V^S\frac{g_*^2}{M^4}|\partial^\mu\phi|^2 V^a_{\nu\rho}V^{a\,\rho\nu} + C_H^S\frac{g_*^2}{M^4}|\partial^\mu\phi|^2|D^\nu H|^2 \tag{6}
$$

$$
+ \; C_V^T\frac{g_*^2}{M^4}\partial^\mu\phi^\dagger\partial^\nu\phi V^a_{\mu\rho}V^{a\,\rho}_{\nu} + C_H^T\frac{g_*^2}{M^4}\partial^\mu\phi^\dagger\partial^\nu\phi D_{\{\mu}H^\dagger D_{\nu\}}H + C_\psi^T\frac{g_*^2}{M^4}\partial^\mu\phi^\dagger\partial^\nu\phi\psi^\dagger\bar\sigma_\mu D_\nu\psi\,, \tag{7}
$$

with $V^a_{\mu\nu} = B_{\mu\nu}, W^a_{\mu\nu}, G^a_{\mu\nu}$ for $U(1)_Y \times SU(2)_L \times SU(3)_C$ gauge bosons, and ψ, H the SM fermions and Higgs. We use a notation based on left-handed Weyl fermions, which carry additional internal indices to differentiate left-handed ψ and right-handed $(\psi^c)^\dagger$ components of Dirac fermions; the Wilson coefficients c, C, associated to the $D=6,8$ Lagrangians respectively, carry these indices, and are expected to be $O(1)$, unless otherwise stated, see table below.

Of course there are more operators that contribute to $2 \to 2$ scattering, but these can either be eliminated through partial integration, field redefinitions (that eliminate operators proportional to the equations of motion), Bianchi or Fierz identities, or they violate some of the linearly realized symmetries that we assume (CP, custodial). For instance, operators antisymmetric in the Higgs field, such as

$$
c_H^{\not{cust}}\frac{g_*^2}{M^2}\phi^\dagger\overleftrightarrow{\partial}_\mu\phi\, H^\dagger \overleftrightarrow{D}^\mu H \tag{8}
$$

transform as $(\mathbf{1},\mathbf{3})$ under custodial symmetry $SU(2)_L \times SU(2)_R$: their coefficient is expected to be generated first at loop level by custodial breaking dynamics, involving for instance g', which satisfies the required transformation rules $c_H^{\not{cust}} \sim g'^2/16\pi^2$. On the other hand at D=8,

$$
\partial^\mu\phi^\dagger\overleftrightarrow{\partial}_\nu\partial_\mu\phi H^\dagger\overleftrightarrow{D}^\nu H\,, \qquad \partial^\mu\phi^\dagger\overleftrightarrow{\partial}_\nu\partial_\mu\phi\psi^\dagger\bar\sigma^\nu\psi\,, \tag{9}
$$

share the same symmetries (among the linearly and non-linearly realized ones that we have presented) as operators in ${}_6\mathcal{L}_{\rm eff}^{DM_\phi}$ and contribute to the same observables; for this reason their contribution is expected to be always suppressed by $\sim E^2/M^2 \ll 1$ in the amplitude and we neglect them (a similar logic was followed in ref. [20]) to argue that the Peskin-Takeuchi U-parameter can be neglected, since it shares the same symmetries as the T parameter, but is higher-dimension).

Similarly, $m_\phi^2|\phi|^2|H|^4$ and $\partial_\mu\phi^\dagger\partial^\mu\phi|H|^4$ give a subleading (by a factor $g_*^2v^2/M^2 \lesssim 1$) contribution w.r.t. c_H^S and $c_H^{\not{s}}$, in processes with 2 longitudinal vectors or Higgses and can only be distinguished in processes with three or more external longitudinal vector bosons/Higgses. Finally, operators of the form $|\phi|^2 \times {}_6\mathcal{L}_{\rm eff}^{SM}$, where ${}_6\mathcal{L}_{\rm eff}^{SM}$ is the D=6 SM Lagrangian (see ref. [21])) but also includes total derivatives, are generally further suppressed by m_ϕ^2/M^2 and count as D=10 effects in our perspective. The important novel aspect that is emphasized by our analysis and summarized in the Lagrangians eqs. (5,7) and table 1, is the following. Both the D=6 and D=8 Lagrangians can be important, as symmetries can suppress the expected leading interactions in favor of higher order ones. Indeed, as table 1 shows, the structures c_ψ^V vanishes for antisymmetry if DM has a single real degree of freedom (such as for the $U(1)/\mathcal{Z}_2$ and $SO(6)/SO(5)$ cosets), so that in this case the leading DM-fermion interaction is given by the D=8 operator C_ψ^T. On the other hand the structures $c_\psi^{\not{S}}$ and c_H^S are unsuppressed only when the generators associated with ϕ and H do not commute (such as in the $SO(6)/SO(5)$ model [18, 9]), but will be further suppressed by $\sim m_{\phi,H}^2/M^2$ in other cases. In those cases the leading DM-Higgs interactions are the D=8 C_H^S and C_H^T. Finally, an important source of suppression is represented by the degree of compositeness of the SM particles - either fermions or (transverse) gauge bosons. The most favorable situation is when the SM particles are fully composite since in this case they feature an unsuppressed g^* coupling to the strong sector. On the contrary, if SM fermions and gauge bosons are elementary degrees of freedom, we expect a suppression in the corresponding couplings, as shown in the first two rows of table 2. In models where the DM dominantly couples to gluons only, the leading effects at high-energy, not suppressed by any small parameters, are the D=8 C_V^S and C_V^T. We summarize in table 2 these and other such situations, where some of the above operators are suppressed by additional small parameters (such as symmetry breaking effects), and become therefore less interesting from the point of view of collider searches.

Table 2: $\times$ *denotes suppression of a given EFT coefficient, according to specific properties of the microscopic dynamics:* ψ_{elem} *denotes the limit where SM fermions are not composite,* V_{elem} *denotes instead the familiar case where the transverse polarizations of vectors are elementary (as opposed to strong multipolar interactions* [19]*).*

	c_ψ^V	c_B^{dip}	c_H^S	$c_H^{\not{S}}$	$c_\psi^{\not{S}}$	$C_V^{S,T}$	C_ψ^T
ψ_{elem}	$\times$				$\times$		$\times$
V_{elem}		$\times$				$\times$	
$U(1)/\mathcal{Z}_2$	$\times$	$\times$	$\times$	$\times$	$\times$		
$SU(2)/U(1)$			$\times$	$\times$	$\times$		
$SO(6)/SO(5)$	$\times$	$\times$					

In Fig. 1 we compare the LHC reach (blue region) in the (g_*, M)-plane with relic density (RD) expectations (green band) for $D = 6$ (e.g. DM as a PNGB of $SU(2)/U(1)$), showing that visible LHC effects are compatible with a non-vanishing RD. Here the LHC constraints have been derived from the data of ref. [22], imposing an additional cut in the centre-of-mass energy $\hat{s} < M^2$. This cut, and the representation in the (g_*, M)-plane, help us establishing consistency of the EFT assumption [23, 24]. Indeed, as M is lowered within the LHC kinematic region, the constraints rapidly deteriorate, since less and less data remains available: this signals the fact that, in that region, our EFT assumptions are not verified.

LHC constraints for the examples discussed above, where D=8 represent the leading effect at high-E, are also shown in Fig. 1 with a dashed (red) curve. Notice that here, while the E-growing cross sections implied by our symmetry structure clearly dominate at LHC energies $M \gtrsim E \gg m_{\rm DM}$, they might be comparable to symmetry breaking $m_{\rm DM}$-suppressed interaction at low-E, relevant at freeze-out. In other words, the complementarity between different DM experiments is partially lost in this setup – we discuss this issue further in [4, 5].

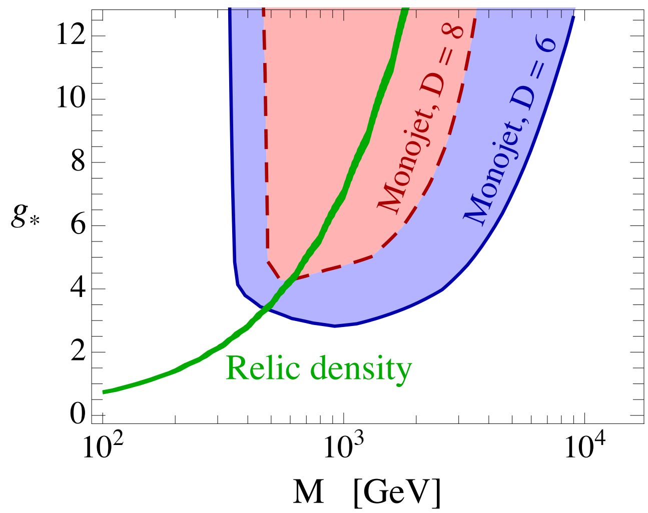

Figure 1: *Constraints on scalar DM with* $m_{\rm DM} = 5$ *GeV. Blue region: excluded by consistent LHC constraints on D=6 operator* c_ψ^V *in eq. (5) (e.g. pseudo-NGB DM from a non-abelian SSB pattern), and comparison with the parameters where the RD is correctly reproduced with the same D=6 operator (solid green). Red region: LHC constraints on D=8,* C_ψ^T *in eq. (7) (e.g. one scalar DM from an abelian SSB).*

3 Outlook

In Summary, we have discussed natural situations in which light DM originates from a strongly-coupled sector but its interactions are small at low-energies because of approximate symmetries, that forbid relevant interactions and allow only irrelevant (higher-derivative) ones. Prime principles dictate that such symmetries are consistent only with D=6 and D=8 operators for $2 \to 2$ scattering. In this article we have identified generic effective Lagrangians at these orders and introduced a power-counting that captures the most well-motivated scenarios that can imply large effects in irrelevant interactions: scalar DM as a PNGB.

These provide a class of models in which the LHC high-E reach plays an important rôle with respect to other types of experiments (such as RD indications and direct detection) and contains genuinely complementary information. Moreover, in these scenarios the DM EFT is not only consistent with LHC analysis (due to the underlying strong coupling, as shown in fig. 1, but also necessary, as the underlying dynamics is uncalculable. Our characterization provides a well-motivated context to model missing transverse-energy distributions at the LHC, in mono-jet, mono-W,Z,γ or mono-Higgs searches, with a handful of relevant parameters and yet a clear and consistent microscopic perspective. To the question of what we have learned from LHC DM searches, these models provide one answer.

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