

Vacuum-Induced Quantum Decoherence and the Entropy Puzzle ¹

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Abstract

Or: “How to generate an ensemble in a single event?” Following recent work on entropy in strong interactions, I explain the concept of environment-induced quantum decoherence in elementary quantum mechanics. The classically chaotic inverted oscillator becomes partially decoherent already in the environment of a single other oscillator performing only vacuum fluctuations. One finds exponential entropy growth in the subsystem with a Lyapunov exponent, which approaches the classical one for weak coupling.

Presented at the Workshops on “Quantum Infrared Physics”, Paris, 6–10.6.94, and “QCD 94”, Montpellier, 7–13.7.94. Invited talk at the NATO Adv. Res. Workshop on “Hot Hadronic Matter”, Divonne, 27.6.–1.7.94. To appear in the Proceedings.

¹Work supported by the Heisenberg Programme (Deutsche Forschungsgemeinschaft).

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Recently the long-standing “entropy puzzle” of high-multiplicity events in strong interactions at ultra-relativistic energies has been analysed from a new point of view [1]. This is related to the concepts of an *open quantum system* and *environment-induced quantum decoherence*. The problem dates back to Fermi and Landau and is intimately connected to understanding the rapid thermalization of high energy density ($\gg 1 \text{ GeV}/\text{fm}^3$) matter [2]. Why do thermal models work so well in reproducing global features of hadronic multiparticle final states? Why do they work at all?

Or, Why does high-energy scattering of pure initial states lend itself to a statistical description characterized by a large apparent *entropy* from a mixed-state density matrix describing intermediate stages in a space-time picture of parton evolution? Effectively, *unitary time evolution* of the observable part of the system breaks down in the transition from a quantum mechanically pure initial state to a highly impure (more or less thermal) high-multiplicity final state. Note that the unitary time evolution operator, $\exp(-i\hat{H}t)$, always transforms a pure state into a pure state, according to the Schrödinger equation, which *cannot* produce entropy under any circumstances (cf. below). This was discussed in detail in Refs. [1], where more references concerning formal aspects of this work can be found. Based on analogies with studies of the quantum measurement process (“collapse of the wave function”) [3] and motivated by related problems in quantum cosmology and by non-unitary non-equilibrium evolution resulting in string theory [4], I argued that **environment-induced quantum decoherence solves the entropy puzzle** of strong interactions.

A complex pure-state quantum system can show a quasi-classical behaviour, i.e. an impure density (sub)matrix together with decoherence of the associated pointer states in an observable subsystem [1, 3, 4]. I will demonstrate in the following that the decoherence process is uniquely correlated with entropy production. Considering strong interactions, in particular, there is a natural *Momentum Space Mode Separation* due to confinement, which is defined in the frame of initial conditions for the time evolution and for the physical (gauge) field degrees of freedom. Thus, almost

constant QCD field configurations form an *unobservable environment*, which interacts with the *observable subsystem* composed of partons. The environment modes are unobservable, since they can neither hadronize nor initiate hard scattering among themselves, whereas the partons are observable in the sense of parton-hadron duality or deep-inelastic scattering; equivalently, low-energy coloured vacuum fluctuations cannot propagate into asymptotic states.

Previously, I studied the induced quantum decoherence and entropy production in a non-relativistic single-particle model resembling an electron coupled to the quantized electromagnetic field, however, with a deliberately enhanced oscillator spectral density in the infrared. The Feynman-Vernon influence functional technique for quantum Brownian motion provided the remarkable result that in the *short-time strong-coupling limit* the model parton behaves like a *classical particle* [1]: Gaussian parton wave packets experience *friction* and *localization*, i.e. no quantum mechanical spreading, and their coherent *superpositions decohere*. The decoherence process has been shown to lead to entropy production in this oversimplified parton model.

It seems somewhat more realistic to consider two coupled scalar fields representing partons and their non-perturbative environment, respectively. In the functional Schrödinger picture employing Dirac's time-dependent variational principle, i.e. a non-perturbative method, I derived a Cornwall-Jackiw-Tomboulis (CJT) type effective action and the equations of motion for renormalizable interactions [1]. Thus, analysis of the entropy puzzle in strong interactions leads to study an observable field (open subsystem) interacting with a dynamically hidden one (unobservable environment), i.e. *quantum field Brownian motion*.

Summarizing, my point of view is that partons feel an unobservable (gluonic) environment, which manifests its strong non-perturbative interactions on a short time scale ($\ll 1 \text{ fm}/c$) through decoherence of suitable partonic pointer states ³, their

³In general, these are not single-particle states but rather coherent (Gaussian) wave functionals, as constructed in the second of Refs. [1].

quasi-classical behaviour, and entropy production. If confirmed in QCD, this will have important consequences for parton-model applications to complex hadronic or nuclear reactions. The emergence of *structure functions* from initial-state wave functions can and will be further studied in this approach.

Instead of representing the formalism and more technical results from Refs. [1], I want to demonstrate here in simple quantum mechanical examples the basic Why and How of the solution to the entropy puzzle.

Consider a system that can be described in terms of two normalized discrete basisstates, $|1\rangle$ and $|2\rangle$. Forming a *pure state*, $|\psi\rangle \equiv a_1|1\rangle + a_2|2\rangle$, by a coherent superposition with amplitudes $a_1 \equiv p^{1/2}$ and $a_2 \equiv (1-p)^{1/2}$, the corresponding density matrix, $\hat{\rho} \equiv |\psi\rangle\langle\psi|$, is

$$\rho_{ij} = \begin{pmatrix} a_1^2 & a_1 a_2 \\ a_1 a_2 & a_2^2 \end{pmatrix} \longrightarrow \rho_{ij}^D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (1)$$

where $\hat{\rho}^D$ is obtained by diagonalization. Note the off-diagonal *interference terms* in ρ_{ij} . Furthermore, observe that $\hat{\rho}^D$ has only one non-vanishing eigenvalue. Introducing the **von Neumann** or **statistical entropy**,

$$S[\hat{\rho}] \equiv -\text{Tr } \hat{\rho} \ln \hat{\rho}, \quad (2)$$

we find $S[\hat{\rho}] = S[\hat{\rho}^D] = -[1 \ln 1 + 0 \ln 0] = 0$, i.e. *no entropy in a pure state*. Secondly, forming a *mixed state* (ensemble) such that the system is in state $|1\rangle$ with probability p and in state $|2\rangle$ with probability $1-p$, the density matrix becomes $\hat{\rho}' = |1\rangle p \langle 1| + |2\rangle (1-p) \langle 2|$, i.e. a decoherent superposition. Hence, we obtain

$$\rho'_{ij} = \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}. \quad (3)$$

The density matrix (3) shows *no interference terms* and is diagonal per se.⁴ Then, $S[\hat{\rho}'] \equiv S(p) = -[p \ln p + (1-p) \ln(1-p)] \neq 0$, generally. In fact, $0 \leq S(p) \leq$

⁴Note that $\text{Tr } \hat{\rho} = \text{Tr } \hat{\rho}' = 1$: the system is in *some* state with total probability 1.

$S(1/2) = \ln 2$. Total ignorance about the state of the system ($p = 1/2$) corresponds to $\ln 2$ units of entropy in a two-state system, i.e. 1 *bit* of information is lost compared to certainty about its state ($p = 0, 1$).

One concludes that *entropy production can only occur if the interference terms of the density matrix representing a more or less pure state of the observed system decay dynamically*.⁵ In a *closed system* evolving unitarily in time, however, there is no way to transform, for example, $\hat{\rho}$ into $\hat{\rho}'$, see Eqs. (1), (3). Only the interaction of the system with an *environment* [1, 3, 4], can have such an effect. The presence (and integrating out) of the environment degrees of freedom essentially changes the dynamics of the observed system. This can lead to the decay of the interference terms in its density matrix, i.e. *environment-induced quantum decoherence*, which is necessary to increase its impurity and, thus, to produce entropy.

Next, consider a non-relativistic particle moving in a one-dimensional double-well potential presenting the observable subsystem, which is coupled translationally invariant to a single environment oscillator. The classical action is

$$\mathcal{S} = \int dt \left\{ \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - \frac{1}{2}m\omega^2(y-x)^2 + \frac{1}{2}M\Omega^2x^2 - \frac{1}{4!}\lambda M\Omega^4x^4 \right\} . \quad (4)$$

For simplicity, let $M = m = \Omega$. Then, properly rescaling by Ω , one obtains

$$\mathcal{S} = \int dt \left\{ \frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{y}^2 - \frac{1}{2}\omega^2(y-x)^2 + \frac{1}{2}x^2 - \frac{1}{4!}\lambda x^4 \right\} , \quad (5)$$

in terms of dimensionless quantities and two coupling constants, $\omega^2, \lambda \geq 0$. For $\omega = 0$ the minima of the doublewell lie at $x_{\pm} = \pm(\lambda/3!)^{-1/2}$, at a depth of $-3/2\lambda$ (the local maximum is zero at $x = 0$). Presently, I want to study the case that the excitation energy of the x-particle (X) is smaller than the level spacing ω of the environment oscillator (Y), which is assumed to be in its ground state. Starting with a given initial state of X, I will calculate the time evolution of the corresponding density matrix $\hat{\rho}_X$

⁵The argument does not depend on particular physical characteristics of the system; it holds for the two-state system as well as for an interacting quantum field.

under the influence of the *vacuum fluctuations* of Y. Excited states of Y contribute only *virtually* here; they cannot become real due to energy conservation.

For illustration, I choose the metastable initial state of X, when classically the particle “rests on top of the hill” ($x = 0$). Quantum mechanically this can be represented by a minimum uncertainty Gaussian wave packet,

$$\psi(x, t = 0) = \pi^{-1/4} w_0^{-1/2} e^{-\frac{1}{2}x^2/w_0^2}, \quad (6)$$

with $w_0 \ll \lambda^{-1/2}$. Also, assume ω to be sufficiently larger than $3/2\lambda$.

First of all, let the system evolve **classically**. Nothing will happen. However, any infinitesimal perturbation of the fine-tuned initial conditions causes X to move “down the hill”, left or right (L or R), dragging Y along. There is local chaos in the sense of extreme sensitivity to the initial conditions at $x, y \approx 0$; arbitrarily small uncertainties in the initial conditions lead to a *loss of predictability*. For an *ensemble of initial conditions* X switches with probabilities $p_L(t')$ and $p_R(t') = 1 - p_L(t')$ between L and R, respectively, if at least one trajectory passes $x = 0$ in a certain interval $[t' - \epsilon, t' + \epsilon]$. This corresponds to a loss of information about the actual binary decision “either L or R” and an entropy $S_X = -\sum_{i=L,R} p_i \ln p_i$.⁶ Note that S_X or $p_{L,R}$ are strongly conditional (“fine-grained”) quantities. In distinction, the usual classical entropy is calculated after “coarse graining”, i.e. by constructing a local probability density $f(t)$ in the phase space of X related to the ensemble average over initial conditions, $S_{c.g.}(t) \equiv \int dx dp f(t) \ln f(t) + C$. A chaotic loss of predictability from strongly diverging trajectories in phase space causes $S_{c.g.}$ to increase: as time passes, more and more cells of the coarse graining contribute – effectively, the phase space volume occupied by the ensemble grows (without violating Liouville’s theorem).

In conclusion, *in a classical system, be it chaotic or not (with or without coarse graining), entropy can only be produced IF there is a physically relevant ensemble of*

⁶If the ensemble of initial conditions is constrained to preserve the reflection symmetry of the action, Eq. (5), then $p_L(t') = p_R(t') = \frac{1}{2}$ and $S_X(t') = \ln 2$ stay constant.

initial conditions. Thus, one cannot explain altogether classically entropy production or thermalization in a single high-multiplicity event in strong interactions.

Secondly, let the system evolve **quantum mechanically**. To begin with, let there be *no* coupling to the environment ($\omega = 0$). Even with the fine-tuned initial condition, Eq. (6), the amplitude ψ to find X at a particular space-time point begins to flow “down the hill” symmetrically (L and R) due to the quantum spreading of the wave packet. For a free particle $w_0 \rightarrow w(t) = (w_0^2 + w_0^{-2}t^2)^{1/2}$ (for $M = 1$); here one expects an accelerated spreading “downhill”, cf. Eq. (12) below. The related probability density $|\psi|^2$ also evolves and stays symmetric; generally, it *cannot* be simulated by the classical evolution starting with an ensemble of initial conditions due to the absence of quantum interference between classical trajectories. In any case, the system remains in a *pure quantum state*. The density matrix is $\hat{\rho}_X(t) = |\psi(t)\rangle\langle\psi(t)|$, $|\psi(t)\rangle = \exp[-i\hat{H}_0t]|\psi(0)\rangle$, where \hat{H}_0 is the Hamiltonian of X from Eq. (5) with $\omega = 0$. Therefore, $S[\hat{\rho}_X(t)] = -1 \ln 1 = 0$, cf. Eq. (2). Quantum mechanically one knows everything there is to know about a *closed system* (X), given any pure initial state and its Hamiltonian, which consistently yields $S[\hat{\rho}_X] = 0$. Even with an ensemble of initial states, i.e. an impure density matrix $\hat{\rho}_X(0)$, there is *no entropy production*, since $S[\hat{\rho}_X(t)] = S[\exp(-i\hat{H}_0t)\hat{\rho}_X(0)\exp(+i\hat{H}_0t)] = S[\hat{\rho}_X(0)]$ stays constant.⁷

The situation changes completely if the subsystem (X) evolves quantum mechanically coupled to the *vacuum fluctuations and virtual excitations of the environment* (Y). With the above assumptions the initial density matrix of the total system is:

$$\hat{\rho}(t=0) \equiv \hat{\rho}_X(0) \otimes \hat{\rho}_Y(0) \ , \quad (7)$$

with matrix elements $\rho_X(x, x'; 0) = \pi^{-1/2}w_0^{-1} \exp[-\frac{1}{2}(x^2 + x'^2)/w_0^2]$ and $\rho_Y(y, y'; 0) = (\omega/\pi)^{1/2} \exp[-\frac{1}{2}\omega(y^2 + y'^2)]$. The time evolution of the density matrix of the observable subsystem, $\hat{\rho}_X(t) = \text{Tr}_Y \hat{\rho}(t)$, can be calculated with the Feynman-Vernon influence functional technique; I will make use of general results obtained in the first of Refs.

⁷The unitary (time evolution) transformation does not change the eigenvalues of $\hat{\rho}_X$. Thus, the statistical entropy, Eq. (2), cannot possibly show a sign of classical chaos in a closed system.

[1]. The idea is to derive a propagator for $\hat{\rho}_X$, which incorporates the influence of the environment degrees of freedom (Y) exactly. This can be achieved, since Y and its coupling to X are at most quadratic in coordinates and momenta, see Eq. (5).

It should be remarked that the *final state* of the environment is not specified; presently, it may contain virtual excitations of Y.⁸ The relevance of this for a high-multiplicity hadronic (or nuclear) reaction is the following: Even though the QCD vacuum “far away” conforms to the usual one before and after, the additionally produced secondary hadrons all require a dressing of their valence quarks by localized virtual excitations of the vacuum or environment, which obviously makes an essential difference as compared to the initial state.

Presently, the resulting density matrix $\hat{\rho}_X(t)$ is (cf. also the first of Refs. [1]):

$$\begin{aligned} \rho_X(z_-, z_+, t) &= \pi^{-1/2} w^{-1}(t) e^{-[z_+ - v(t)t]^2/w^2(t)} \\ &\times e^{-z_-^2 \{C + \frac{1}{4}w_0^2 c^2 - d^{-2}[B + \frac{1}{2}w_0^2 bc]^2/w^2(t)\}} \\ &\times e^{iz_- \{az_+ - 2d^{-1}[B + \frac{1}{2}w_0^2 bc][z_+ - v(t)t]/w^2(t)\}} \quad , \quad (8) \end{aligned}$$

with the effective velocity $v(t) = 0$ for the zero-momentum initial wave packet, the effective width $w(t) \equiv 2\xi|d|^{-1}$, $\xi \equiv (A + \frac{1}{4}w_0^{-2} + \frac{1}{4}w_0^2 b^2)^{1/2}$, and with rather complicated time-dependent coefficients A, B, C, a, b, c, d , to be discussed elsewhere; the coordinates in Eq. (8) are $z_- \equiv x - x'$ and $z_+ \equiv \frac{1}{2}(x + x')$ in terms of ordinary one-dimensional ones. Since we are particularly interested in the decoherence process and entropy production, we consider only the simplest *off-diagonal density matrix elements* here, $\rho_X(x, x' = -x, t) = \rho_X(z_- = 2x, z_+ = 0, t)$. They can be directly related to the **linear entropy** produced in the observable subsystem (X):

$$\begin{aligned} S^{lin} &\equiv \text{Tr} [\hat{\rho}_X - \hat{\rho}_X^2] = 1 - \int_{-\infty}^{\infty} dz_- \int_{-\infty}^{\infty} dz_+ \rho_X(z_-, z_+, t) \rho_X(-z_-, z_+, t) \\ &= 1 - \frac{1}{2} c_1^{-1/2} w^{-1} \quad , \quad (9) \end{aligned}$$

⁸As a corollary to the *Schmidt decomposition* [1] it is easy to prove that starting with an overall pure state of the complex system, cf. Eq. (7), IF the final state of the environment is a pure state, THEN the observable subsystem ends up in a pure state too (without entropy production).

with $c_1 \equiv C + \frac{1}{4}w_0^2c^2 - (B + \frac{1}{2}w_0^2bc)^2/(dw)^2$ and independently of the initial wave packet momentum ($p = 0$ at present). Thus, inserting (9) into (8), one obtains:

$$\rho_X(x, -x, t) = \pi^{-1/2}w^{-1}(t) \exp\left\{-x^2w^{-2}(t)[1 - S^{lin}(t)]^{-2}\right\} , \quad (10)$$

$$\int_{-\infty}^{\infty} dx \rho_X(x, -x, t) = 1 - S^{lin}(t) \geq e^{-S(t)} . \quad (11)$$

The inequality results from the fact that the linear entropy provides a *lower bound* for the relevant statistical entropy, cf. Eq. (2), as shown in [1]. Note that Eqs. (9)–(11) are completely independent of the time-dependent functions entering there, which are specific for a particular dynamical system. They are based, however, on the Gaussian structure of the subsystem density matrix, Eq. (8).

At this point the attentive reader might wonder what happened to the non-linear interaction $\propto \lambda x^4$ of the double-well potential, see Eq. (5). Of course, it cannot be treated exactly. I employed a mean-field-type approximation, replacing $\frac{1}{4!}\lambda x^4$ by $\frac{1}{2}\lambda\langle x^2\rangle x^2 \equiv \frac{1}{2}\Lambda^2(t)x^2$. As long as one studies only the initial time-evolution over short periods, as compared to the time a classical particle would need to “roll down the hill”, one may even set $\Lambda \approx 0$. For the following qualitative considerations, Λ plays the role of an adiabatically changing parameter. However, a more accurate approximation is necessary (and feasible) to follow the truly long-time quasi-periodic motions of the system. Then, one expects periods of increasing decoherence and entropy production, cf. below, followed by periods of *quantum revival* in the observed subsystem. The more complex the environment becomes, the more unlikely quantum revivals will be, since the total system including the environment finds more and more ways to evolve before a reconstruction of the subsystem initial-state wave function.⁹

To begin with, it can be checked explicitly that there is no entropy production for a vanishing coupling to the environment, $S_{\omega=0}^{lin}(t) = 0$, cf. Eq. (9). Next, calculating

⁹Such effects have been experimentally observed in even simpler systems involving a two-state subsystem of one Rydberg atom coupled to a single mode of the electromagnetic field [5].

the effective width in the *long-time limit*¹⁰, one finds:

$$w(t) = \left(w_0^2 + w_0^{-2} f_-^{-2} + \omega^3 f_-^{-2} [f_-^2 + \omega^2]^{-1} \right)^{1/2} \frac{f_-^2 + \omega^2}{f_-^2 + f_+^2} \exp t_- \quad , \quad (12)$$

with $t_- \equiv f_- t$, $f_{\pm} \equiv [\pm \frac{1}{2} \omega_+^2 + (\frac{1}{4} \omega_+^4 - \omega^2 \omega_-^2)^{1/2}]^{1/2}$, and $\omega_{\pm}^2 \equiv \pm \omega^2 + \Lambda^2 - 1$. Assuming a sufficiently small coupling, $\omega^2 < 1$, note that ω_{\pm}^2 is *negative* as long as $\Lambda^2(t) < 1 - \omega^2$. Thus, the *width grows exponentially* with an *effective Lyapunov exponent* f_- . For vanishing coupling to the environment, it reduces to the classical Lyapunov exponent of the inverted oscillator, $f_-^{\omega=0} = (1 - \Lambda^2)^{1/2}$, while the width becomes $w_{\omega=0}(t) = (w_0^2 + [w_0 f_-^{\omega=0}]^{-2})^{1/2} \exp(f_-^{\omega=0} t)$. This suggests quite generally that the time-dependent widths of suitable (Gaussian) wave packets may serve as “quantum indicators” of chaotic behaviour in the corresponding classical system.

It is remarkable how the Lyapunov exponent reflects the dynamics: as the wave packet spreads “downhill”, $\Lambda^2(t) \propto \langle x^2 \rangle$ increases until f_- reaches zero (becoming purely imaginary afterwards), when $\omega_- = 0$. At this point the behaviour becomes regular in the sense of being governed by harmonic motions close to the minima of the double-well potential with a correspondingly milder time-dependence of the width (cf. the model studied in the first of Refs. [1]).

The second dynamical time scale f_+^{-1} always stays real. It is relevant for certain non-Markovian effects generated by the interaction with the environment ($f_+^{\omega=0} = 0$). These become clearly visible in the entropy evaluated in the same limit as Eq. (12). Using $c_1 = C + O(w_0^2)$, one obtains in leading order:

$$S^{lin}(t) = 1 - w^{-1}(t) \frac{f_-^2 + \omega^2}{\omega^{3/2} f_-} \times \quad (13)$$

$$\left[\left(\frac{f_- [f_+^2 - \omega^2]}{f_+ [f_-^2 + f_+^2]} + \frac{f_+ [f_-^2 + \omega^2]}{f_- [f_-^2 + f_+^2]} \right) \sin t_+ + \frac{2\omega^2 \cos t_+}{f_-^2 + f_+^2} \right]^2 + \omega^2 \left[\frac{\sin t_+}{f_+} - \frac{\cos t_+}{f_-} \right]^2 \right]^{-1/2} ,$$

with $t_+ \equiv f_+ t$. Thus, the linear entropy approaches exponentially its saturation value 1 on the time scale set by the Lyapunov exponent, see Eq. (12), and the

¹⁰Here, hyperbolic functions dominate over trigonometric ones in the time-dependent coefficients in Eq. (8) for times such that a classical particle would still be “rolling down the hill” of the potential; this restriction is presently assumed for simplicity.

von Neumann entropy grows exponentially according to eq. (11), at least as fast. Note that the periodic function multiplying $w^{-1}(t)$ in Eq. (13) is approximately $\propto \omega^{-1/2}[1 + 2^{1/2} + \sin^2(2^{1/4}\omega t)]^{-1/2}$ for $f_- \approx f_+$ and $\propto \omega^{-1/2}[1 + \cos^2(2^{1/2}\omega t)]^{-1/2}$ for $f_- \approx 0$; i.e. it persists qualitatively even until the effective Lyapunov exponent becomes imaginary, when the stabilizing effect of the x^4 -term in the potential is felt.

To summarize, in the above specified long-time limit and for the chosen initial conditions, Eq. (7), one obtains the observable subsystem (X) density matrix,

$$\rho_X(z_-, z_+, t) = \pi^{-1/2} w^{-1}(t) e^{-\{z_+^2 + \frac{1}{4}z_-^2[1 - S^{lin}(t)]^{-2}\}} w^{-2}(t) + iz_- z_+ f_- . \quad (14)$$

Even though this density matrix describes the exponential entropy production and its diagonal matrix elements with $z_- = 0$ grow rapidly, apart from the overall normalization factor, the (simplest) off-diagonal matrix elements ($z_+ = 0$) do *not* really decay here as in usual models of quantum decoherence [1, 3]. This is no surprise in view of the “poor environment” considered at present, which has only one degree of freedom frozen in its ground state (modulo virtual excitations). Loosely speaking, it is unable to accommodate all the phase information contained in the off-diagonal density matrix elements of the subsystem.

In conclusion, a strong observable entropy production in a quantum system, which shows a chaotic behaviour in the classical limit with exponentially growing modes, requires only a minimal decohering effect due to an environment of vacuum fluctuations coupled to it from a higher energy scale.

In particular, complementary to previous studies [3, 6], one observes here that the environment does not necessarily have to be at any finite temperature for this effect of *partial decoherence* to work. Furthermore, it should be realized that the Schmidt decomposition reveals the remarkable fact that the density submatrices of “subsystem” and “environment” always have identical non-zero eigenvalues [1]. Thus, from the point of view of calculating the entropy, Eq. (2), their roles can be interchanged and it is a matter of practicability to decide which part of the total Hilbert space is integrated out to find the entropy of the *physically observed subsystem*. The

environment-induced quantum decoherence and its relevance for entropy production have presenly been illustrated by an elementary example, which, however, points out to interesting consequences for the quantum evolution of classically chaotic non-linear field theories.

I thank P. Carruthers, M. Danos, N. E. Mavromatos, B. Müller, O. Bertolami, and J. Rafelski for stimulating criticism and helpful discussions.

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