ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAK004

# ACCURATE AND EFFICIENT TRACKING IN ELECTROMAGNETIC QUADRUPOLES

T. Pugnat*, B. Dalena, CEA, Irfu, DACM, Université Paris-Saclay, F-91191, Gif-sur-Yvette, France
A. Simona and L. Bonaventura, MOX, Politecnico di Milano, Milano, Italy
R. De Maria, J. Molson, CERN, Geneva, Switzerland

*Abstract*

Accelerator physics needs advanced modeling and simulation techniques, in particular for beam stability studies. A deeper understanding of the effects of magnetic fields nonlinearities will greatly help in the improvement of future colliders design and performance. This paper presents a study of quadrupole tracking using realistic field maps and measured or simulated longitudinal harmonics. The main goal is to describe the effect of the longitudinal dependence of high order non-homogeneity of the field in the case of the HL-LHC inner triplet.

## INTRODUCTION

In Ref. [1], a method to evaluate the non-linear fringe field effect on the long term beam dynamics has been presented.

Following Ref. [2], the magnetic field map or the field harmonics are used to compute first a representation of the vector potential, which enters in the expression of the Hamiltonian, after the non-linear transfer map of the quadrupole is derived using Lie algebra techniques for tracking simulations.

This paper presents further studies and improvements of the same procedure. First, a specific gauge for the representation of the vector potential is employed in order to reduce computational cost [3]. Then, a summary of the comparison with other integrators than Lie of second order performed in [3] is given. The implementation of the method in SixTrack and further improvement to the code presented in [1] are described. Finally, tracking results are discussed.

## METHOD

The motion of a charged particle in a magnetic field is described in terms of its position and its canonical momentum by the Hamilton equations. Ref. [4] have shown that the relativistic Hamiltonian can be expressed as an 8 D equivalent Hamiltonian written as follows (see also pages 5 to 8 in Ref. [3]):

$$\begin{aligned} K &= p_z - \delta \quad - a_z \quad + \frac{(p_x - a_x)^2}{2(1+\delta)} \quad + \frac{(p_y - a_y)^2}{2(1+\delta)} \\ &= K_1 \quad + K_2 \quad + K_3 \quad + K_4 \end{aligned} \tag{1}$$

where $\mathbf{p}$ is the normalised momentum vector, $\delta$ is the momentum deviation, $\mathbf{P}_0$ is the nominal momentum vector and $\mathbf{a} = q\mathbf{A}/P_0$ is the normalised vector potential.

As shown in Eq. (1), the equivalent paraxial Hamiltonian is a sum of 4 parts that are used to generate transfer map for the tracking of charged particles using the Lie Algebra, as explained in Ref. [4], [5] and [6]. It can also be shown that between the formulation of SixTrack (Ref. [7]) and the one used here, the longitudinal deviation for the position (respectively $\sigma$ and $l$) and momentum (respectively $p_\sigma$ and $\delta$) are connect by $\Delta\sigma = L_f + \Delta l \beta\beta_0$ and $\delta + 1 = \beta\beta_0 p_\sigma + \beta/\beta_0$ where $L_f$ is the length of integration.

### *Generalized Gradient*

Up to now, all the studies on beam dynamics used hard-edge approximation in order to consider the effect of the magnet. This paper explores the effect of longitudinal dependency of magnetic field on beam stability's observable, such as Dynamic Aperture and detuning with amplitude. In particular, the effect of the generalized gradient $C_m^{[n]}$ (see Eq. (20) at the page 15 in Ref. [1]) can be cancelled in the Hard Edge approximation.

The generalized gradients dump the high longitudinal frequency components of the harmonics. Then, using more generalized gradients derivatives means considering higher frequency components of the harmonics and a better reconstruction of the magnetic field longitudinally. Only considering the integrated strength (equivalent to $\int C_m^{[0]} dz$) is a fast and simple approach, but it generates an approximation of the magnetic field effect in the fringe field's regions.

### *Vector Potential Representation*

In Ref. [1], the vector potential is reconstructed starting from the field harmonics using the AF gauge to derive the expression of the vector potential components (see Eq. (22) in Ref. [3]). In Ref. [3], a new expression has been derived using a horizontal free gauge HFC (see Eq. (28) in Ref. [3]). Table 1 summarizes the evaluation's cost in terms of number of operations needed to reconstruct the vector potential monomials $\mathbf{A}(x, y, z) = \sum_{i,j} \mathbf{a}_{i,j}(z) x^i y^j$, where $\mathbf{a}_{i,j}(z)$ are coefficients that only depend on z and ND is the number of derivatives.

Table 1: Vector Potential Evaluation's Cost

| | ND=2 | | ND=16 | |
|---|---|---|---|---|
| | **Normal** | **Skew** | **Normal** | **Skew** |
| **AF** | 80 | 68 | 352 | 330 |
| **HFC** | 64 | 52 | 251 | 225 |
| **HFC/AF** | 0.80 | 0.76 | 0.72 | 0.68 |

### *Comparison of Integrators*

The Hamilton equations that describe the motion of the charged particle in the quadrupole system have been solved

* thomas.pugnat@cea.fr

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAK004



using different symplectic and non symplectic integrators: mid-point, $4^{th}$ order and $6^{th}$ order Gauss, $4^{th}$ order non-symplectic Runge-Kutta and Lie maps of equivalent order (including the one presented in Ref. [1]), as discussed in Ref. [3]. A comparison of the different integrators is reported in Ref. [3], in terms of speed and accuracy, using analytic as well as more realistic fields, generated with the ROXIE code [8] for the inner triplet of HL-LHC.

The main conclusions of the study are:

- For a dz greater than 40 mm, the longitudinal information is greatly deteriorated (Fig. 1).
- Lie methods profit more from the change of gauge than the other methods.
- Lie methods are faster with respect to other symplectic methods. The explicit, non-symplectic Runge-Kutta method is the fastest.
- All the methods display the same low accuracy for step size bigger than 4 cm for the realistic field considered.

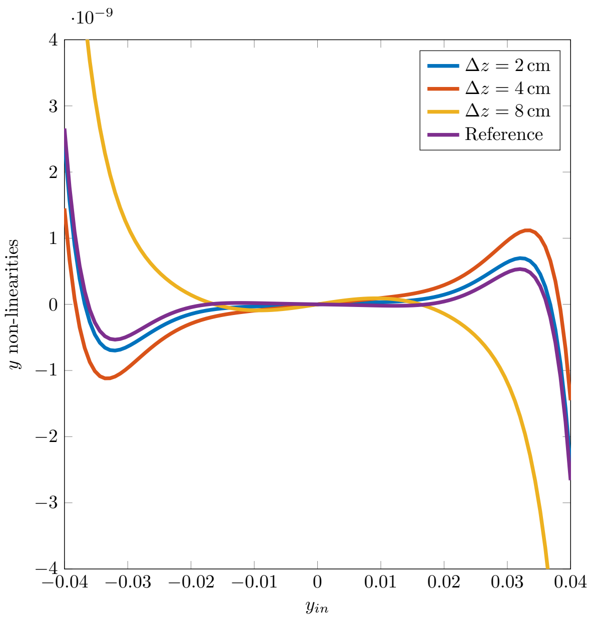


Figure 1: Information losses on the position with dz.

## IMPLEMENTATION IN SIXTRACK

A $2^{nd}$-order Lie map (Lie2, described in Ref. [1]), has been integrated in SixTrack (Ref. [7]). Historically, it was chosen for its speed, symplecticity and, as a $2^{nd}$ order tracking method like the thin lens used by SixTrack, real measured harmonic (consisting on the mean value by sections dz) can be used without interpolations. Other maps are also considered to increase the accuracy as shown in Ref. [3] but require more accurate measurement. Here, the code has been implemented as a 4 D Tracking.

As input, it needs a config file and files (**A**-files) containing the vector potential's coefficients discretised by section of same length, dz. Figure 2 shows the implementation in SixTrack with the aim to do not change the internal structure of the main code. In Fig. 2, it is assumed that $Q_{SixTrack}$ is the normal SixTrack routine for one full quadrupole and for each **A**-file (in or out), $I$ is a $2^{nd}$ order Lie integrator, $D$ is a drift of length $L_f - L_q$ (with the integration length $L_f$ and the equivalent magnetic length $L_q = \int B_2(z)dz/B_{2,center}$ in the config file) and $Q^{-1} = (\prod Q_i)^{-1}$ is the Anti-Quad with $Q_i$ a quadrupole's thin matrix with length $dz$ and strength $K_i = 2C_2^{[0]}(z_i)$ for the section $z_i$ in the **A**-file (see Eq. (20) in Ref [3]).

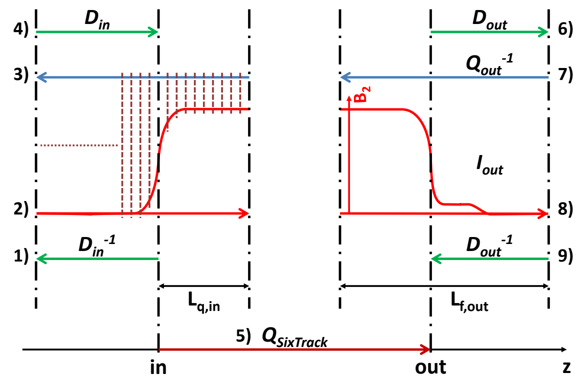


Figure 2: Implementation in SixTrack.

The routines that compute $\mathbf{A}(x, y)$ and its derivatives, are improved using specific tables (Matrix Market format, Ref. [9]) for the coefficient of **A**, which reduces the computation time by almost 50%.

## TRACKING RESULTS USING HL-LHC

The Dynamic Aperture (DA) is computed using the HLLHCV1.0 optics, simulating the particles' motion over $10^4$ revolutions. A set of initial conditions distributed on a polar grid is considered in such a way as to have 30 particles (different initial conditions) for each interval of 2 sigma from 0 to 28. Five phase space angles and 60 different machines (also called seeds), according to dipole field errors, are considered. The initial momentum offset $\delta$ is set to 2.7e-4. The inner triplet is modelled using 8 different vector potential files with only the natural harmonics of the quadrupole (2-6-10-14), according to the connector side or the non-connector side, and to the polarity.

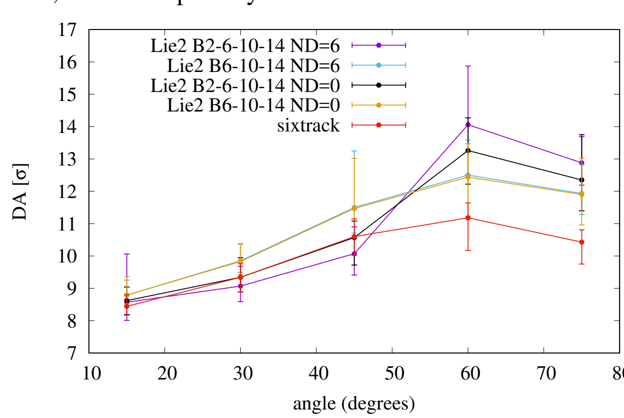


Figure 3: Dynamic aperture comparison for $\delta = 2.7e-4$.

As shown in Fig. 3, the DA computed using B6-10-14 is on average systematically better than the one using the multipoles model of SixTrack but compatible in the horizontal plan considering the minimum and maximum values

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAK004

(error bars). If B2 is added in the tracking, the DA is almost the same for the horizontal plane, but the discrepancy in the vertical plane increases, showing the impact of the longitudinal dependency of the field, among other effects, like interference between B2 and the others multipoles or incertitudes of the method (interface procedure for example). Fig. 3 also show the influence of ND on the DA. One would expect the DA to converge to the SixTrack value as ND tend to 0. But in the case of B6-10-14, the DA doesn't have major change and when adding B2, the convergence can be seen, but if the DA is the same for the x-plane, the discrepancy reduce in the y-plane but still significative. The origin of this difference has yet to be explained quantitatively.

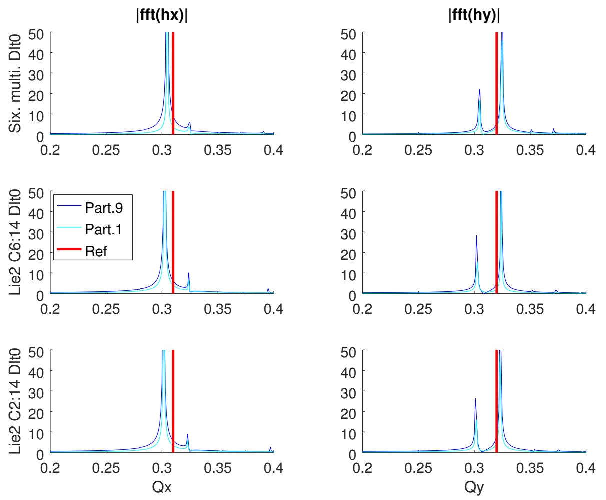


Figure 4: Tunes for 1000 revolutions for $\delta = 0$.

By extracting the positions and momenta of the particles at one point on the ring (here, at ip3), the fractional part of the tunes can be computed using the DFT. Figures 4 and 5 shows the tunes computed over 1000 revolutions for particles of different $\delta$ with a horizontal amplitude equal to 0.1993 mm (Part.1) and 0.4599 mm (Part.9) and a ratio of 0.19281 between horizontal and vertical phase space.

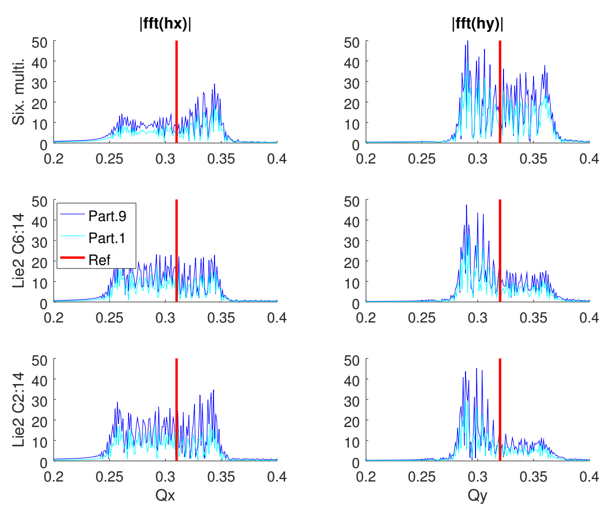


Figure 5: Tunes for 1000 revolutions for $\delta = 2.7e - 4$.

As shown in Fig. 4, the tunes measured are very close to the reference ones ($Q_x = 62.31$ and $Q_y = 60.32$) with a precision of $10^{-3}$. Showing that the linear part introduced by our method called Lie2 is well compensated by the combination of anti-drift, anti-quadrupoles and drifts described in the previous section. The amplitude of the frequency distributions is well preserved in our method (Lie2) with respect to nominal Sixtrack ones. The tune spread in Fig. 5 caused by the $\delta$-oscillation (periodicity $\approx$ 600 revolutions) does not change, while the amplitude seems affected. Statistical studies and full 6D Tracking need to be done to confirm or not the phenomena.

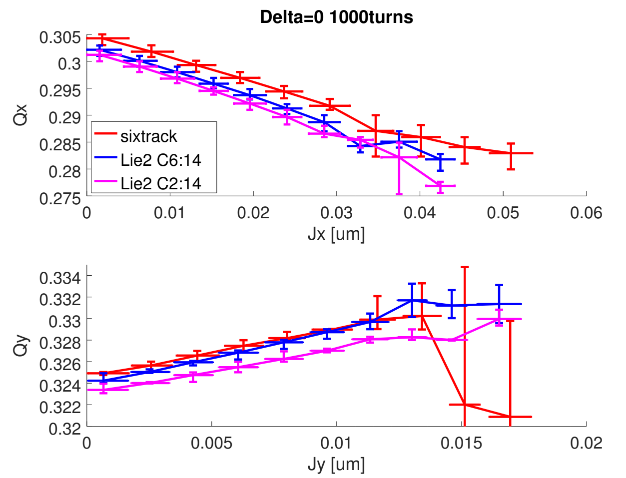


Figure 6: Tune vs action (J) (1000 revolutions with seed 1).

Finally, Fig. 6 shows the evolution of the tunes with the action variables (J), using positions and momenta of particles with the same phase space angle. The tunes are computed on the first 1000 revolutions with $\delta = 0$. The small tune shift between the multipole and our method is probably caused by some small difference on the field error used for the tracking. The detuning observed appear to be the same whatever the method used.

## CONCLUSION

We report the first results of the impact of the longitudinal description of the HL-LHC inner triplet field harmonics on the long term tracking. A method, called Lie2, is described and implemented into SixTrack. DA computed with the two methods are pretty similar in the horizontal plane and slightly different in the vertical one. First order detuning with amplitude is also very consistent between the two methods, proving the robustness of the SixTrack present model. The full 6 D Tracking is still to be investigated to have a more quantitative comparison between the models. Further studies on the uncertainties introduced by the methods and on the second order detuning with amplitude are planned as well.

## ACKNOWLEDGEMENT

The authors would like to thanks S. Izquierdo-Bermudez and E. Todesco for providing the field harmonics computed with ROXIE of the inner triplet prototype for the HL-LHC project, K.N. Sjobak for the help and comment during the implementation in SixTrack and A. Chancé for useful discussions and for the first implementation of the Horner routines.

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAK004



## REFERENCES

[1] B. Dalena *et al.*, "Fringe Field Modeling for the High Luminosity LHC Large Aperture quadrupole", in *Proc. IPAC'14*, Dresden, Germany, June 2014, paper TUPRO002, pp. 993-996.

[2] M. Venturini, A. J. Dragt,, "Accurate computation of transfer maps from magnetic field data", *Nucl. Instr. Meth.*, vol. 427, pp. 387-392, May 1999.

[3] A. Simona, L. Bonaventura, T. Pugnat, B. Dalena, *High order time integrators for the simulation of charged particle motion in magnetic quadrupoles*, arXiv:1802.08157, submitted for publication.

[4] Y. K. Wu, E. Forest and D. S. Robin, "Explicit symplectic integrator for s-dependent static magnetic field", *Phys. Rev. E*, vol. 68, pp. 046502, Oct. 2003.

[5] A. J. Dragt, *Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics*, University of Maryland, MD, USA, 1997.

[6] H. Yoshida, "Construction of higher order symplectic integrators", *Phys. Lett. A*, vol. 150, no. 5, pp. 262-268, 1990.

[7] SixTrack, http://sixtrack.web.cern.ch/SixTrack

[8] ROXIE, https://espace.cern.ch/roxie

[9] Matrix Market Exchange Formats, https://math.nist.gov/MatrixMarket/formats.html